JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

Factorization Violation and Glauber Gluons

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Factorization breaking and collinear functions

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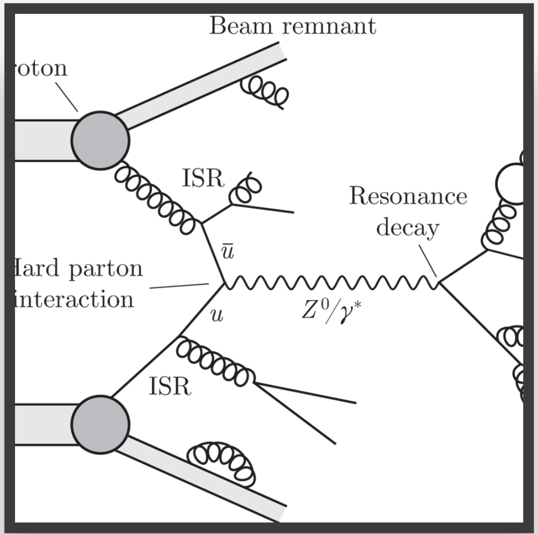
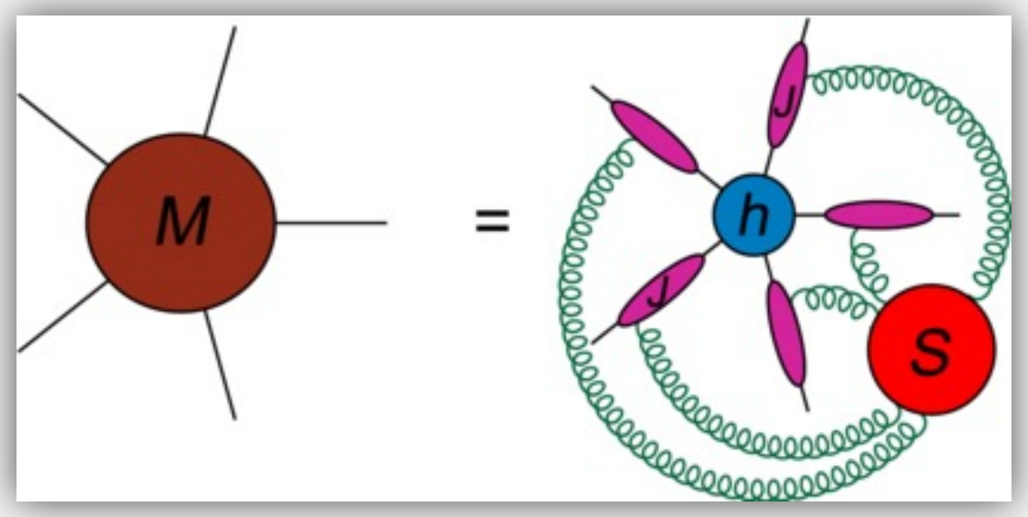
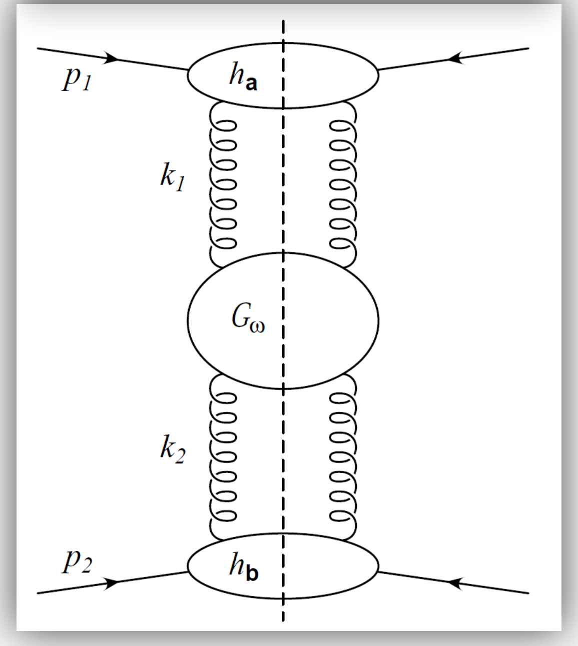
Catani, de Florian, GR, JHEP07 (2012) 026



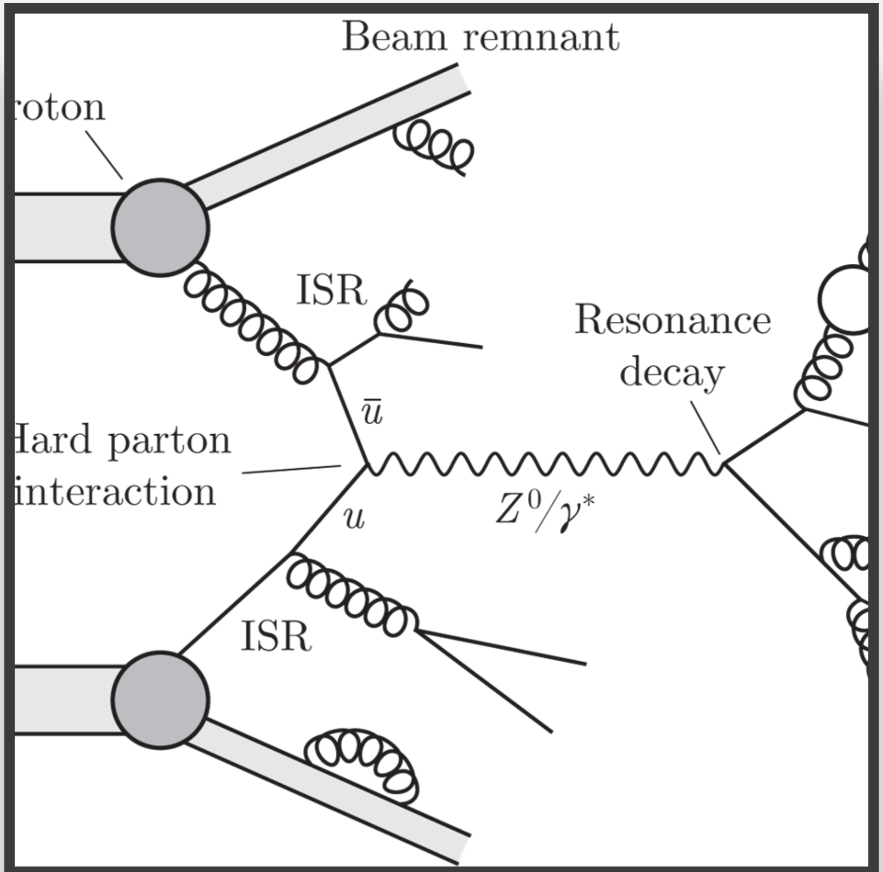




Keywords of (any kind of) factorization

- Universal behaviour
- Process independent objects

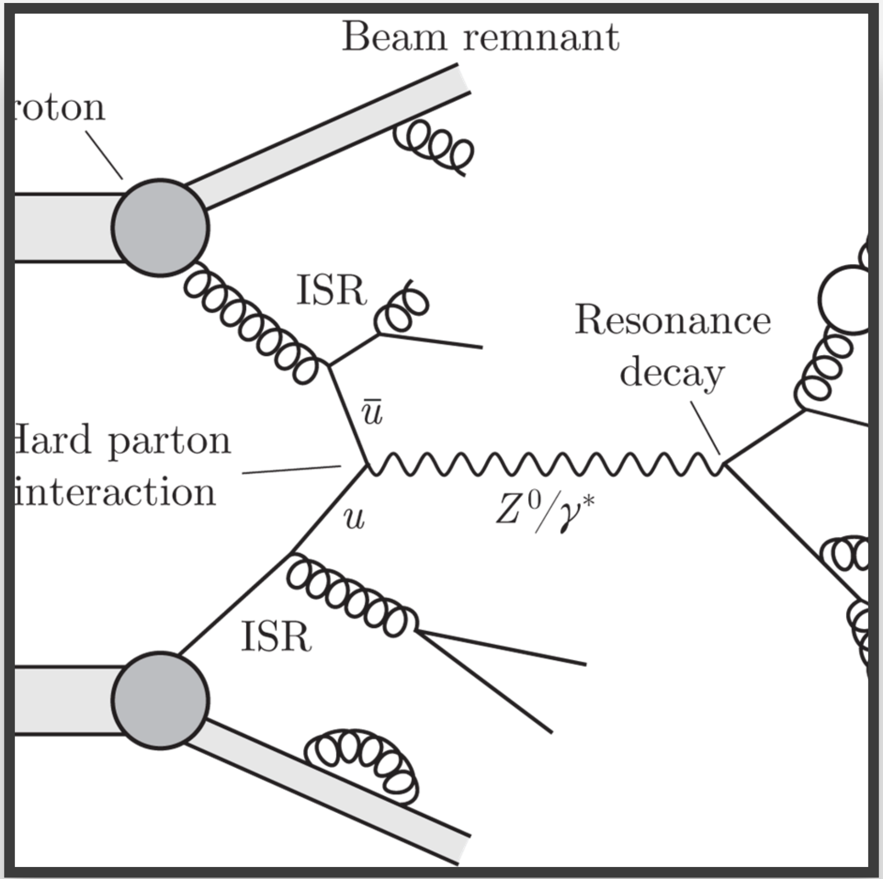
collinear factorization at higher orders ?



pQCD for hard-scattering processes based on **universality of collinear factorization**:

- the sole un-cancelled IR divergences are due to partonic states whose momenta are collinear to the collider partons
- removed by redefinition of bare parton densities
- process independent: e+e-, DIS, hadron-hadron (universal)

collinear factorization at higher



Collinear factorization theorem proven for **sufficiently inclusive** observables in the final state of the scattering of colorless hadrons [Collins, Soper, Sterman]

- Offen assumed that partonic scattering amplitudes factorize: fixed order and resummations
- Monte Carlo event generators are based on factorization
- In neither of these cases factorization is guaranteed.

Multiple collinear limit

- Momenta $p_1, \ldots, p_m$ of m partons become parallel
- Sub-energies $s_{ij} = (p_i + p_j)^2$ of the same order and vanish simultaneously

The momentum of the m partons in terms of two back-to-back light-like momenta $\tilde{P}^2 = 0, n^2 = 0$:

$$(p_1 + \cdots + p_m)^\mu = \tilde{P}^\mu + \frac{s_{1\ldots m} n^\mu}{2n \cdot \tilde{P}}$$

$\tilde{P}^\mu$: collinear direction

n^μ: describes how the collinear limit is approached

$z_i = \frac{n \cdot p_i}{n \cdot \tilde{P}}$: longitudinal momentum fraction, $\sum z_i = 1$

Singular behaviour

- Matrix element in perturbation theory (QCD)

$$M = M^{(0)} + M^{(1)} + M^{(2)} + \cdots$$

- At tree-level ($s = s_{ij}, s_{ijk}$, or any sub-energy)

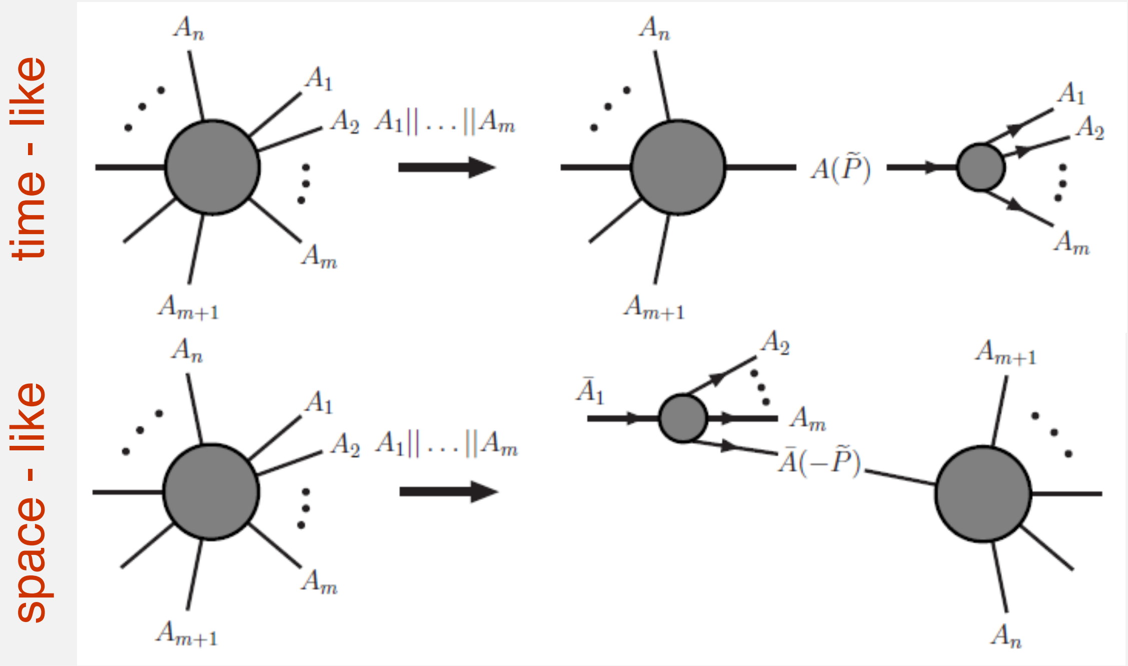
$$M^{(0)}(p_1, \ldots, p_m, \ldots, p_n) \simeq \left(\frac{1}{\sqrt{s}}\right)^{m-1}$$

- At one-loop (scaling violation)

$$M^{(1)}(p_1, \ldots, p_m, \ldots, p_n) \simeq \left(\frac{1}{\sqrt{s}}\right)^{m-1} \left(\frac{s}{\mu^2}\right)^{-\epsilon}$$

Collinear factorization at tree - level

- External legs on-shell with physical polarisations
- factorization in colour-space [Catani, de Florian, GR]
- vs colour stripped (Split function of colour-subamplitudes) [Bern, Chalmers, Dixon, Kosower, Catani, Grazzini, Glover, Campbell, del Duca, …]



Collinear limit

- Singular behaviour captured by universal (process independent) factorisation properties
- Splitting matrix depends on the collinear partons only.
- Space-like and time-like related by crossing

$$
|M^{(0)}(p_1, \ldots, p_n)\rangle \simeq \boldsymbol{Sp}^{(0)}(p_1, \ldots, p_m; \tilde{P}) \, |\overline{M}^{(0)}(\tilde{P}, p_{m+1}, \ldots, p_n)\rangle
$$

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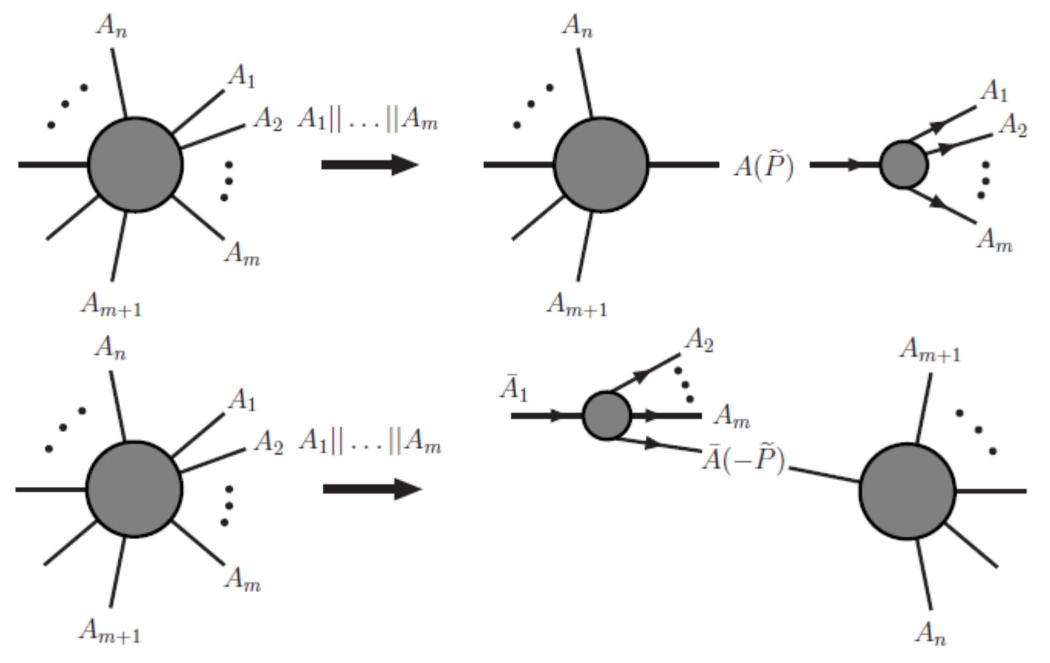
time - like

space - like



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$$\mathbf{Sp}^{(0)(\alpha_1 a_2;\alpha)}_{q_1 g_2}(p_1, p_2, \widetilde{P}) = \frac{1}{s_{12}}\, \mu^\epsilon\, g_S\, t^{a_2}_{\alpha_1 \alpha}\, \bar{u}(p_1)\, \not\varepsilon(p_2) u(\widetilde{P})$$

At two loops: time - like

$$|M^{(2)}(p_1,\ldots,p_n)\rangle$$
$$\simeq \mathbf{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{M}^{(2)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle$$
$$+\mathbf{Sp}^{(1)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{M}^{(1)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle$$
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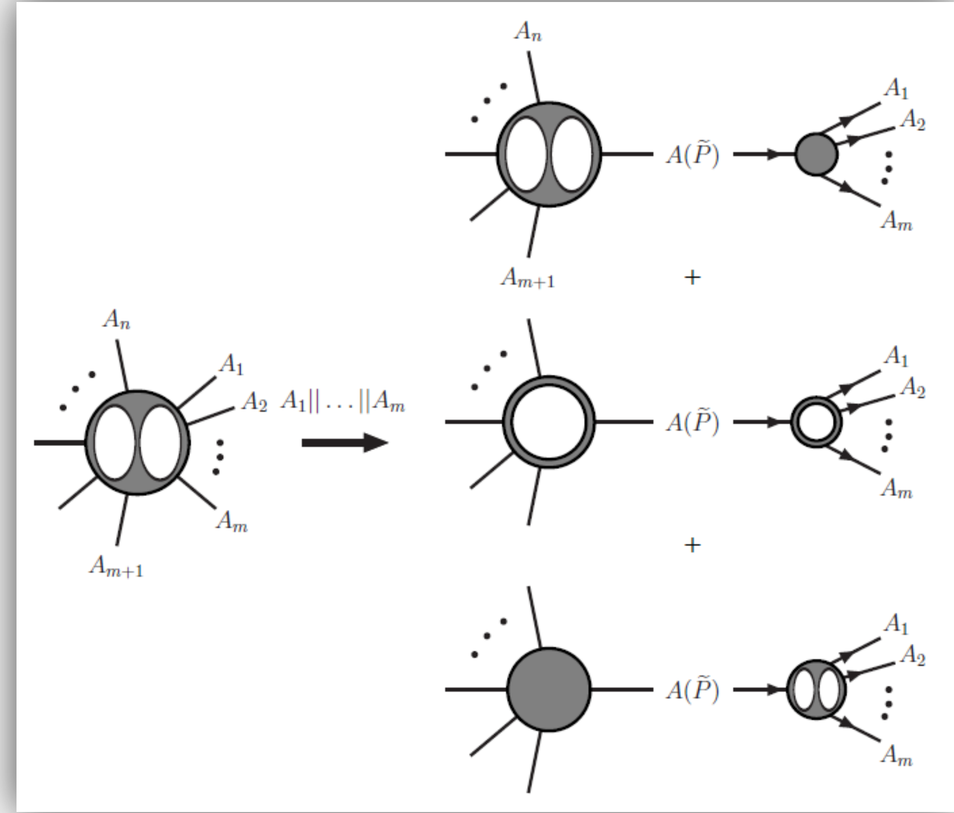

At two loops: time - like

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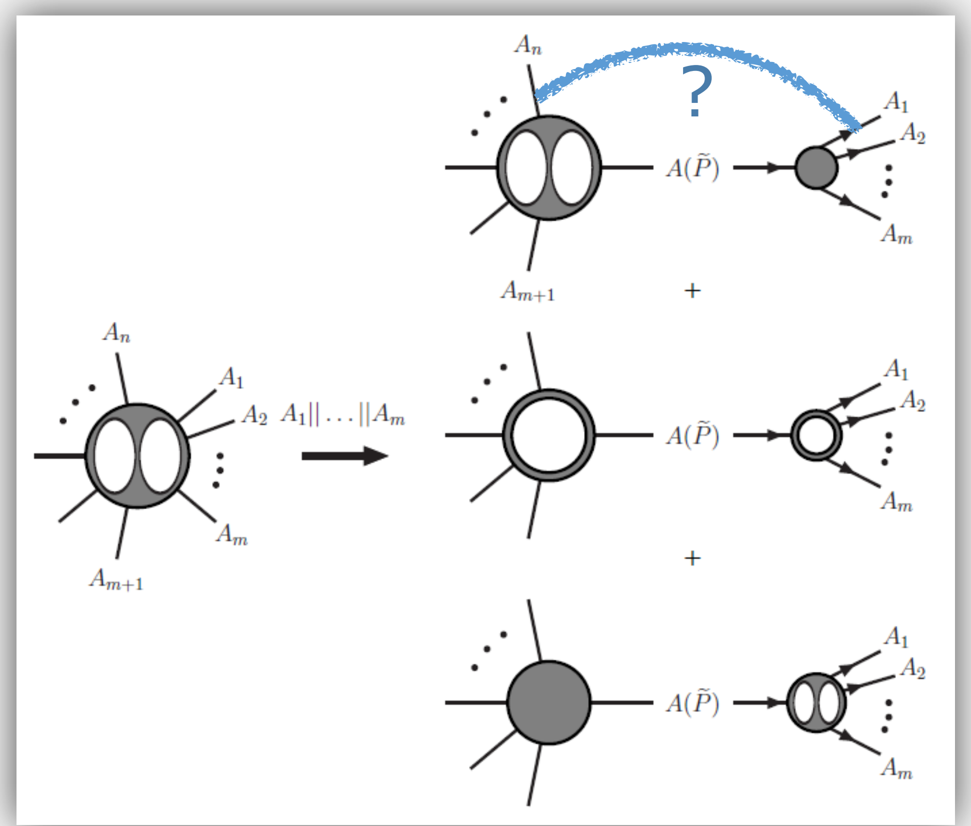
\end{aligned}
$$



Qualitative interpretation: m=2

- **Tree level:**
  - Two-scale problem: collinear subenergy $s_{12} \ll$ any other subernergy (large- vs short-distance interactions)

- **Loops:**
  - Gauge interactions are long-range
  - Interactions separately spoil factorization,
    but $\theta_{j1} \cong \theta_{j2} \cong \theta_{j\tilde{P}}$ and $\boldsymbol{T}_j \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2) = \boldsymbol{T}_j \cdot \boldsymbol{T}_{\tilde{P}}$
    Colour coherence restores factorization
  - Both collinear partons in the final- or intial-state, otherwise color coherence limited by *causality*

Factorization breaking in the space-like collinear region at higher orders ?

Generalized collinear factorization at all orders

- Multiparton collinear limit at **all orders**

$$|M(p_1,\dots,p_n)\rangle \simeq \boldsymbol{Sp}(p_1,\dots,p_m;\tilde{P};\boldsymbol{p_{m+1},\dots,p_n})\,|\overline{M}(\tilde{P},\dots,p_n)\rangle$$

- At two-loops

$$\begin{aligned}|M^{(2)}(p_1,\dots,p_n)\rangle \simeq\ & \boldsymbol{Sp}^{(0)}(p_1,\dots,p_m;\tilde{P})\,|\overline{M}^{(2)}(\tilde{P},\dots,p_n)\rangle \\ & + \boldsymbol{Sp}^{(1)}(p_1,\dots,p_m;\tilde{P};\boldsymbol{p_{m+1},\dots,p_n})\,|\overline{M}^{(1)}(\tilde{P},\dots,p_n)\rangle \\ & + \boldsymbol{Sp}^{(2)}(p_1,\dots,p_m;\tilde{P};\boldsymbol{p_{m+1},\dots,p_n})\,|\overline{M}^{(0)}(\tilde{P},\dots,p_n)\rangle\end{aligned}$$

- Time-like:

$$\boldsymbol{Sp}(p_1,\dots,p_m;\tilde{P};\boldsymbol{p_{m+1},\dots,p_n}) = \boldsymbol{Sp}(p_1,\dots,p_m;\tilde{P})$$

- Matrix elements invariant under renormalization (scattering amplitudes with external partons on-shell and physical polarizations): $\boldsymbol{Sp}$ also renorm. invariant $\boldsymbol{Sp} \to \boldsymbol{Sp}^{(R)}$

One-loop two-parton splitting matrix

- To all orders in ϵ (unrenormalized)

$$
\begin{aligned}
&\boldsymbol{Sp}^{(1)}(p_1,p_2;\tilde{P};\boldsymbol{p_3,\dots,p_n}) \\
&\quad = \boldsymbol{Sp}_H^{(1)}(p_1,p_2;\tilde{P}) + \boldsymbol{I}_C(p_1,p_2;\tilde{P};\boldsymbol{p_3,\dots,p_n})\,\boldsymbol{Sp}^{(0)}(p_1,p_2;\tilde{P})
\end{aligned}
$$

where

$$
\begin{aligned}
&\boldsymbol{I}_C(p_1,p_2;\tilde{P};\boldsymbol{p_3,\dots,p_n}) \\
&\quad = g_S^2\, c_\Gamma \left(\frac{-s_{12}-i0}{\mu^2}\right)^{-\epsilon} \left\{\frac{1}{\epsilon^2}(C_{12}-C_1-C_2)\right. \\
&\quad \left. + \frac{1}{\epsilon}(\gamma_{12}-\gamma_1-\gamma_2+b_0) + \frac{2}{\epsilon}\sum_{\substack{i=1,2\\ j=3,\dots,n}} \boldsymbol{T}_i\cdot\boldsymbol{T}_j\; f(\epsilon;z_i - \boldsymbol{i0\,s_{ij}})\right\}
\end{aligned}
$$

- Casimir coefficients:
  $C_q = C_{\bar{q}} = C_F$ and $C_g = C_A$
- $\gamma_q = \gamma_{\bar{q}} = \frac{3}{2} C_F$ and $\gamma_g = b_0$

IR structure of the splitting matrix

- The IR structure of scattering amplitudes at one- [Catani, Seymour] and two-loops is known [Catani]

$$|M^{(1,R)}\rangle = \boldsymbol{I}_M^{(1)}(\epsilon)\,|M^{(0,R)}\rangle + |M^{(1)\mathrm{fin}}\rangle$$

$$|M^{(2,R)}\rangle = \boldsymbol{I}_M^{(2)}(\epsilon)\,|M^{(0,R)}\rangle + \boldsymbol{I}_M^{(1)}(\epsilon)\,|M^{(1,R)}\rangle + |M^{(2)\mathrm{fin}}\rangle$$

- Can be applied to M and to the reduced matrix element $\overline{M}$

$$\boldsymbol{Sp}^{(1,R)} \simeq \boldsymbol{I}_M^{(1)}(\epsilon)\boldsymbol{Sp}^{(0,R)} - \boldsymbol{Sp}^{(0,R)}\boldsymbol{I}_{\overline{M}}^{(1)}(\epsilon) + \mathcal{O}(\epsilon^0) \simeq \boldsymbol{I}_{mC}^{(1)}(\epsilon)\,\boldsymbol{Sp}^{(0,R)}$$

$$\boldsymbol{Sp}^{(2,R)} \simeq \boldsymbol{I}_{mC}^{(2)}(\epsilon)\,\boldsymbol{Sp}^{(0,R)} + \boldsymbol{I}_{mC}^{(1)}(\epsilon)\,\boldsymbol{Sp}^{(1,R)} + \overline{\boldsymbol{Sp}}^{(2)\mathrm{div}}$$

Where $\overline{\boldsymbol{Sp}}^{(2)\mathrm{div}}$ is $\mathcal{O}(1/\epsilon)$ purely non-abelian and non-vanishing in SL (requires to know $\boldsymbol{Sp}^{(1,R)}$ at $\mathcal{O}(\epsilon^0)$ in SL, known in the two-parton collinear limit)

Factorization violation: two coll partons one-loop

The one-loop splitting matrix depends on the momenta and quantum numbers of the **non-collinear partons (no longer universal)**

- If $z_2 < 0$ $(z_1 > 0)$, the factorization violation is proportional to $\delta(p_1, p_2; p_3, \ldots, p_n) = \frac{2}{\epsilon} \sum_{j=3}^{n} \boldsymbol{T}_2 \cdot \boldsymbol{T}_j \, f(\epsilon; z_2 - i0\, s_{2j})$ $f\left(\epsilon, \frac{1}{x}\right) = \frac{1}{\epsilon}\left[{}_2F_1(1, -\epsilon; 1-\epsilon; 1-x) - 1\right] \simeq \ln x - \epsilon\, Li_2(1-x) + \ldots$
- Main physical effect: ***colour correlations*** between collinear and non-collinear partons

Time-like $(z_2 > 0)$

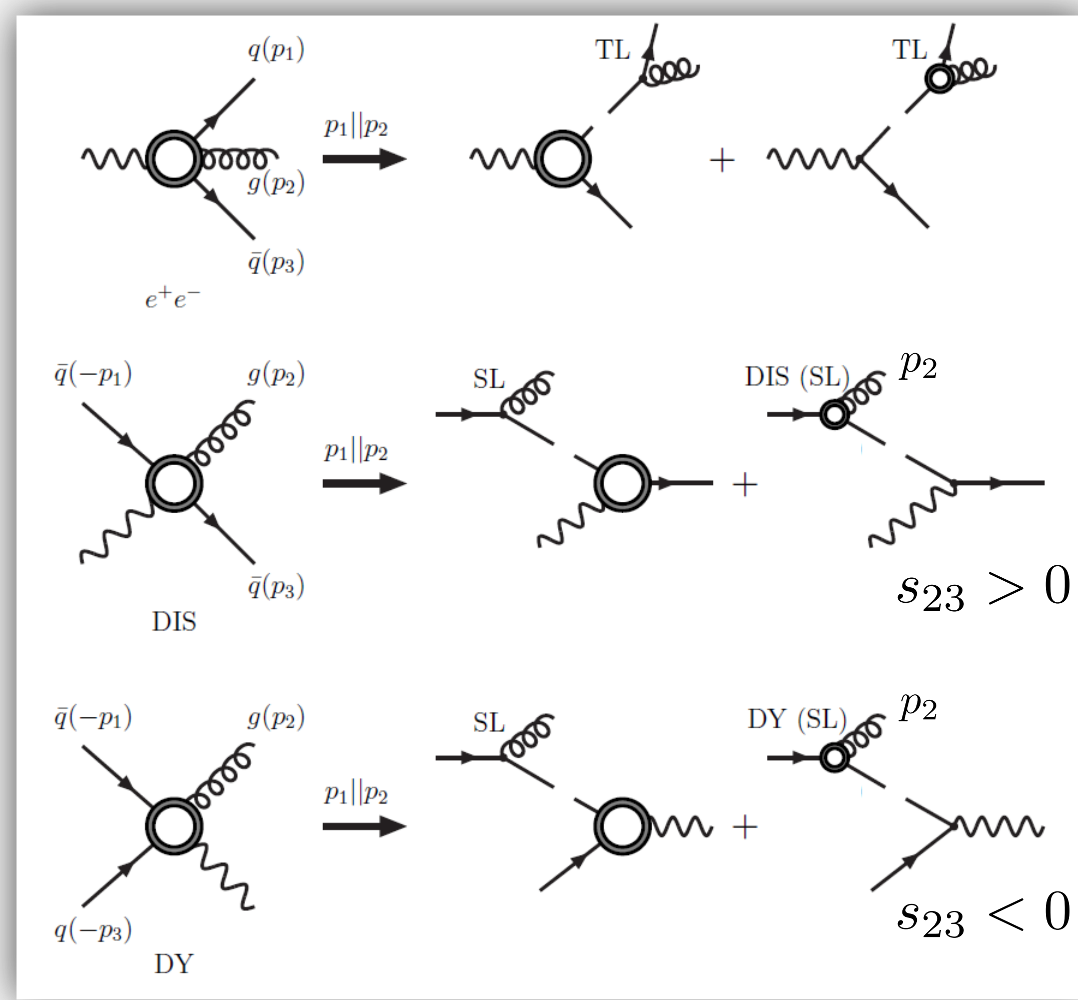
- $\delta\,(p_1, \ldots) = \frac{2}{\epsilon} f(\epsilon; z_2)\, \boldsymbol{T}_2 \cdot \sum_{j=3}^{n} \boldsymbol{T}_j = -\frac{2}{\epsilon} f(\epsilon; z_2)\, \boldsymbol{T}_2 \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2)$
- Colour coherence (colour conservation): each interaction is separately not factorized, but the global effect is a single interaction with the parent parton
- Color charge: $T_{ij} = t^a{}_{ij}$ $(-t^a{}_{ji})$ for a quark (antiquark), $T_{bc} = i f_{abc}$ for a gluon

n=3 QCD partons

e+e-

DIS

Drell-Yan



time-like: $z_2 = 1 - z > 0$

$$\delta^{(e+e-)}(p_1, p_2; p_3) = -\frac{1}{\epsilon} C_A\, f(\epsilon; z_2)$$

space-like: $z_2 = 1 - \frac{1}{z} < 0$

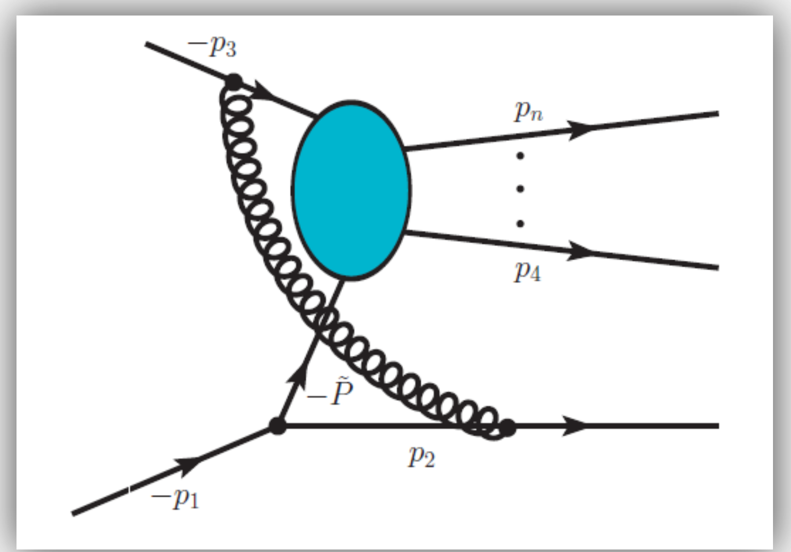
$$\delta^{(DIS)}(p_1, p_2; p_3) = -\frac{1}{\epsilon} C_A\, f(\epsilon; z_2 - i0)$$

$$\delta^{(DY)}(p_1, p_2; p_3) = -\frac{1}{\epsilon} C_A\, f(\epsilon; z_2 + i0)$$

Lepton-hadron DIS: n≥4 QCD partons

- All the non-collinear QCD partons are final-state partons, same-sign subenergies
  $$\boldsymbol{\delta}(p_1, p_2; p_3, \ldots, p_n) = -\frac{1}{\epsilon}\left(C_{12} - C_1 - C_2\right) f(\epsilon; \boldsymbol{z_2} - \boldsymbol{i0}) \qquad s_{2j} > 0$$
- No explicit dependence on the colours and momenta of the non-collinear partons
- "Effectively" factorized

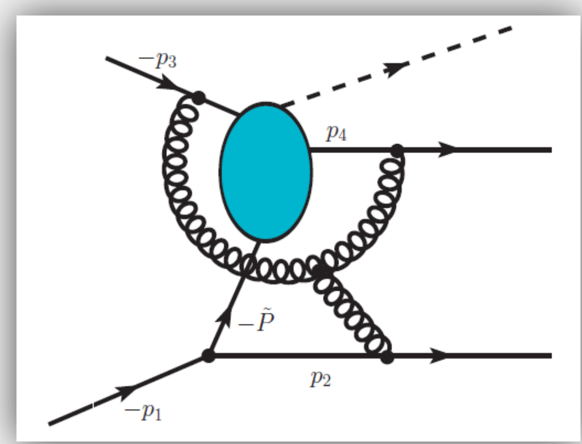
Hadron-hadron: n≥4 QCD partons

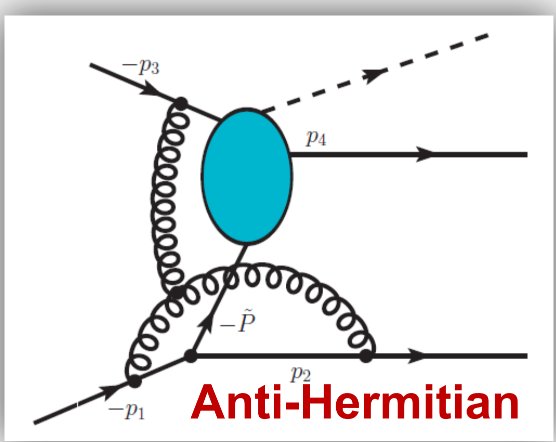


- Colour correlations between the collinear and non-collinear partons remain

$$\delta(p_1, p_2; p_3, \dots, p_n)$$
$$= -\frac{1}{\epsilon}(C_{12} - C_1 - C_2)\, f(\epsilon; \boldsymbol{z_2 - i0})$$
$$+ \frac{i}{\epsilon}\, 4\, \boldsymbol{T_2} \cdot \boldsymbol{T_3}\, f_I(\epsilon; z_2) \qquad s_{23} < 0 < s_{2j}$$

Two-loops (IR structure)





- Non-Abelian operator $\mathcal{O}\,(1/\epsilon^2)$
- Three-parton correlations
- n≥4 QCD partons (not in DIS)

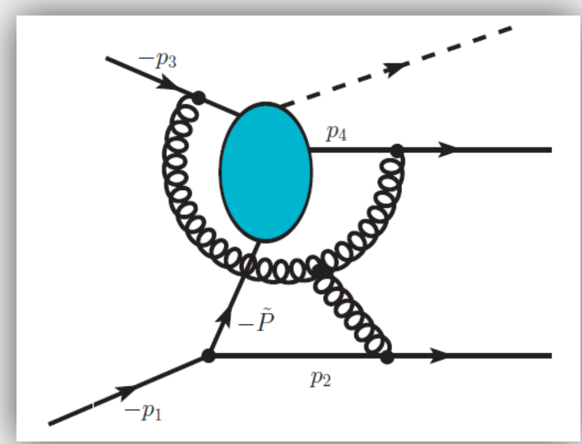
$$\Delta_{mC}^{(2;2)}(\epsilon) = \left(\frac{\alpha_S}{2\pi}\right)^2 \left(-\frac{\pi}{2\epsilon^2}\right) \sum_{i\epsilon C} \sum_{j,k\epsilon NC} f_{abc}\, \boldsymbol{T}_i^a\, \boldsymbol{T}_j^b\, \boldsymbol{T}_k^c$$

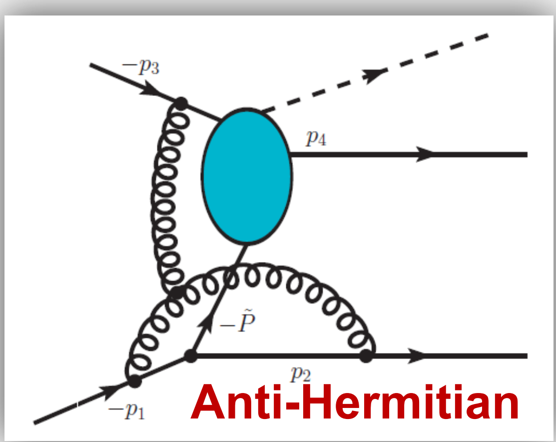
$$\boldsymbol{\theta}(-\boldsymbol{z}_i)\, \boldsymbol{\theta}(-\boldsymbol{s}_{jk})\, \mathrm{sign}(s_{ij})\, \ln\left(-\underline{\frac{s_{j\tilde{P}}\, s_{k\tilde{P}}}{s_{jk}\ \mu^2}} - i0\right)$$

positive

- One-collinear + two non-collinear partons

Two-loops (IR structure)





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$$\boldsymbol{\theta}(-\boldsymbol{z}_i)\, \boldsymbol{\theta}(-\boldsymbol{s}_{jk})\, \mathrm{sign}(s_{ij})\, \ln\left(-\underbrace{\frac{s_{j\tilde{P}}\, s_{k\tilde{P}}}{s_{jk}\ \mu^2}}_{\text{positive}} - i0\right)$$

- One-collinear + two non-collinear partons
- Hermitian part depends also on the size of the non-collinear momenta
- Anti-Hermitian part can be rewritten in terms of two-collinear + one non-collinear parton correlations

Summarizing: amplitude level

Factorization Breaking (FB)

- At **one-loop** factorization breaking is purely **anti-Hermitian** and depends only on the sign of the energy of non-collinear partons (initial vs final state)
- At **two-loops** both **Hermitian and anti-Hermitian** contributions, and the Hermitian part depends also on the size of the momenta of the non-collinear

Squared amplitudes and cross-sections

- Squared matrix element summed over colours and spins of external partons

$$|M|^2 \simeq \langle \overline{M} | \mathbf{P} | \overline{M} \rangle \qquad \mathbf{P} \equiv [\boldsymbol{Sp}]^\dagger \boldsymbol{Sp}$$

The matrix $\mathbf{P}$ is the square of the all-order splitting matrix

$$\mathbf{P}^{(0,R)} = \left(\boldsymbol{Sp}^{(0,R)}\right)^\dagger \boldsymbol{Sp}^{(0,R)}$$
$$\mathbf{P}^{(1,R)} = \left(\boldsymbol{Sp}^{(0,R)}\right)^\dagger \boldsymbol{Sp}^{(1,R)} + \mathrm{h.c.}$$
$$\mathbf{P}^{(2,R)} = \left(\boldsymbol{Sp}^{(1,R)}\right)^\dagger \boldsymbol{Sp}^{(1,R)} + \left[\left(\boldsymbol{Sp}^{(0,R)}\right)^\dagger \boldsymbol{Sp}^{(2,R)} + \mathrm{h.c.}\right]$$

Generalized Altarelli-Parisi kernels

e.g. tripple collinear at one-loop [Sborlini et al.]

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Partially cancel FB

Generalized Altarelli-Parisi kernels

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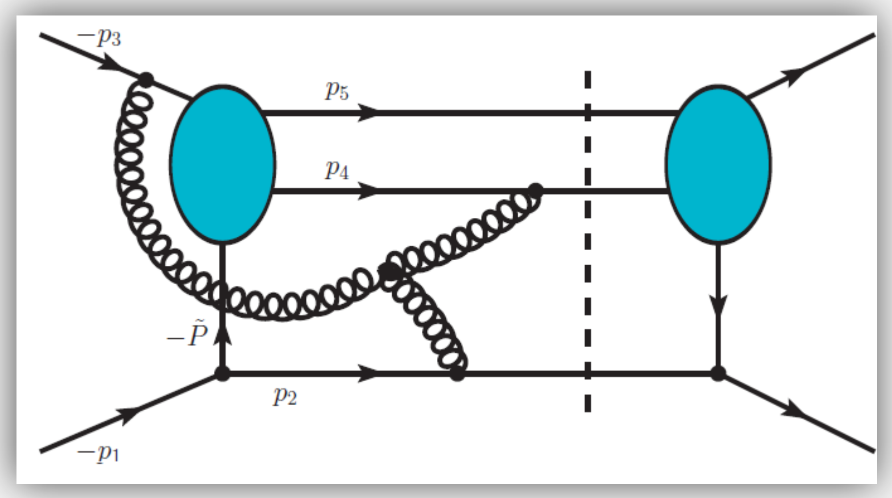
Two-parton collinear limit of squared amp.

- $\mathbf{P}^{(1,R)}$ **is strictly factorized** in spite the fact that $Sp^{(1,R)}$ violates collinear factorization ▶ NNLO ✓
- Analytic continuation from TL to SL [Bern, Dixon, Kosower] works
- Three parton correlations survive for $n \geq 5$ in $\mathbf{P}^{(2,R)}$ ▶ N^{3}LO ?



Physical effect at N3LO

- The expectation value over the reduced matrix element

$$|M|^2 \simeq \langle \bar{M} | \mathbf{P} | \bar{M} \rangle$$

can cancel non-factorized contributions [Seymour, Forshaw, Siodmok]

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can cancel non-factorized contributions [Seymour, Forshaw, Siodmok]

  - True in pure QCD: $\overline{M}^{(0)} = (\overline{M}^{(0)})^*$
  - **N^{4}LO needed:** super-leading logs (non-global) in hadroproduction of a pair of jets with a rapidity gap [Forshaw, Kyrieleis, Seymour 06]
  - Still posible at N^{3}LO if $\overline{M}^{(0)}$ is EW (CPV and/or finite W/Z width) or includes QED corrections at one-loop

Conclusions

- Beyond tree-level strict (process-independent) collinear factorization is violated in the SL collinear region
- IR structure calculable (two-parton collinear limit all orders in ϵ)
- DIS: all the non-collinear partons are final state, "effectively" factorized
- Hadron-hadron: non-Abelian factorization breaking terms in the two-loop SL splitting matrix
- Partly cancel in squared amplitudes

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⇒

- Can lead to logarithmic enhanced contributions at higher orders
- Challenges the validity of mass-singularity factorization in jet and hadron production from N^3LO