Resummation techniques for heavy-quark hadroproduction and Dark Matter annihilation

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MITP Workshop HQHCAP Mainz 2.10.2019

Outline

- Basics of threshold resummation: the Drell-Yan case
- Threshold factorization and resummation for heavy quark production (PIM, 1PI kinematics, boosted regime)
- Top quark pair + Higgs/W/Z production at the LHC
- Sudakov resummation for WIMP dark matter annihilation

Introduction: threshold limit

- Threshold resummation and fixed-order expansions have been applied to many different processes: Drell-Yan, Higgs production, ttbar, ttbar+V, squarks and gluinos production…

- Goal: achieve better behaved predictions and estimate the size of the higher order corrections

- First case studied: production of a single uncoloured particle (Drell-Yan process) $A(p_A)B(p_B) \to \mathrm{DY(Q)} + X$ (G. Sterman '87, S. Catani, L. Trentadue '89, L.Magnea '91, G.Sterman, W. Vogelsang '00, S. Moch, A. Vogt '05, T. Becher, M. Neubert, G. Xu '08, M.Bonvini, S.Forte, G.Ridolfi '12,…)

$$\frac{d\sigma}{dM^2} \sim \sum_{ab} \int_\tau^1 \frac{dz}{z} ff_{ab}(\tau/z)\hat{\sigma}_{ab}(z)$$

When real radiation is present in the final state $\rightarrow$ $\hat{s} \neq M^2$, $z = M^2/\hat{s} \to 1$

$$\hat{\sigma}_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \Bigg[\underbrace{c_n \delta(1-z) + \sum_{m=0}^{2n-1} \Bigg(c_{nm} \left[\frac{\ln^m(1-z)}{1-z}\right]_+}_{\text{LP}} + \underbrace{d_{nm} \ln^m(1-z) \Bigg)}_{\text{NLP}} + \dots \Bigg]$$

The relevance of the threshold region arises dynamically due to the steeply falling behaviour of the parton luminosity

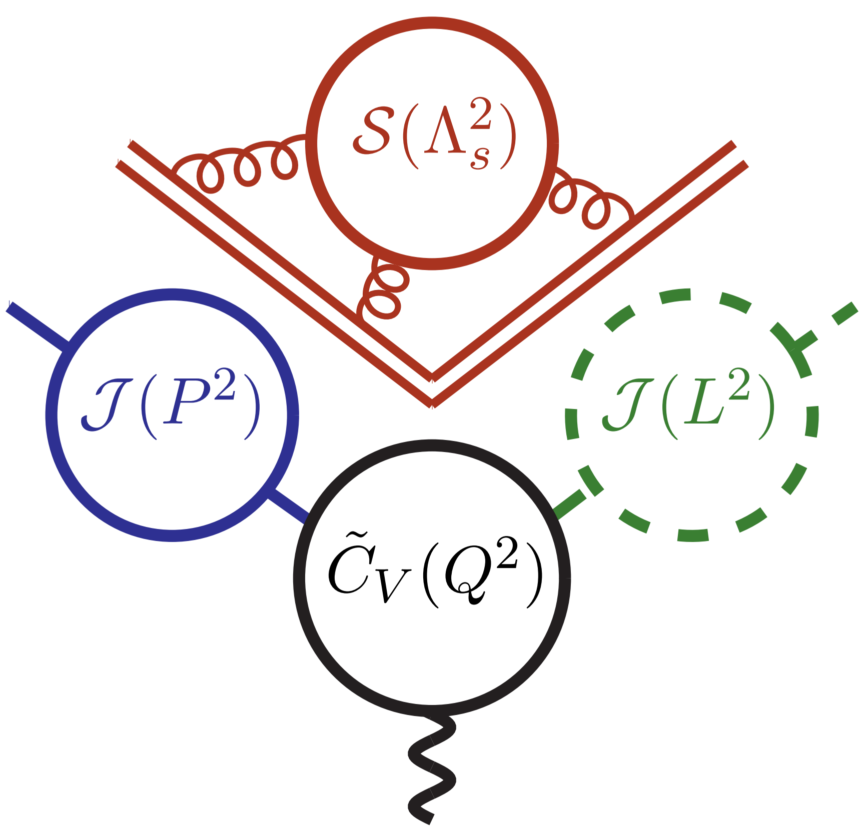
Factorization & Resummation

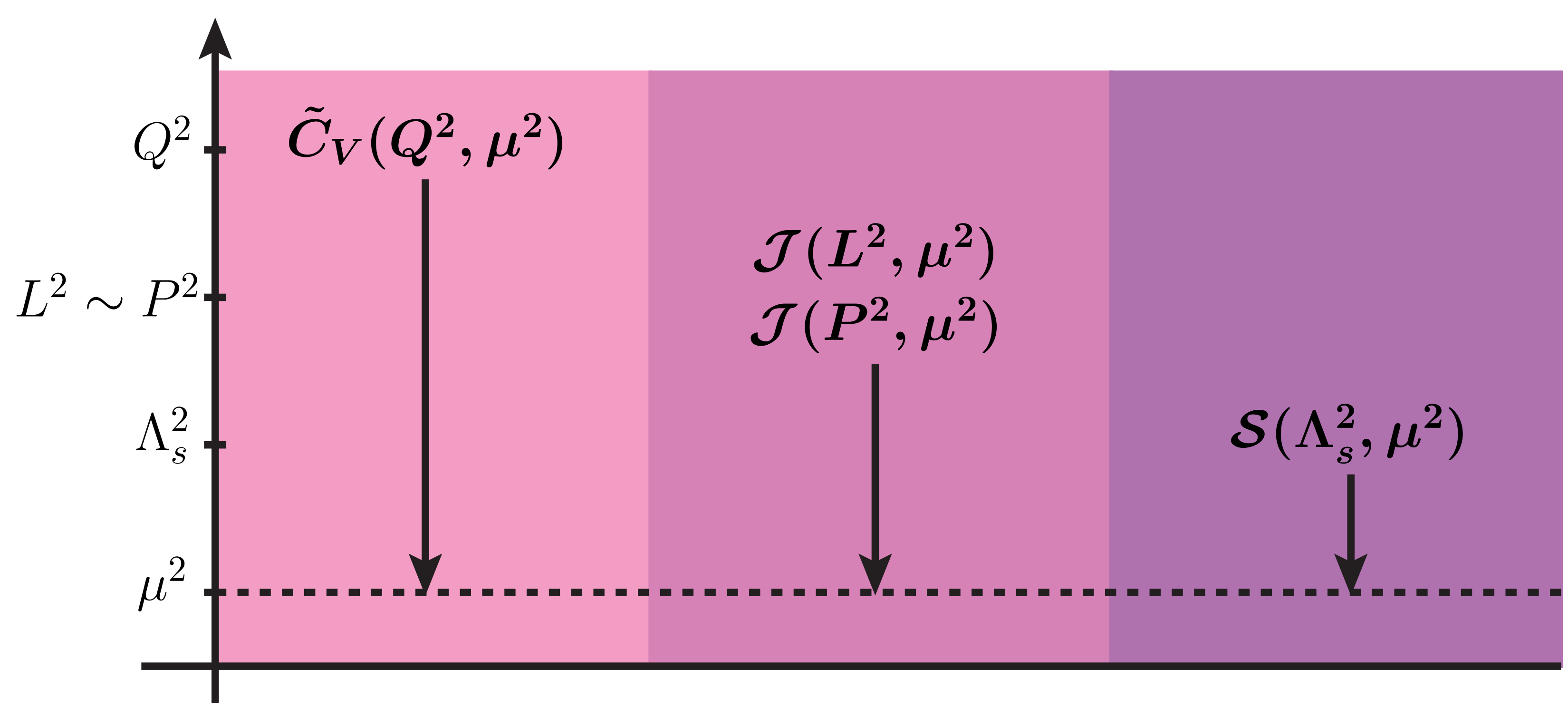
- Resummation program schematically:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

$$\alpha_s L \sim 1$$

  - large logarithmic corrections from soft emissions
  - separation of scales (factorization)
  - evaluate each (single scale) factor in fixed order perturbation theory at a scale for which it is free of large logs
  - use Renormalization Group equations to evolve the factors to a common scale





RG equations - Drell-Yan case

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^2 s} \boxed{|C_V(-M^2,\mu_f)|^2} \sum_q e_q^2 \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} \left[f_{q/N_1}(x_1,\mu_f)\, f_{\bar{q}/N_2}(x_2,\mu_f) + (q \leftrightarrow \bar{q}) \right]$$

$$\times \boxed{\sqrt{\hat{s}}\, W_{\rm DY}(\sqrt{\hat{s}}\,(1-z),\mu_f)\,,} \qquad z \equiv \frac{M^2}{\hat{s}}$$

[Becher, Neubert & Xu '07]

Renormalization Group equations for the hard and the soft functions

- for example for the soft function (in Laplace space)

$$\frac{d\,\widetilde{s}_{\rm DY}(\kappa,\mu)}{d\ln\mu} = \left[-4C_F\gamma_{\rm cusp}(\alpha_s)\ln\left(\frac{\kappa}{\mu}\right) - 2\gamma_W(\alpha_s) \right] \widetilde{s}_{\rm DY}(\kappa,\mu)$$

- its solution (back) in momentum space

$$\boxed{W_{\rm DY}(\omega,\mu) = \exp\left[-4C_F S(\mu_s,\mu) + 2A_{\gamma_W}(\mu_s,\mu)\right] \widetilde{s}_{\rm DY}\left(\partial_\eta,\mu_s\right) \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \frac{1}{\omega} \left(\frac{\omega}{\mu_s}\right)^{2\eta}}$$

Resummation

- Resummed formula for the hard-scattering kernel

$$C(z, M, \mu_f) = \left|\tilde{C}_V(-M^2, \mu_h)\right|^2 \boxed{U(M, \mu_h, \mu_f, \mu_s)} \frac{\sqrt{\hat{s}}}{\omega} \left(\frac{M}{\omega}\right)^{-2\eta} \widetilde{s}_{\rm DY}\left(\ln\frac{\omega^2}{\mu_s^2} + \partial_\eta, \mu_s\right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

$$\omega = M(1-z)/\sqrt{z}$$

Evolution function which evolves the soft μ_s and the hard μ_h scale to a common factorization scale μ_f

$$U(M, \mu_h, \mu_f, \mu_s) = \exp\left[4C_F \boxed{S(\mu_h, \mu_s)} + 4A_{\gamma_{f_q}}(\mu_s, \mu_f) - 2A_{\gamma_V}(\mu_h, \mu_s)\right] \left(\frac{M^2}{\mu_h^2}\right)^{-2C_F A_{\gamma_{\rm cusp}}(\mu_h, \mu_s)}$$

LL NLL

$$S(\nu, \mu) = \frac{\gamma_0^{\rm cusp}}{4\beta_0^2}\left[\boxed{\frac{4\pi}{\alpha_s(\nu)}\left(\frac{r-1}{r} - \ln r\right)} + \boxed{\left(\frac{\gamma_1^{\rm cusp}}{\gamma_0^{\rm cusp}} - \frac{\beta_1}{\beta_0}\right)(1 - r + \ln r)} \boxed{+ \frac{\beta_1}{2\beta_0}\ln^2 r}\right] + \mathcal{O}(\alpha_s), \quad r = \alpha_s(\mu)/\alpha_s(\nu).$$

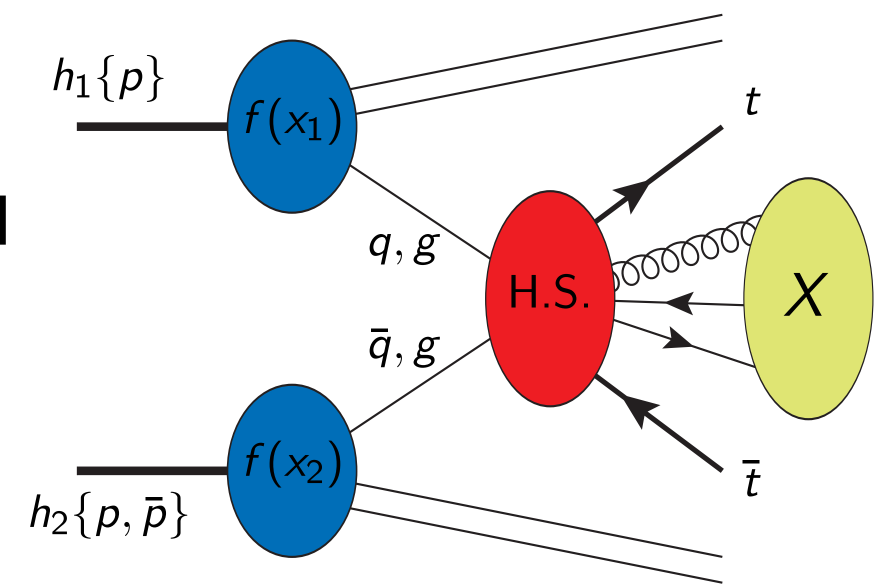
- Wilson coefficients and soft functions are free of large logarithms and can be computed in perturbation theory

- singular terms (plus distributions and delta functions) in fixed-order perturbation theory can be obtained by expanding in α_s (approximate result)

Heavy quark production

$$N_1(P_1) + N_2(P_2) \to t(p_t) + \bar{t}(p_{\bar{t}}) + X$$

- The top quark is the heaviest particle of the SM
- the LHC is a Top factory ~3x10^7 top quark pairs produced
- at LO two partonic production channels



$$\begin{aligned} q(p_1) + \bar{q}(p_2) &\to t(p_t) + \bar{t}(p_{\bar{t}}) \\ g(p_1) + g(p_2) &\to t(p_t) + \bar{t}(p_{\bar{t}}) \end{aligned}$$

- $q\bar{q}$ channel is the dominant at the Tevatron (85%), gg channel is the dominant at the LHC (90% for the LHC 14 TeV)
- the method/formalism outlined for DY can be extended to the case of 4 external coloured particles:
  - the hard and soft functions are ***matrices in colour space***
  - the anomalous dimensions are non-diagonal ***matrices in colour space***
  - more complicated kinematics: different soft limits can be studied (PIM, 1PI)

Heavy quark production

- Many groups contributed to this, ***be careful*** many different soft limits and kinematics *[Catani,Mangano,Nason,Trentadue '96, Bonciani, Catani, Mangano, Nason '98, Kidonakis, Sterman '96-'97,Kidonakis, Laenen, Moch, Vogt '01, Kidonakis, Vogt '03-'08, Moch, Uwer '08, Czakon, Mitov '09, Langenfeld, Moch, Uwer '09, Beneke, Falgari, Schwinn '10, Czakon, Mitov, Sterman '09, Beneke, Czakon, Falgari, Mitov, Schwinn '09,…]*

- Soft-gluon resummation in SCET at NNLL accuracy and approximate NNLO results for PIM and 1PI kinematics were computed here in Mainz by [Ahrens, Ferroglia, Neubert, Pecjak, Yang, '10, '11]

- Computation of the two-loop soft anomalous dimensions [Ferroglia, Neubert, Pecjak, Yang '09]

- Improved predictions for several observables: total cross section, invariant mass distribution, p_T and rapidity distribution of (anti-)top, asymmetry

PIM & 1PI kinematics

[Kidonakis, Laenen, Moch, Sterman] [Ahrens, Ferroglia, Neubert, Pecjak, Yang]

The Pair Invariant Mass kinematics (PIM)

$$N_1(P_1) + N_2(P_2) \to (t+\bar{t})(p_t + p_{\bar{t}}) + X(p_X)$$

$$M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$$

to study the invariant mass distribution

$$\boxed{\frac{d^2\sigma}{dM\, d\cos\theta}}$$

$$(1-z) = 1 - \frac{M_{t\bar{t}}^2}{s} \to 0$$

Soft gluon Energy $\quad E_s = \dfrac{(1-z)M_{t\bar{t}}}{2\sqrt{z}}$

One Particle Inclusive kinematics (1PI)

$$N_1(P_1) + N_2(P_2) \to t(p_t) + (\bar{t}+X)(p_{\bar{t}} + p_X)$$

$$s_4 = (p_{\bar{t}} + p_X)^2 - m_t^2$$

to study the transverse momentum and rapidity distributions

$$\boxed{\frac{d^2\sigma}{dp_T\, dy}}$$

$$s_4 \to 0$$

Soft gluon Energy $\quad E_s = \dfrac{s_4}{\left(2\sqrt{m_t^2 + s_4}\right)}$

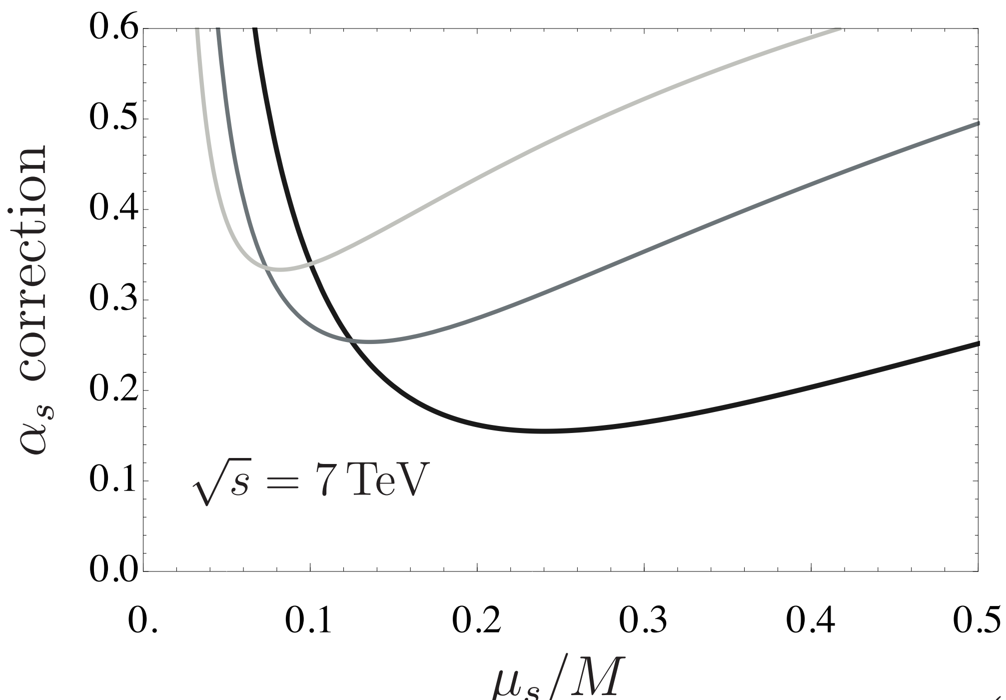
- large logarithms appear in the partonic cross section when the "soft variable" is small
- the difference between PIM and 1PI comes from which combinations of momenta are counted as "small". In PIM $(p_3 + p_4)\cdot k \ll M^2$, in 1PI $p_4 \cdot k \ll m_{\tilde{t}_1}^2$

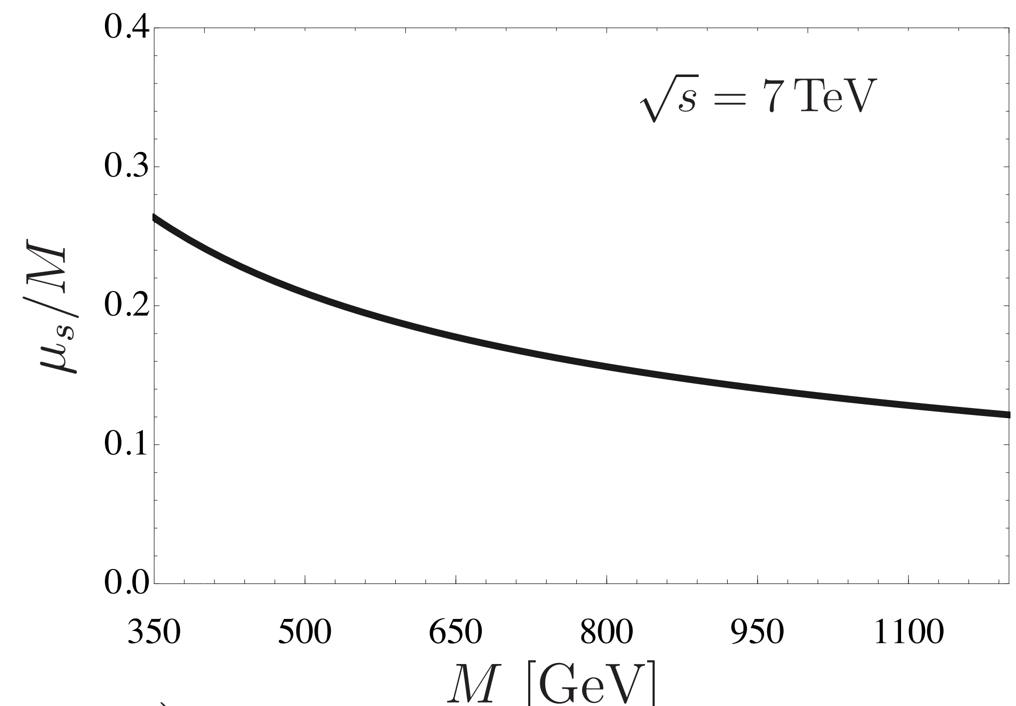
Choice of the soft scale (PIM)

Ahrens, Ferroglia, Neubert, Pecjak, Yang, '10 [arXiv:1003.5827]

$$C(z, M, m_t, \cos\theta, \mu_f) = \exp\left[4a_{\gamma\phi}(\mu_s, \mu_f)\right] \,\mathrm{Tr}\Big[\boxed{\boldsymbol{U}(M, m_t, \cos\theta, \mu_h, \mu_s)}\,\boxed{\boldsymbol{H}(M, m_t, \cos\theta, \mu_h)}\,\boldsymbol{U}^\dagger(M, m_t, \cos\theta, \mu_h, \mu_s)$$

$$\times\, \boxed{\tilde{\boldsymbol{s}}\left(\ln\frac{M^2}{\mu_s^2} + \partial_\eta, M, m_t, \cos\theta, \mu_s\right)}\Big] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$





$$\mu_s^{\mathrm{def}} = \frac{M(1-\tau)}{(a + b\,\tau^{1/4})^c}$$

- Numerical results for μ_s has a function of M can be fitted by this function with $a = -1.3, b = 23, c = 0.98$ for the LHC 7 TeV
- This is a difference with the "Direct QCD" approach, one can choose the soft scale in Mellin space (see later)

Approximate results

By solving iteratively the RG equations one can obtain approximate NNLO predictions

$$\frac{d}{d\ln\mu}\tilde{\mathbf{s}} = \mathbf{\Gamma}_{\mathbf{s}}^{\dagger}\tilde{\mathbf{s}} + \tilde{\mathbf{s}}\mathbf{\Gamma}_{\mathbf{s}}$$

$$C_{ij}^{(2)} = \mathrm{Tr}\left[\mathbf{H}_{ij}^{(1)}\mathbf{S}_{ij}^{(1)}\right] + \mathrm{Tr}\left[\mathbf{H}_{ij}^{(0)}\mathbf{S}_{ij}^{(2)}\right] + \mathrm{Tr}\left[\mathbf{H}_{ij}^{(2)}\mathbf{S}_{ij}^{(0)}\right]$$

α_s^2 expansion $$C^{(2)}(z,\mu) = \sum_{i=0}^{3} \boxed{D_i(\mu)}\,\boxed{P_i(z)} + C_0(\mu)\delta(1-z) + R(z,\mu)$$

$$P_n(z) \equiv \left[\frac{\ln^n(1-z)}{1-z}\right]_+$$

Matching $$\sigma^{\text{NLO+ approx NNLO}} = \sigma^{\text{NLO}} + \sigma^{\text{approx. NNLO}} - \sigma^{\text{approx. NLO}}$$

Boosted tT production at NNLO+NNLL'

M. Czakon, A. Ferroglia, D. Heymes, A. Mitov, B.D. Pecjak, D.J. Scott, X. Wang, L.L. Yang,
[arXiv:1803.0723]

- The LHC enables us to study the kinematic regime where the energy of the produced top quark is much larger than the top quark mass

$$\hat{s},\, M_{t\bar{t}}^2 \gg m_t^2 \gg \hat{s}(1-z)^2$$

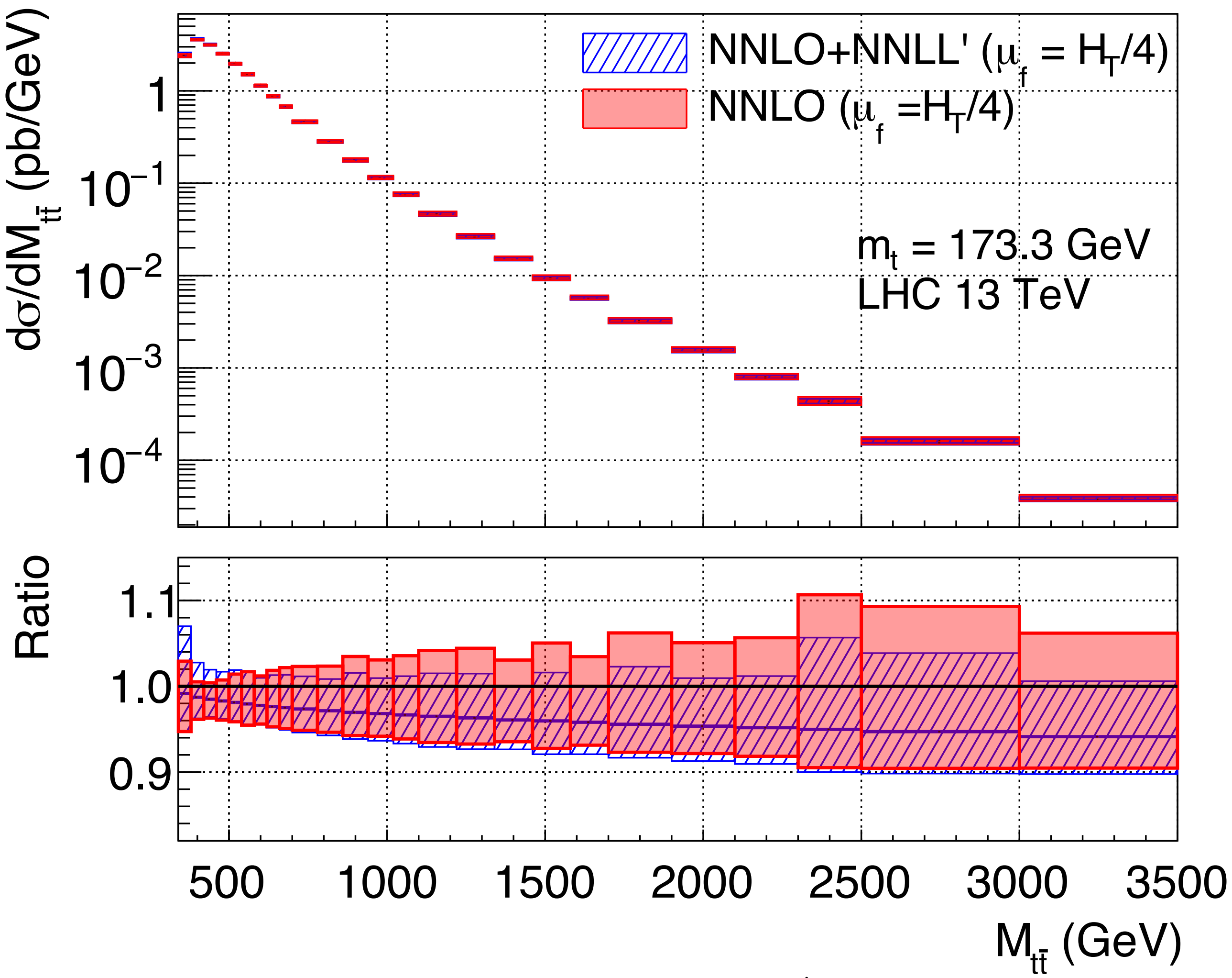
- For $m_t \to 0$ large logs of the type $\ln m_t/M_{t\bar{t}}$ appear in the soft resummation formula. One can resum soft and small mass logs (joint soft-small mass resummation)

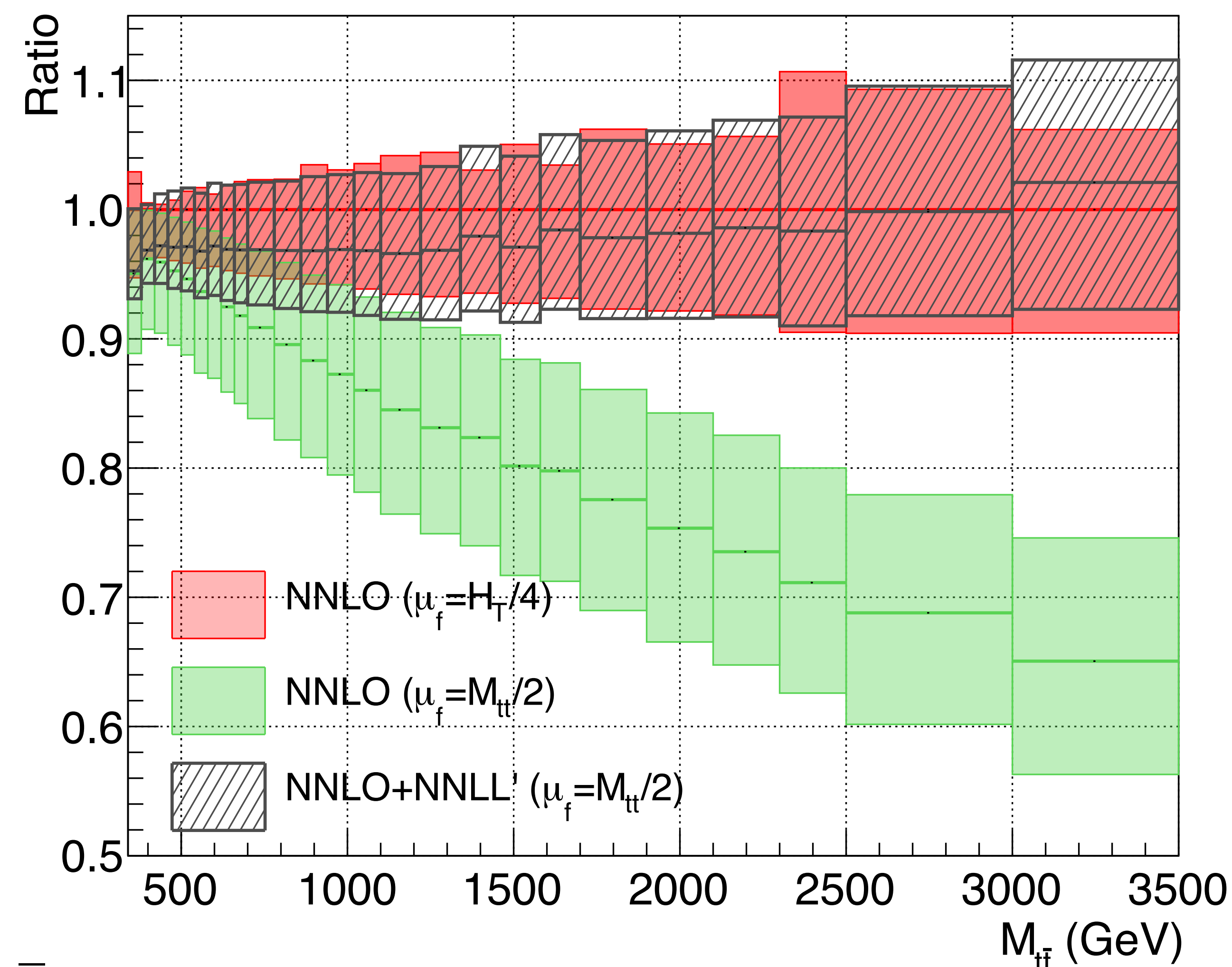
- In this framework the resummation can be pushed to NNLL' accuracy (NNLO soft and hard functions known for zero top quark mass)

- One can match the NNLL' to the NNLO calculation of Czakon, Fiedler, Heymes, Mitov.

NNLO+NNLL` top pair invariant mass

M. Czakon, A. Ferroglia, D. Heymes, A. Mitov, B.D. Pecjak, D.J. Scott, X. Wang, L.L. Yang,
[arXiv:1803.0723]





$$\mu_h = H_T/2 \quad \mu_s = H_T/\bar{N} \quad \mu_f = \{H_T/4,\, M_{t\bar{t}}/2\}$$

In the tail of the invariant mass distribution the relevant hard scale is proportional to H_T

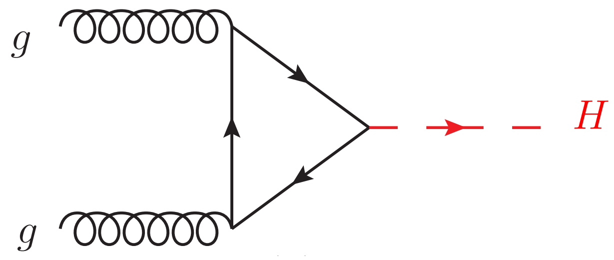
The NNLO+NNLL' prediction is significantly less sensitive to the choice of factorization scale compared to fixed order predictions, even at NNLO

Top quark and Higgs boson

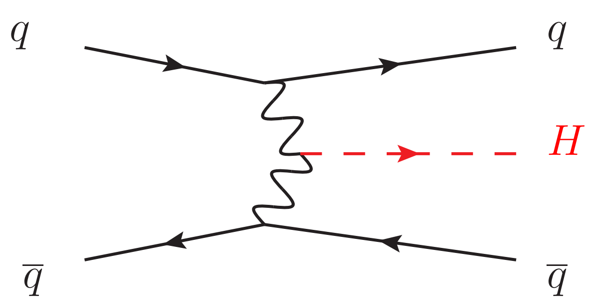
the two heaviest Standard Model (SM) particles
mt~173 GeV, mH~125 GeV

Higgs production channels

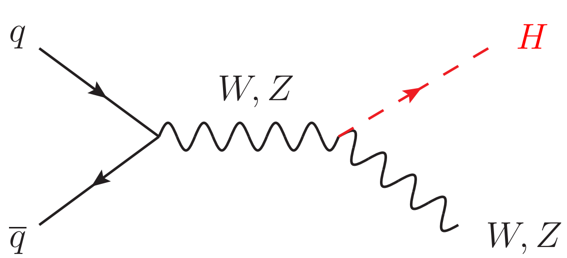
LHC @ 14 TeV



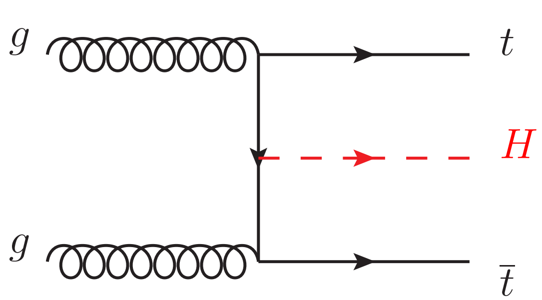
54.67 pb



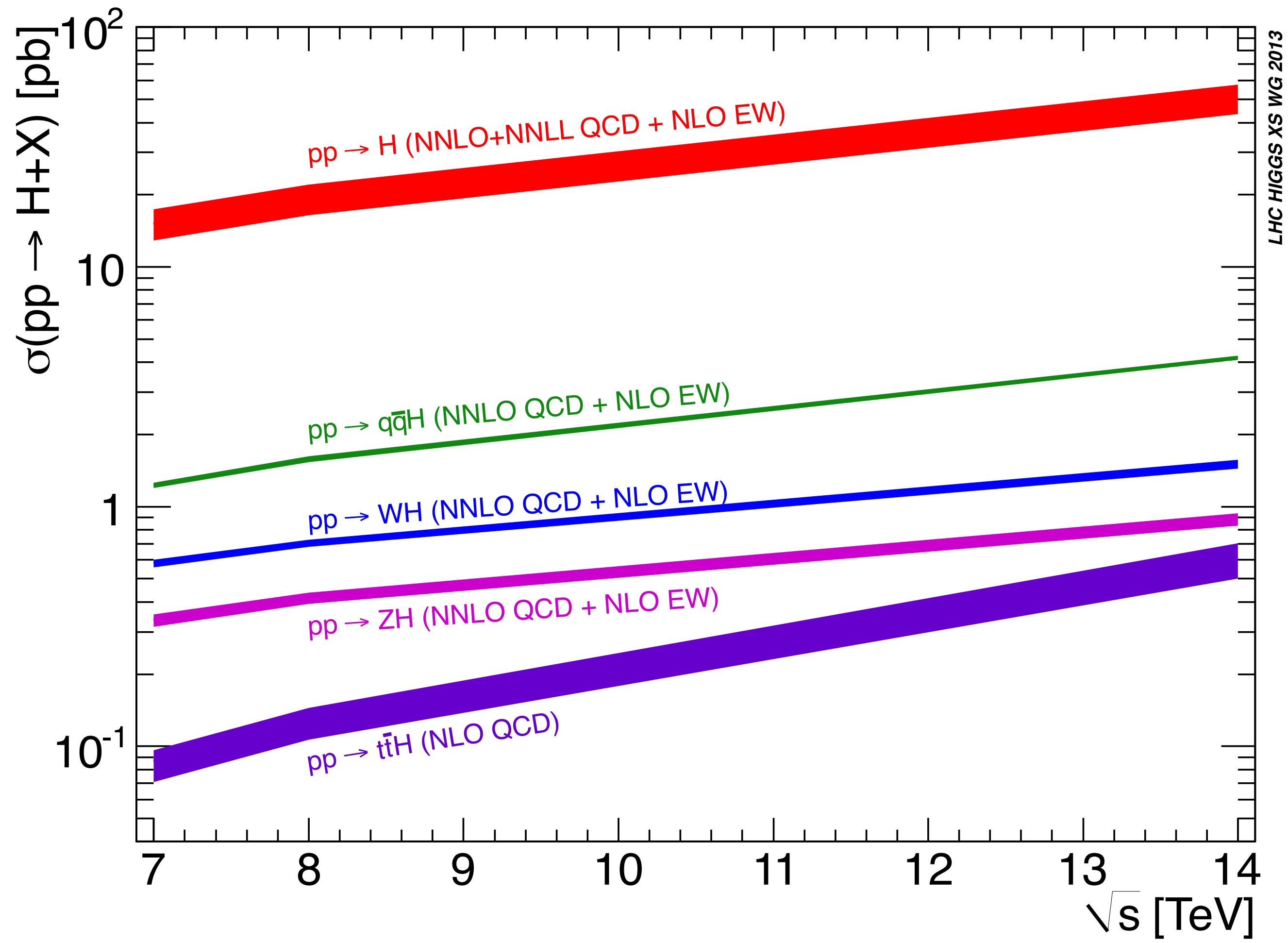
4.18 pb



1.50 pb (WH)
0.88 pb (ZH)



~ 0.61 pb



Observation ATLAS [arXiv:1806.00425]

$670^{+90+110}_{-90-100}$ fb

Observation CMS: [arXiv:1804.02610]

direct measurement of the top Yukawa coupling

$$\sigma \propto g^2_{ttH}$$

top pair + Higgs calculations

- Cross section and some distributions computed to NLO QCD (Beenakker, Dittmaier, Kraemer, Plumper, Spira, Zerwas '01-'02 and Dawson, Reina, Wackeroth, Orr, Jackson '01-'03)
- top pair + Higgs benchmark process to test automated NLO multileg codes (Frixione et al. '11; Hirschi et al '11; Garzelli et al '11; Bevilacqua et al. '11)
- EW corrections to the parton level cross section are known (Frixione, Hirshi, Pagani, Shao, Zaro '14; Zhang, Ma, Chen, Guo '14; Frixione, Hirshi, Pagani, Shao, Zaro '15)
- NLO QCD corrections were interfaced with SHERPA and POWHEG BOX (Gleisberg, Hoeche, Krauss, Schonherr, Schaumann '09; Hartanto, Jaeger, Reina, Wackeroth '15)
- NLO QCD corrections to $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu \mathrm{b}\bar{\mathrm{b}}H$ (Denner, Feger '15)
- NLO+NLL resummation of soft gluon emissions for the total cross section (production threshold limit) (Kulesza, Motyka, Stebel, Theeuwes '15)
- nNLO in the "PIM" threshold limit from NNLL resummation formula (AB, A. Ferroglia, B. Pecjak, A. Signer, L. Yang '15)

and the list continues…

- NLO+NNLL resummation in "TIM" kinematics, RG-evolution in Mellin space (AB, A. Ferroglia, B. Pecjak, A. Signer, L. Yang '16)
- NLO EW and QCD corrections with off-shell top-antitop pairs (A.Denner, J. Lang, M. Pellen, S. Uccirati '16)
- NLO+NNLL resummation in "TIM" kinematics with direct QCD approach (invariant mass distribution of the triplet) (Kulesza, Motyka, Stebel, Theeuwes '17)

“Triplet” Invariant Mass kinematics

Tree Level subprocesses

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

$$\hat{s} = (p_1 + p_2)^2 = 2p_1 \cdot p_2$$

Partonic center of mass energy squared

$$M^2 = (p_3 + p_4 + p_5)^2$$

Invariant mass of the tTH final state

When real radiation is present in the final state → $\hat{s} \neq M^2$ $\quad z = \dfrac{M^2}{\hat{s}} \to 1$ “TIM” soft limit

In the soft emission limit a scale hierarchy emerges

$$\boxed{\hat{s},\ M^2,\ m_t^2,\ m_H^2} \gg \boxed{\hat{s}(1-z)^2} \gg \Lambda^2_{\mathrm{QCD}}$$

Hard scales $\qquad$ Soft scale

Mellin space

- Resummation can also be carried out in Mellin space by taking the Mellin transform of the factorized cross section, similar to "direct QCD" resummation

$$\widetilde{f}(N) \equiv \mathcal{M}[f](N) = \int_0^1 dx x^{N-1} f(x)\,, \quad f(x) = \mathcal{M}^{-1}[\widetilde{f}](x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN x^{-N} \tilde{f}(N)$$

- The total cross section can be recovered with an inverse Mellin transform

$$\sigma(s, m_t, m_H) = \frac{1}{2s}\int_{\tau_{\min}}^1 \frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sum_{ij} \widetilde{ff}_{ij}(N,\mu) \int d\mathrm{PS}_{t\bar{t}H}\, \widetilde{c}_{ij}(N,\mu)$$

- Hard and soft functions are evaluated at values of the scale where the large corrections are absent $\mu_h = M, \qquad \mu_s = M/\bar{N}$

Mellin space

- RG evolution to obtain the hard-scattering kernels at the factorization scale

Hard function (color matrix), obtained using self-modified versions of Openloops and Gosam

$$\widetilde{c}_{ij}(N,\mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}_{ij}(\bar{N},\{p\},\mu_f,\mu_h,\mu_s)\,\boxed{\mathbf{H}_{ij}(\{p\},\mu_h)}\,\widetilde{\mathbf{U}}^{\dagger}_{ij}(\bar{N},\{p\},\mu_f,\mu_h,\mu_s)\right.$$
$$\left.\times\,\boxed{\widetilde{\mathbf{s}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},\{p\},\mu_s\right)}\right]. \qquad (2.10)$$

Soft scale is set in Mellin space

Soft function (color matrix)

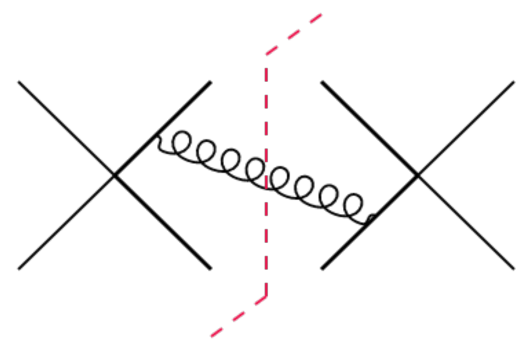
$$\widetilde{\mathbf{U}}\left(\bar{N},\{p\},\mu_f,\mu_h,\mu_s\right) = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)}g_1\left(\lambda,\lambda_f\right) + g_2\left(\lambda,\lambda_f\right) + \frac{\alpha_s(\mu_h)}{4\pi}g_3\left(\lambda,\lambda_f\right) + \cdots\right\}$$
$$\times\,\mathbf{u}(\{p\},\mu_h,\mu_s)\,, \qquad (2.12)$$

$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0\ln\frac{\mu_h}{\mu_s}\,, \qquad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0\ln\frac{\mu_h}{\mu_f}$$

- Differences with the "Direct QCD" approach can still be present in the evolution of the hard function

Ingredients of the calculation

- NLO soft function: similar to the ttbar soft function, we had to recompute it for a general kinematics



- Two loop soft anomalous dimension, adapted to a 2 to 3 kinematics from the t-tbar calculation (more general kinematics) [Ferroglia, Neubert, Pecjak, Yang 09']

- The calculation of the NLO hard function requires the one loop amplitudes for a 2 to 3 process, separating out the various color components

  - It is convenient to take advantage of the automated tools available on the market

  - However none provided the hard functions out of the box and all require a certain level of customization

Modified versions of Gosam, Openloops (+ Collier), Madloop
(with the help of the developers)

Total cross section tTH

$$\check{\Sigma}^{t\bar{t}V}_{\mathrm{NLO}}(\alpha_s,\alpha) = \alpha_s^3\alpha\Sigma^{t\bar{t}V}_{4,0} + \alpha_s^2\alpha^2\Sigma^{t\bar{t}V}_{4,1} + \alpha_s\alpha^3\Sigma^{t\bar{t}V}_{4,2} + \alpha^4\Sigma^{t\bar{t}V}_{4,3}$$
$$\equiv \check{\Sigma}_{\mathrm{NLO}_1} + \check{\Sigma}_{\mathrm{NLO}_2} + \check{\Sigma}_{\mathrm{NLO}_3} + \check{\Sigma}_{\mathrm{NLO}_4}\,.$$

Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos [arXiv:1907.04343]

$$\boxed{\mu_f^0 = \frac{m(t\bar{t}V)}{2}\,, \qquad \mu_h^0 = m(t\bar{t}V)\,, \qquad \mu_s^0 = \frac{m(t\bar{t}V)}{\bar{N}}}$$

$$\boxed{\mu_f^0 = \frac{H_T}{2}\,, \qquad \mu_h^0 = \frac{H_T}{2}\,, \qquad \mu_s^0 = \frac{H_T}{\bar{N}}}$$

Combined scales

Order	σ [fb]		
$\mathrm{LO}_{\mathrm{QCD}}$	336.25(3)	$^{+109.98(+32.7\%)}_{-77.07(-22.9\%)}$	$^{+7.42(+2.2\%)}_{-7.42(-2.2\%)}$
$\mathrm{NLO}_{\mathrm{QCD}}$	467.96(5)	$^{+45.57(+9.7\%)}_{-53.98(-11.5\%)}$	$^{+11.31(+2.4\%)}_{-11.31(-2.4\%)}$
NLO	479.99(5)	$^{+47.46(+9.9\%)}_{-55.42(-11.5\%)}$	$^{+11.45(+2.4\%)}_{-11.45(-2.4\%)}$
$\mathrm{nNLO}_{\mathrm{QCD}}$	490.27(6)	$^{+18.56(+3.8\%)}_{-9.50(-1.9\%)}$	$^{+11.93(+2.4\%)}_{-11.93(-2.4\%)}$
nNLO	502.31(6)	$^{+20.32(+4.0\%)}_{-10.95(-2.2\%)}$	$^{+12.06(+2.4\%)}_{-12.06(-2.4\%)}$
$\mathrm{NLO}_{\mathrm{QCD}}$+NNLL	484.33(7)	$^{+39.60(+8.2\%)}_{-29.43(-6.1\%)}$	$^{+11.78(+2.4\%)}_{-11.78(-2.4\%)}$
NLO+NNLL	496.36(7)	$^{+38.64(+7.8\%)}_{-29.35(-5.9\%)}$	$^{+11.92(+2.4\%)}_{-11.92(-2.4\%)}$

complete NLO QCD+EW

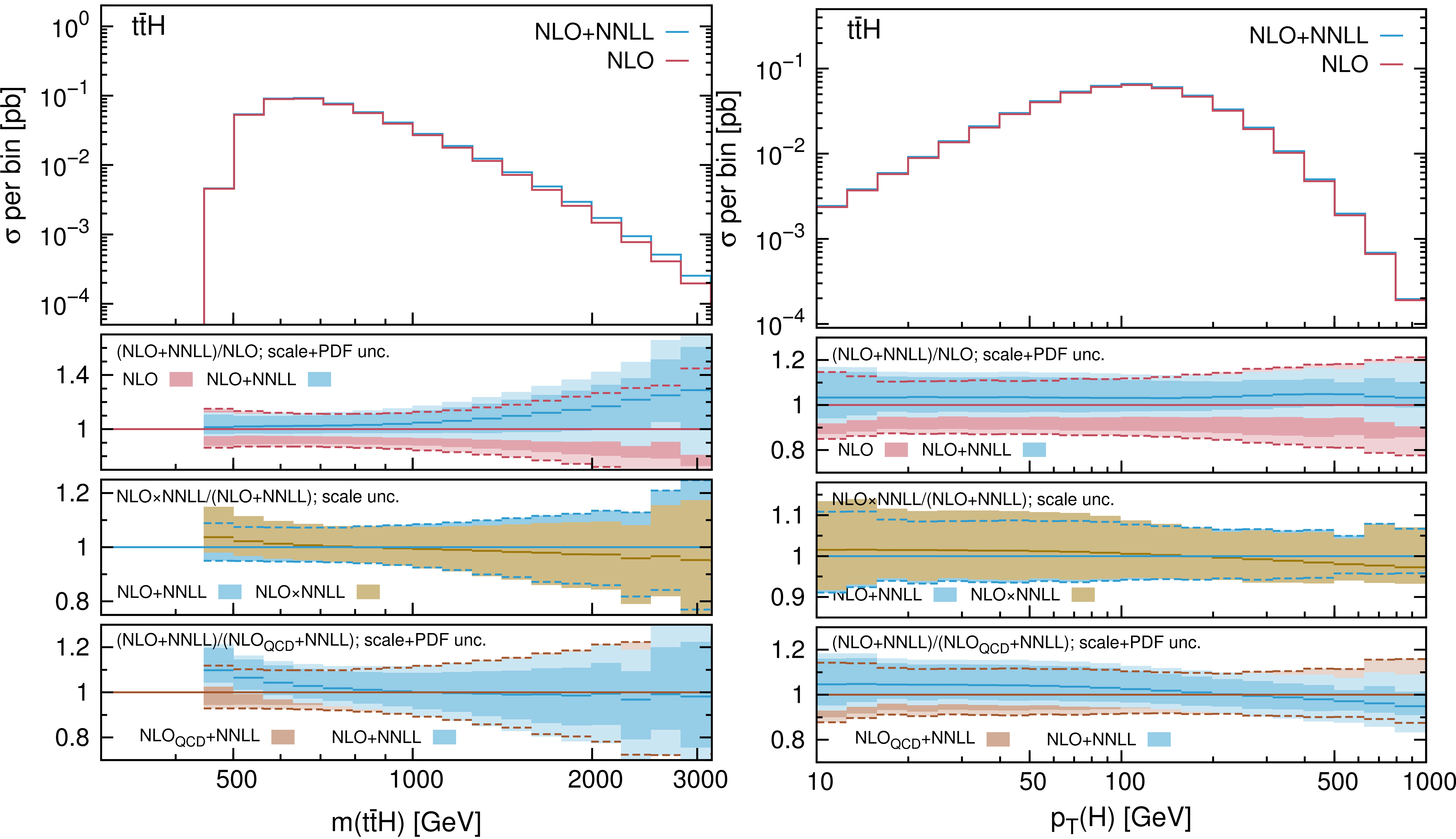
LUX QED PDFs

Uncertainties

$$\Delta O_i^+ = \max\{O\,(\kappa_i = 1/2)\,, O\,(\kappa_i = 1)\,, O\,(\kappa_i = 2)\} - \bar{O}\,,$$
$$\Delta O_i^- = \min\{O\,(\kappa_i = 1/2)\,, O\,(\kappa_i = 1)\,, O\,(\kappa_i = 2)\} - \bar{O}\,,$$

Distributions tTH

Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos [arXiv:1907.04343]



Top pair + W or Z boson

- tTW and tTZ are the two heaviest set of particles measured at the LHC with c.o.m. energy of 7,8,13 TeV

- Important to detect anomalies in the top couplings of the Z boson, and can be considered background processes in new physics searches

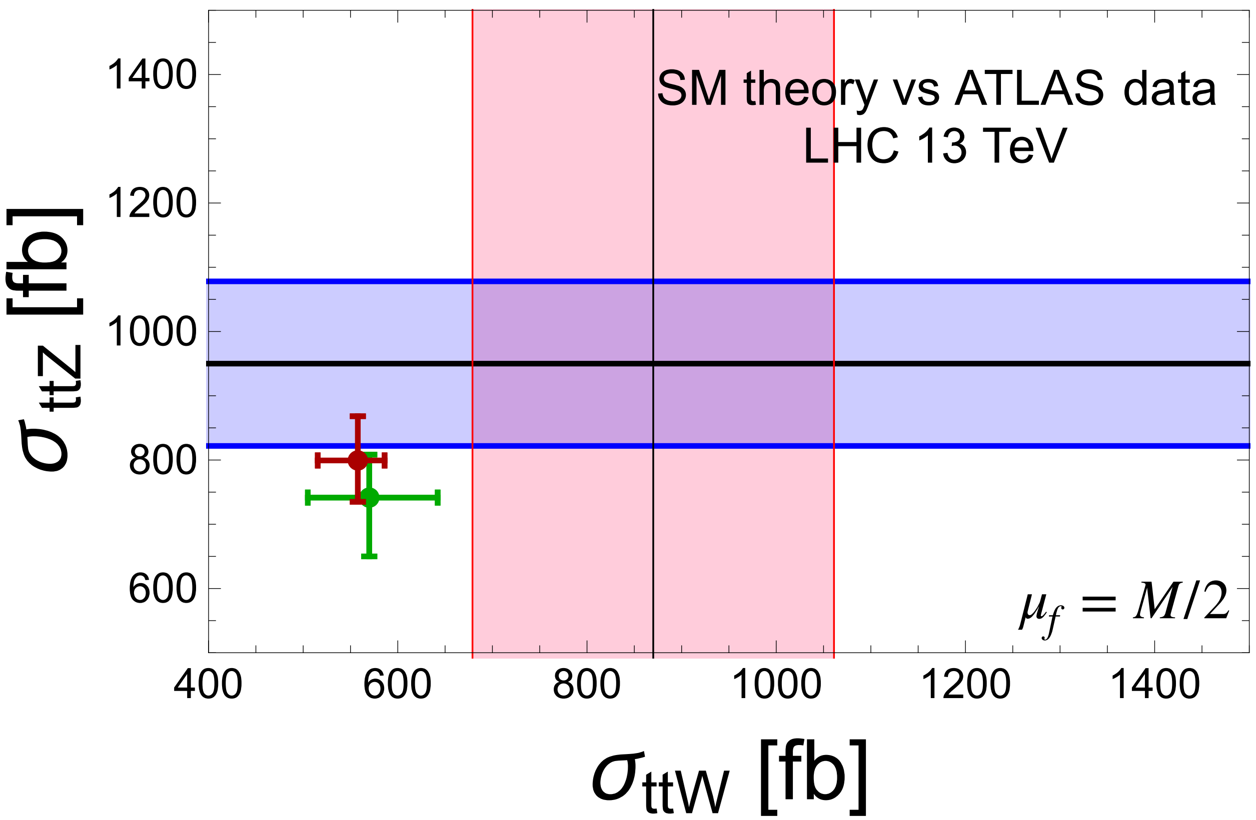
- Both processes were calculated to NLO QCD accuracy by several groups (A. Lazopoulos, T. McElmurry, K. Melnikov, F. Petriello ’07 - ’08, M.V. Garzelli, A. Kardos, C.G. Papadopoulos, Z. Trocsanyi ’12, J.M. Campbell, R.K. Ellis ’12, F. Maltoni, M.L. Mangano, I. Tsinikos, M. Zaro ’14, R. Roentsch and M. Schulze ’14 - ’15)

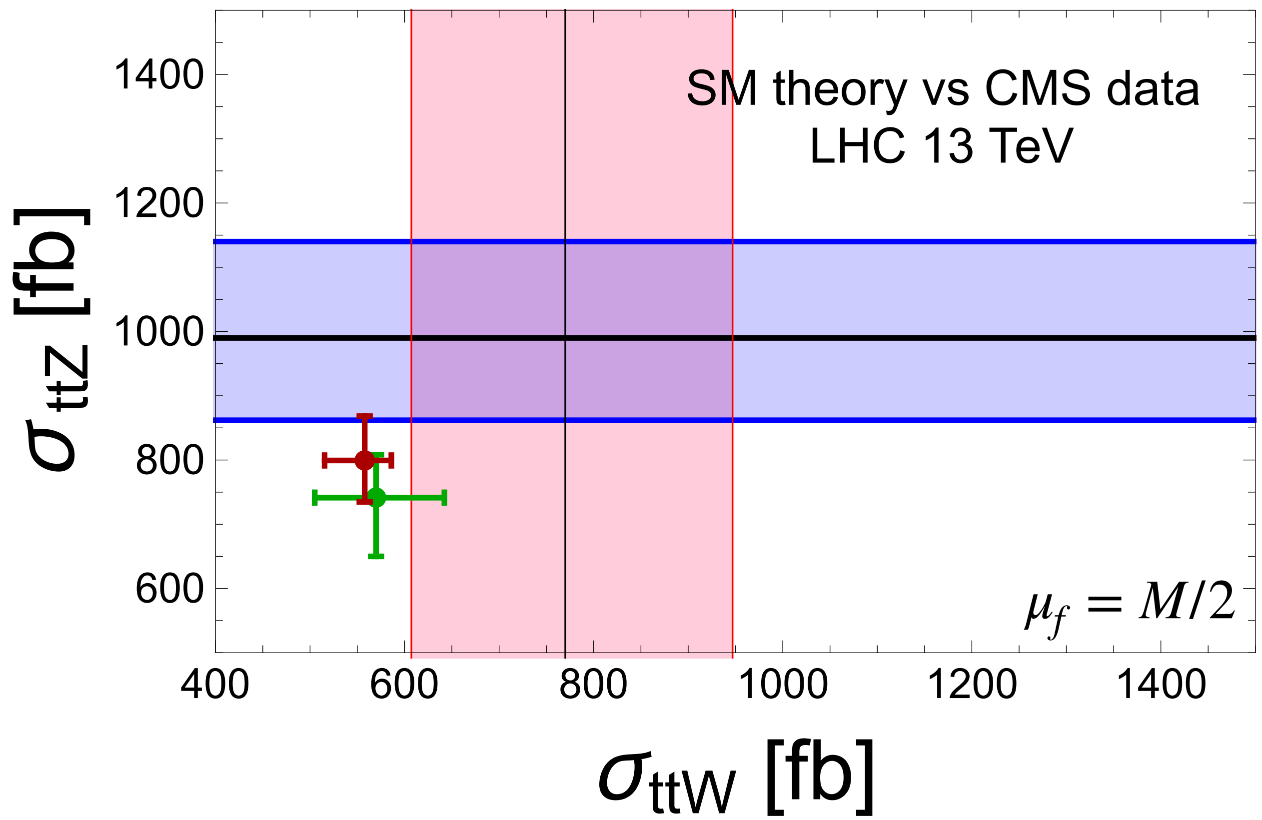
- EW corrections are also known (Frixione, Hirshi, Pagani, Shao, Zaro ’15)

- NLO+NNLL for tTW in momentum space (Li, Li and Li ’14)

- NLO+NNLL for ttW and ttZ in the direct QCD approach (A. Kulesza, L. Motyka, D. Schwartlaender, T. Stebel, V. Theeuwes ‘18)

Top pair + W or Z production





arXiv:1901.03584

arXiv:1711.02547

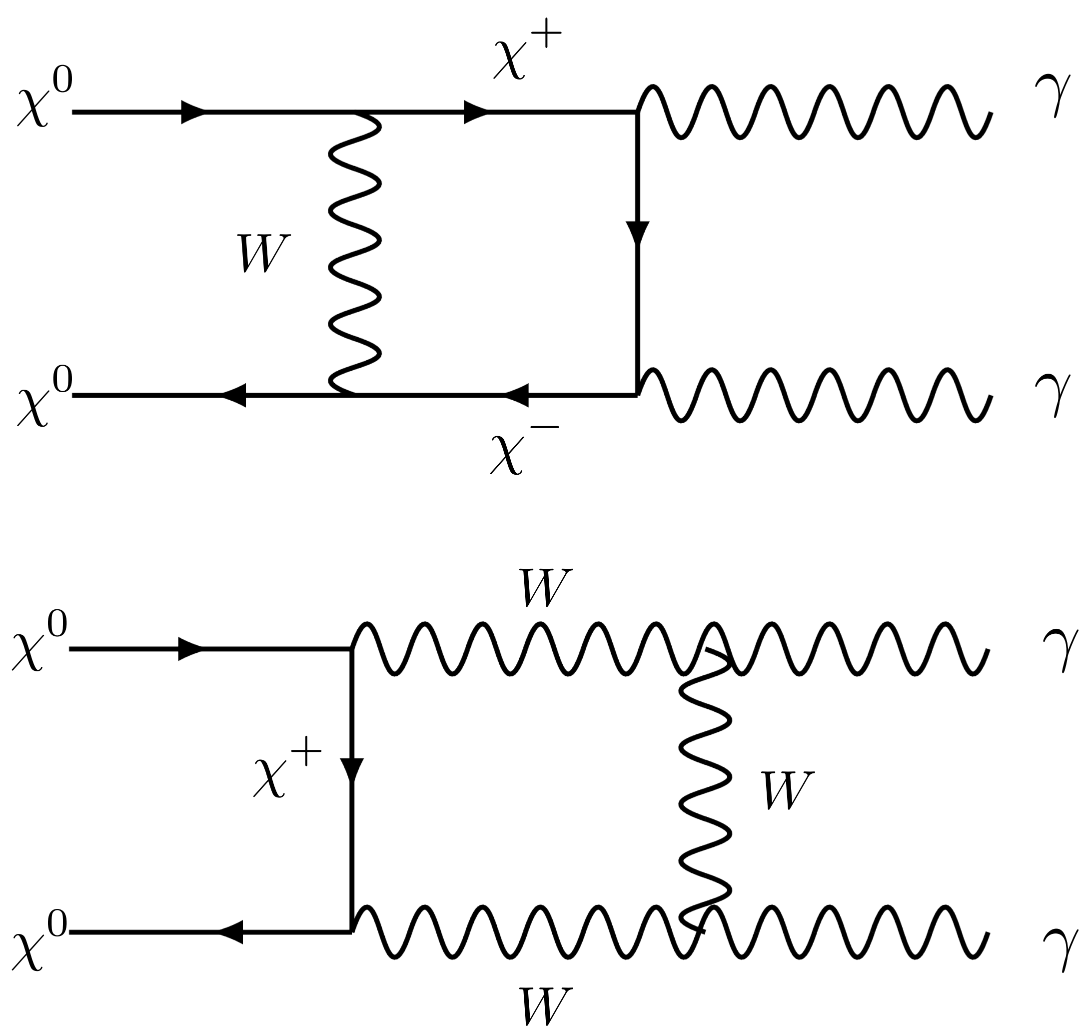
Total cross section at complete NLO
(mg5_aMC@NLO)
and at complete NLO+NNLL.
The crosses reflect only scale uncertainty
and not PDFs uncertainty

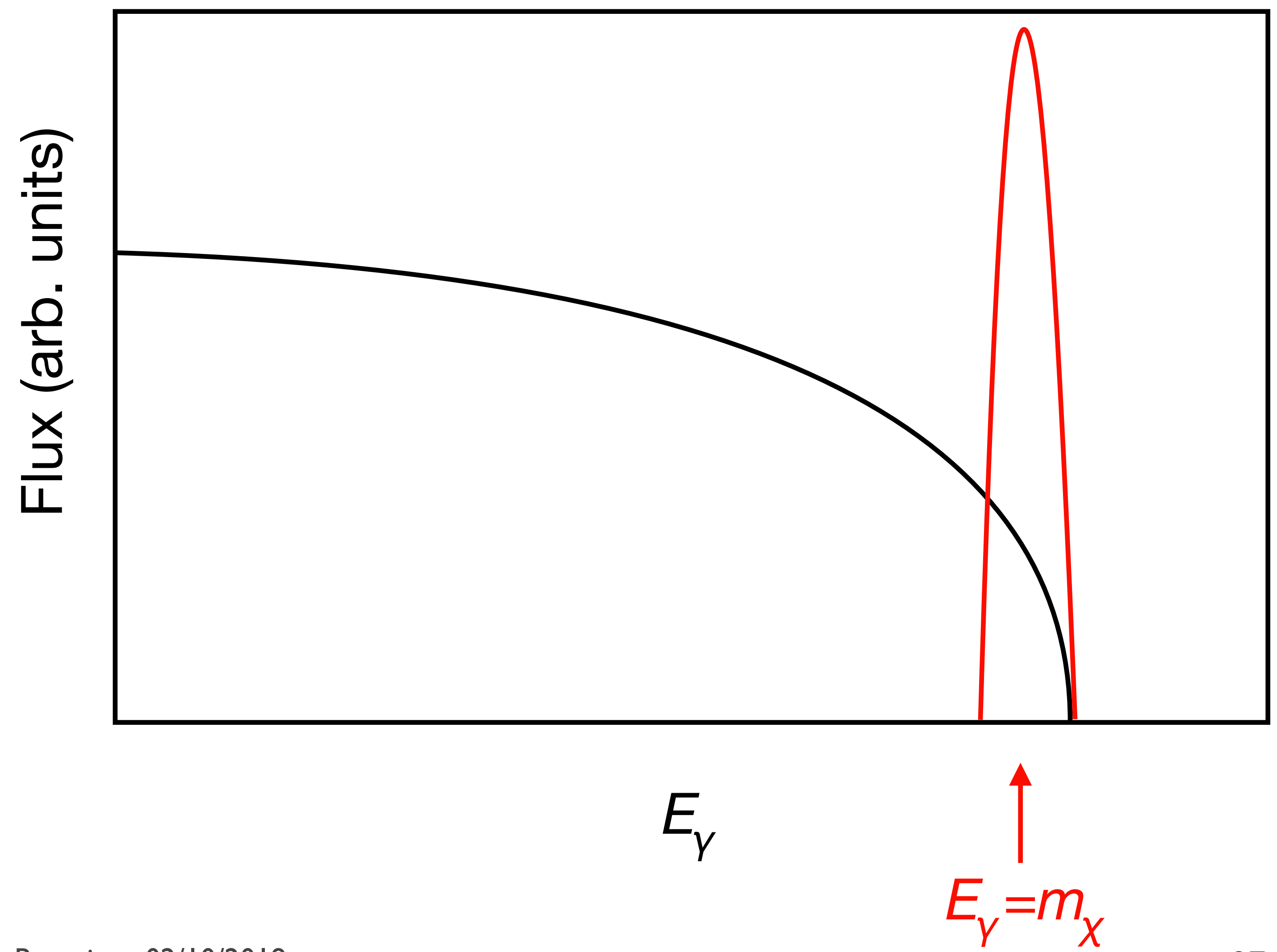
Sudakov resummation for WIMP dark matter annihilation

Introduction

- Dark matter (DM) candidate as Weakly Interacting Particle with mass in the 100 GeV to 10 TeV range (WIMP)

- The pair annihilation of WIMPs into two photons or a photon + Z boson is loop suppressed but provides a clear signature as a monochromatic component of high-energy cosmic gamma rays





Introduction

- TeV-scale DM annihilation is not accurately described by the LO rate, it is modified by the Sommerfeld effect generated by the EW Yukawa force on the DM particles before their annihilation

$$\mathcal{O}((m_\chi \alpha_2 / m_W)^n)$$

- In addition to the Sommerfeld effect, large logarithmically enhanced quantum corrections (Sudakov logarithms) arise due to restrictions on the emission of soft radiation

$$\mathcal{O}((\alpha_2 \ln^2(m_\chi / m_W))^n)$$

- EW Sudakov logarithms in DM annihilation into photons have been identified as potential source of large corrections [Hryczuk, Iengo '12] and need to be resummed to all orders in perturbation theory [Baumgart, Rothstein, Vaidya '15], [Bauer, Cohen, Hill, Solon '14], [Baumgart, Vaydia '15], [Ovanesyan, Slatyer, Stewart '14], [Ovanesyan, Rodd, Slatyer, Stewart '16], [Baumgart, Cohen, Moult, Rodd, Slatyer, Solon, Stewart, Vaidya '17]

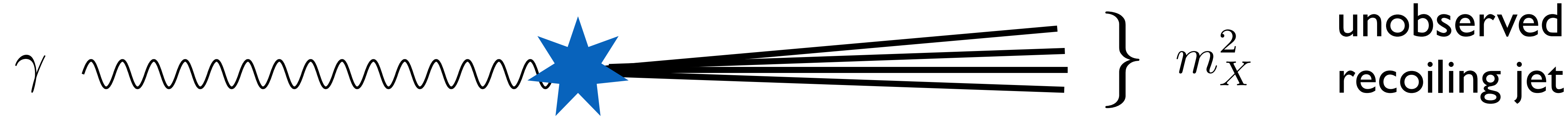
The wino-like triplet model

Add to the SM Lagrangian a fermionic multiplet χ (of Majorana or Dirac type) with arbitrary isospin-j representation of the EW SU(2) gauge group and zero hypercharge (Y=0)

Dirac $\mathcal{L} = \mathcal{L}_{\rm SM} + \overline{\chi}(i\not{D} - m_\chi)\chi \qquad D_\mu = \partial_\mu - i g_2 A_\mu^C T^C$

The DM particle is the electrically neutral member of the 2j+1 multiplet

We consider the process $\chi(p_1) + \chi(p_2) \to \gamma(p_\gamma) + X(p_X)$



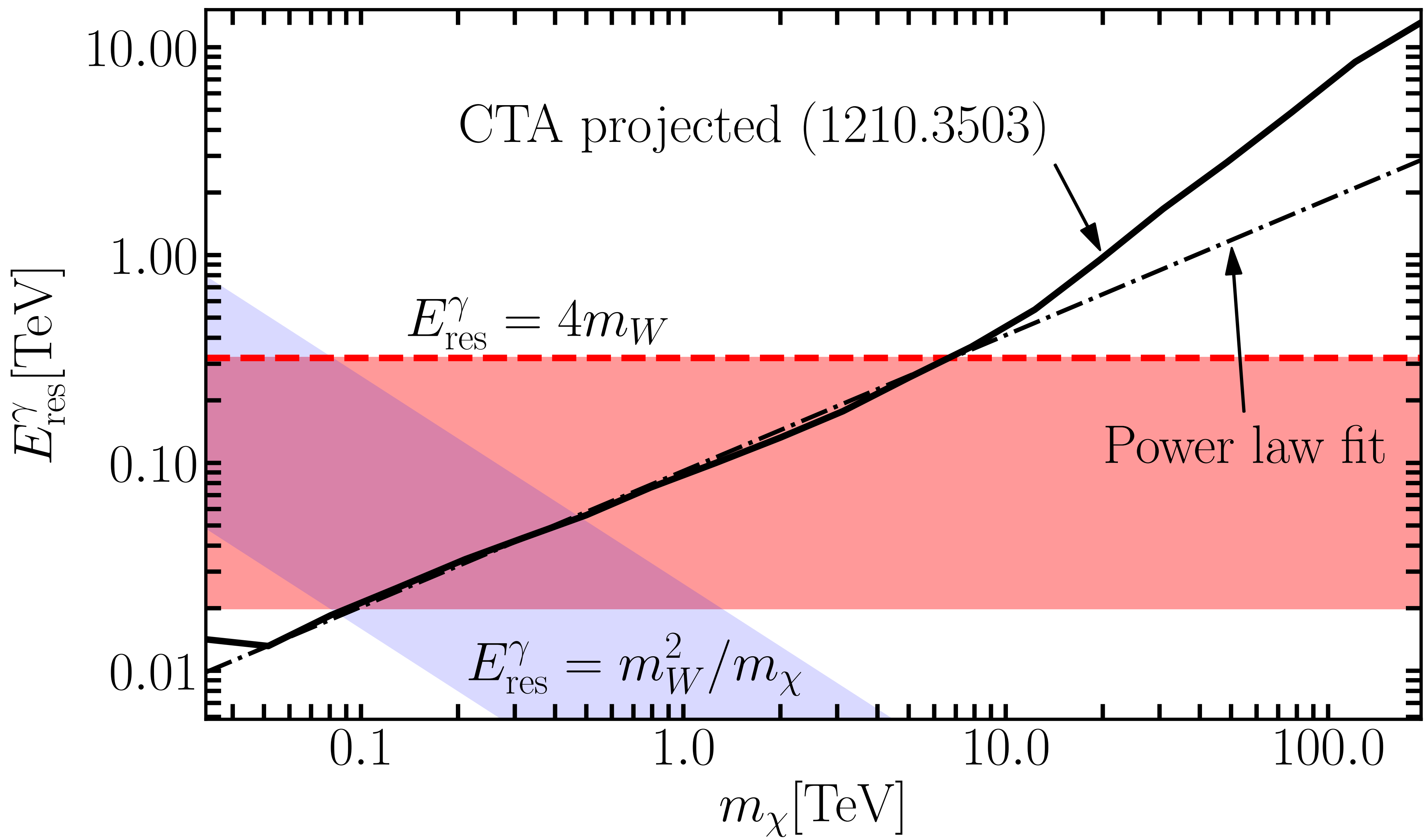
$$\boxed{\langle \sigma v\rangle(E^\gamma_{\rm res}) = \int_{m_\chi - E^\gamma_{\rm res}}^{m_\chi} dE_\gamma \, \frac{d(\sigma v)}{dE_\gamma}}$$

The photon endpoint spectrum depends on 4 scales: m_χ (hard scale), the small invariant mass $m_X = \sqrt{4 m_\chi E^\gamma_{\rm res}}$ of the unobserved energetic final state, the EW scale m_W and the energy resolution scale $E^\gamma_{\rm res}$

Narrow vs Wider resolution

Details of the resummation of EW Sudakov logs differ according to the scaling of E^{γ}_{res} and m_W with respect to each other

Narrow:	$E^{\gamma}_{\text{res}} \sim m_W^2/m_\chi$	Beneke, Broggio, Hasner, Vollmann. [arxiv:1805.07367]
Intermediate:	$E^{\gamma}_{\text{res}} \sim m_W$	Beneke, Broggio, Hasner, Urban, Vollmann. [arxiv:1903.08702]
Wide:	$E^{\gamma}_{\text{res}} \gg m_W$	Baumgart et al. [arxiv:1808.08956]

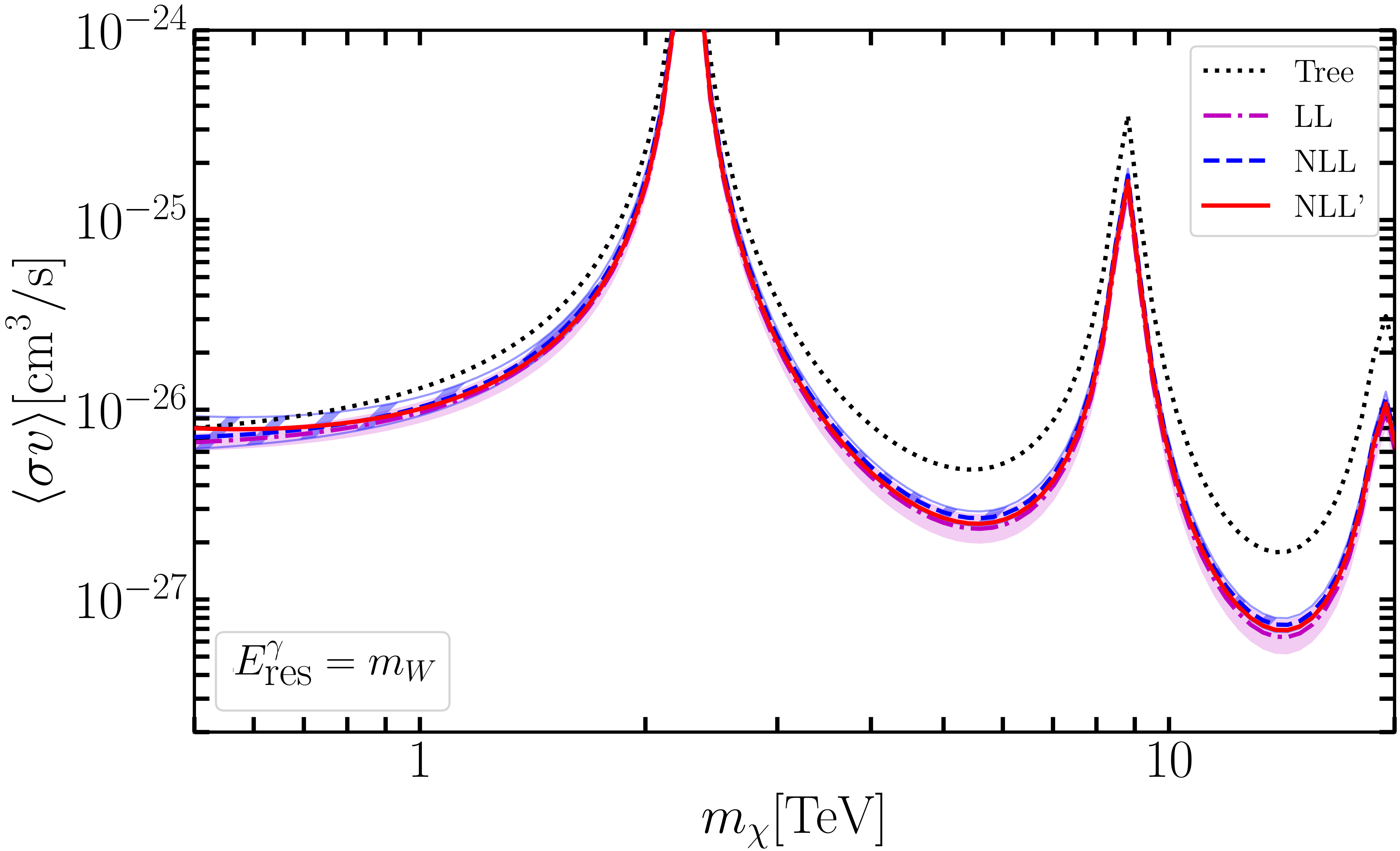


Results

Integrated photon energy spectrum

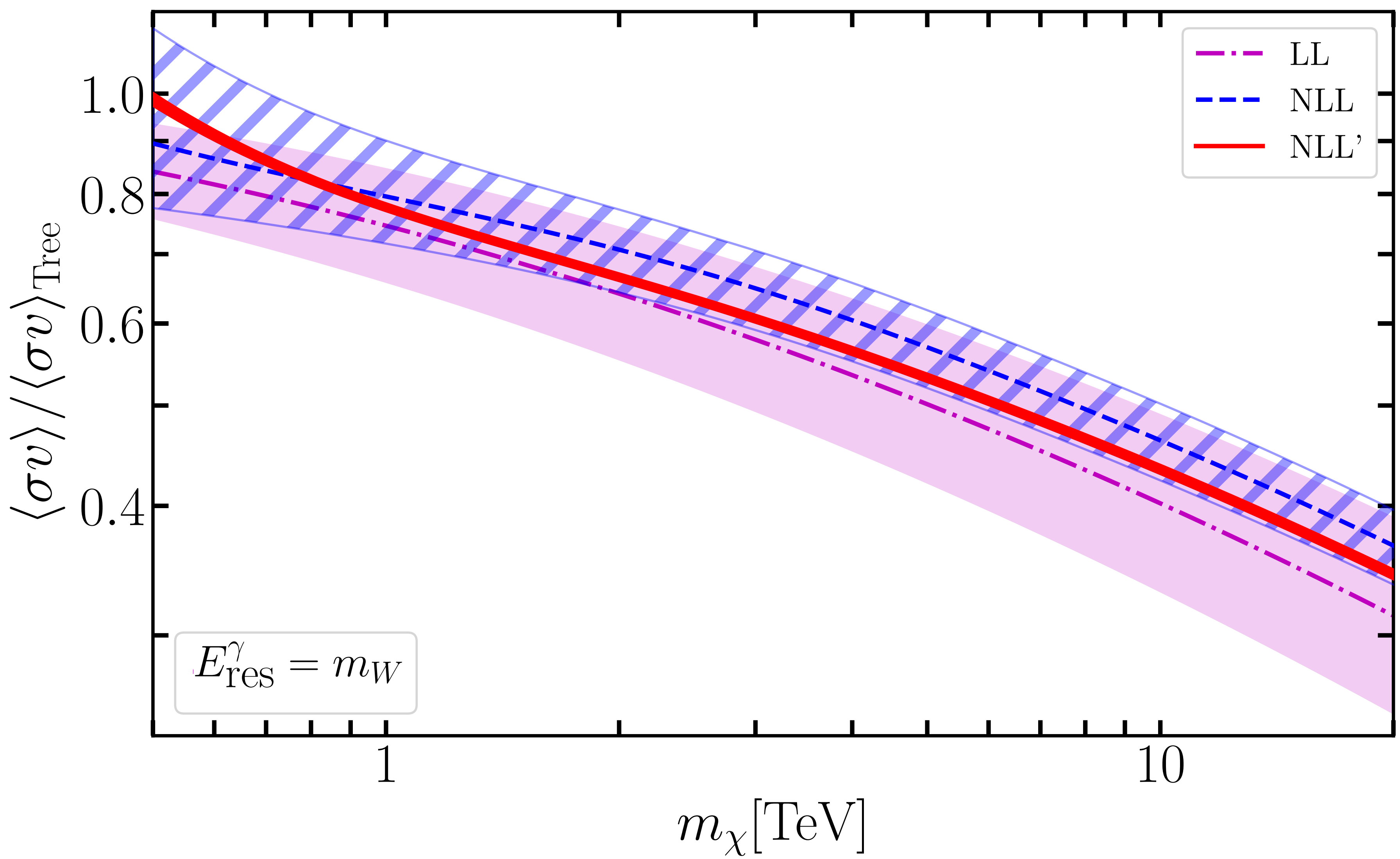
$$\langle \sigma v \rangle (E^{\gamma}_{\text{res}}) = \int_{m_\chi - E^{\gamma}_{\text{res}}}^{m_\chi} dE_\gamma \, \frac{d(\sigma v)}{dE_\gamma}$$

$c = 1$



Results

Results normalized to the Sommerfeld-only result, resummation reduces the annihilation rate, in the interesting mass range around 3 TeV, the rate is suppressed by a factor of ~ 2



- The scale uncertainty reduces from 15% (LL) to 9% (NLL) to 1% (NLL') for $m_\chi = 2$ TeV
- At $m_\chi = 2$ TeV $(10$ TeV$)$ the ratio of the resummed at NLL' to the Sommerfeld-only rate is $0.665^{+0.008}_{-0.007}\,(0.434^{+0.006}_{-0.005})$

Summary

- Basics of threshold resummation for Drell-Yan and top pair production
- Threshold + small mass resummation for boosted ttbar production
- Associated production ttbar+H (or W/Z) at complete NLO+NNLL
- Sommerfeld effect and resummation of Sudakov logarithms for dark matter annihilation processes

Thank you!

Backup slides

PIM & 1PI factorization

Factorization of the cross sections studied in these limits by
QCD: [Kidonakis, Laenen, Moch, Sterman,…], SCET: [Ahrens, Ferroglia, Neubert, Pecjak, Yang, '10, '11]

PIM

$$\frac{d^2\hat{\sigma}}{dM\, d\cos\theta} = \frac{\pi\beta_t}{sM}\sum_{i,j} C_{\mathrm{PIM},\,ij}(z, M, m_t, \cos\theta, \mu_f)$$

$$C_{\mathrm{PIM},\,ij}(z, M, m_t, \cos\theta, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}(M, m_t, \cos\theta, \mu_f)\,\mathbf{S}_{\mathrm{PIM},\,ij}(\sqrt{s}(1-z), M, m_t, \cos\theta, \mu_f)\right]$$

$$P_m(z) = \left[\frac{\ln^m(1-z)}{1-z}\right]_+ \; ; \quad m = 0, \ldots, 2n-1$$

1PI

$$\frac{d^2\hat{\sigma}}{dp_T dy} = \frac{2\pi p_T}{s}\sum_{i,j} C_{\mathrm{1PI},\,ij}(s_4, s, t_1, u_1, m_t, \mu_f)$$

$$C_{\mathrm{1PI},\,ij}(s_4, s, t_1, u_1, m_t, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}(s, t_1, u_1, m_t, \mu_f)\,\mathbf{S}_{\mathrm{1PI},\,ij}(s_4, s, t_1, u_1, m_t, \mu_f)\right]$$

$$\bar{P}_m(s_4) = \left[\frac{\ln^m(s_4/m_t^2)}{s_4}\right]_+ = \frac{1}{m_t^2} P_m\left(1 - \frac{s_4}{m_t^2}\right) \; ; \quad m = 0, \ldots, 2n-1$$

- **H** and **S** satisfy RG equations
- By knowing **H** and **S** at NLO in both kinematics, we can solve explicitly the RG equations for **H** and **S** at NNLO

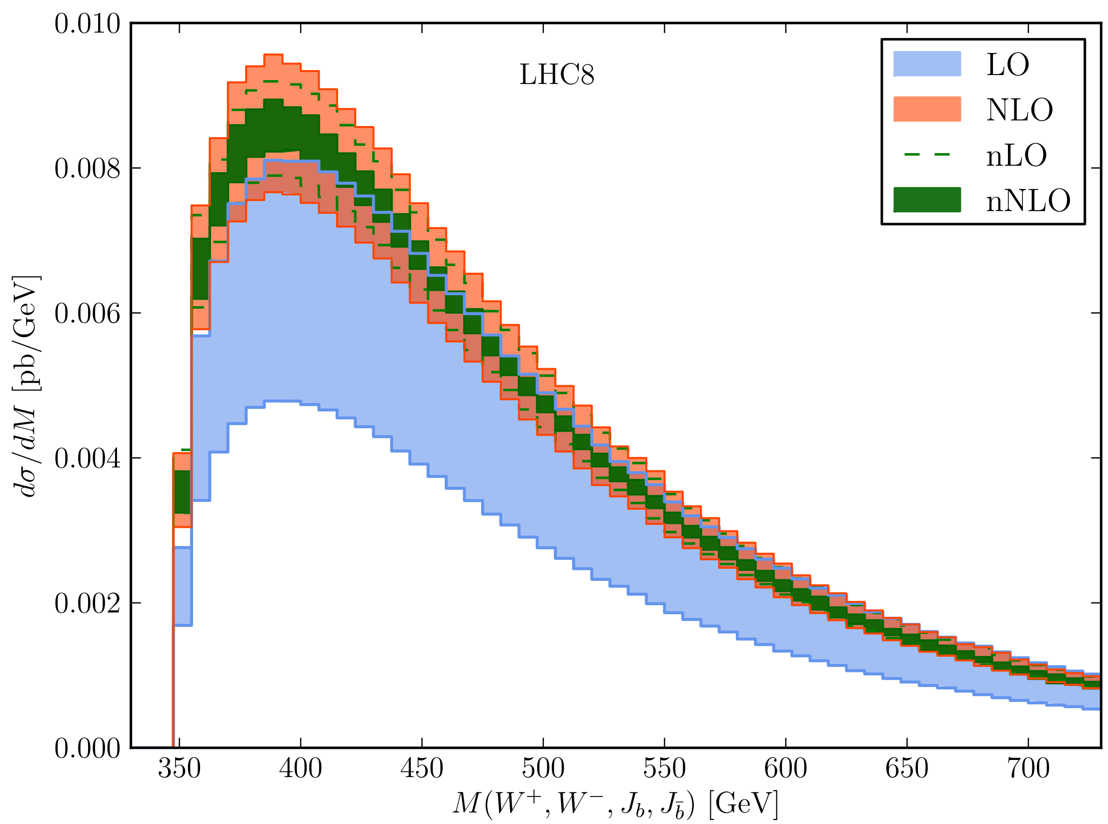
Distributions with final state cuts

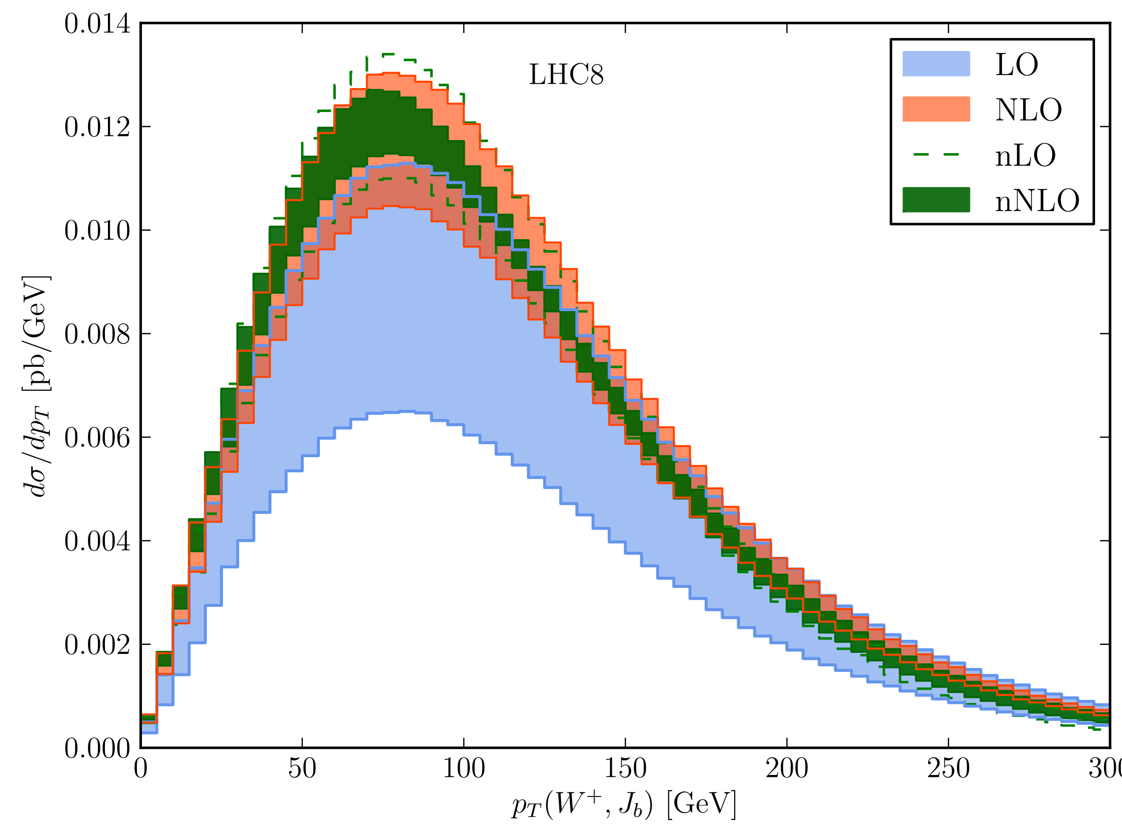
- Cluster final state partons into jets
- Reconstruct top $p(t) \equiv p(W^+) + p(J_b) \neq p_t$
- Cuts

$$p_T(J_b) > 15 \text{ GeV} \qquad p_T(J_{\bar{b}}) > 15 \text{ GeV} \qquad M(W^+, W^-, J_b, J_{\bar{b}}) > 350 \text{ GeV}$$

$$E_T(e^+) > 15 \text{ GeV} \qquad E_T(e^-) > 15 \text{ GeV} \qquad \not{E}_T > 20 \text{ GeV}$$

- Top decay included in NWA at NLO





- uncertainty bands of nNLO: scale variation+kinematics (envelope of PIM and 1PI)
- stable perturbative behaviour, reduction of theoretical uncertainty

Factorization and Resummation

$$\sigma(s, m_t, m_H) = \frac{1}{2s}\int_{\tau_{\min}}^{1}\frac{d\tau}{\tau}\int_{\tau}^{1}\frac{dz}{\sqrt{z}}\sum_{ij}\boxed{ff_{ij}\left(\frac{\tau}{z},\mu\right)}$$

parton luminosity

$$\int d\mathrm{PS}_{t\bar{t}H}\,\mathrm{Tr}\left[\boxed{\mathbf{H}_{ij}(\{p_i\},\mu)}\boxed{\mathbf{S}_{ij}\left(\frac{M(1-z)}{\sqrt{z}},\{p_i\},\mu\right)}\right] + \mathcal{O}(1-z)$$

Hard function (color matrix), obtained using self-modified versions of Openloops and Gosam

Soft function (color matrix)

$$\tau = \frac{M^2}{s}, \qquad \tau_{\min} = \frac{(2m_t + m_H)^2}{s}$$

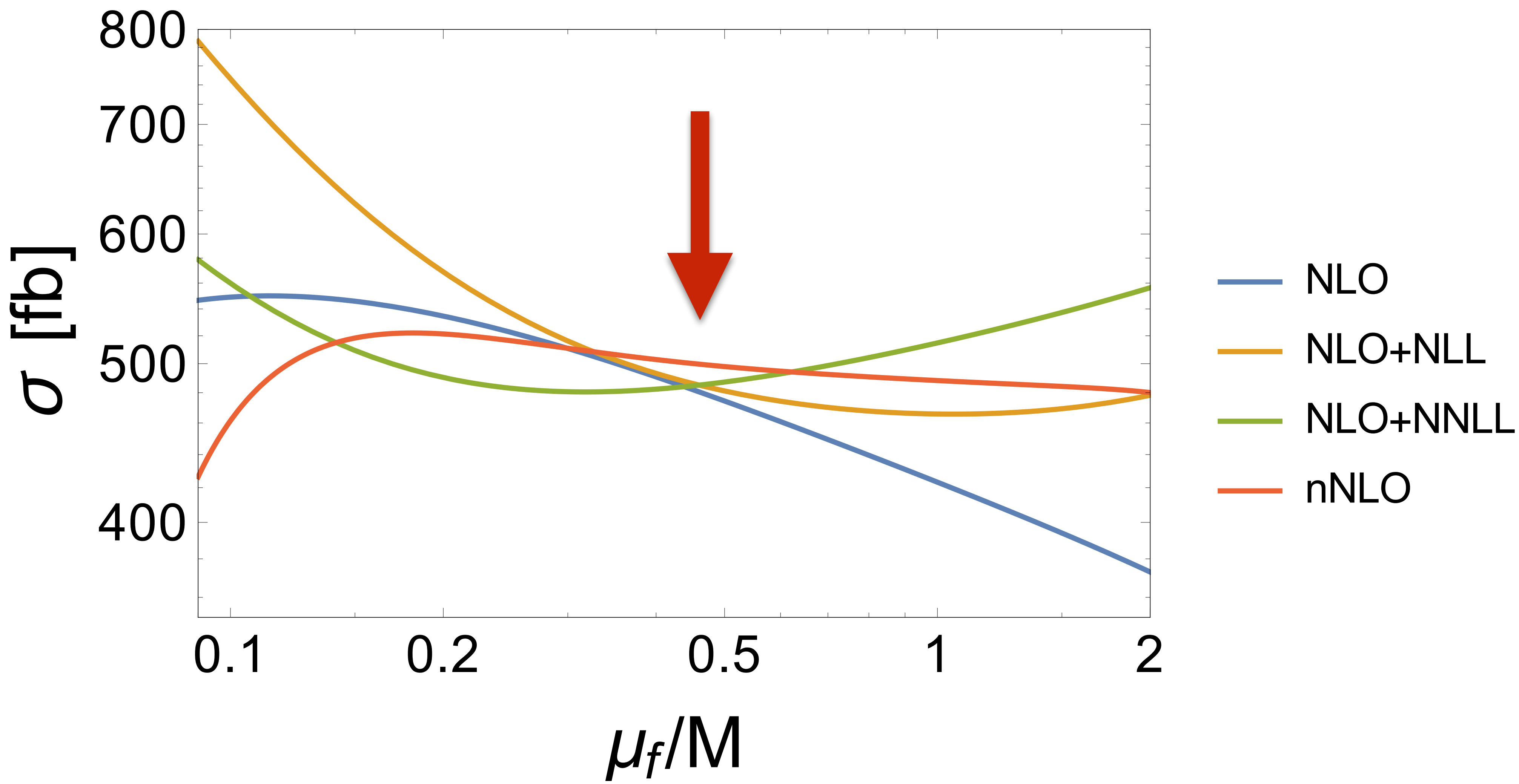
$$P_n(z) \equiv \left[\frac{\ln^n(1-z)}{1-z}\right]_+$$

The hard and soft functions satisfy RG equations that can be solved to obtain the resummed hard-scattering kernels

$$C_{ij}(z,\mu_f) = \exp\left[4a_{\gamma_\phi}(\mu_s,\mu_f)\right]\mathrm{Tr}\Big[\mathbf{U}_{ij}(\{p\},\mu_h,\mu_s)\,\mathbf{H}_{ij}(\{p\},\mu_h)$$
$$\times\,\mathbf{U}^{\dagger}_{ij}(\{p\},\mu_h,\mu_s)\,\tilde{\mathbf{s}}_{ij}\left(\ln\frac{M^2}{\mu_s}+\partial_\eta,\{p\},\mu_s\right)\Big]\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\frac{z^{1/2-\eta}}{(1-z)^{1-2\eta}}\,.$$

tTH: total cross section

Factorization scale choice



Matching

$$\sigma^{\mathrm{NLO+NNLL}} = \sigma^{\mathrm{NLO}} + \left[\sigma^{\mathrm{NNLL}} - \sigma^{\mathrm{approx.\ NLO}}\right]$$

$$\sigma^{\mathrm{NLO+NLL}} = \sigma^{\mathrm{NLO}} + \left[\sigma^{\mathrm{NLL}} - \sigma^{\mathrm{NLL\ expanded\ to\ NLO}}\right]$$

Resummation Accuracy

The large logarithms count as $1/\alpha_s$, it is always possible to rewrite a log of a ratio of two scales as

$$\ln\frac{\nu}{\mu'} = \int_{\alpha_s(\mu')}^{\alpha_s(\nu)} \frac{d\alpha}{\beta(\alpha)} \qquad \beta(\alpha_s) = -2\alpha_s\left[\beta_0\left(\frac{\alpha_s}{4\pi}\right)+\beta_1\left(\frac{\alpha_s}{4\pi}\right)^2+\mathcal{O}(\alpha_s^3)\right]$$

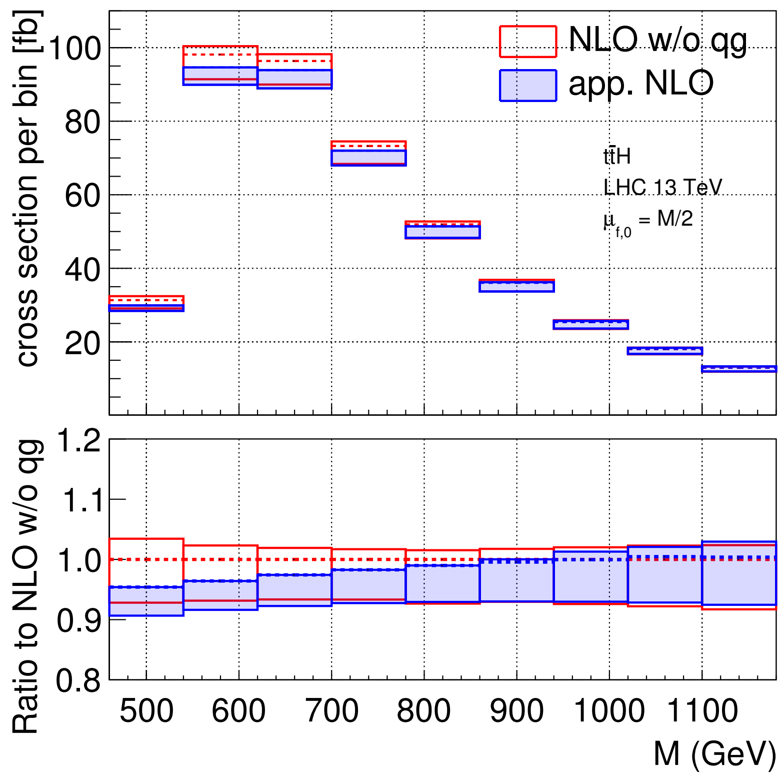
$$\beta(\alpha_s)\sim\alpha_s^2$$

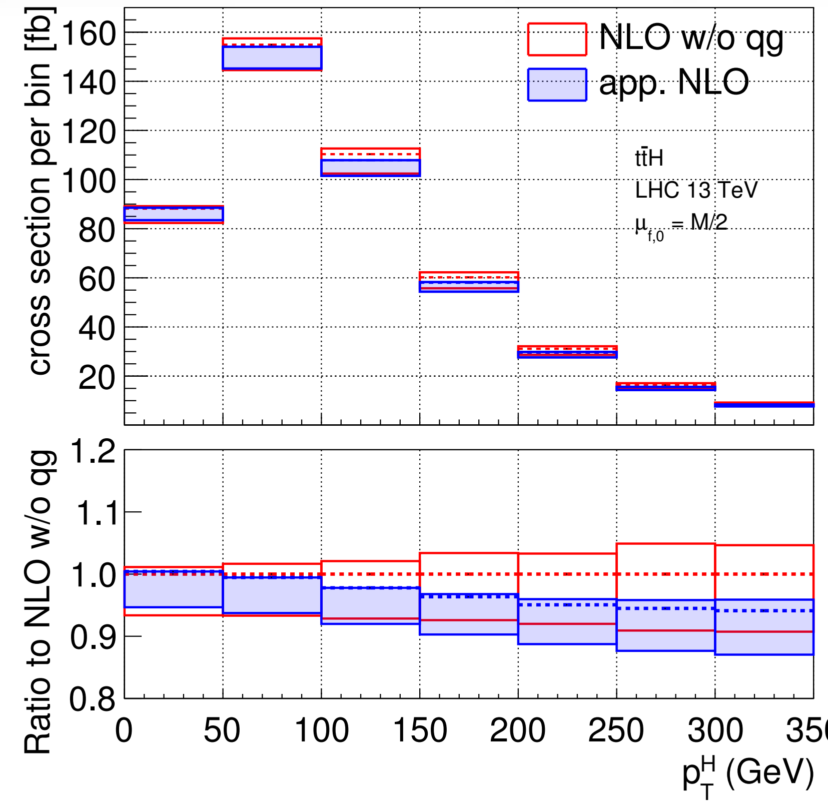
RG-impr. PT	Log. approx.	Accuracy $\sim \alpha_s^n L^k$	Γ_{cusp}	$\gamma^V,\ \gamma^\phi$	$C_V,\ \widetilde{s}_{\text{DY}}$
—	LL	$k=2n$	1-loop	tree-level	tree-level
LO	NLL	$2n-1\le k\le 2n$	2-loop	1-loop	tree-level
NLO	NNLL	$2n-3\le k\le 2n$	3-loop	2-loop	1-loop
NNLO	NNNLL	$2n-5\le k\le 2n$	4-loop	3-loop	2-loop

- the counting is organized in the exponent because the LL and NLL terms count as $\mathcal{O}(1/\alpha_s)$ and $\mathcal{O}(1)$ and cannot be expanded
- N^{m}LL accuracy predicts the first 2m logarithms in the cross section
- It is not uncommon to include the Wilson coefficient and the soft function one order higher than in the table above (N^{m}LL', one L more in Laplace space)

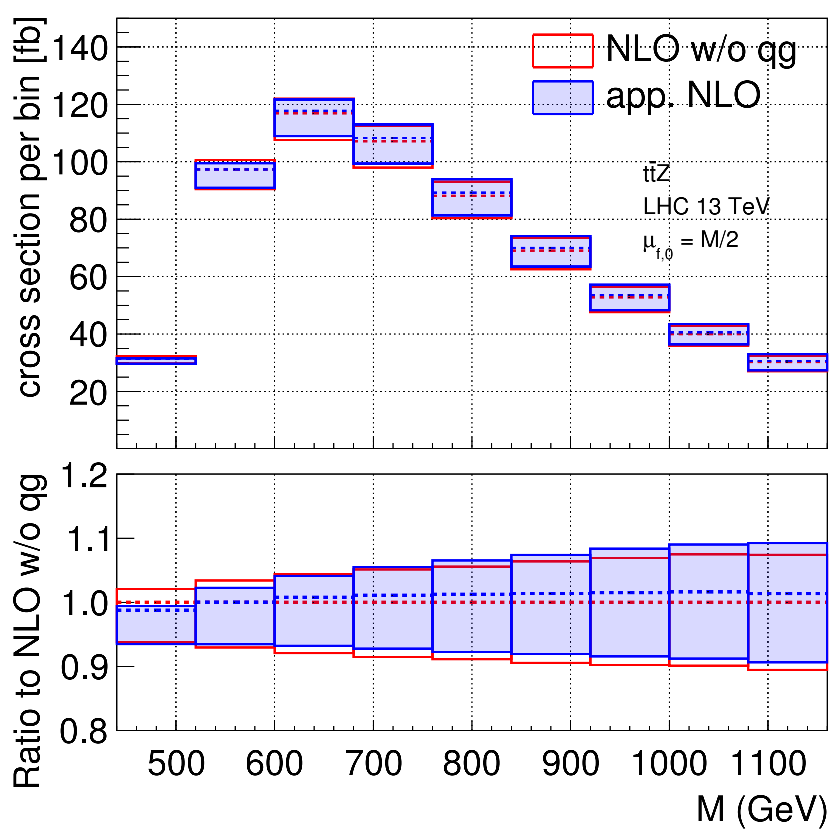
Distributions: nLO vs NLO

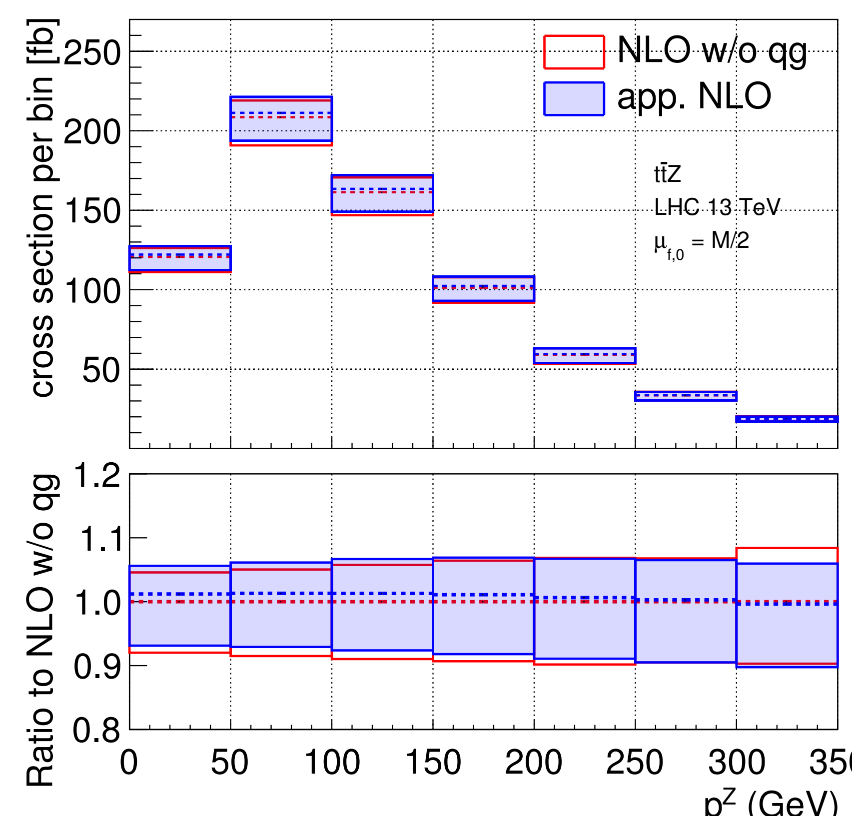
tTH:nLO vs NLO
without qg channel



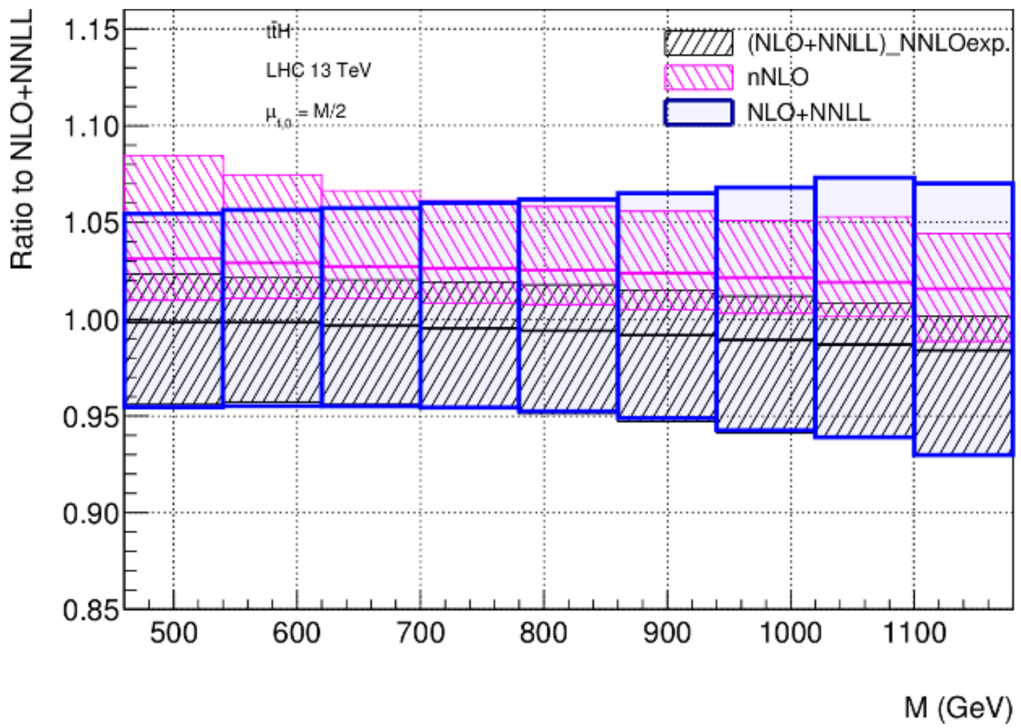


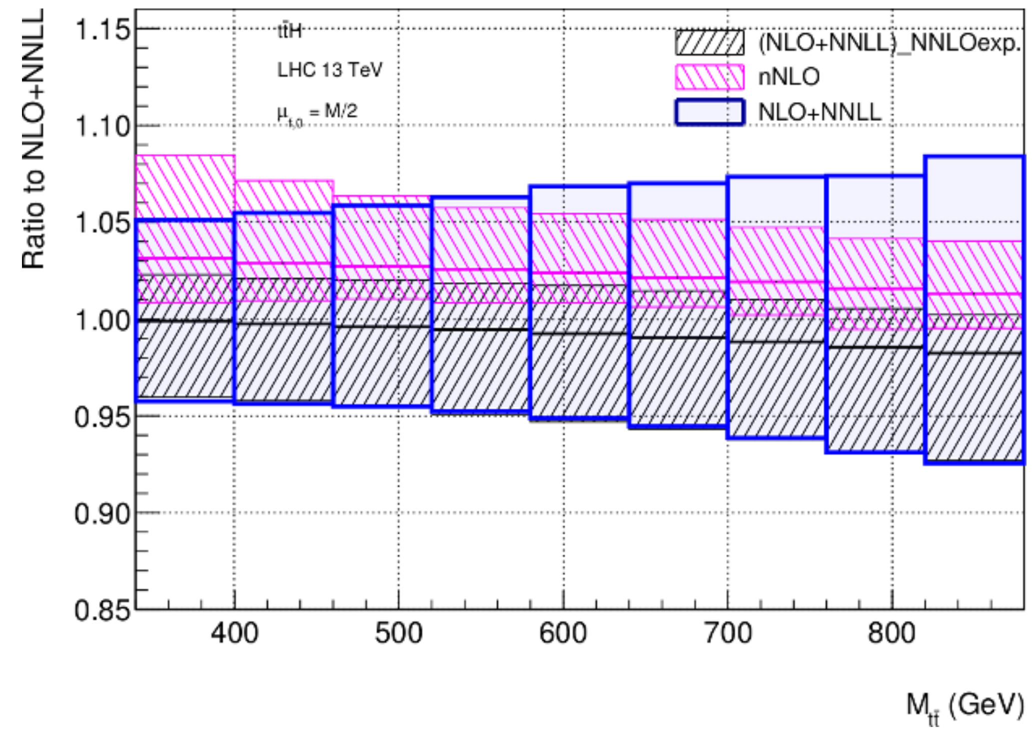
tTZ: nLO vs NLO
without qg channel

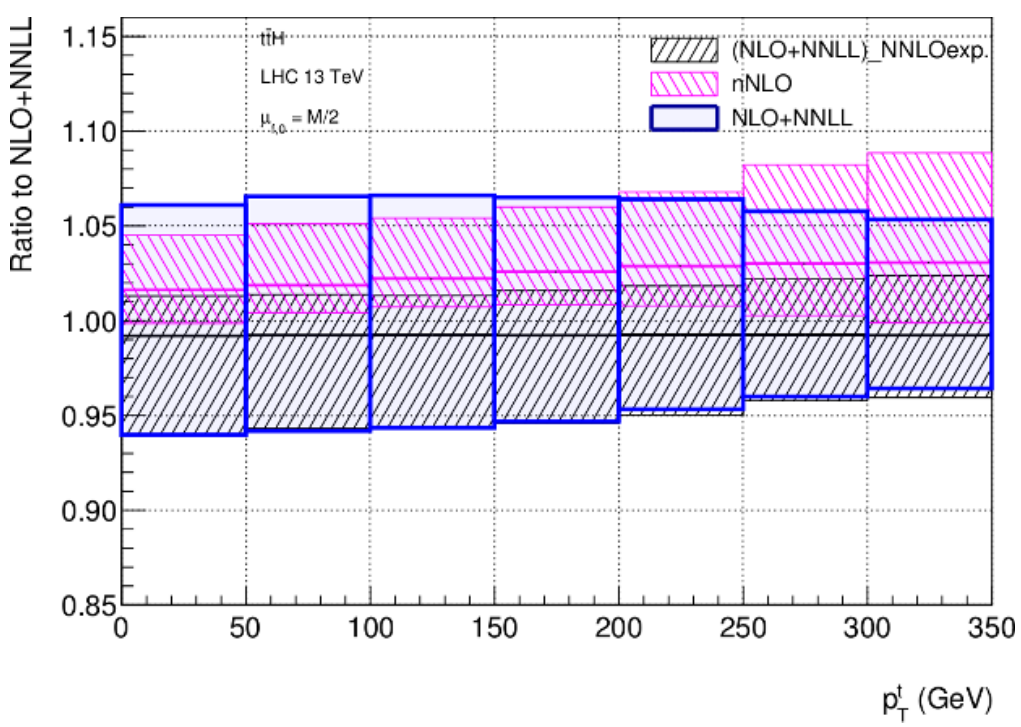


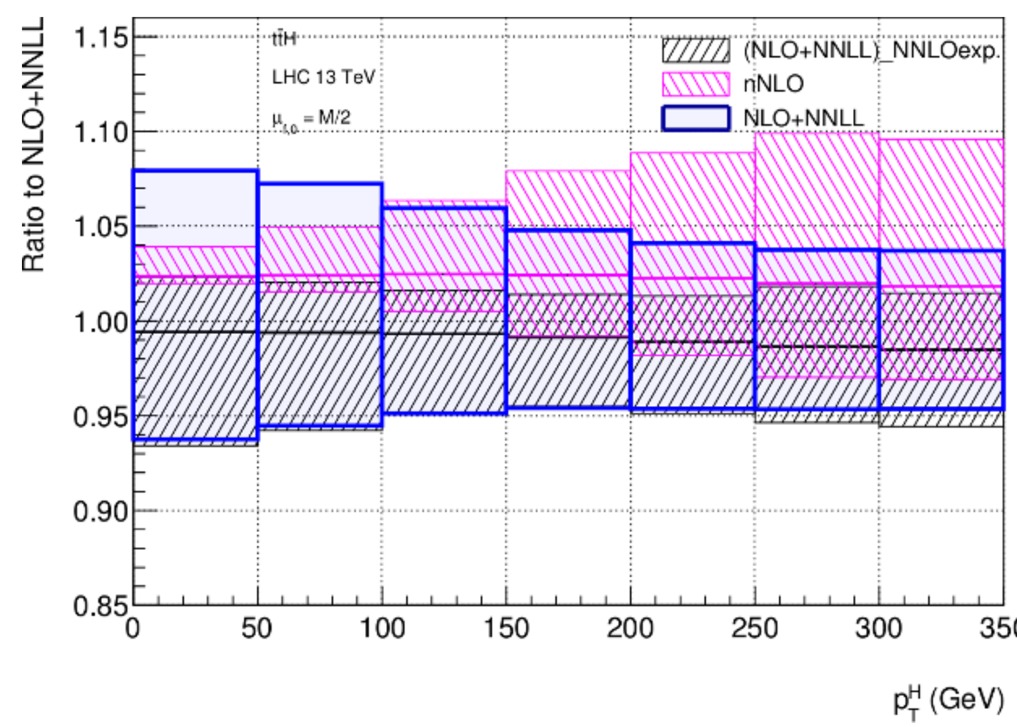


Distributions tTH: NNLL vs expansions









g functions

$$g_1(\lambda_s,\lambda_f) = \frac{\Gamma_0}{2\beta_0^2}\left\lfloor \lambda_s + (1-\lambda_s)\ln(1-\lambda_s) + \lambda_s \ln(1-\lambda_f)\right\rfloor ,$$

$$g_2(\lambda_s,\lambda_f) = \frac{\Gamma_0\beta_1}{2\beta_0^3}\left[\ln(1-\lambda_s) + \frac{1}{2}\ln^2(1-\lambda_s)\right] - \frac{\Gamma_1}{2\beta_0^2}\ln(1-\lambda_s) + \frac{\gamma_0^\phi}{\beta_0}\ln\frac{1-\lambda_s}{1-\lambda_f}$$
$$+ \frac{\Gamma_0}{2\beta_0}L_s \ln\frac{1-\lambda_s}{1-\lambda_f} + \frac{\Gamma_0}{2\beta_0}L_h\ln(1-\lambda_f) + \frac{1}{1-\lambda_f}\left\{\frac{\Gamma_0\beta_1}{2\beta_0^3}\lambda_s\left[1+\ln(1-\lambda_f)\right] - \frac{\Gamma_1}{2\beta_0^2}\lambda_s\right\},$$

$$\lambda_i = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_i}$$