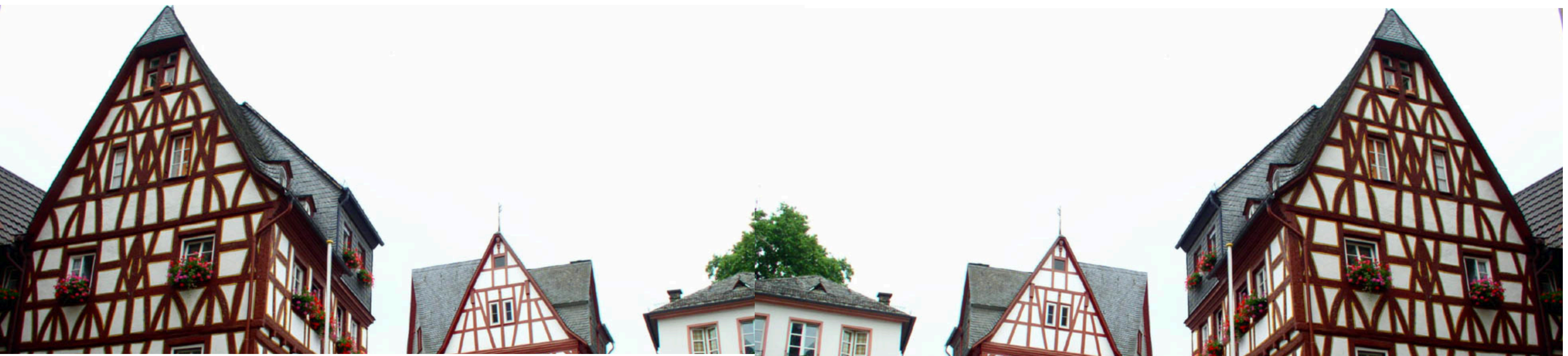
Composite models on the lattice
— in and out of the conformal window —

Anna Hasenfratz

University of Colorado Boulder

MITP Workshop
Aug 30 2019



Why (near-) conformal?

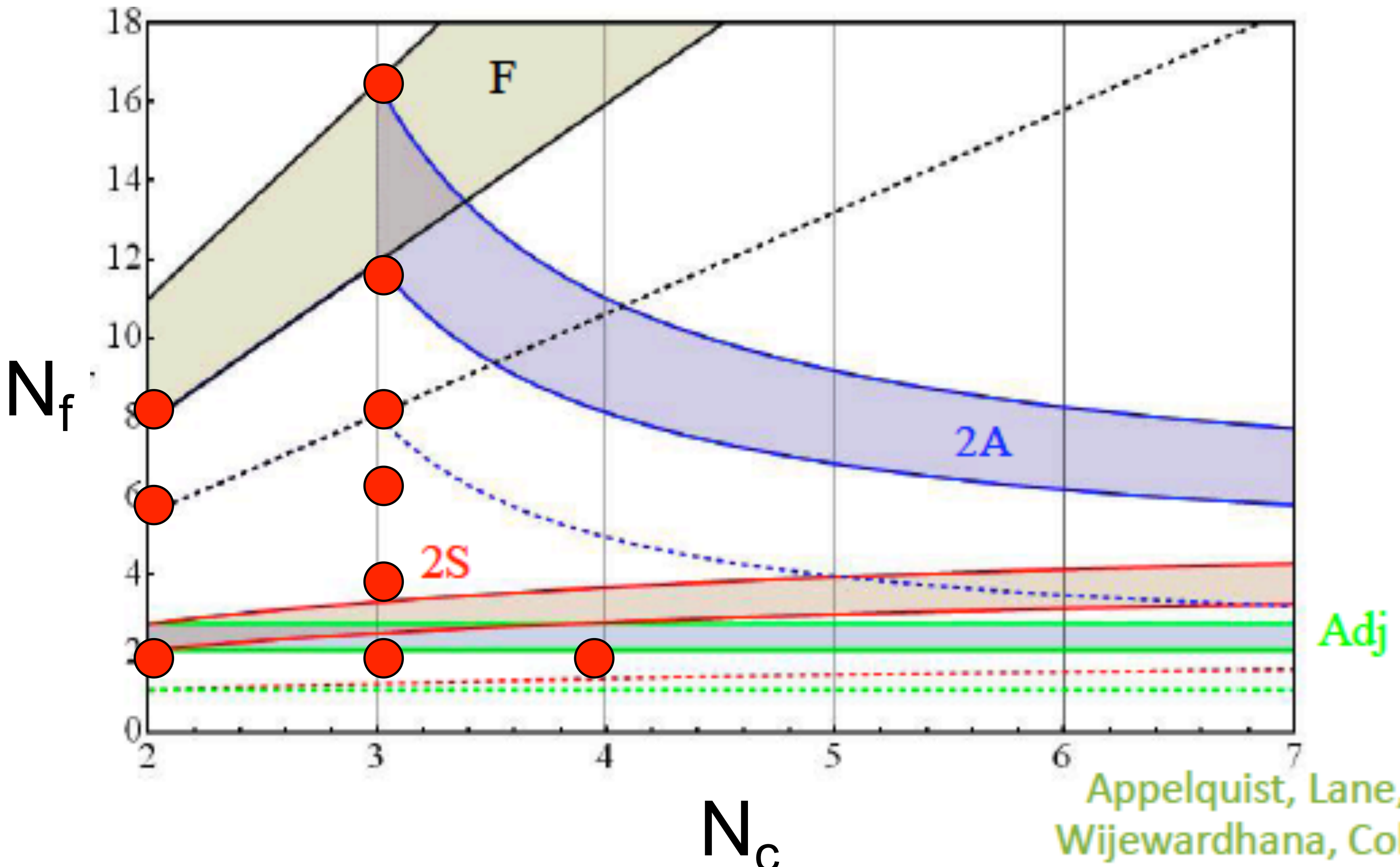
Composite Higgs BSM models are chirally broken but not QCD-like:

- large scale separation "walking"
- large anomalous dimension
- light 0^{++} if dilaton-like Higgs

Strongly coupled (near-)conformal models could've these properties

Sketch of theory space

S-D type calculations



Shaded: conformal
Below : confining
Above: IR free
Dotted lines: 2-loop PT

fermion representation:
Fundamental
Adjoint
2Symmetric
2Antisymm

Appelquist, Lane, Mahanta, Wijewardhana, Cohen, Georgi, Yamawaki, Shrock, Dietrich, Sannino, Tuominen

Red dots: systems investigated on the lattice (incomplete !)

Why? We are looking for generic properties & understanding

Questions for lattice studies:

Opening of conformal window:

Difficult, still controversial but there is hope …

Anomalous dimensions:

New RG method is promising for any hadronic operator

Spectrum:

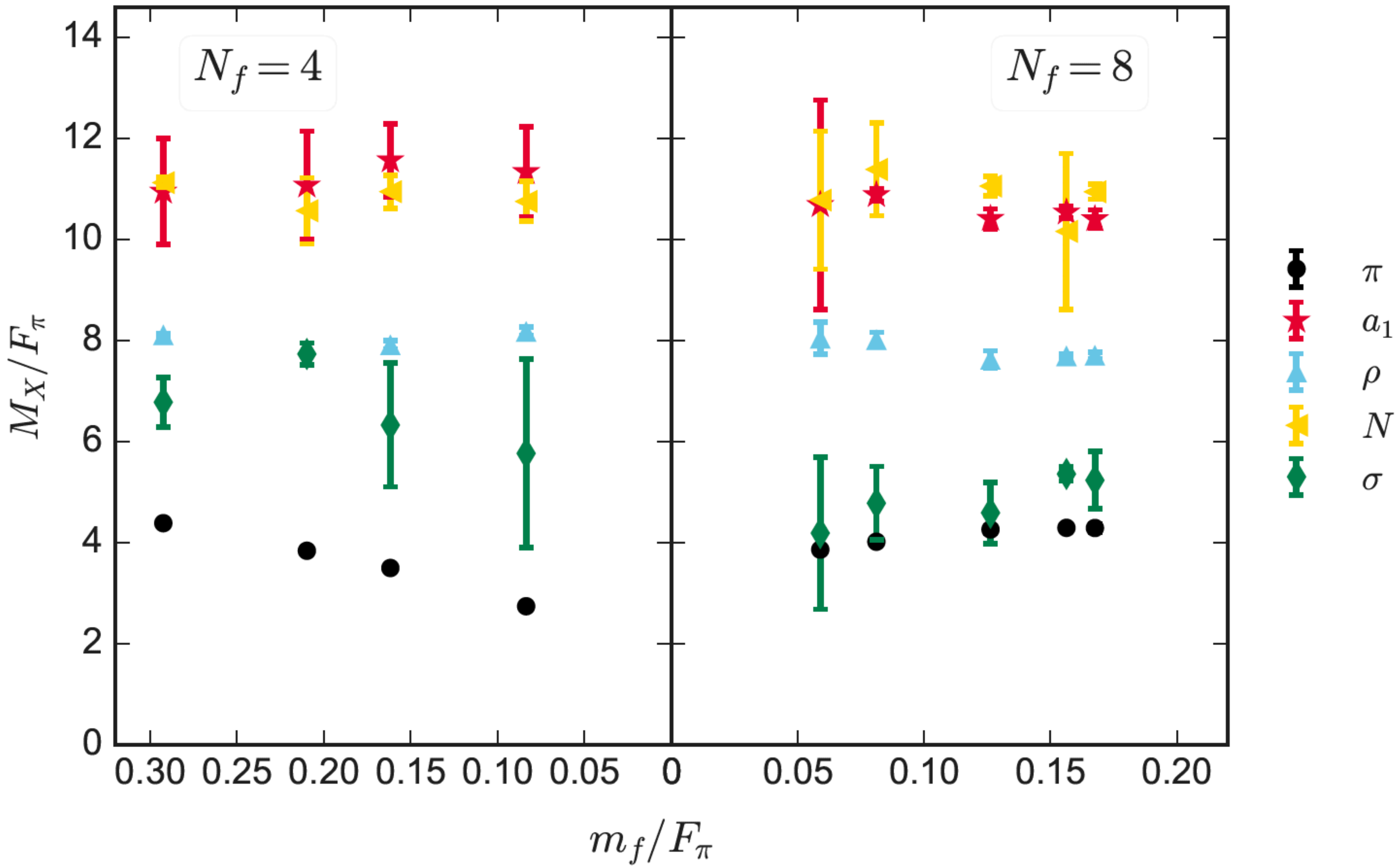
Most hadrons look similar to QCD ($m_\rho / m_{F_\pi} \approx 8$) except the 0^{++} sigma that is light in every near-conformal model

A few examples :

Spectrum

Spectrum of a walking system

SU(3) gauge, 8 fundamental flavors (LSD coll) : Dilaton-like Higgs



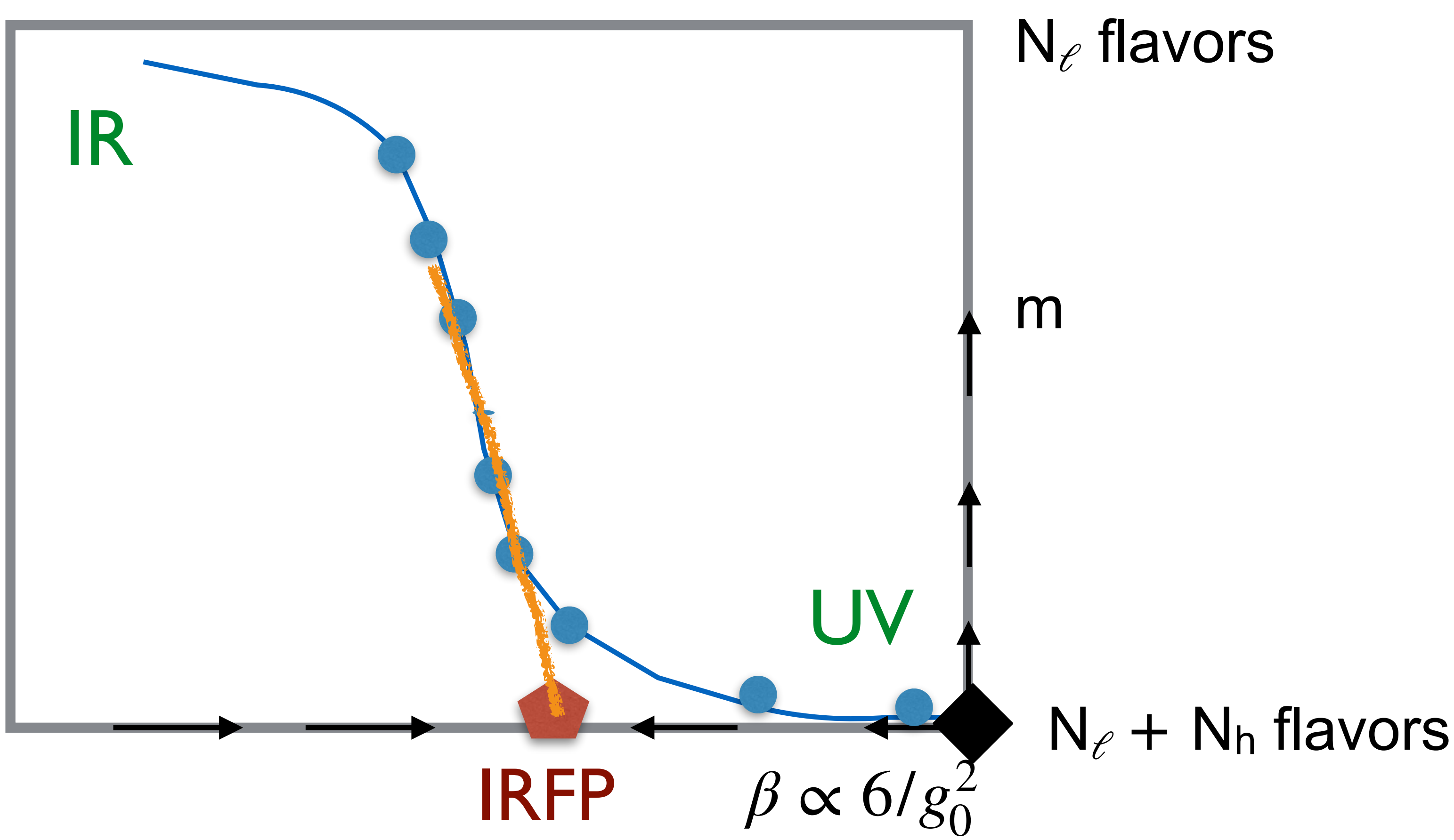
Mass-split model

Recall Wedn discussion
of the 10 flavor model

- Take N_f above the conformal window
- Split the masses: $N_f = N_\ell + N_h$

  N_h flavors are massive, m_h varies → decouple in the IR

  N_ℓ (= 2 - 4) flavors are massless, $m_\ell = 0$ → chirally broken



How predictive
is this model?

$$g^2, m_h, m_\ell$$

Hyperscaling in mass-split models

In conformal systems Wilson RG considerations predict the mass dependence of all dimensional quantities (hyperscaling)

If the scale changes as $\mu \to \mu' = \mu / b, \; b > 1$

the couplings run as

$$\hat{m}(\mu) \to \hat{m}(\mu') = b^{y_m} \hat{m}(\mu) \quad \text{(increases)}$$

$$g \to g^{\star}$$

Any 2-point correlation function at large b scales as

$$C_H(t; g_i, \hat{m}_i, \mu) \to b^{-2y_H} C_H(t/b; g^{\star}, b^{y_m} \hat{m}_h, b^{y_m} \hat{m}_\ell, \mu)$$

$$\equiv b^{-2y_H} C_H(t/b; g^{\star}, b^{y_m} \hat{m}_h, \hat{m}_\ell / \hat{m}_h, \mu)$$

since

$$C_H(t) \propto e^{-M_H t} \longrightarrow aM_H \propto (\hat{m}_h)^{1/y_m} F_H(m_\ell / m_h)$$

where $F_H(m_\ell / m_h)$ is a universal function

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Hyperscaling in mass-split models

All physical masses scale as

$$aM_H \propto (\hat{m}_h)^{1/y_m} F_H(m_\ell / m_h)$$

Dimensionless ratios are universal functions of m_ℓ/m_h

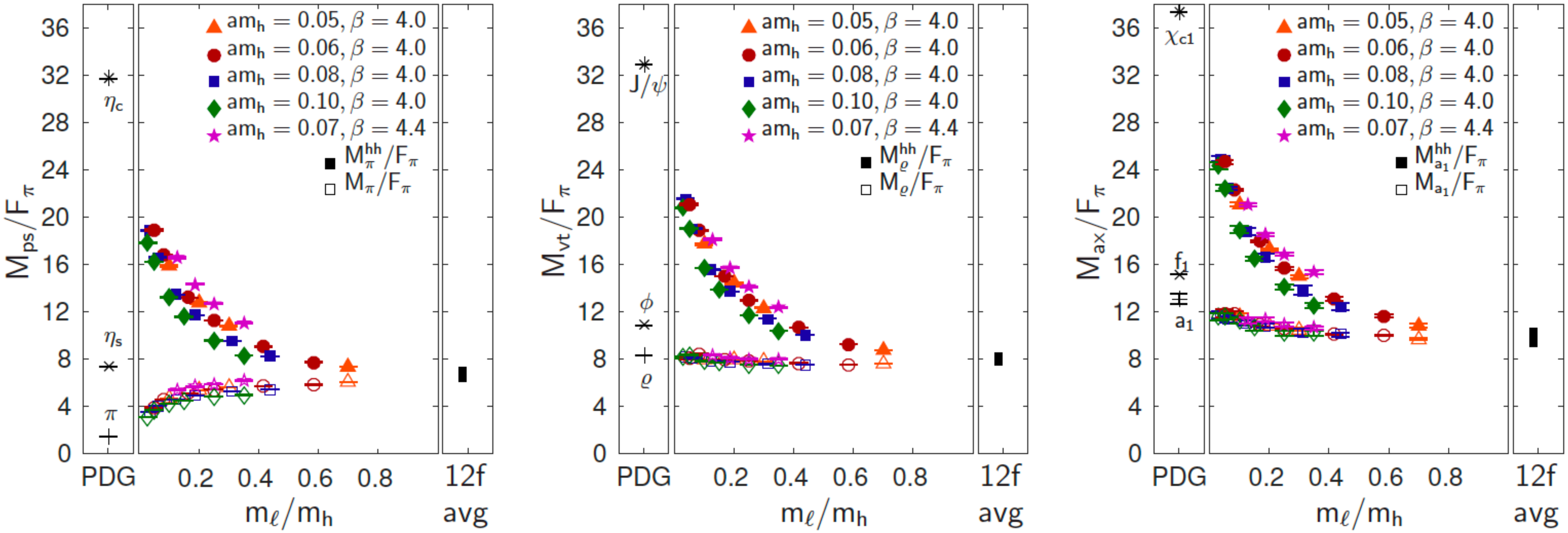
$$M_{H_1}/M_{H_2} = \Phi_H(m_\ell/m_h),$$
$$M_{H_1}/F_\pi = \tilde{\Phi}_H(m_\ell/m_h)$$

In the m_ℓ=0 chiral limit dimensionless ratios are independent of m_h

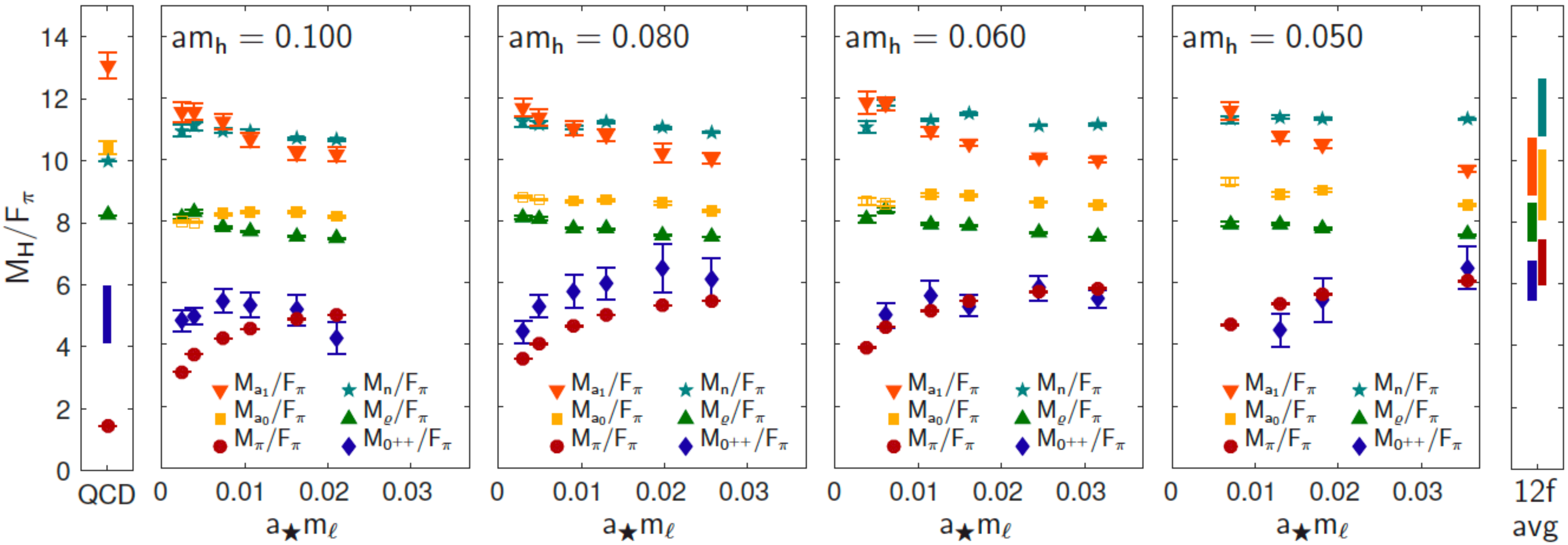
If F_π is known, the rest of the spectrum is predicted - no free parameters

- True for light-light, heavy-light and heavy-heavy spectrum
- This is very different from QCD!

Spectrum of a mass-split model ($4\ell + 8h$)



Still a light 0^{++} $(4\ell + 8h)$



The 10-flavor model is much more predictive if conformal

A few examples :

Opening the conformal window

Opening of the conformal window

A. Carosso, AH, E.Neal, PRL121,(2018)201601

Need the RG β function :

- Step scaling calculations are standard
- Wilsonian RG “blocking” is more promising

Use a continuous smoothing transformation to define blocked fields

GF is just what we need!

→ GF defines RG blocking with **continuous** scale change

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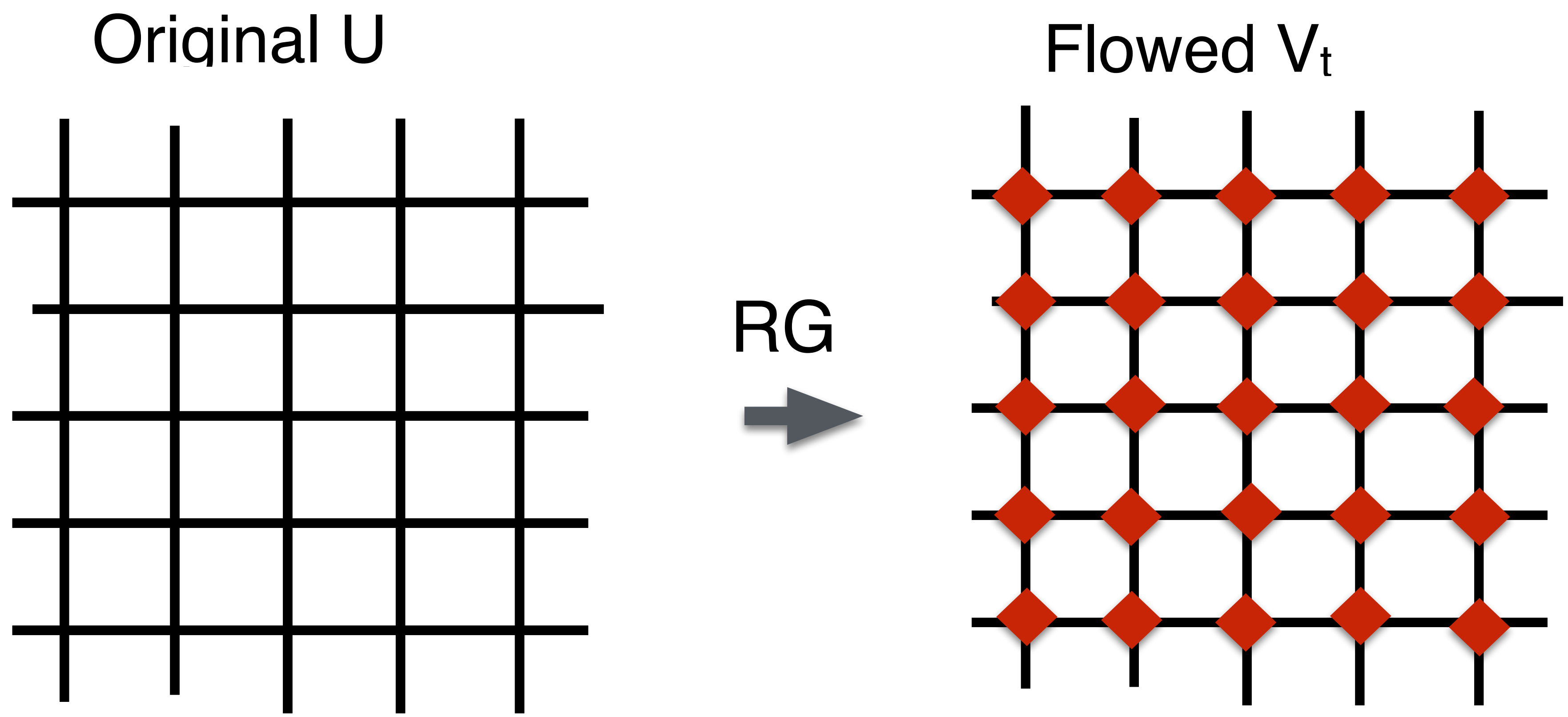

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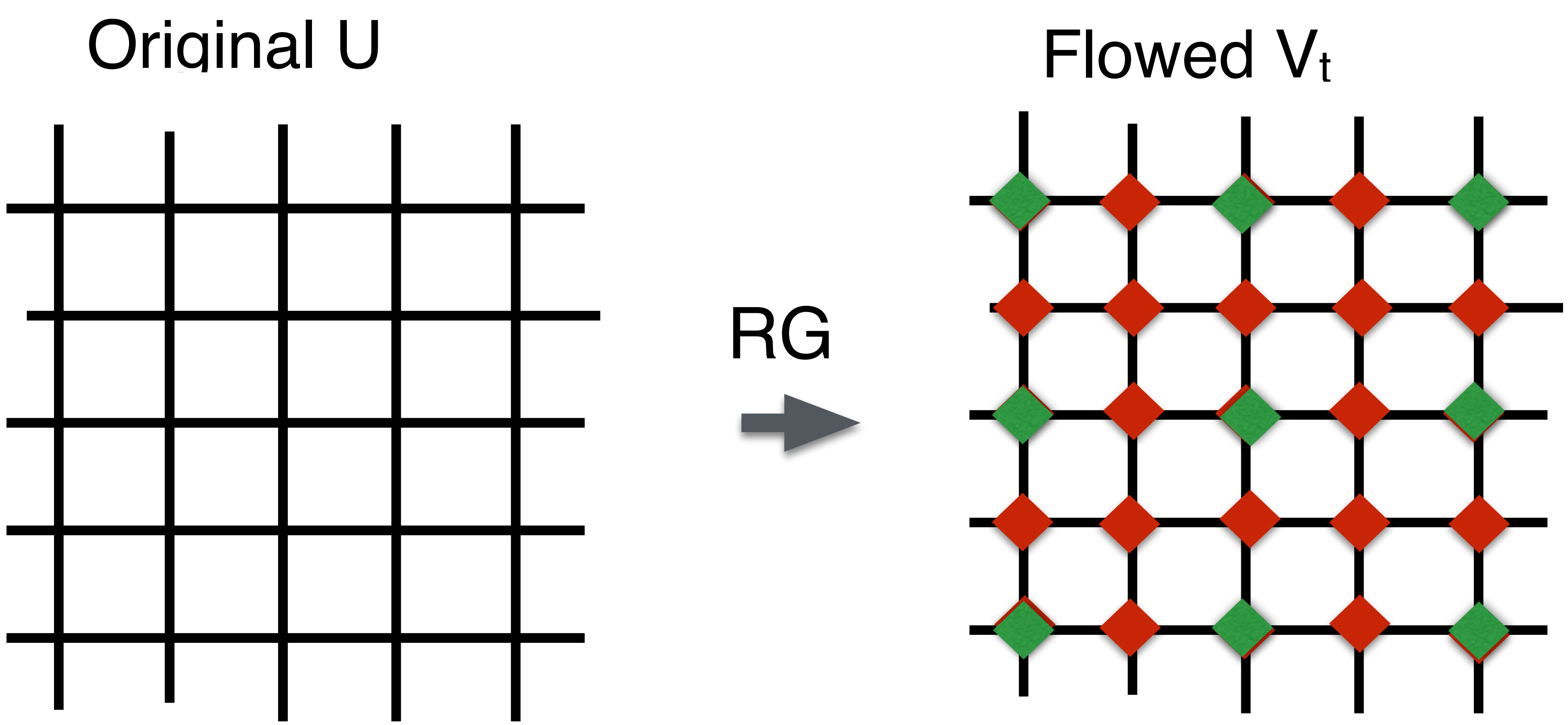

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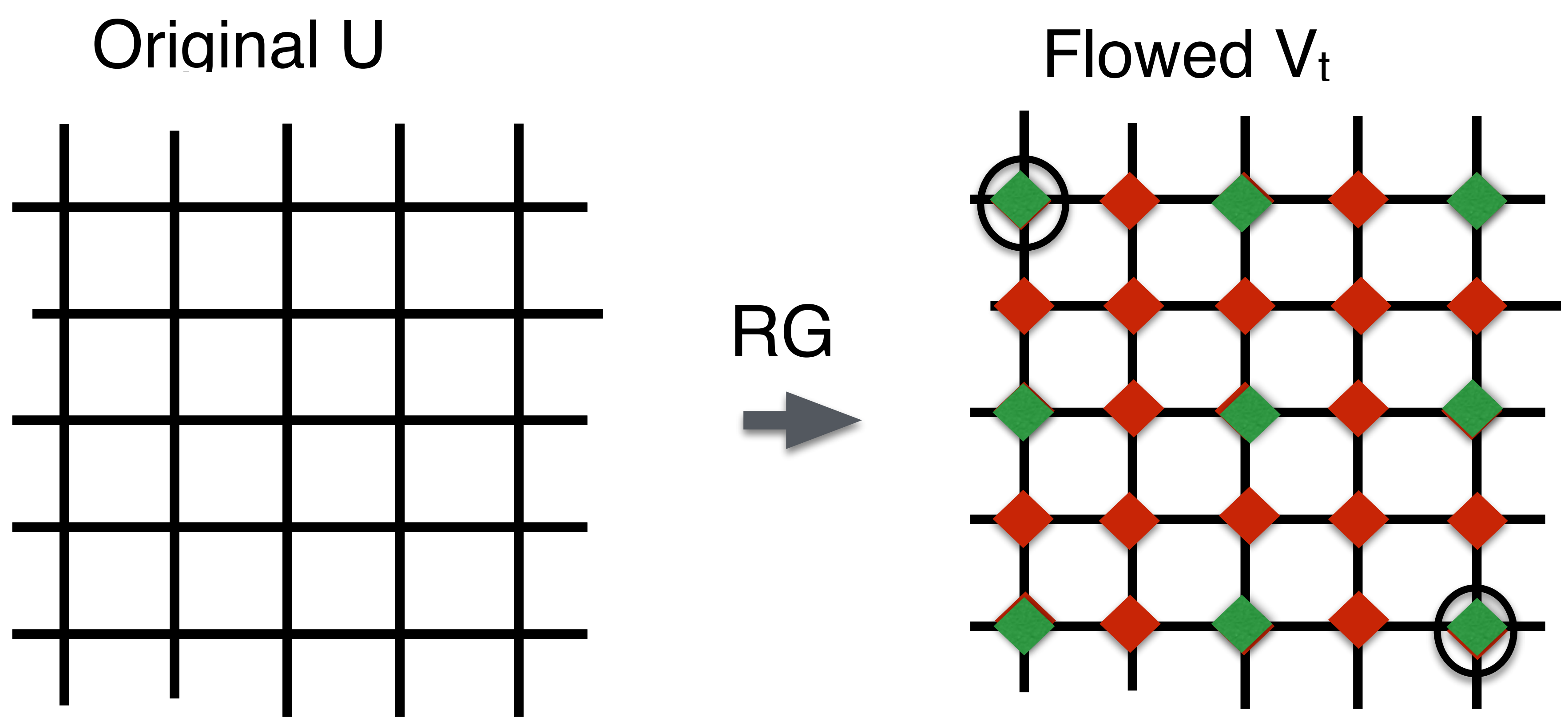

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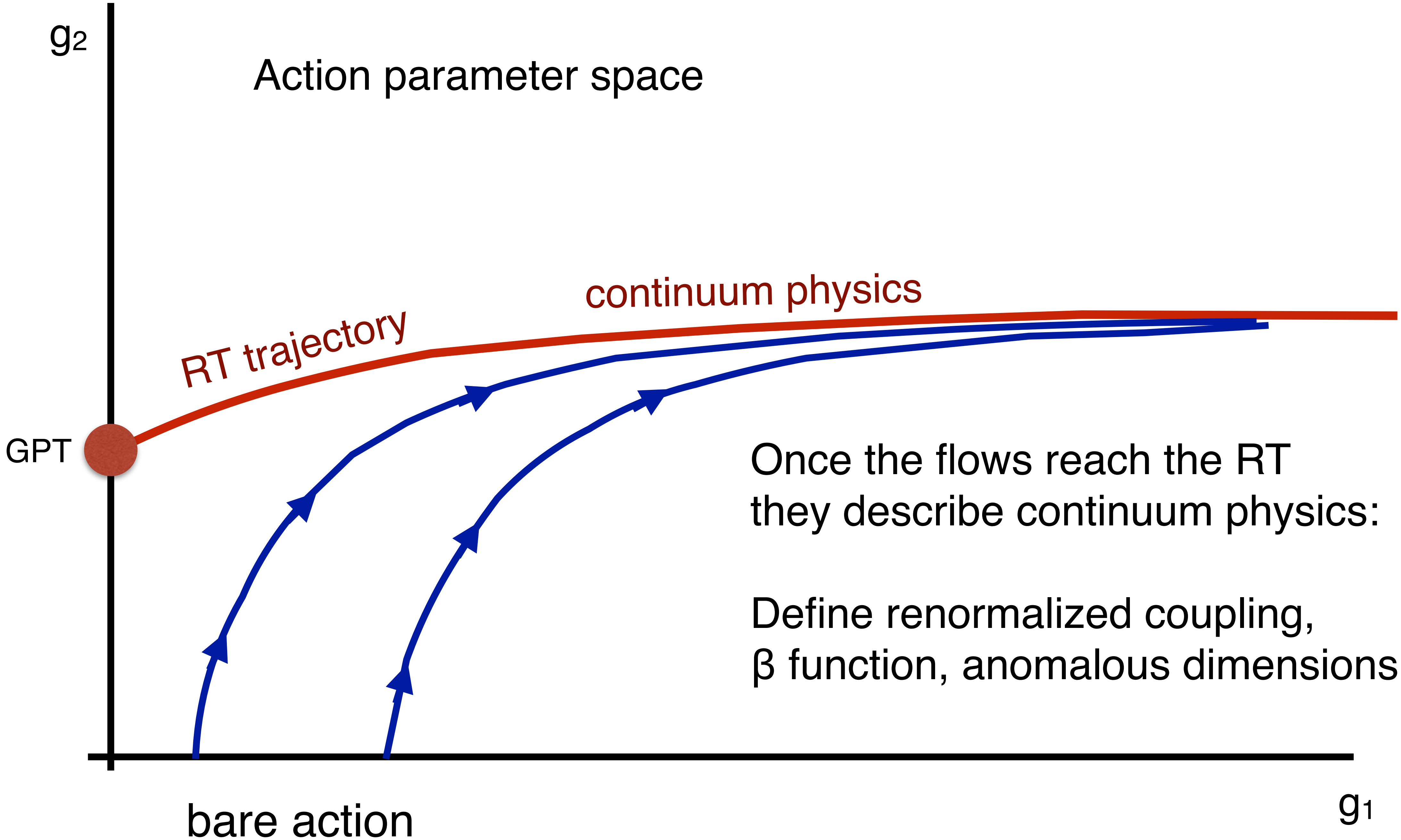
GF is just what we need!

$\rightarrow$ GF defines RG blocking with **continuous** scale change

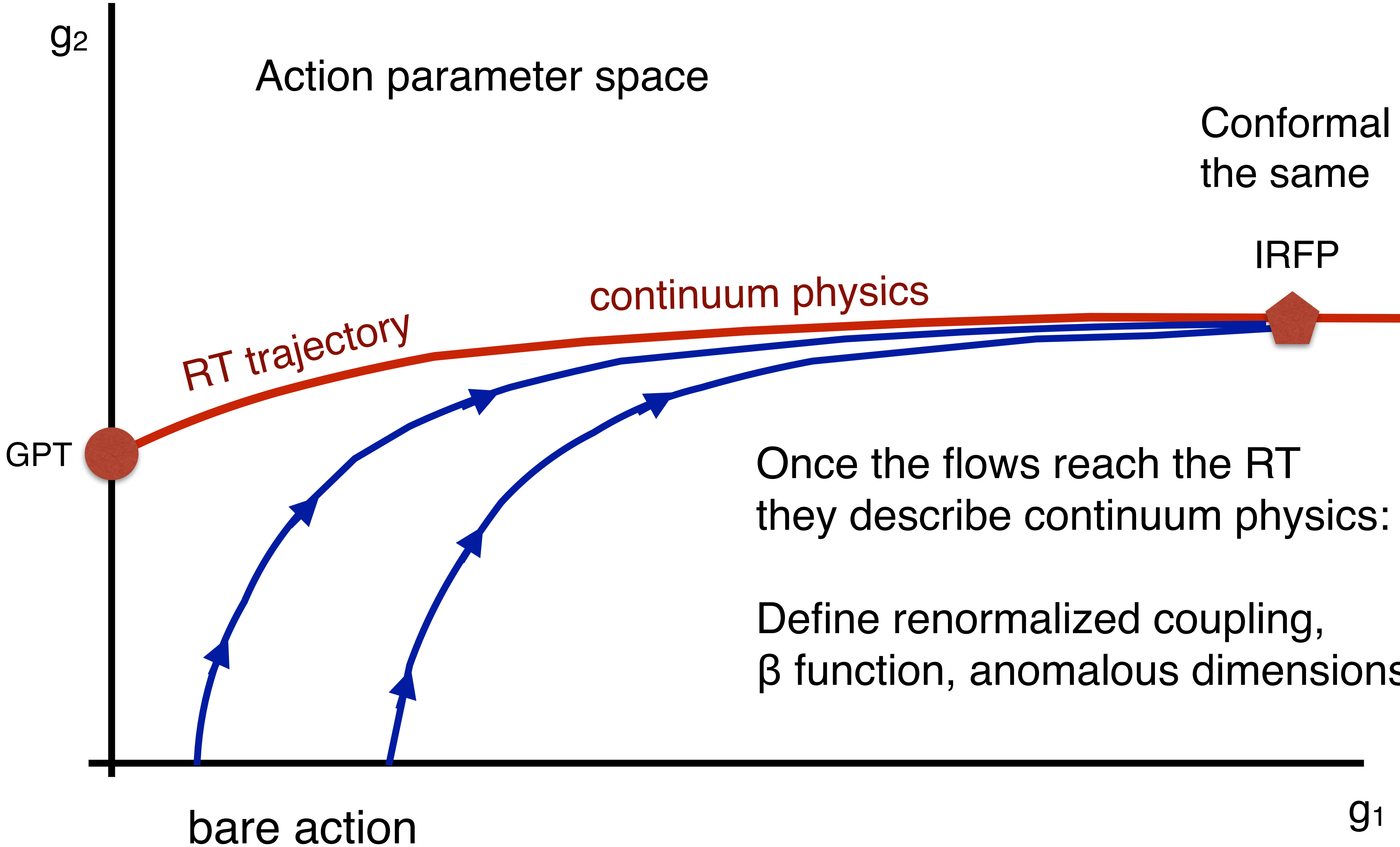
Topology of RG flows



Once the flows reach the RT
they describe continuum physics:

Define renormalized coupling,
β function, anomalous dimensions

Topology of RG flows



Once the flows reach the RT they describe continuum physics:

Define renormalized coupling, β function, anomalous dimensions

Continuous β function

On a single ensemble (any bare coupling, infinite volume) calculate

$$\beta = \mu \frac{dg^2}{d\mu} = -2t \frac{dg^2}{dt}$$

$t \propto \mu^2$ is the GF flow time and g^2 is the GF coupling

$$g_c^2(L) = \frac{128\pi^2}{3(N^2-1)} \frac{1}{C(c,L)} \langle t^2 E(t) \rangle$$

This should give (part) of the RG β function as $t \to \infty$

Equivalent to step scaling when $c = \sqrt{8t}/L = 0$

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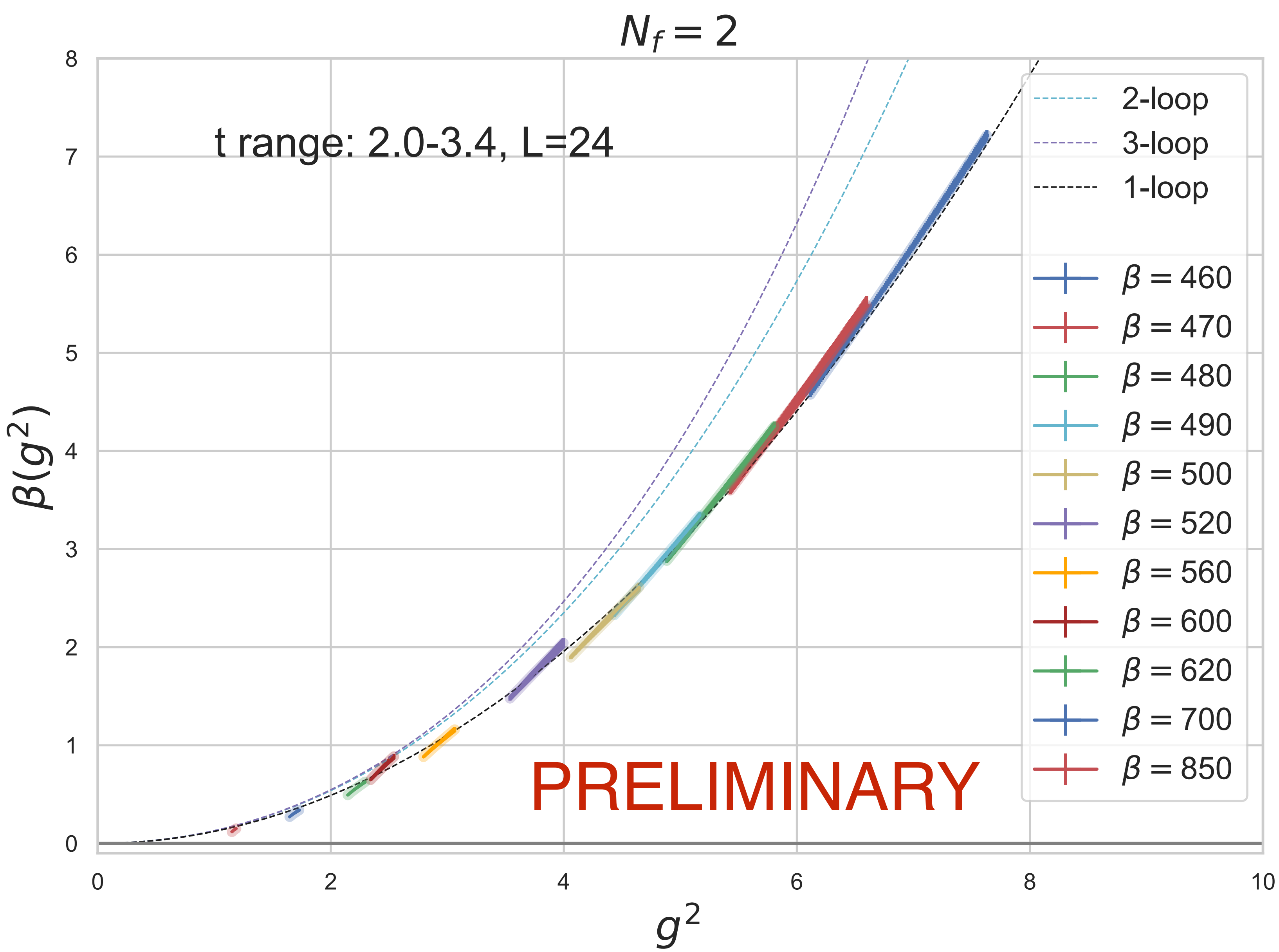
Different bare couplings will overlap on RT!

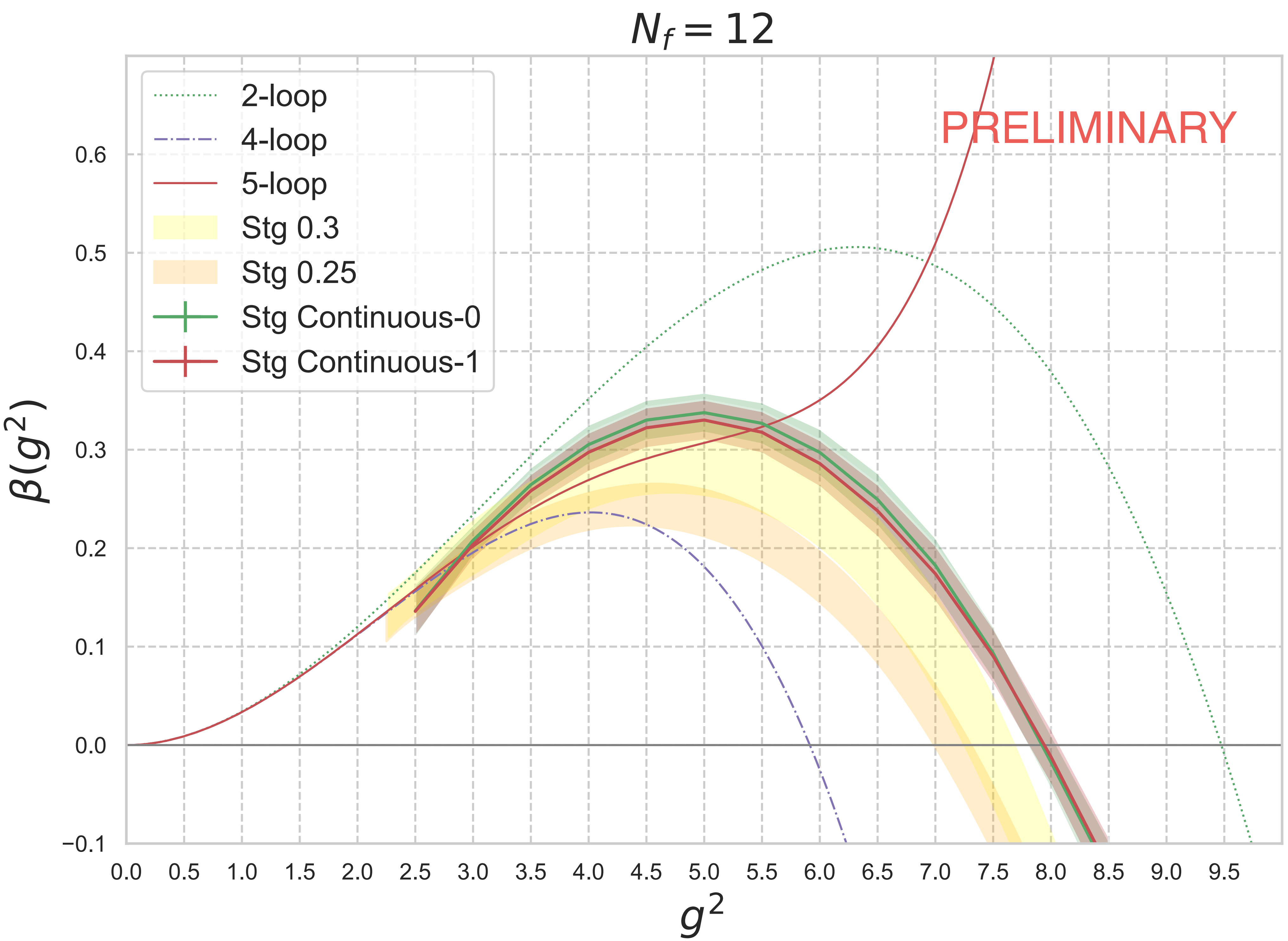
Continuous β function with 2-flavors (QCD)



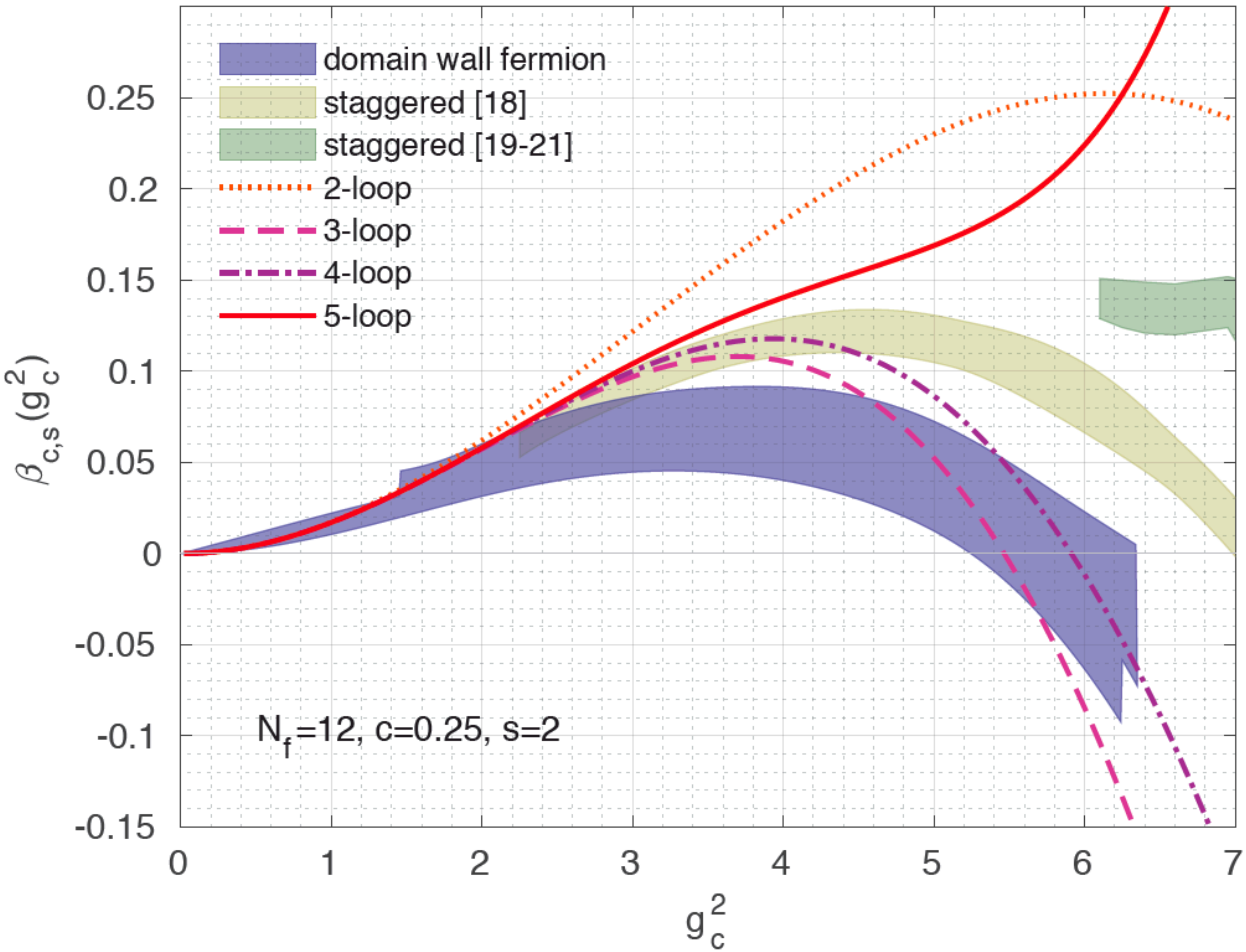
color bands:
predictions of the β function from various single ensembles

Predictions of different bare coupling values overlap, as RG considerations suggests

SU(3) with N_f=12 : controversial and difficult



SU(3) with N_f=12 : step scaling fn. with DW



5-loop beta function: what's wrong with it?

A) Not convergent, needs analytic continuation, it is really close to 4-loop

B) It signals 2 FPs in the conformal regime and 2 complex FPs just below the conformal window (Gorbenko, Rychkov, Zan)
   - the extra FPs could explain all the scaling violations lattice studies observed
   - continuous beta fn can handle the new exponents

A few examples :

Anomalous dimensions

Anomalous dimensions

Compensate for wave function renormalization by an operator that does not have an anomalous dimension — vector

Ratio

$$R(t,x_0)=\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\Big(\frac{\langle A(0)A(x_0)\rangle}{\langle A_t(0)A_t(x_0)\rangle}\Big)^{n_O/n_A}=b^{\gamma_O}\propto t^{\gamma_O/2}$$

independent of $x_0 >> b$ and predicts γ

Anomalous dimensions

Ratio of flowed & unflowed hadronic correlators

$$\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle} = b^{2\Delta_O - 2n_O\Delta_\phi}$$

$$x_0 \gg b$$

$$\Delta_O = d_O + \gamma_O$$

$$\Delta_\phi = d_\phi + \eta/2$$

Compensate for wave function renormalization by an operator that does not have an anomalous dimension — vector

Ratio

$$R(t,x_0) = \frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A_t(0)A_t(x_0)\rangle}\right)^{n_O/n_A} = b^{\gamma_O} \propto t^{\gamma_O/2}$$

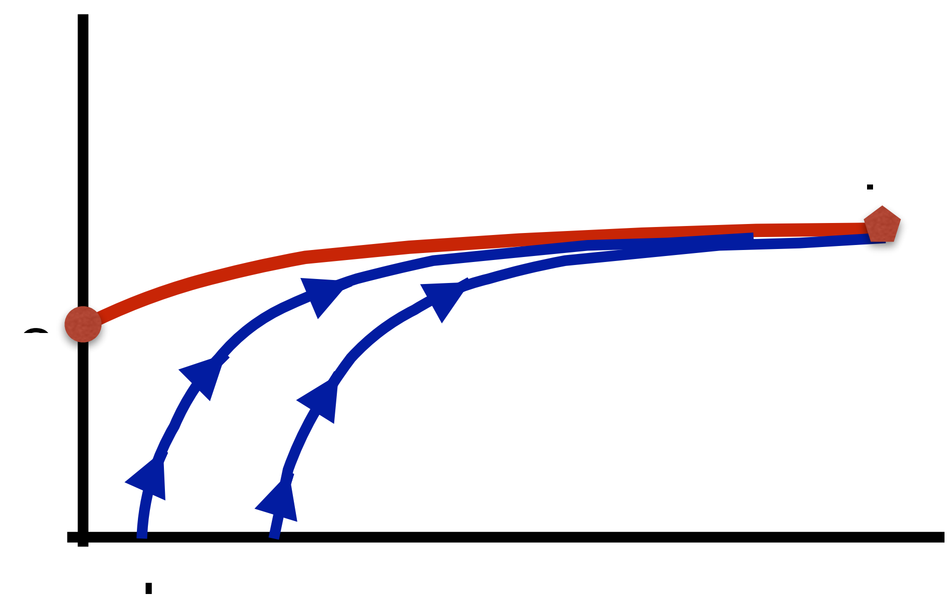
independent of $x_0 >> b$ and predicts γ

N_f=2 flavors

QCD : chirally broken, no IRFP
but anomalous dimension is still defined

$$\gamma_m(g^2) = \frac{d\log m}{d\log\mu} = \gamma_0 g^2 + \gamma_1 g^4 + \ldots$$

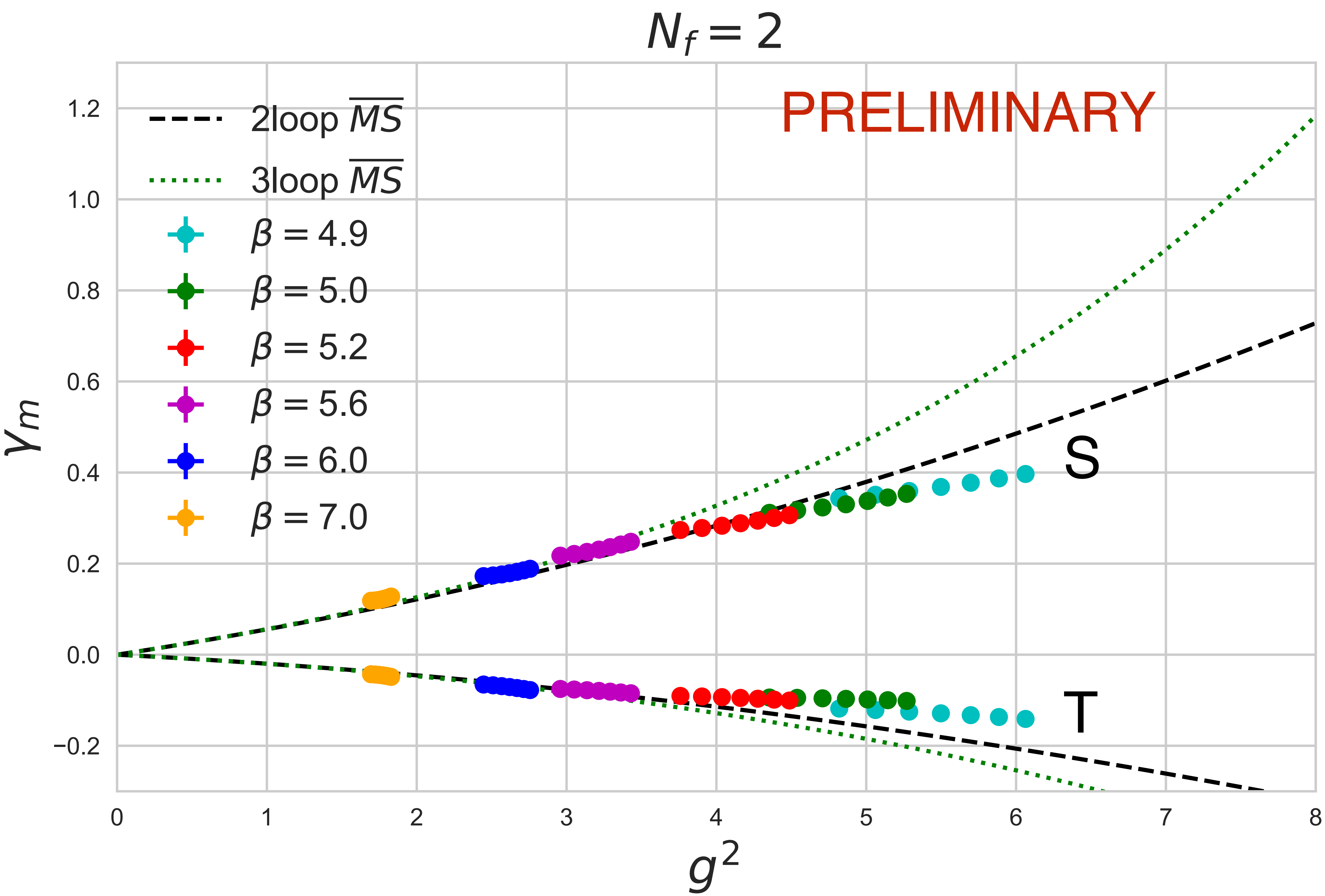
and the coefficients are known to 4 loop.



Gradient flow defines a running coupling $g^2(t)$;
Combine with $\gamma(t)$ to predict the (scale dependent) $\gamma(g^2)$

Simulations : 24^3 x64, weak coupling so remains deconfined
Easy to extend to other volumes, finite temperature

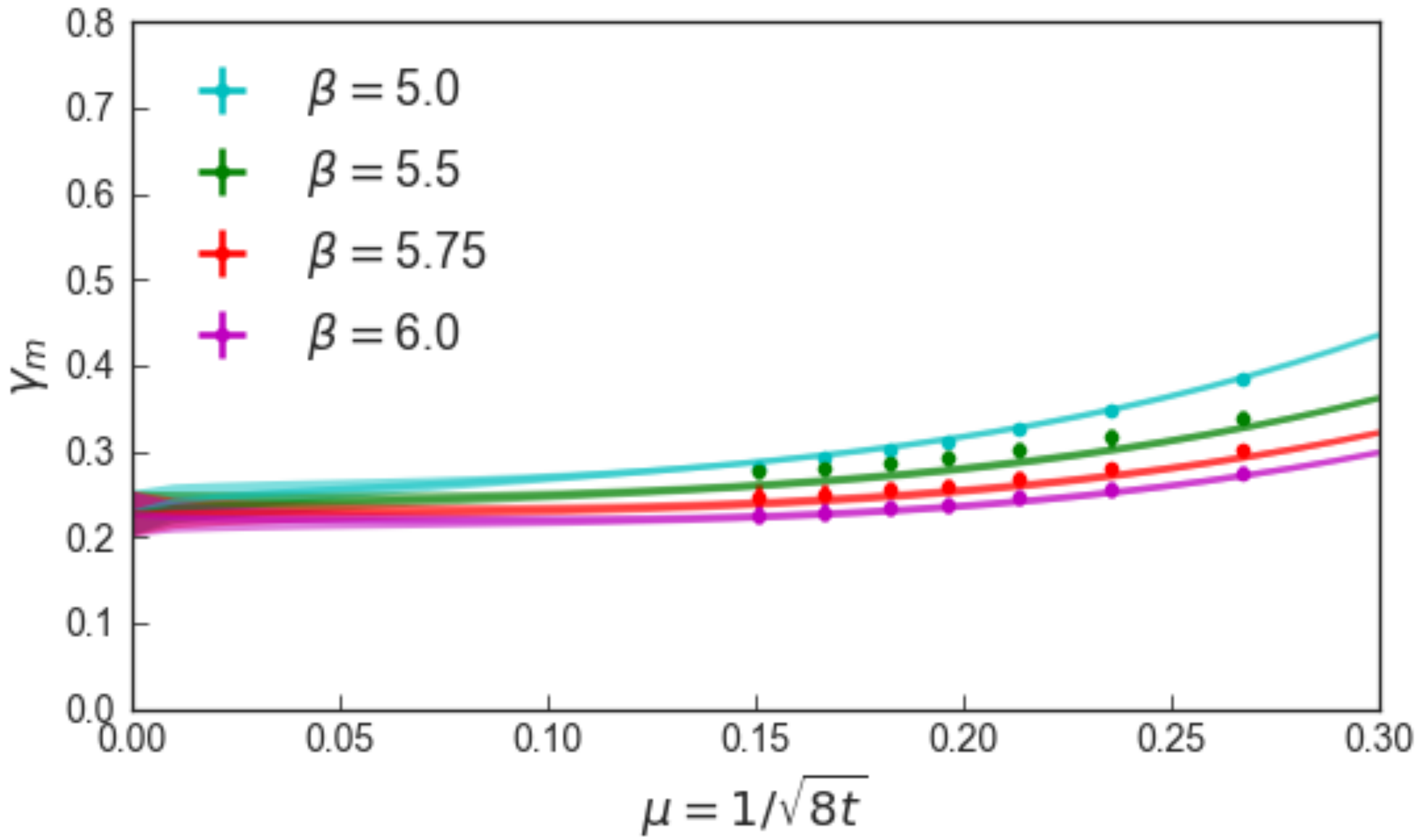
Scalar and Tensor



Daisy-chain together many bare coupling values to cover a wide range of renormalized couplings

Agrees well with PT - first non-perturbative calculation for T

N_f=12, Pseudo scalar:



Staggered

$\gamma_m = 0.24(3)$, $t \to \infty$

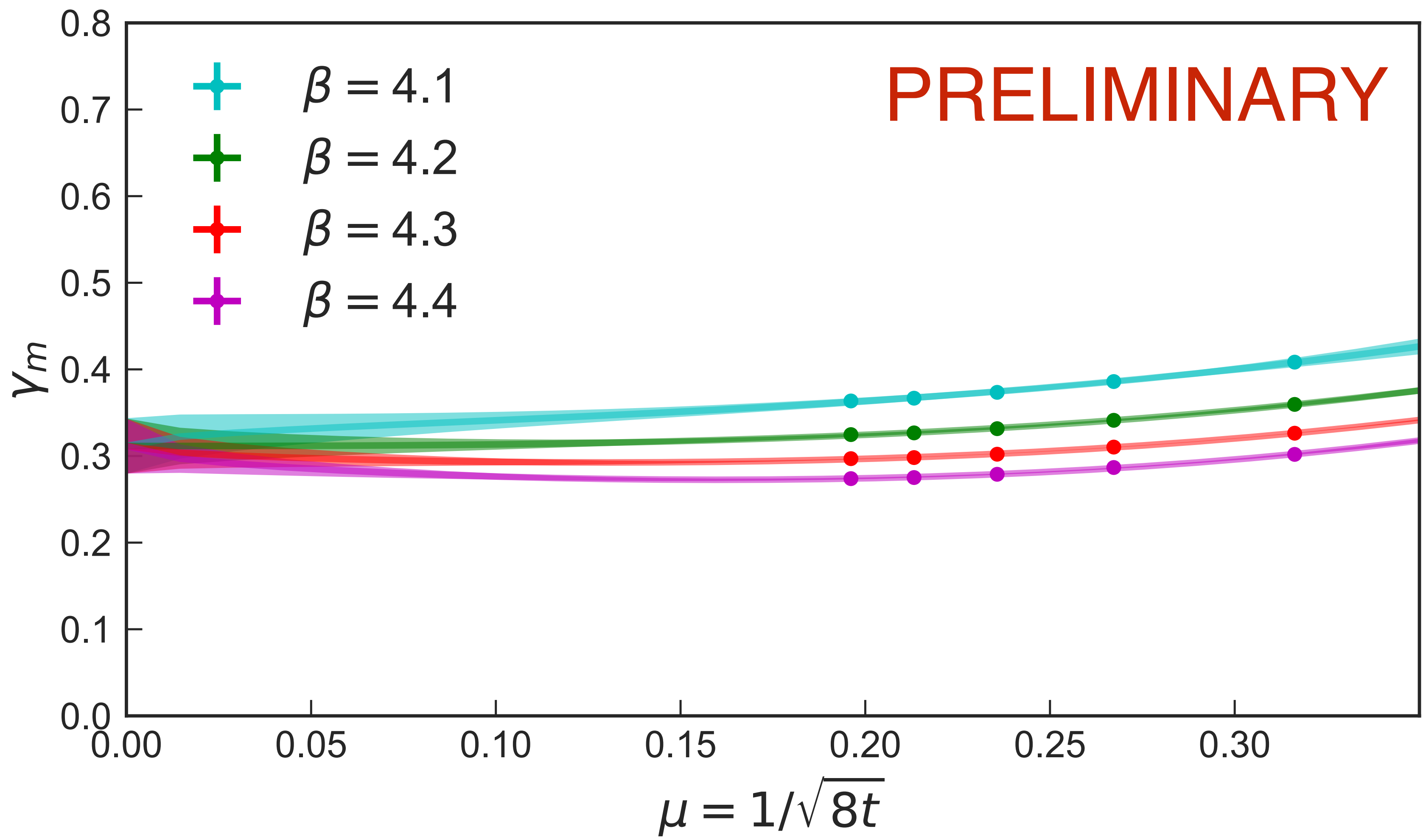
extrapolate to $t \to \infty$:

$$\gamma_m(\beta,t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

error: systematic + statistical

result consistent with other methods

N_f=12, Pseudo scalar:



Domain wall

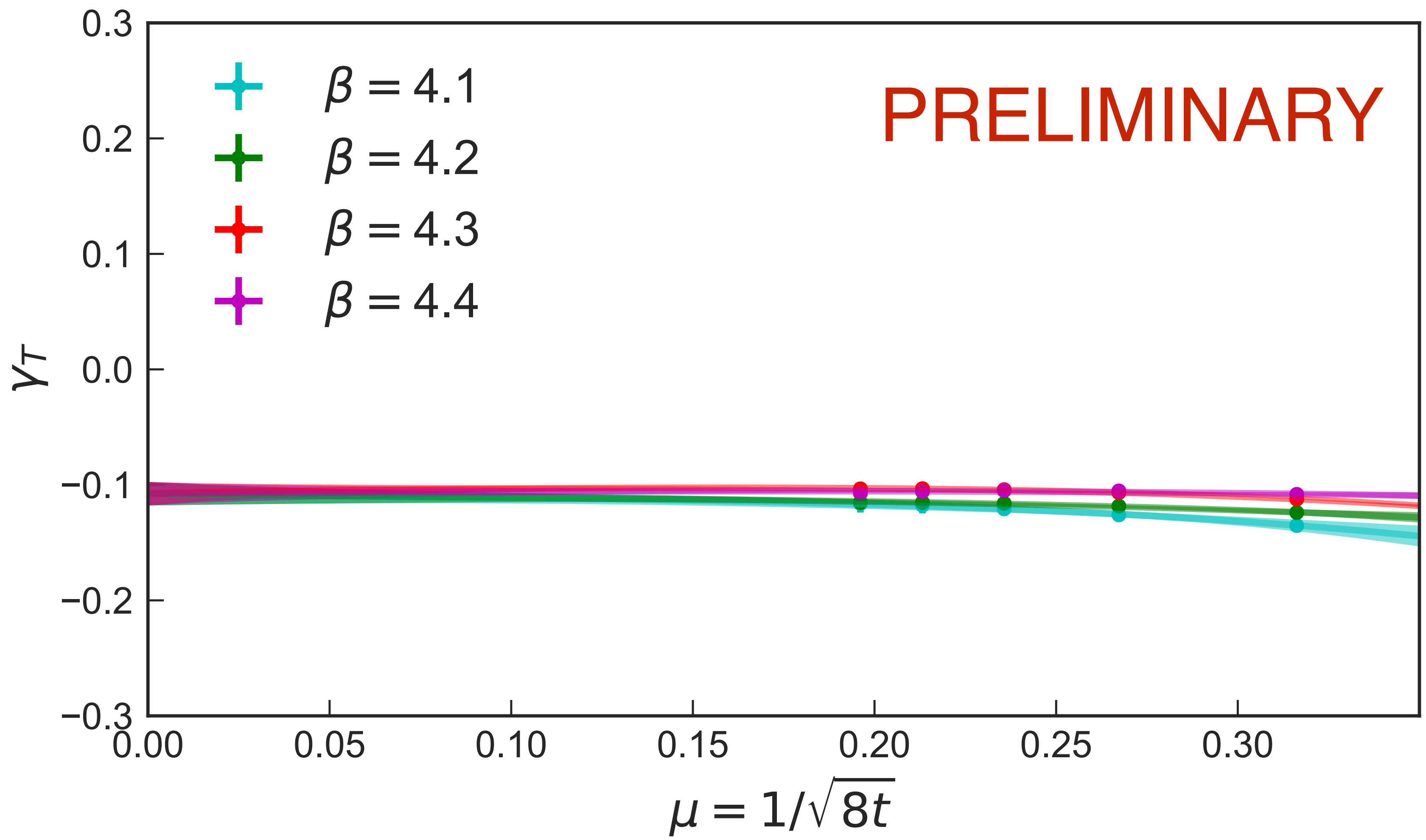
$\gamma_m = 0.31(3), \quad t \to \infty$

(needs finite volume extrapolation)

extrapolate to t → ∞ :

$$\gamma_m(\beta, t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

N_f=12, Tensor:



Domain wall

$\gamma_T = -0.11(1)$, t→∞

extrapolate to t → ∞ :

$$\gamma_m(\beta,t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

Lattice BSM

Lattice calculations can predict non-perturbative properties of strongly coupled systems

- specific models
- generic properties

What is the most useful?