Electroweak phase transition in composite Higgs models: calculability, gravitational waves and collider searches

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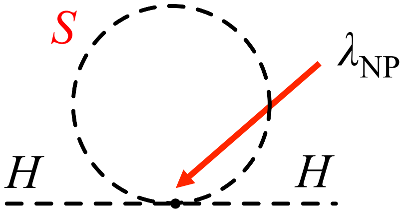
In collaboration with Ligong Bian and Yongcheng Wu, 1908.xxxxx.

Two problems of the Standard Model

- The hierarchy problem

The mass of an elementary Higgs boson is sensitive to the quantum corrections of high scale.



$$\delta M_h^2 \sim \left(\frac{\lambda_{\rm NP}}{16\pi^2}\right)^n \left[M_{\rm NP}^2 \frac{1}{\epsilon} + \text{finite terms}\right]$$

Hierarchy!

"Naturally" $M_h \sim M_{\rm Planck} \sim 10^{19}$ GeV

Reality $M_h =$ **125.09** GeV!

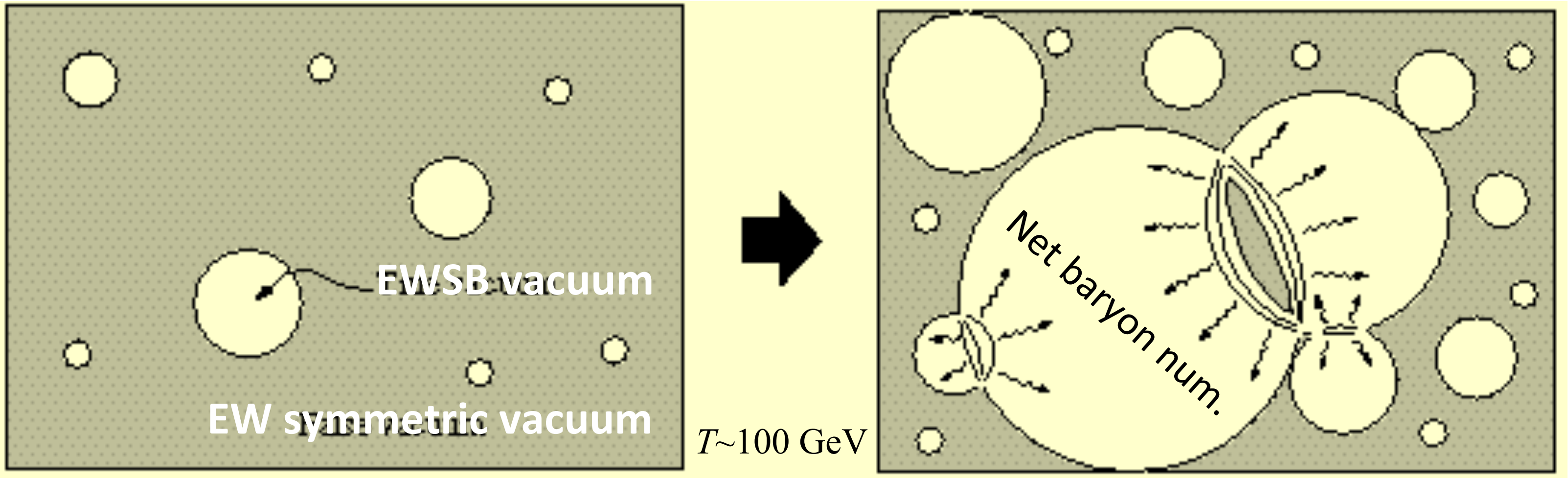
- Baryon-antibaryon asymmetry of the universe (BAU)

The three conditions for BAU:

A.D. Sakharov, Pisma Zh.Eksp.Teor.Fiz. 5 (1967) 32-35

1. Departure from the thermal equilibrium;
2. Baryon number violation;
3. C and CP violation.

EW baryogenesis: based on strong 1st order EW phase transition and CP vialation interactions with bubble wall



Baryon-antibaryon asymmetry of the universe (BAU)

The three conditions for BAU:

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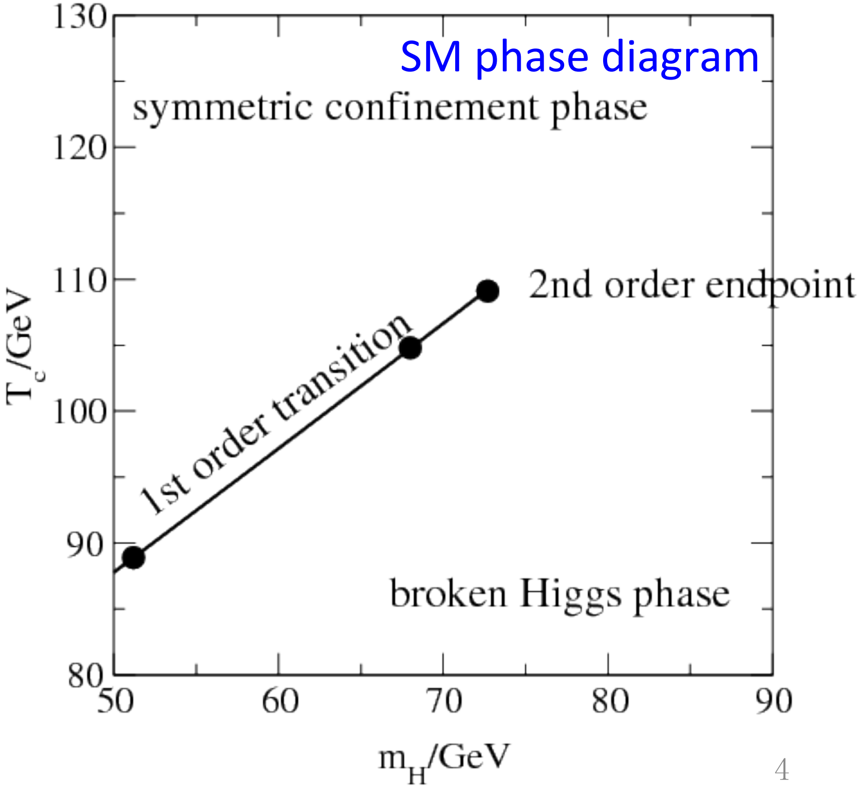
1. Departure from the thermal equilibrium;
2. Baryon number violation;
3. C and CP violation.

However, in the SM:

1. The EW phase transition is a smooth crossover;
2. The CP phase from CKM is too small.

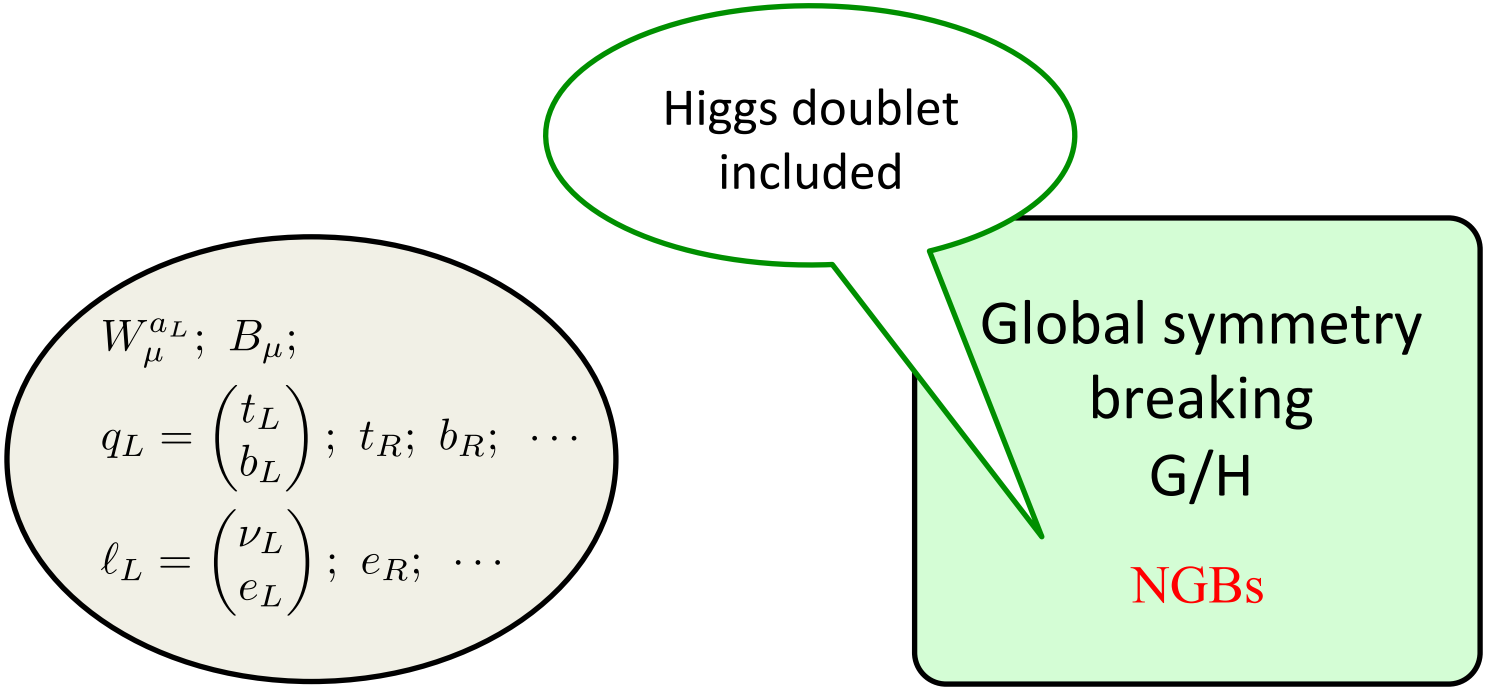
BAU cannot be realized!

Figure from hep-ph/0010275



A composite solution to two problems

- Solving hierarchy problem Kaplan *et al* (1984) and Agashe *et al* (2005)



$$W_\mu^{a_L};\ B_\mu;$$

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix};\ t_R;\ b_R;\ \cdots$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};\ e_R;\ \cdots$$

Higgs doublet included

Global symmetry breaking G/H

NGBs

The elementary sector (SM fermions and EW bosons)

The strong sector (new physics)

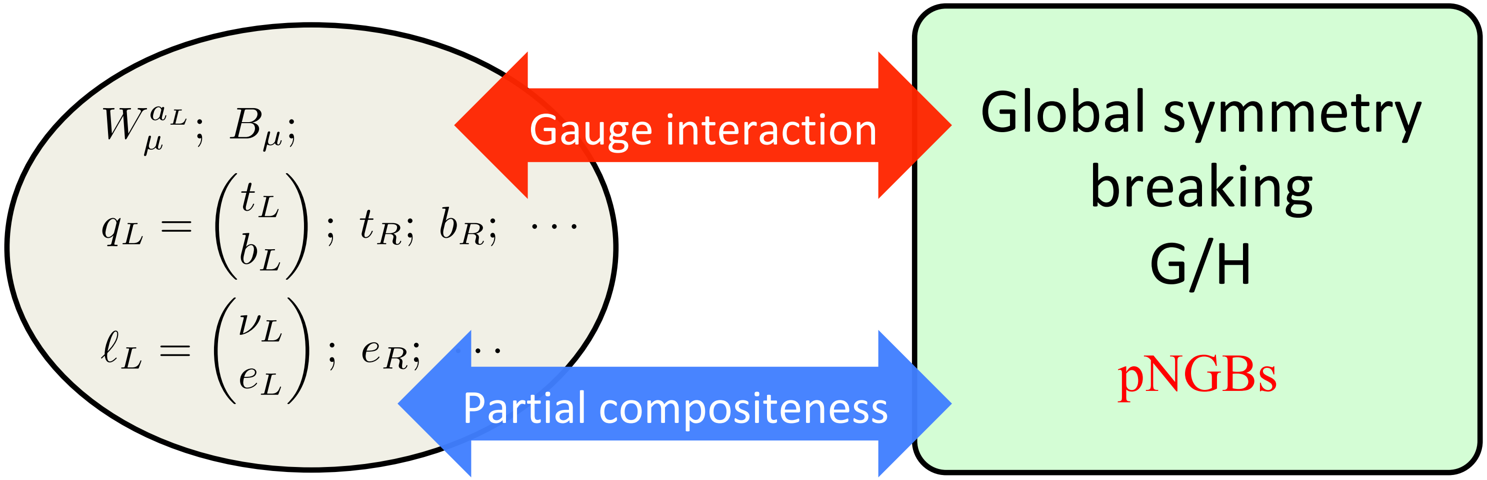
- Solving hierarchy problem Kaplan *et al* (1984) and Agashe *et al* (2005)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm strong}$$

$$+ \mathcal{J}^a_\mu W^\mu_a + \mathcal{J}_{Y\,\mu} B^\mu + y_L \bar{q}_L \mathcal{O}_R + y_R \bar{t}_R \mathcal{O}_L$$

EW gauge coupling:
Subgroup SU(2)$_{\rm L}$×U(1)$_{\rm Y}$ gauged

Partial compositeness:
$q_{\rm L}$ and $u_{\rm R}$ fill in the incomplete representation of G



$W^{a_L}_\mu;\ B_\mu;$

$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix};\ t_R;\ b_R;\ \cdots$

$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};\ e_R;\ \cdots$

Gauge interaction

Partial compositeness

Global symmetry breaking
G/H

pNGBs

Interactions **Explicitly** break G !!
Higgs potential generated; EWSB triggered

- Generating the matter-antimatter asymmetry

J. R. Espinosa *et al*, JCAP 1201 (2012) 012

Choose G/H = SO(6)/SO(5);
1 Higgs doublet & 1 real singlet. The scalar potential

$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2.$$

Thermal correction:

$$\Delta V_T(h,\eta) = \frac{c_h T^2}{2}h^2 + \frac{c_\eta T^2}{2}\eta^2,$$

with

$$c_h = \frac{3g^2+g'^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_h}{2} + \frac{\lambda_{h\eta}}{12}, \quad c_\eta = \frac{\lambda_\eta}{4} + \frac{\lambda_{h\eta}}{3}$$

May trigger Strong 1st order EWPT (departure from equilibrium) for suitable parameter choices;

And the *t*-*t*-*η* interaction gives CP phase. EW baryogenesis!!

Generating the matter-antimatter asymmetry

Realizing strong 1st order EWPT

$$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$$

$$V_T(h,\eta) = \frac{\mu_h^2 + c_h T^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\mu_\eta^2 + c_\eta T^2}{2} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

- # Generating the matter-antimatter asymmetry

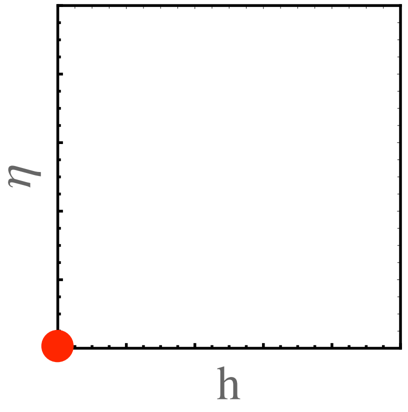
Realizing strong 1st order EWPT

$$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$$

$$V_T(h,\eta) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{\text{Positive}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{\text{Positive}} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

At the very first

$$T \gg \text{EW scale}$$



Minimum: $(0,0)$

- Generating the matter-antimatter asymmetry

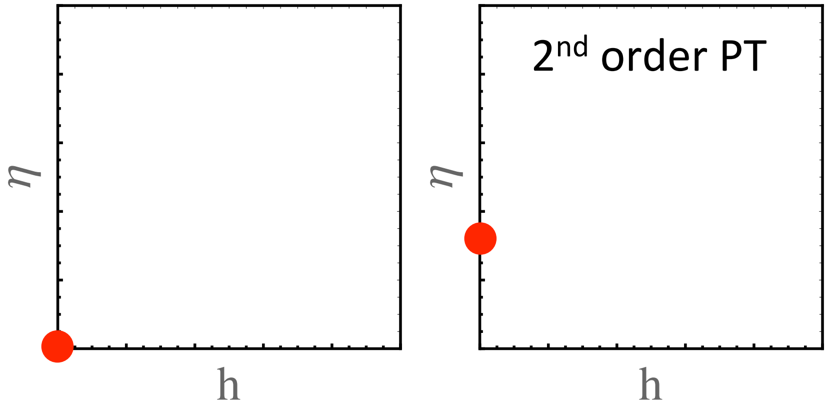
Realizing strong 1st order EWPT

$$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$$

$$V_T(h,\eta) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{\text{Positive}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{\text{Negative}} \eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

When the universe cooled down to

$$\sqrt{-\mu_h^2/c_h} < T < \sqrt{-\mu_\eta^2/c_\eta}$$



Minimum: $(0, w_T) = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T^2)/\lambda_\eta}\right)$

- Generating the matter-antimatter asymmetry

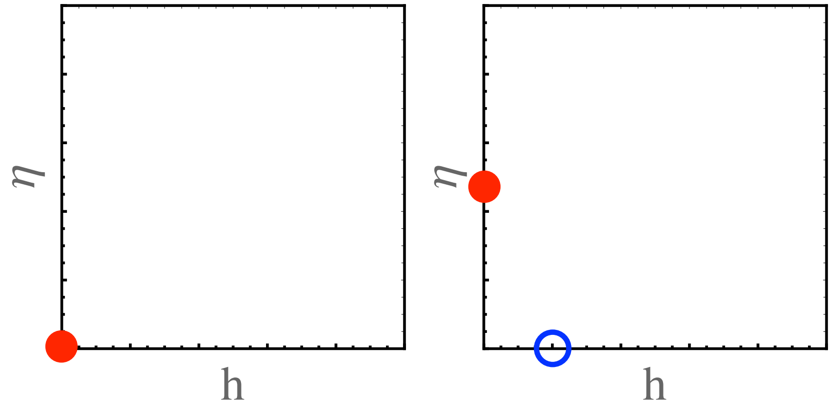
Realizing strong 1st order EWPT

$$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$$

$$V_T(h,\eta) = \underset{\text{Negative}}{\frac{\mu_h^2 + c_h T^2}{2}} h^2 + \frac{\lambda_h}{4} h^4 + \underset{\text{Negative}}{\frac{\mu_\eta^2 + c_\eta T^2}{2}} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

When the universe cooled down to

$$T_c < T < \sqrt{-\mu_h^2 / c_h}$$



Minima: $\underbrace{(0, w_T)}_{\text{Global}} = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T^2)/\lambda_\eta}\right), \ \underbrace{(v_T, 0)}_{\text{Local}} = \left(\sqrt{-(\mu_h^2 + c_h T^2)/\lambda_h}, 0\right)$

- Generating the matter-antimatter asymmetry

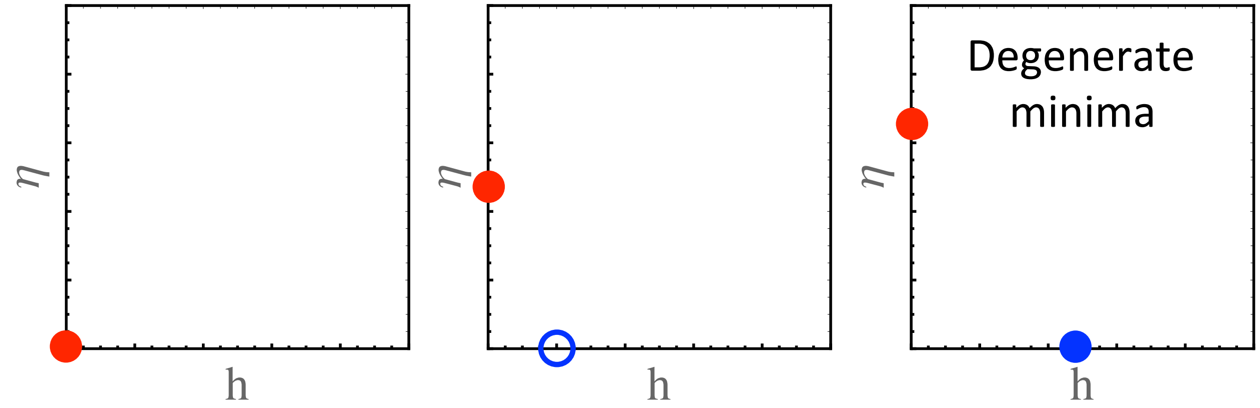
Realizing strong 1st order EWPT

$$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$$

$$V_T(h,\eta) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{\text{Negative}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{\text{Negative}} \eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

When the universe cooled down to critical temperature

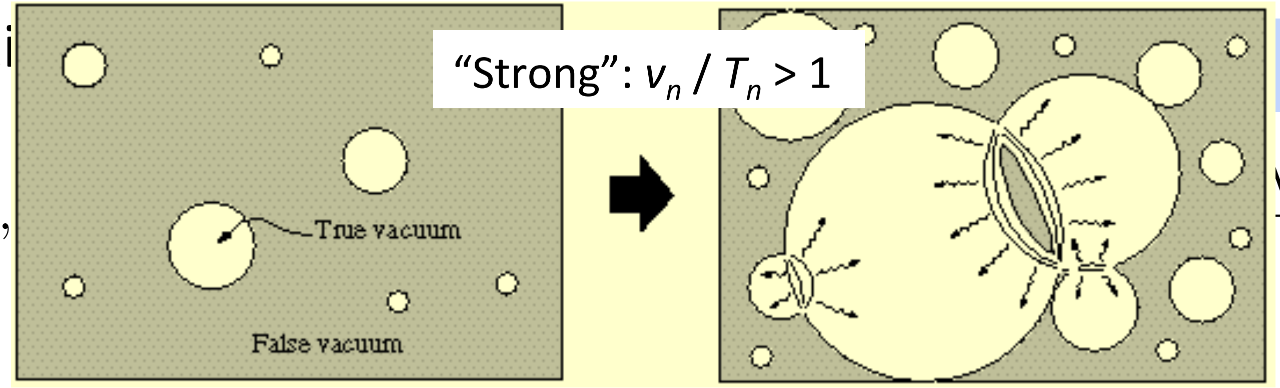
$$T = T_c$$



$$\text{Minima: } \underbrace{(0, w_c)}_{\text{Global}} = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T_c^2)/\lambda_\eta}\right), \quad \underbrace{(v_c, 0)}_{\text{Global}} = \left(\sqrt{-(\mu_h^2 + c_h T_c^2)/\lambda_h}, 0\right)$$

- Generating the matter-antimatter asymmetry

Realizi



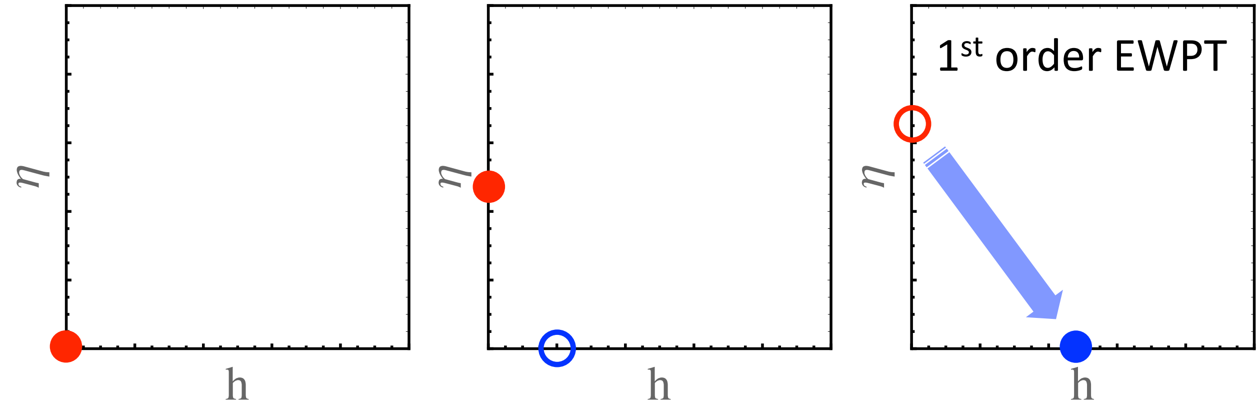
“Strong”: $v_n / T_n > 1$

$, c_\eta > 0,$

$V_T(h,$ $\frac{\lambda_{h\eta}}{2}h^2\eta^2$

When the universe cooled down to nucleation temperature

$$T = T_n \lesssim T_c$$



1st order EWPT

Minima: $\underbrace{(0, w_n)}_{\text{Local}} = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T_n^2)/\lambda_\eta}\right), \underbrace{(v_n, 0)}_{\text{Global}} = \left(\sqrt{-(\mu_h^2 + c_h T_n^2)/\lambda_h}, 0\right)$

- Generating the matter-antimatter asymmetry

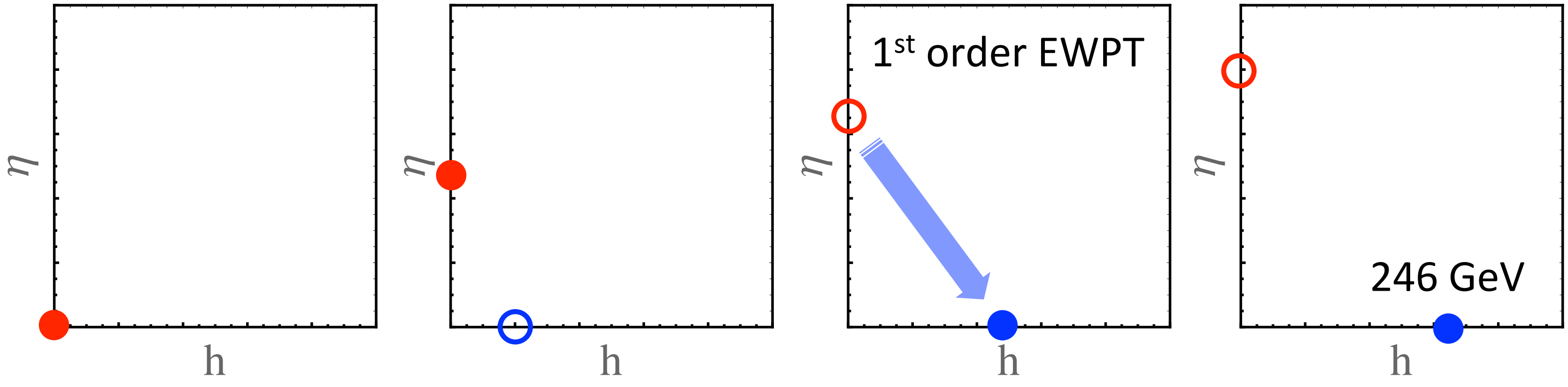
Realizing strong 1st order EWPT

$\mu_h^2, \mu_\eta^2 < 0, \quad c_h, c_\eta > 0,$

$$V_T(h,\eta) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{\text{Negative}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{\text{Negative}} \eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

When the universe cooled down to today

$$T = 0$$



Minima: $\underbrace{(0,w)}_{\text{Local}} = \left(0, \sqrt{-\mu_\eta^2/\lambda_\eta}\right), \ \underbrace{(v,0)}_{\text{Global}} = \left(\sqrt{-\mu_h^2/\lambda_h}, 0\right)$

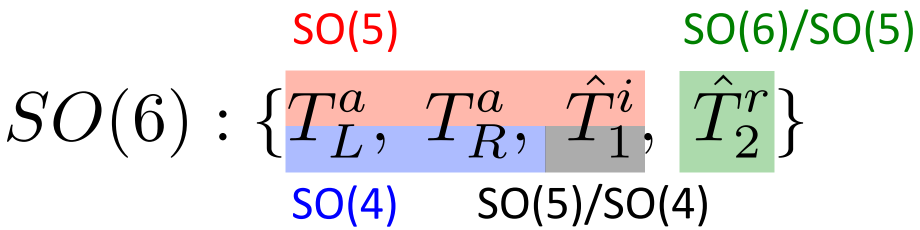
New!!

A detailed study about *V*(*h*,*η*) and EWPT

Y.Wu, L.Bian and K.-P.Xie, 1908.xxxxx (This talk)

- The scalar sector

Symmetry sturcture:



$$SO(6):\{T_L^a,\ T_R^a,\ \hat{T}_1^i,\ \hat{T}_2^r\}$$

a: 1,2,3;
i: 1,…,4;
r: 1,…,5.

The pNGBs: $U(\vec{\pi})=\exp\left\{i\frac{\sqrt{2}}{f}\pi_r\hat{T}_2^r\right\},\quad\Rightarrow\quad\begin{cases}H=\frac{1}{\sqrt{2}}\begin{pmatrix}\pi_2+i\pi_1\\ \pi_4-i\pi_3\end{pmatrix},\\ \pi_5.\end{cases}$

Unitary gauge: $\pi_{1,2,3}=0;$

$$\frac{h}{f}=\frac{\pi_4}{\sqrt{\pi_4^2+\pi_5^2}}\sin\frac{\sqrt{\pi_4^2+\pi_5^2}}{f},\qquad\frac{\eta}{f}=\frac{\pi_5}{\sqrt{\pi_4^2+\pi_5^2}}\sin\frac{\sqrt{\pi_4^2+\pi_5^2}}{f}.$$

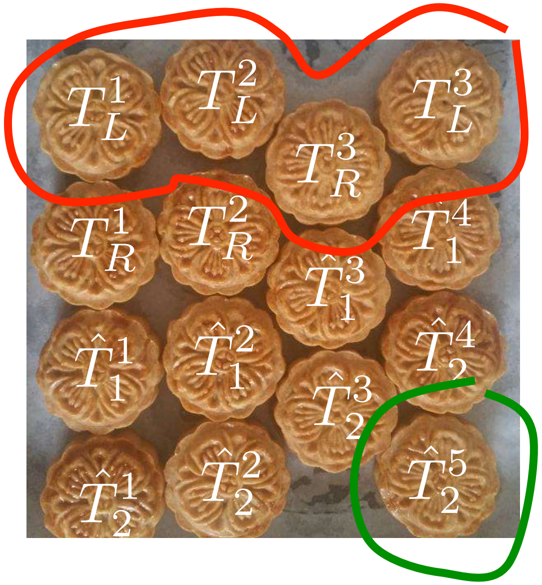
The gauge sector

Subgroup $SU(2)_L \times U(1)_Y$ gauged:

$$SO(6) \xrightarrow[\text{breaking}]{\text{explicit}} \underline{SU(2)_L \times U(1)_Y} \times \underline{U(1)_\eta},$$

T_2^5 : associated with *η*.

Gauge interactions only generate $V(h)$!!

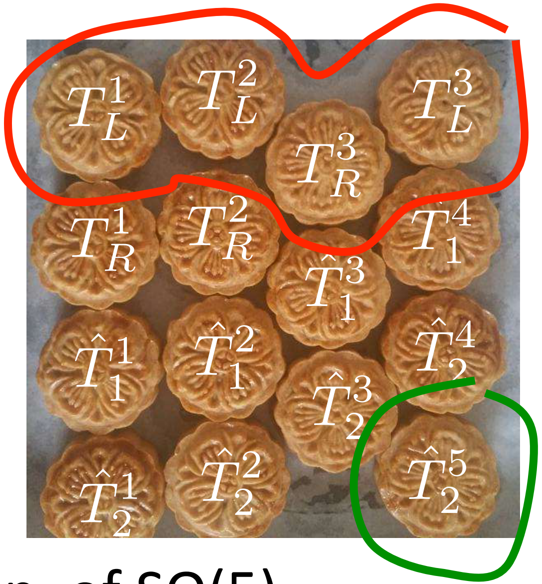


- The gauge sector

Subgroup SU(2)$_L$ × U(1)$_Y$ gauged:

$$SO(6)\ \xrightarrow[\text{breaking}]{\text{explicit}}\ \underline{SU(2)_L\times U(1)_Y}\times \underline{U(1)_\eta},$$

T_2^5 : associated with *η*.
Gauge interactions only generate *V*(*h*) !!



Vector resonances: *ρ* and *a*, in 10 and 5 rep. of SO(5).

$$\mathbf{10}\to\mathbf{3}_0\oplus\mathbf{1}_1\oplus\mathbf{1}_0\oplus\mathbf{1}_{-1}\oplus\mathbf{2}_{1/2}\oplus\mathbf{2}_{-1/2}:$$

$$\rho^{\bar{A}}\to\rho_L\oplus\rho_R^+\oplus\rho_R^0\oplus\rho_R^{-1}\oplus\rho_D\oplus\tilde{\rho}_D;$$

$$\mathbf{5}\to\mathbf{2}_{1/2}\oplus\mathbf{2}_{-1/2}\oplus\mathbf{1}_0:$$

$$a^r\to a_D\oplus\tilde{a}_D\oplus a_S$$

Interactions: depend only on the SO(6)/SO(5) structure

$$\mathcal{L}_\rho = -\frac{1}{4}\mathrm{tr}\left[\rho_{\mu\nu}\rho^{\mu\nu}\right]+\frac{M_\rho^2}{2g_\rho^2}\mathrm{tr}\left[(g_\rho\rho_\mu-e_\mu)^2\right]-\frac{1}{4}\mathrm{tr}[a_{\mu\nu}a^{\mu\nu}]+\frac{M_a^2}{2}\mathrm{tr}\left[a_\mu a^\mu\right],$$

$$\supset\frac{M_\rho^2}{2g_\rho^2}\left[\left(g_\rho\rho_{L\mu}^a-g_0W_\mu^a+\frac{i}{2f^2}H^\dagger\sigma^a\overleftrightarrow{D}_\mu H\right)^2+\left(g_\rho\rho_{R\mu}^3-g_0'B_\mu+\frac{i}{2f^2}H^\dagger\overleftrightarrow{D}_\mu H\right)^2\right],$$

The gauge-induced potential

Integrating out the resonances:

$$\mathcal{L}_{\rm EW} \supset \frac{1}{2} P_T^{\mu\nu} \Big\{ \left(-p^2 + \frac{g_0'^2}{g_0^2}\Pi_0(p^2)\right) B_\mu B_\nu + \left(-p^2 + \Pi_0(p^2)\right) W_\mu^a W_\nu^a$$

$$+ \frac{\Pi_1(p^2)}{4} \frac{h^2}{f^2} \left[W_\mu^1 W_\nu^1 + W_\mu^2 W_\nu^2 + \left(W_\mu^3 - \frac{g_0'}{g_0} B_\mu\right)\left(W_\nu^3 - \frac{g_0'}{g_0} B_\nu\right)\right] \Big\}$$

$$+ \text{Higher order operators,}$$

- # The gauge-induced potential

Integrating out the resonances:

$$\mathcal{L}_{\rm EW} \supset \frac{1}{2}P_T^{\mu\nu}\Big\{\left(-p^2+\frac{g_0'^2}{g_0^2}\Pi_0(p^2)\right)B_\mu B_\nu + \left(-p^2+\Pi_0(p^2)\right)W^a_\mu W^a_\nu$$
$$+\frac{\Pi_1(p^2)}{4}\frac{h^2}{f^2}\left[W^1_\mu W^1_\nu + W^2_\mu W^2_\nu + \left(W^3_\mu - \frac{g_0'}{g_0}B_\mu\right)\left(W^3_\nu - \frac{g_0'}{g_0}B_\nu\right)\right]\Big\}$$
$$+\,\text{Higher order operators},$$

Agashe *et al*, Nucl.Phys. B719 (2005) 165-187

The IR contributions: form factors

$$V_g^{\rm IR}(h) \approx \frac{6}{2}\int\frac{d^4Q}{(2\pi)^4}\ln\left(1+\frac{\Pi_1}{4\Pi_W}\frac{h^2}{f^2}\right)+\frac{3}{2}\int\frac{d^4Q}{(2\pi)^4}\ln\left[1+\left(\frac{g_0'^2}{g_0^2}\frac{\Pi_1}{4\Pi_B}+\frac{\Pi_1}{4\Pi_W}\right)\frac{h^2}{f^2}\right],$$

$$\Pi_W = Q^2+\Pi_0,\quad \Pi_B = Q^2+(g_0'^2/g_0^2)\Pi_0$$

Calculable. *Expressed* in terms of resonances masses and interactions.

$$\Pi_0 = \sum_{n=1}^{N_\rho} g_0^2\frac{Q^2 f^2_{\rho(n)}}{Q^2+M^2_{\rho(n)}},\quad \Pi_1 = g_0^2 f^2 + 2g_0^2\left(\sum_{n=1}^{N_a}\frac{Q^2 f^2_{a(n)}}{Q^2+M^2_{a(n)}} - \sum_{n=1}^{N_\rho}\frac{Q^2 f^2_{\rho(n)}}{Q^2+M^2_{\rho(n)}}\right),$$

- The gauge-induced potential

Integrating out the resonances:

$$\mathcal{L}_{\rm EW} \supset \frac{1}{2} P_T^{\mu\nu} \Big\{ \left(-p^2 + \frac{g_0'^2}{g_0^2} \Pi_0(p^2) \right) B_\mu B_\nu + \left(-p^2 + \Pi_0(p^2) \right) W_\mu^a W_\nu^a$$
$$+ \frac{\Pi_1(p^2)}{4} \frac{h^2}{f^2} \left[W_\mu^1 W_\nu^1 + W_\mu^2 W_\nu^2 + \left(W_\mu^3 - \frac{g_0'}{g_0} B_\mu \right) \left(W_\nu^3 - \frac{g_0'}{g_0} B_\nu \right) \right] \Big\}$$
$$+ \text{Higher order operators},$$

Marzocca *et al*, JHEP 1208 (2012) 013

The UV contributions: physics from higher scale

Incalculable. *Estimated* by Naïve Dimensional Analysis.

Written down by spurions

$$g A_\mu = g T_L^a W_\mu^a + g' T_R^3 B_\mu \equiv \mathcal{G}_{\bar{A}a} T^{\bar{A}} W_\mu^a + \mathcal{G}'_{\bar{A}} T^{\bar{A}} B_\mu,$$

$$c_g f^4 \Sigma^\dagger \mathcal{G}_a \mathcal{G}_a \Sigma \to c_g \frac{3}{4} g^2 f^2 h^2, \;\; \frac{d_g}{16\pi^2} f^4 \left(\Sigma^\dagger \mathcal{G}_a \mathcal{G}_a \Sigma \right)^2 \to d_g \frac{9 g^4}{256\pi^2} h^4, \;\; \cdots,$$
$$c_g, d_g \sim \mathcal{O}(1).$$

- The fermion sector

* U(1)$_X$ is introduced: $Y = X + T_R^3$.

Elementary quarks: incomplete rep. of SO(6)

$$\mathbf{6}_{2/3} \to \mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} \oplus \mathbf{1}_{2/3} \oplus \mathbf{1}_{2/3},$$

$$q_L^{\mathbf{6}} = \frac{1}{\sqrt{2}} \begin{pmatrix} ib_L & b_L & it_L & -t_L & 0 & 0 \end{pmatrix}^T,$$

$$t_R^{\mathbf{6}} = \begin{pmatrix} 0 & 0 & 0 & 0 & it_R c_\theta & t_R s_\theta \end{pmatrix}^T,$$

$$\mathbf{1}_{2/3} : t_R$$

$$\mathbf{15}_{2/3} \to \mathbf{3}_{2/3} \oplus \mathbf{1}_{5/3} \oplus \mathbf{1}_{2/3} \oplus \mathbf{1}_{-1/3}$$
$$\oplus \mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} \oplus \mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} \oplus \mathbf{1}_{2/3},$$

$$q_L^{\mathbf{15}} = (q_L^{\mathbf{6}})_j \hat{T}_1^j, \; t_R^{\mathbf{15}} = T_R^3 t_R c_\theta + \hat{T}_2^5 t_R e^{I\phi} s_\theta.$$

Composite top parnters: complete rep. of SO(5)

$$\mathbf{1}_{2/3} : \Psi_{\mathbf{1}}$$

$$\mathbf{5}_{2/3} \to \mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} \oplus \mathbf{1}_{2/3} :$$

$$\Psi_{\mathbf{5}} \to Q_X \oplus Q \oplus \widetilde{T}$$

$$\mathbf{10}_{2/3} \to \mathbf{3}_{2/3} \oplus \mathbf{1}_{5/3} \oplus \mathbf{1}_{2/3} \oplus \mathbf{1}_{-1/3}$$
$$\oplus \mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} :$$

$$\Psi_{\mathbf{10}} \to Y \oplus K_{5/3} \oplus K_{2/3} \oplus K_{-1/3} \oplus J_X \oplus J_Q,$$

Partial compositeness:

$$SO(6) \times U(1)_X \xrightarrow[\text{breaking}]{\text{explicit}} SU(2)_L \times U(1)_Y$$

Contributes to *V*(*h*,*η*) generally.

2 × 3 = 6 models.

- The fermion sector

2 × 3 = 6 models labled by (q_L embedding + t_R embedding).

$$\mathcal{L}_{\mathbf{6}+\mathbf{1}} \supset y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}} + \text{h.c.} \ ;$$

$$\begin{aligned}\mathcal{L}_{\mathbf{6}+\mathbf{6}} \supset\ & y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} \\ & + y_R^{\mathbf{5}} f (\bar{t}_R^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f (\bar{t}_R^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + \text{h.c.} \ ;\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\mathbf{6}+\mathbf{15}} \supset\ & y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} \\ & + y_R^{\mathbf{10}} f (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_R^{\mathbf{5}} f \Sigma_I^\dagger (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + \text{h.c.} \ ;\end{aligned}$$

$$\mathcal{L}_{\mathbf{15}+\mathbf{1}} \supset y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}} + \text{h.c.} \ ;$$

$$\begin{aligned}\mathcal{L}_{\mathbf{15}+\mathbf{6}} \supset\ & y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r \\ & + y_R^{\mathbf{5}} f (\bar{t}_R^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f (\bar{t}_R^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + \text{h.c.} \ ;\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\mathbf{15}+\mathbf{15}} \supset\ & y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r \\ & + y_R^{\mathbf{10}} f (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_R^{\mathbf{5}} f \Sigma_I^\dagger (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + \text{h.c.} \ .\end{aligned}$$

- The fermion-induced potential

Integrating out the resonances (6+6 as an example):

$$
\begin{aligned}
\mathcal{L}_{\mathbf{6+6}} \supset\ & \bar{t}_L \not{p} \left(\Pi_0^q + \frac{\Pi_1^q}{2}\frac{h^2}{f^2}\right) t_L + \bar{t}_R \not{p} \left[\Pi_0^t + \Pi_1^t \left(c_\theta^2 \frac{\eta^2}{f^2} + s_\theta^2 \left(1 - \frac{h^2+\eta^2}{f^2}\right)\right)\right] t_R \\
& - \frac{M_1^t}{\sqrt{2}} \frac{h}{f} \left(s_\theta \sqrt{1 - \frac{h^2+\eta^2}{f^2}} + i c_\theta \frac{\eta}{f}\right) \bar{t}_L t_R + \text{h.c.} \\
& + \text{Higher order operators}
\end{aligned}
$$

- The fermion-induced potential

Integrating out the resonances (6+6 as an example):

$$\mathcal{L}_{\mathbf{6+6}} \supset \bar{t}_L \not{p} \left(\Pi_0^q + \frac{\Pi_1^q}{2} \frac{h^2}{f^2} \right) t_L + \bar{t}_R \not{p} \left[\Pi_0^t + \Pi_1^t \left(c_\theta^2 \frac{\eta^2}{f^2} + s_\theta^2 \left(1 - \frac{h^2+\eta^2}{f^2} \right) \right) \right] t_R$$

$$- \frac{M_1^t}{\sqrt{2}} \frac{h}{f} \left(s_\theta \sqrt{1 - \frac{h^2+\eta^2}{f^2}} + i c_\theta \frac{\eta}{f} \right) \bar{t}_L t_R + \text{h.c.}$$

$$+ \text{Higher order operators}$$

The IR contributions: form factors

$$V_f^{\mathrm{IR}}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4} \left\{ \ln\left(1 + \frac{\Pi_1^q}{2\Pi_0^q} \frac{h^2}{f^2}\right) + \ln\left[1 + \frac{\Pi_1^t}{\Pi_0^t} \left(s_\theta^2 \left(1 - \frac{h^2}{f^2}\right) + c_{2\theta} \frac{\eta^2}{f^2} \right)\right] \right.$$
$$\left. + \ln\left[1 + \frac{1}{Q^2} \frac{|M_1^t|^2}{2\Pi_0^q \Pi_0^t} \frac{h^2}{f^2} \left(s_\theta^2 \left(1 - \frac{h^2}{f^2}\right) + c_{2\theta} \frac{\eta^2}{f^2} \right)\right] \right\},$$

$$\Pi_0^{q,t} = 1 + \sum_{n=1}^{N_{\mathbf{5}}} \frac{|y_{L,R}^{\mathbf{5}(n)}|^2 f^2}{Q^2 + M_{\mathbf{5}(n)}^2}, \quad \Pi_1^{q,t} = -\sum_{n=1}^{N_{\mathbf{5}}} \frac{|y_{L,R}^{\mathbf{5}(n)}|^2 f^2}{Q^2 + M_{\mathbf{5}(n)}^2} + \sum_{n=1}^{N_{\mathbf{1}}} \frac{|y_{L,R}^{\mathbf{1}(n)}|^2 f^2}{Q^2 + M_{\mathbf{1}(n)}^2},$$

$$M_0^t = \sum_{n=1}^{N_{\mathbf{5}}} \frac{y_L^{\mathbf{5}(n)} (y_R^{\mathbf{5}(n)})^* f^2 M_{\mathbf{5}(n)}}{Q^2 + M_{\mathbf{5}(n)}^2}, \quad M_1^t = -\sum_{n=1}^{N_{\mathbf{5}}} \frac{y_L^{\mathbf{5}(n)} (y_R^{\mathbf{5}(n)})^* f^2 M_{\mathbf{5}(n)}}{Q^2 + M_{\mathbf{5}(n)}^2} + \sum_{n=1}^{N_{\mathbf{1}}} \frac{y_L^{\mathbf{1}(n)} (y_R^{\mathbf{1}(n)})^* f^2 M_{\mathbf{1}(n)}}{Q^2 + M_{\mathbf{1}(n)}^2}.$$

Calculable.

The fermion-induced potential

Integrating out the resonances (6+6 as an example):

$$\mathcal{L}_{\mathbf{6}+\mathbf{6}} \supset \bar{t}_L \not{p} \left(\Pi_0^q + \frac{\Pi_1^q}{2}\frac{h^2}{f^2}\right) t_L + \bar{t}_R \not{p} \left[\Pi_0^t + \Pi_1^t \left(c_\theta^2 \frac{\eta^2}{f^2} + s_\theta^2 \left(1 - \frac{h^2+\eta^2}{f^2}\right)\right)\right] t_R$$

$$- \frac{M_1^t}{\sqrt{2}}\frac{h}{f}\left(s_\theta \sqrt{1 - \frac{h^2+\eta^2}{f^2}} + i c_\theta \frac{\eta}{f}\right) \bar{t}_L t_R + \text{h.c.}$$

$$+\ \text{Higher order operators}$$

The UV contributions: operators

Incalculable. *Estimated* by Naïve Dimensional Analysis.

Written down by spurions $q_L^{\mathbf{6}} = \mathcal{Q}^{\mathbf{6}} q_L, \quad t_R^{\mathbf{6}} = \mathcal{T}^{\mathbf{6}} t_R,$

$$c_f^L \Lambda^2 f^2 \frac{|y_L|^2}{16\pi^2} \Sigma^\dagger \mathcal{Q}^{\mathbf{6}} \mathcal{Q}^{\mathbf{6}\dagger} \Sigma \to c_f^L \frac{|y_L|^2}{2} f^2 h^2, \quad \cdots$$

$$c_f^R \Lambda^2 f^2 \frac{|y_R|^2}{16\pi^2} \Sigma^\dagger \mathcal{T}^{\mathbf{6}} \mathcal{T}^{\mathbf{6}\dagger} \Sigma \to c_f^R |y_R|^2 f^2 \left(\eta^2 c_{2\theta} + (f^2 - h^2) s_\theta^2\right), \quad \cdots$$

$$c_f^{L,R} \sim \mathcal{O}(1)$$

- The minimal Higgs potential hypothesis (MHP)

A summary: sources of the potential

	Gauge-induced	Fermion-induced
IR contributions (calculable)	Form factors of vector bosons $\Pi_{0,1}(p^2)$	Form factors of fermions $\Pi_0^{q,t}(p^2)$, $\Pi_1^{q,t}(p^2)$ and $M_{0,1}^t(p^2)$
UV contributions (estimated by NDA)	Local operators involved $g^{(\prime)}$	Local operators involved $y_{L,R}$

* NDA: Naïve Dimentional Analysis

• The minimal Higgs potential hypothesis (MHP)

A summary: sources of the potential

	Gauge-induced	Fermion-induced
IR contributions (calculable)	Form factors of vector bosons $\Pi_{0,1}(p^2)$	Form factors of fermions $\Pi_0^{q,t}(p^2)$, $\Pi_1^{q,t}(p^2)$ and $M_{0,1}^t(p^2)$

MHP: assume the UV contributions to be zero due to some unknown mechanism.

Then the potential is calculable!!

General Composite Higgs Models

David Marzocca (INFN, Trieste & SISSA, Trieste), Marco Serone (INFN, Trieste & SISSA, Trieste & ICTP, Trieste), Jing Shu (INFN, Trieste & SISSA, Trieste)

May 2012 - 51 pages

First proposed by Ref. **JHEP 1208 (2012) 013** DOI: 10.1007/JHEP08(2012)013 and then generally adopted by other studies [1205.6434, 1404.7419, 1703.08011, etc].

- Question: is MHP compatible with 1st order EWPT?

Y.Wu, L.Bian and K.-P.Xie, 1908.xxxxx (This talk)

Triggering strong 1st order EWPT via IR contributions alone

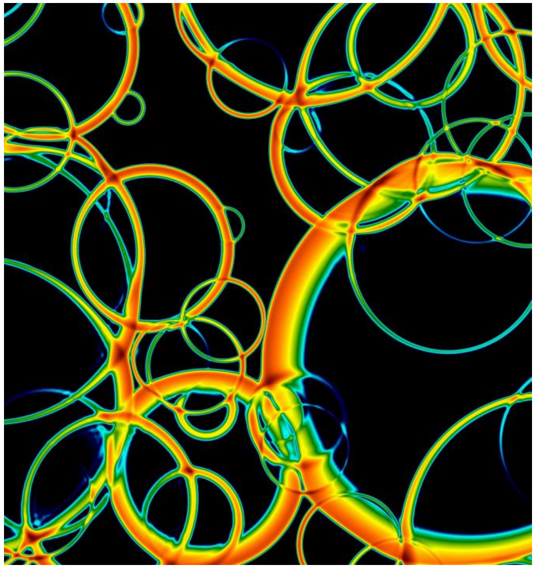
	Gauge-induced	Fermion-induced
IR contributions (calculable)	Form factors of vector bosons $\Pi_{0,1}(p^2)$	Form factors of fermions $\Pi_0^{q,t}(p^2)$, $\Pi_1^{q,t}(p^2)$ and $M_{0,1}^t(p^2)$

* MHP: Minimal Higgs potential hypothesis

IR contributions from the 6 models

v.s.

Strong 1st order EWPT



- The 15+1 model

Lagrangian…

$$\mathcal{L}_{\mathbf{15+1}} \supset \underline{y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi^{rs}_{\mathbf{10}} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi^r_{\mathbf{5}}} + \underline{y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}}} + \mathrm{h.c.}$$

Fails to give a mass to the top quark!

- ## The 15+1 model

Lagrangian…

$$\mathcal{L}_{\mathbf{15+1}} \supset \underline{y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r} + \underline{y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}}} + \mathrm{h.c.}$$

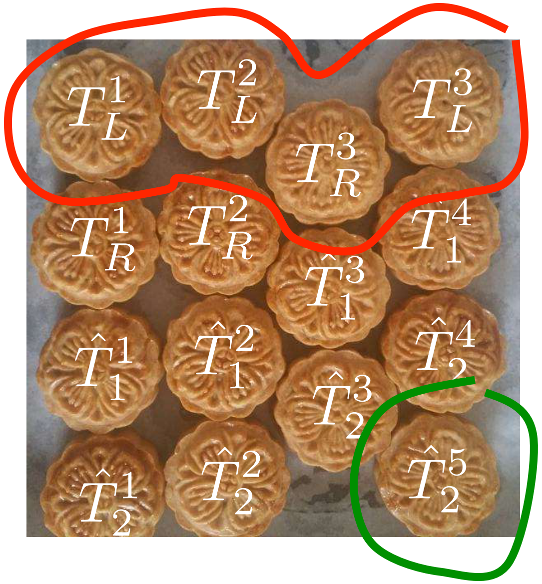
Fails to give a mass to the top quark!

- ## The 6+1 and 6+15 models

The embeddings have an accidental U(1)$_\eta$ symmetry: under T_2^5,

$$\delta q_L^{\mathbf{6}} = 0, \quad \delta t_R^{\mathbf{1}} = 0, \quad \delta t_R^{\mathbf{15}} = 0.$$

Hence U(1)$_\eta$ is preserved even in fermion sector!
No η potential is generated!!



Gauged

- The 6+6 model

The potential via IR contributions

$$V_f^{\mathrm{IR}}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4} \Bigg\{ \ln\left(1+\frac{\Pi_1^q}{2\Pi_0^q}\frac{h^2}{f^2}\right) + \ln\left[1+\frac{\Pi_1^t}{\Pi_0^t}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right] + \ln\left[1+\frac{1}{Q^2}\frac{|M_1^t|^2}{2\Pi_0^q\Pi_0^t}\frac{h^2}{f^2}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right]\Bigg\},$$

Matching to

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

yields

$$(\mu_f^2)^{\mathrm{IR}} = -2\alpha_q + 4s_\theta^2\alpha_t - 4s_\theta^4 f^2\beta_t - 2s_\theta^2 f^2\epsilon,$$

$$(\mu_\eta^2)^{\mathrm{IR}} = -4c_{2\theta}\alpha_t + 4c_{2\theta}s_\theta^2 f^2\beta_t,$$

$$(\lambda_f)^{\mathrm{IR}} = \beta_q + 4s_\theta^4\beta_t + 4s_\theta^2\epsilon,$$

$$(\lambda_\eta)^{\mathrm{IR}} = 4c_{2\theta}^2\beta_t,$$

$$(\lambda_{h\eta})^{\mathrm{IR}} = -4c_{2\theta}s_\theta^2\beta_t - 2c_{2\theta}\epsilon.$$

$$\alpha_{q,t} = \frac{N_c}{f^2}\int\frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}},$$

$$\beta_{q,t} = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2,$$

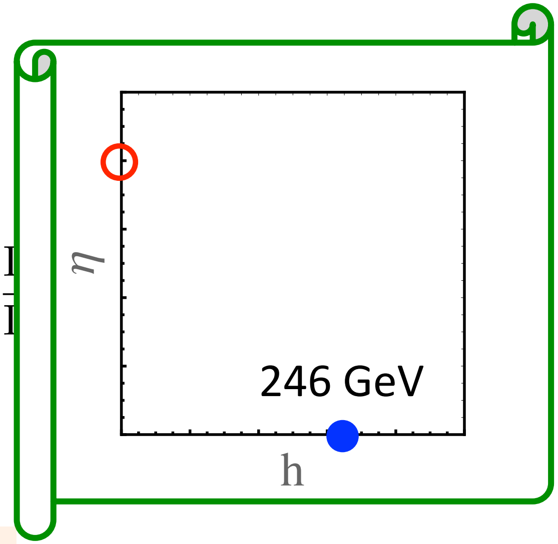
$$\epsilon = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t}$$

- The 6+6 model

The potential via IR contributions

$$V_f^{\rm IR}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4}\Big\{ \ln\Big(1+\frac{\Pi_1^q}{2\Pi_0^q}\frac{h^2}{f^2}\Big) + \ln\Big[1+$$

$$+\ln\Big[1+\frac{1}{Q^2}\frac{|M_1^t|^2}{2\Pi_0^q\Pi_0^t}\frac{h^2}{f^2}\Big(s_\theta^2$$



Matching to

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

yields

$$(\mu_f^2)^{\rm IR} = -2\alpha_q + 4s_\theta^2\alpha_t - 4s_\theta^4 f^2\beta_t - 2s_\theta^2 f^2\epsilon,$$

$$(\mu_\eta^2)^{\rm IR} = -4c_{2\theta}\alpha_t + 4c_{2\theta}s_\theta^2 f^2\beta_t,$$

$$(\lambda_f)^{\rm IR} = \beta_q + 4s_\theta^4\beta_t + 4s_\theta^2\epsilon,$$

$$(\lambda_\eta)^{\rm IR} = 4c_{2\theta}^2\beta_t,$$

$$(\lambda_{h\eta})^{\rm IR} = -4c_{2\theta}s_\theta^2\beta_t - 2c_{2\theta}\epsilon.$$

$$\alpha_{q,t} = \frac{N_c}{f^2}\int\frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}},$$

$$\beta_{q,t} = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2,$$

$$\epsilon = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t}$$

Hence

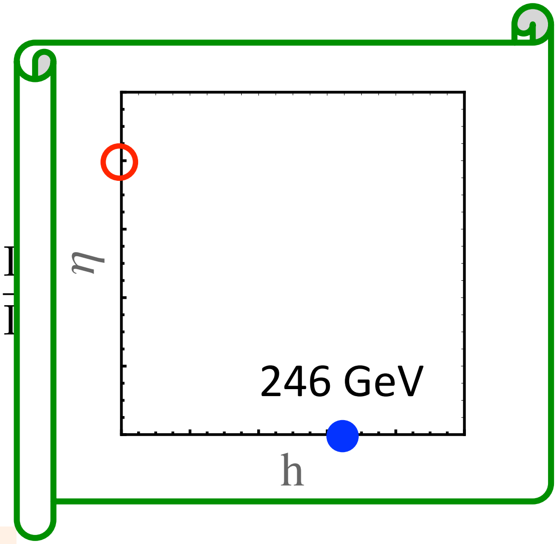
$$\alpha_t \gg \beta_t f^2 \;\Rightarrow\; \langle\eta\rangle_{\rm local} = \sqrt{-\mu_\eta^2/\lambda_\eta} \gg f.$$

- # The 6+6 model

The potential via IR contributions

$$V_f^{\rm IR}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4}\Big\{ \ln\Big(1+\frac{\Pi_1^q}{2\Pi_0^q}\frac{h^2}{f^2}\Big) + \ln\Big[1+$$

$$+\ln\Big[1+\frac{1}{Q^2}\frac{|M_1^t|^2}{2\Pi_0^q\Pi_0^t}\frac{h^2}{f^2}\Big(s_\theta^2$$



Matching to

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

yields

However, by definition *η* < *f* :

$$\frac{h}{f} = \frac{\pi_4}{\sqrt{\pi_4^2+\pi_5^2}}\sin\frac{\sqrt{\pi_4^2+\pi_5^2}}{f}, \quad \frac{\eta}{f} = \frac{\pi_5}{\sqrt{\pi_4^2+\pi_5^2}}\sin\frac{\sqrt{\pi_4^2+\pi_5^2}}{f}.$$

Thus 6+6 fails to get a local minimum for *η* !

Hence

$$\alpha_t \gg \beta_t f^2 \;\Rightarrow\; \langle\eta\rangle_{\rm local} = \sqrt{-\mu_\eta^2/\lambda_\eta} \gg f.$$

- The 15+15 model

Fails for similar reasons:

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

and

$$\begin{aligned}
(\mu_f^2)^{\mathrm{IR}} &= -\alpha_q - \alpha_t(1-3s_\theta^2) + \frac{\beta_t f^2}{2}(1-3s_\theta^2)s_\theta^2 - \frac{\epsilon f^2}{2}s_\theta^2,\\
(\mu_\eta^2)^{\mathrm{IR}} &= -4\alpha_q,\\
(\lambda_f)^{\mathrm{IR}} &= \frac{\beta_q}{4} + \frac{\beta_t}{4}(1-3s_\theta^2)^2 + \epsilon s_\theta^2,\\
(\lambda_\eta)^{\mathrm{IR}} &= 2\beta_q,\\
(\lambda_{h\eta})^{\mathrm{IR}} &= \frac{\beta_q}{2} - \frac{1-3s_\theta^2}{4}\epsilon.
\end{aligned}$$

$$\begin{aligned}
\alpha_{q,t} &= \frac{N_c}{f^2}\int\frac{d^4Q}{(2\pi)^4}\,\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}},\\
\beta_{q,t} &= \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2,\\
\epsilon &= \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\,\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t}
\end{aligned}$$

Hence we get

$$\alpha_q \gg \beta_q f^2 \;\Rightarrow\; \langle\eta\rangle_{\mathrm{local}} = \sqrt{-\mu_\eta^2/\lambda_\eta} \gg f.$$

which can never be achieved.

The 15+6 model

Match to the polynomial form

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

gives

$$\begin{aligned}
(\mu_f^2)^{\rm IR} &= -\alpha_q + 4\alpha_t s_\theta^2 - 4s_\theta^4 f^2\beta_t - c_\theta^2 f^2\epsilon, \\
(\mu_\eta^2)^{\rm IR} &= -4\alpha_q - 4\alpha_t c_{2\theta} + 4c_{2\theta}s_\theta^2\beta_t f^2, \\
(\lambda_f)^{\rm IR} &= \frac{\beta_q}{4} + 4s_\theta^4\beta_t, \\
(\lambda_\eta)^{\rm IR} &= 2\beta_q + 4c_{2\theta}^2\beta_t, \\
(\lambda_{h\eta})^{\rm IR} &= \frac{\beta_q}{2} - 4c_{2\theta}s_\theta^2\beta_t.
\end{aligned}$$

$$\begin{aligned}
\alpha_{q,t} &= \frac{N_c}{f^2}\int\frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}, \\
\beta_{q,t} &= \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2, \\
\epsilon &= \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t}
\end{aligned}$$

Hence we get

$$\begin{aligned}
(\lambda_{h\eta}^2)^{\rm IR} - (\lambda_h)^{\rm IR}(\lambda_\eta)^{\rm IR} &\approx (\lambda_{h\eta}^2)^{\rm IR} - (\lambda_f)^{\rm IR}(\lambda_\eta)^{\rm IR} \\
&= -\frac{1}{4}\beta_q\left[\beta_q + 8(1-c_{2\theta})\beta_t + 2(1+c_{4\theta})\beta_t\right] < 0,
\end{aligned}$$

- The 15+6 model

Match to the polynomial form

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

However, the local minima (v,0) and (0,w) require

$$\mu_h^2 < 0, \quad \lambda_h\mu_\eta^2 > \lambda_{h\eta}\mu_h^2, \quad \mu_\eta^2 < 0, \quad \lambda_\eta\mu_h^2 > \lambda_{h\eta}\mu_\eta^2$$

$$\Rightarrow \; \lambda_{h\eta}^2 > \lambda_h\lambda_\eta,$$

Thus the 15+6 model fails to trigger 1st order EWPT.

$$\epsilon = \frac{\quad}{f^4}\int\frac{\quad}{(2\pi)^4}\frac{\quad}{Q^2\Pi_0^q\Pi_0^t}$$

Hence we get

$$(\lambda_{h\eta}^2)^{\mathrm{IR}} - (\lambda_h)^{\mathrm{IR}}(\lambda_\eta)^{\mathrm{IR}} \approx (\lambda_{h\eta}^2)^{\mathrm{IR}} - (\lambda_f)^{\mathrm{IR}}(\lambda_\eta)^{\mathrm{IR}}$$

$$= -\frac{1}{4}\beta_q\left[\beta_q + 8(1-c_{2\theta})\beta_t + 2(1+c_{4\theta})\beta_t\right] < 0,$$

- A summary

* MHP: Minimal Higgs potential hypothesis

All 6 models fail to trigger 1st order EWPT under MHP!

15+1: no mass term for top quark;

6+1 & 6+15: no η potential because of the $U(1)_\eta$ symmetry;

6+6 & 15+15: local minimum of $\eta > f$;

15+6: cannot give two local minima at zero temperature.

- A summary

* MHP: Minimal Higgs potential hypothesis

All 6 models fail to trigger 1st order EWPT under MHP!

15+1: no mass term for top quark;

6+1 & 6+15: no η potential because of the $U(1)_\eta$ symmetry;

Hopeless due to the structure of the model.

6+6 & 15+15: local minimum of $\eta > f$;

15+6: cannot give two local minima at zero temperature.

Still hopefull if the we go beyond the minimal Higgs potential hypothesis and include the incalculable UV contributions!

New!!

Beyond MHP: the 6+6 model and EWPT

Y.Wu, L.Bian and K.-P.Xie, 1908.xxxxx (This talk)

- The UV contributions (incalculable)

$\mathbf{6}+\mathbf{6}$	Gauge-induced	Fermion-induced
UV contributions Estimated by NDA	$c_g f^4 \Sigma^\dagger \mathcal{G}_a \mathcal{G}_a \Sigma$	$c_f^L \lvert y_L\rvert^2 f^4 \Sigma^\dagger \mathcal{Q}^{\mathbf{6}} \mathcal{Q}^{\mathbf{6}\dagger} \Sigma$
	$c_{g'} f^4 \Sigma^\dagger \mathcal{G}' \mathcal{G}' \Sigma$	$c_f^R \lvert y_R\rvert^2 f^4 \Sigma^\dagger \mathcal{T}^{\mathbf{6}} \mathcal{T}^{\mathbf{6}\dagger} \Sigma$
	$\frac{d_g}{16\pi^2} f^4 \left(\Sigma^\dagger \mathcal{G}_a \mathcal{G}_a \Sigma\right)^2$	$\frac{d_f^L}{16\pi^2} \lvert y_L\rvert^4 f^4 \left(\Sigma^\dagger \mathcal{Q}^{\mathbf{6}} \mathcal{Q}^{\mathbf{6}\dagger} \Sigma\right)^2$
	$\frac{d_{g'}}{16\pi^2} f^4 \left(\Sigma^\dagger \mathcal{G}' \mathcal{G}' \Sigma\right)^2$	$\frac{d_f^R}{16\pi^2} \lvert y_R\rvert^4 f^4 \left(\Sigma^\dagger \mathcal{T}^{\mathbf{6}} \mathcal{T}^{\mathbf{6}\dagger} \Sigma\right)^2$

Matching to

$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2.$$

gives

$$(\mu_h^2)^{\rm UV} = c_g\frac{3g^2}{2}f^2 + c_{g'}\frac{g'^2}{2}f^2 + c_f^L|y_L|^2f^2 - 2c_f^R|y_R|^2f^2s_\theta^2 - \frac{d_f^R}{4\pi^2}|y_R|^4f^2s_\theta^4,$$

$$(\mu_\eta^2)^{\rm UV} = 2c_f^R|y_R|^2f^2c_{2\theta} + \frac{d_f^R}{4\pi^2}|y_R|^4f^2s_\theta^2c_{2\theta},$$

$\cdots$

- The IR contributions (calculable)

Gauge induced part

$$V_g^{\rm IR}(h) \approx \frac{6}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left(1+\frac{\Pi_1}{4\Pi_W}\frac{h^2}{f^2}\right) + \frac{3}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left[1+\left(\frac{g_0'^2}{g_0^2}\frac{\Pi_1}{4\Pi_B}+\frac{\Pi_1}{4\Pi_W}\right)\frac{h^2}{f^2}\right],$$

$$\Pi_W = Q^2 + \Pi_0, \quad \Pi_B = Q^2 + (g_0'^2/g_0^2)\Pi_0$$

$$\Pi_0 = \sum_{n=1}^{N_\rho} g_0^2 \frac{Q^2 f_{\rho(n)}^2}{Q^2 + M_{\rho(n)}^2}, \quad \Pi_1 = g_0^2 f^2 + 2g_0^2 \left(\sum_{n=1}^{N_a} \frac{Q^2 f_{a(n)}^2}{Q^2+M_{a(n)}^2} - \sum_{n=1}^{N_\rho} \frac{Q^2 f_{\rho(n)}^2}{Q^2+M_{\rho(n)}^2}\right),$$

Convergence require $\Pi_1 \sim Q^{-4}$, i.e. sum rules

$$\sum_{n=1}^{N_\rho} f_{\rho(n)}^2 = \frac{f^2}{2} + \sum_{n=1}^{N_a} f_{a(n)}^2; \quad \sum_{n=1}^{N_\rho} f_{\rho(n)}^2 M_{\rho(n)}^2 = \sum_{n=1}^{N_a} f_{a(n)}^2 M_{a(n)}^2,$$

Assume lightest resonances dominate, $N_\rho = N_a = 1$ then sum rules reduce to

$$f_\rho^2 = \frac{f^2}{2} + f_a^2, \quad f_\rho^2 M_\rho^2 = f_a^2 M_a^2;$$

$$\Pi_0(Q^2) = g^2 Q^2 \frac{f_\rho^2}{Q^2+M_\rho^2}, \quad \Pi_1(Q^2) = \frac{g^2 f^2 M_\rho^2 M_a^2}{(Q^2+M_\rho^2)(Q^2+M_a^2)},$$

The IR contributions

Gauge induced part

$$V_g^{\rm IR}(h) \approx \frac{6}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left(1 + \frac{\Pi_1}{4\Pi_W}\frac{h^2}{f^2}\right) + \frac{3}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \left(\frac{g_0'^2}{g_0^2}\frac{\Pi_1}{4\Pi_B} + \frac{\Pi_1}{4\Pi_W}\right)\frac{h^2}{f^2}\right],$$

$$\Pi_W = Q^2 + \Pi_0, \quad \Pi_B = Q^2 + (g_0'^2/g_0^2)\Pi_0$$

$$V_g(h) = \frac{\mu_g^2}{2}h^2 + \frac{\lambda_g}{4}h^4$$

$$(\mu_g^2)^{\rm IR} = \frac{3(3g^2 + g'^2)}{64\pi^2}\frac{M_\rho^2 M_a^2}{M_a^2 - M_\rho^2}\ln\frac{M_a^2}{M_\rho^2},$$

$$(\lambda_g)^{\rm IR} = \frac{3\left[2g^4 + (g^2 + g'^2)^2\right]}{256\pi^2(M_a^2 - M_\rho^2)^2}\left[M_a^4 + \frac{M_\rho^4(M_\rho^2 - 3M_a^2)}{M_a^2 - M_\rho^2}\ln\frac{M_a^2}{M_W^2} + (a \leftrightarrow \rho)\right].$$

$$f_\rho^2 = \frac{f^2}{2} + f_a^2, \quad f_\rho^2 M_\rho^2 = f_a^2 M_a^2;$$

$$\Pi_0(Q^2) = g^2 Q^2 \frac{f_\rho^2}{Q^2 + M_\rho^2}, \quad \Pi_1(Q^2) = \frac{g^2 f^2 M_\rho^2 M_a^2}{(Q^2 + M_\rho^2)(Q^2 + M_a^2)},$$

- The IR contributions (fermion-induced)

$$V_f^{\rm IR}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4}\left\{ \ln\left(1+\frac{\Pi_1^q}{2\Pi_0^q}\frac{h^2}{f^2}\right) + \ln\left[1+\frac{\Pi_1^t}{\Pi_0^t}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right] \right.$$
$$\left. + \ln\left[1+\frac{1}{Q^2}\frac{|M_1^t|^2}{2\Pi_0^q\Pi_0^t}\frac{h^2}{f^2}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right]\right\},$$

$$\Pi_0^{q,t} = 1+\sum_{n=1}^{N_\mathbf{5}}\frac{|y_{L,R}^{\mathbf{5}(n)}|^2 f^2}{Q^2+M^2_{\mathbf{5}(n)}}, \quad \Pi_1^{q,t} = -\sum_{n=1}^{N_\mathbf{5}}\frac{|y_{L,R}^{\mathbf{5}(n)}|^2 f^2}{Q^2+M^2_{\mathbf{5}(n)}} + \sum_{n=1}^{N_\mathbf{1}}\frac{|y_{L,R}^{\mathbf{1}(n)}|^2 f^2}{Q^2+M^2_{\mathbf{1}(n)}},$$
$$M_0^t = \sum_{n=1}^{N_\mathbf{5}}\frac{y_L^{\mathbf{5}(n)}(y_R^{\mathbf{5}(n)})^* f^2 M_{\mathbf{5}(n)}}{Q^2+M^2_{\mathbf{5}(n)}}, \quad M_1^t = -\sum_{n=1}^{N_\mathbf{5}}\frac{y_L^{\mathbf{5}(n)}(y_R^{\mathbf{5}(n)})^* f^2 M_{\mathbf{5}(n)}}{Q^2+M^2_{\mathbf{5}(n)}} + \sum_{n=1}^{N_\mathbf{1}}\frac{y_L^{\mathbf{1}(n)}(y_R^{\mathbf{1}(n)})^* f^2 M_{\mathbf{1}(n)}}{Q^2+M^2_{\mathbf{1}(n)}}.$$

Convergence requires $\Pi_1^{q,t}\sim Q^{-6},\ M_1^t\sim Q^{-2}$, with sum rules

$$\sum_{n=1}^{N_\mathbf{5}}\left|y_{L,R}^{\mathbf{5}(n)}\right|^2 = \sum_{n=1}^{N_\mathbf{1}}\left|y_{L,R}^{\mathbf{1}(n)}\right|^2, \quad \sum_{n=1}^{N_\mathbf{5}}\left|y_{L,R}^{\mathbf{5}(n)}\right|^2 M^2_{\mathbf{5}(n)} = \sum_{n=1}^{N_\mathbf{1}}\left|y_{L,R}^{\mathbf{1}(n)}\right|^2 M^2_{\mathbf{1}(n)}.$$

Assuming N_5 = 1, N_1 = 2 then

$$\Pi_0^{q,t}(Q^2) = 1+\frac{|y^\mathbf{5}_{L,R}|^2 f^2}{Q^2+M^2_\mathbf{5}}, \quad \Pi_1^{q,t}(Q^2) = \frac{\left|y^{\mathbf{1}'}_{L,R}\right|^2 f^2\left(M^2_{\mathbf{1}'}-M^2_\mathbf{5}\right)\left(M^2_{\mathbf{1}'}-M^2_\mathbf{1}\right)}{(Q^2+M^2_\mathbf{5})(Q^2+M^2_\mathbf{1})(Q^2+M^2_{\mathbf{1}'})},$$
$$M_1^t(Q^2) = \frac{y_L^\mathbf{1}(y_R^\mathbf{1})^* f^2 M_\mathbf{1}}{Q^2+M^2_\mathbf{1}} + \frac{y_L^{\mathbf{1}'}(y_R^{\mathbf{1}'})^* f^2 M_{\mathbf{1}'}}{Q^2+M^2_{\mathbf{1}'}} - \frac{y_L^\mathbf{5}(y_R^\mathbf{5})^* f^2 M_\mathbf{5}}{Q^2+M^2_\mathbf{5}}.$$

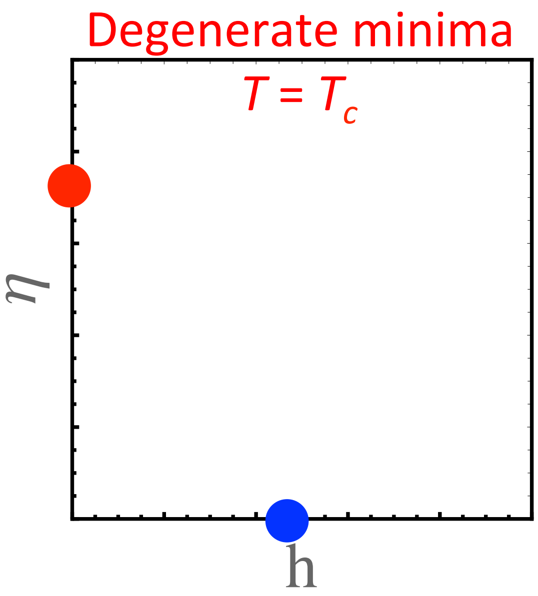
- Combining UV and IR contributions

We further require the potential

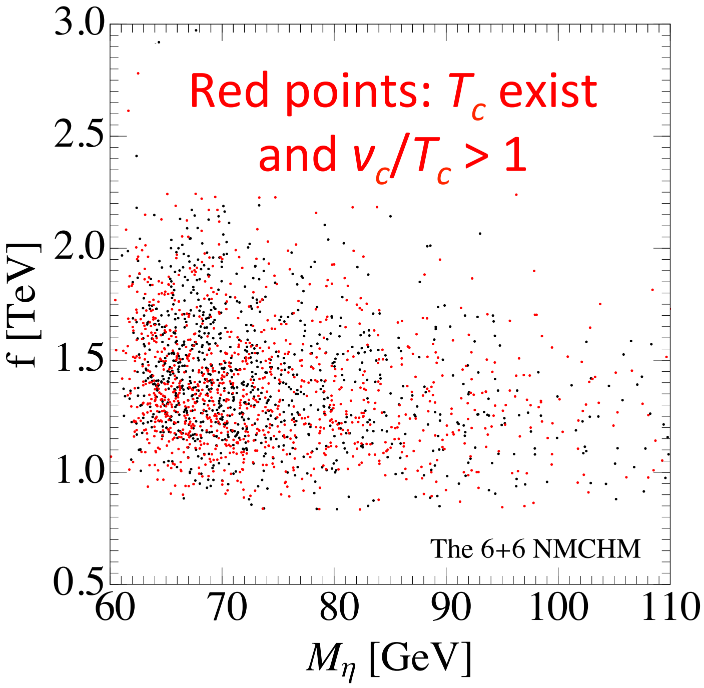
$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2.$$

to give correcct masses for SM particles such as $W^\pm$, Z, top.

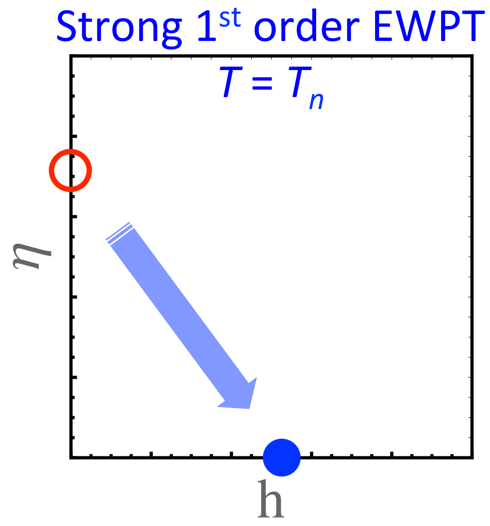
For 1st order EWPT, there should be a T_c for degenerate vacuums.


Degenerate minima

$T = T_c$


Red points: T_c exist and $v_c/T_c > 1$

• Bubble nucleation A. D. Linde, Nucl. Phys. B216 (1983) 421.



To really achieve a strong 1^{st} order EWPT, one needs to calculate vacuum decay rate

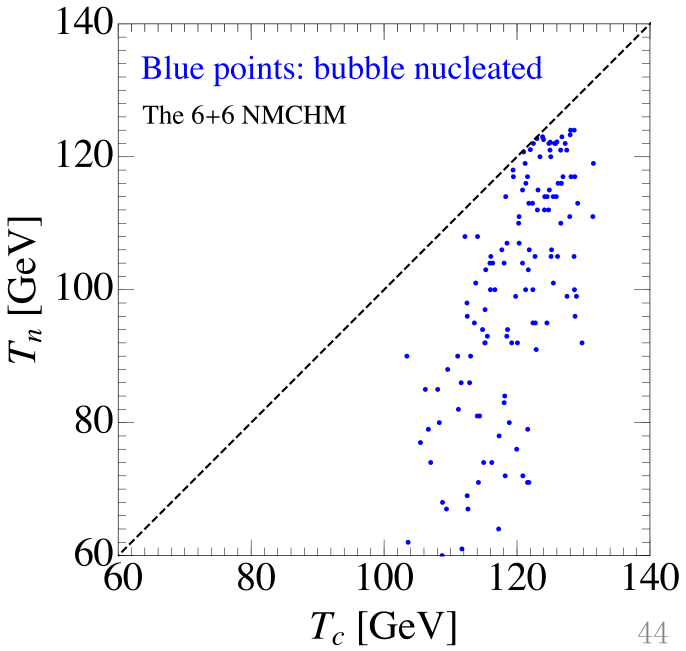
$$\Gamma/V \approx T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3(T)/T},$$

with $S_3(T)/T$ being the action of the O(3) bounce solution.

Bubble nucleation condition: at T_n

$$\frac{S_3(T_n)}{T_n} \sim 4 \ln \frac{\xi M_{\rm Pl}}{T_n} \sim 140,$$

Typically T_n is slightly lower than T_c.



- Phase transition gravitational waves (GWs)

Described by energy density C. Grojean *et al*, Phys. Rev. D75 (2007) 043507

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f},$$

After the redshift, the frequency today is O(mHz).
A two-parameter problem:

Latent heat: $\alpha = \frac{\epsilon}{\rho_{\rm rad}}, \quad \epsilon = -\Delta V_T + T_n \Delta \frac{\partial V_T}{\partial T}\Big|_{T_n}, \quad \rho_{\rm rad} = \frac{\pi^2}{30} g_* T_n^4,$

Time inverse: $\beta = \frac{d}{dt}\left(\frac{S_3}{T}\right)\Big|_{t=t_n}, \quad \frac{\beta}{H_n} = T_n \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T=T_n}$

Strength of GWs can be derived using the numerical formulae in [C. Caprini *et al*. JCAP 1604 (2016) 001].

Sources of the GWs: bubble collision (negligible), sound waves in fluid (leading), and turbulence in plasma (sub-leading).

- Phase transition gravitational waves (GWs)

Described

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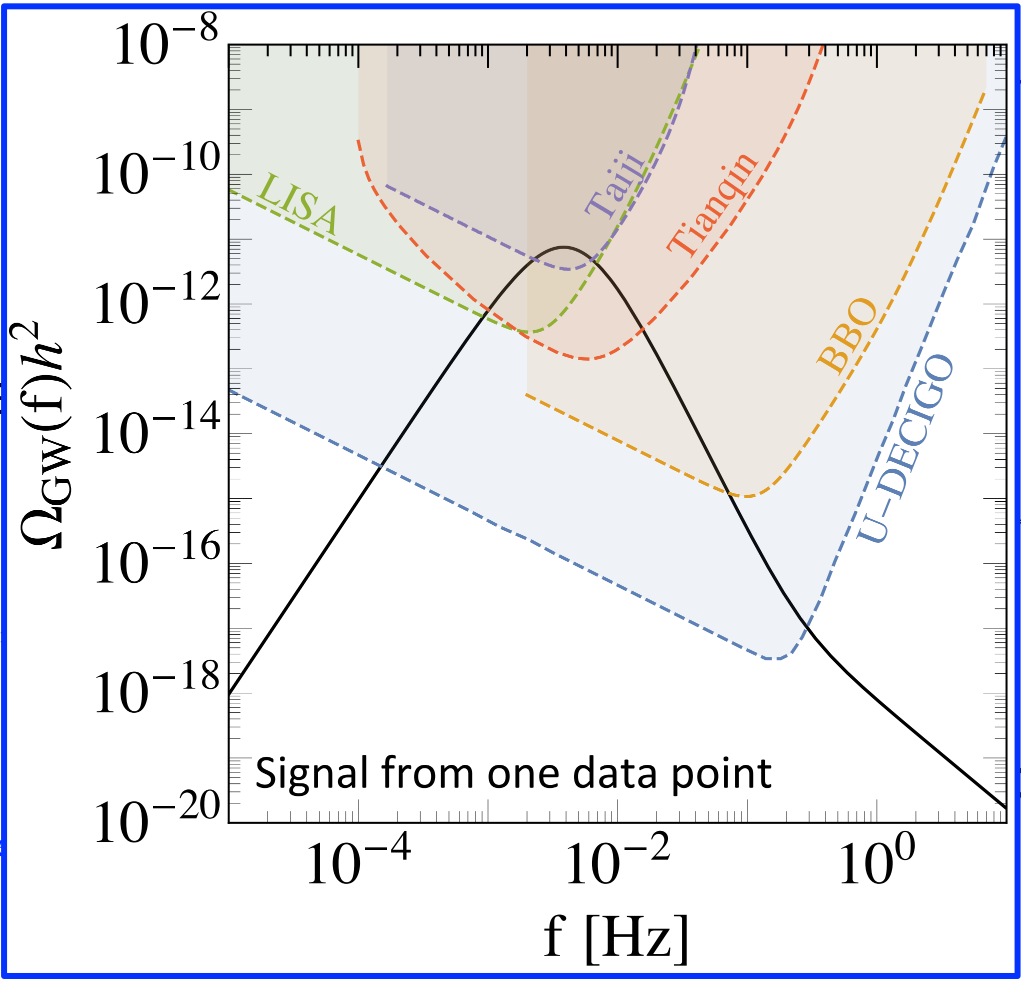

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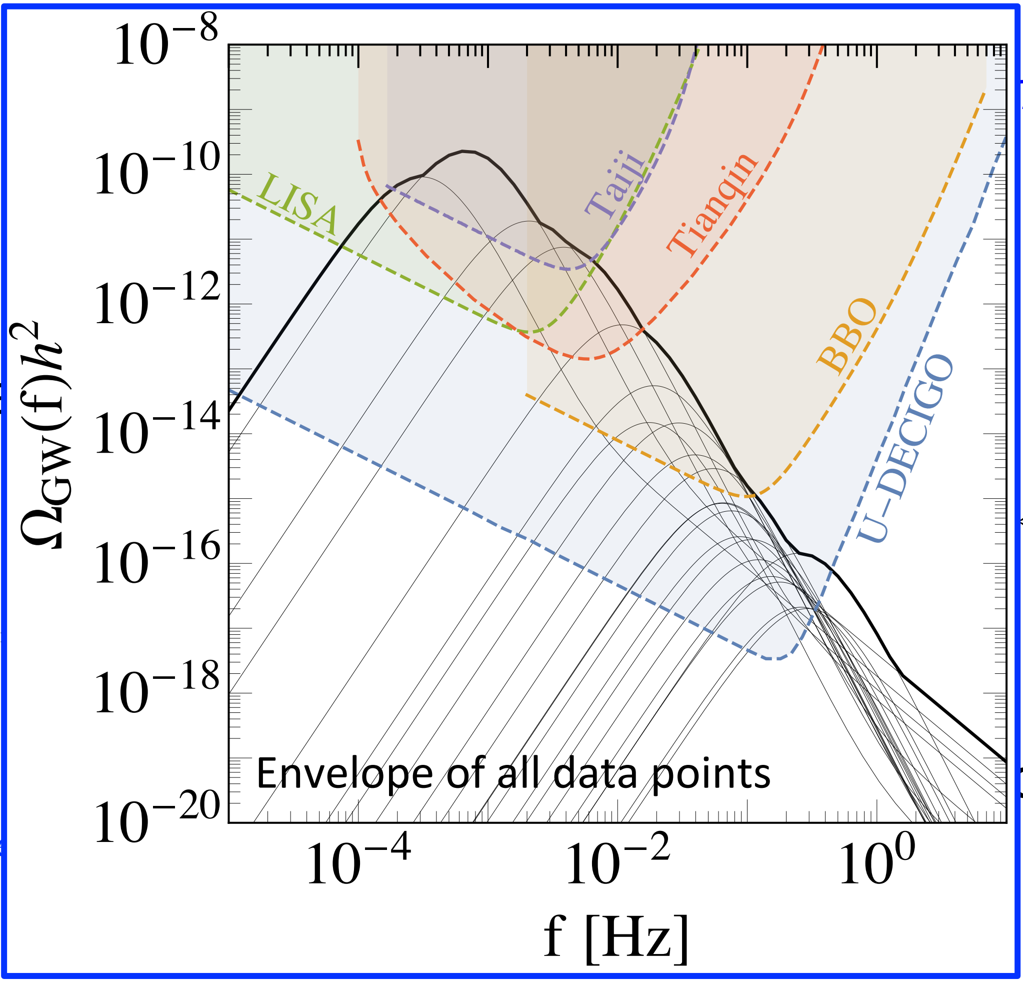
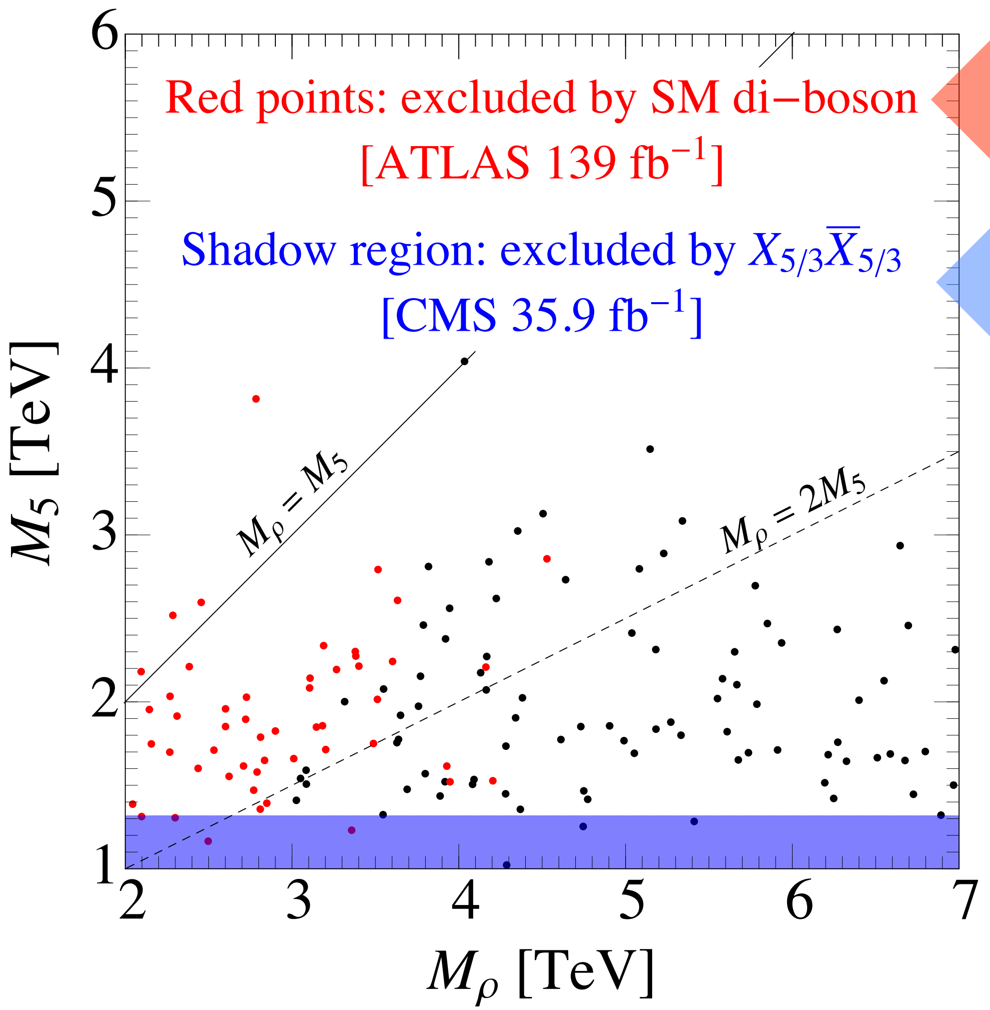


Sources of the GWs: bubble collision (negligible), sound waves in fluid (leading), and turbulence in plasma (sub-leading).

- Collider searches

The data points allowed by strong 1st order EWPT

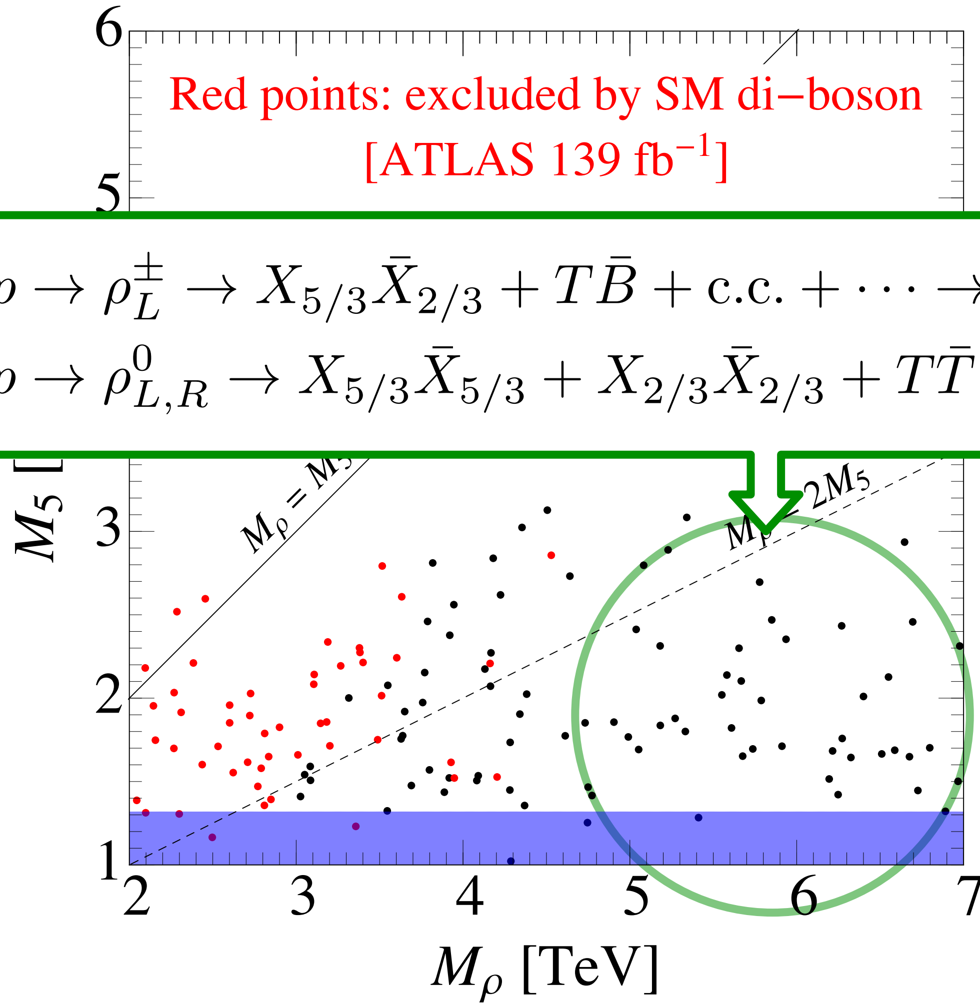


$$pp \to \rho_L^\pm, \rho_R^0 \to W^\pm Z, W^+W^-, \cdots$$

$$pp \to X_{5/3}\bar{X}_{5/3} \to t\bar{t}W^+W^-$$

- Collider searches

The data points allowed by strong 1st order EWPT



No dedicated searches yet!

$$pp \to \rho_L^{\pm} \to X_{5/3}\bar{X}_{2/3} + T\bar{B} + \text{c.c.} + \cdots \to t\bar{t}W^{\pm}Z/t\bar{t}W^{\pm}h,$$

$$pp \to \rho_{L,R}^{0} \to X_{5/3}\bar{X}_{5/3} + X_{2/3}\bar{X}_{2/3} + T\bar{T} + \cdots \to t\bar{t}W^{+}W^{-}/t\bar{t}Zh,$$

Conclusion

- The minimal Higgs potential hypothesis (MHP) is incompatible with 1st order EWPT in the SO(6)/SO(5) composite Higgs models with fermion embeddings up to 15.

- If we abandon the calculability and introduce the UV contributions, strong 1st order EWPT can be realized and the corresponding GW signals are detectable.

Thank you!