Mainz Institute for Theoretical Physics

John Collins "Glaubers in QCD factorization"
(12 Aug 2019 at MITP)

Say "Glauber region refers to a particular region of momentum on line(s) of Feynman graphs. It is a region that gives particular difficulty in obtaining factorization for processes with a large scale."

"Glauber" in title means "line in Feynman graph with momentum in Glauber region."

Overall questions & motivations

1. What is Glauber region & why?
2. Why is it important? Where?
3. Consequences.

(All in context of getting tractable, predictive approximations for interesting processes)

Technical summary of talk

1. Processes with large scale ('Q').
2. Determination of important regions of momentum (Libby-Sterman etc)
3. Standard power counting & its critical role in arguments leading to factorization
4. Glauber: beyond standard power scaling of momenta
   - existence v. non-existence
   - cancelation v. non-cancellation
   - implications

My aim here is to give an overview of the logic leading to the topic of Glaubers with an accurate account of key points & pitfalls.

(See my QCD book for details of my way of organizing things & viewing them.)

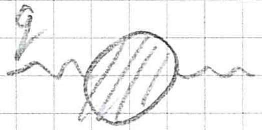
1. Processes with a large scale

E.g. $e^+e^- \to$ hadrons, total xsect
DIS
Drell Yan
jets at high p_T
...

Reminder: Predictiveness of std factorization is because:
(a) Hard scattering estimable from first principles in expansion in small $\alpha_s(Q)$
(b) Similarly DGLAP kernel.
(c) pdfs universal (modulo evolution): infer from subset of experiments, & use broadly.

#2 Determination of important regions

[N.B. 'region' has 2 meanings:
(a) region in 4-momentum space
(b) region in space of loop momenta of graph with lines having momentum in particular regions in first sense]

a. Basic case:  Green function of 2 currents at high q

(Discontinuity + coupling to leptons gives total cross section of $e^+e^- \longrightarrow$ hadrons.)

Can show: dominated by internal momenta of virtuality Q^2 or larger. (Renormalization cancels UV divergences.)

$$\Pi(Q, \mu, \underline{m}, \alpha_s(\mu)) = \Pi(Q, Q, \underline{m}, \alpha_s(Q))$$
$$\longrightarrow \Pi(Q, Q, 0, \alpha_s(Q))$$

Renormalization at scale $Q \Rightarrow$ internal scales Q (to leading power)

$\Rightarrow$ Can neglect masses

& use perturbation theory (PT) at low order in $\alpha_s(Q)$ to get useful predictions.

b. In general case, we need the location of regions with lines with $|k^2 - m^2| \ll Q^2$

Key point #1

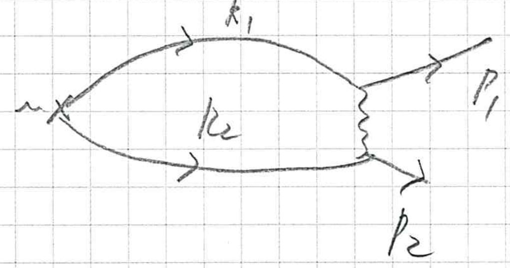
Libby & Sterman: these are neighborhoods where internal momenta are trapped (pinched) at on-shell values in the massless theory.

Non-pinch case: can deform contour away from on shell values & lines are effectively far off shell.

Use Landau method: locations of trapping of contour correspond to classical scattering.

Example

NB. in

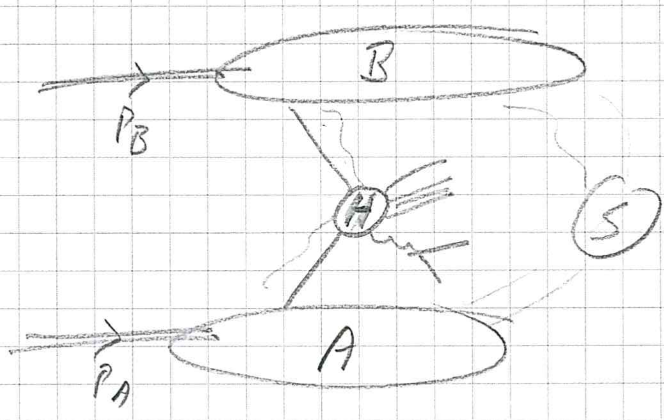


with p_1 & p_2 in $\pm z$ direction, there's a region where k_1 & k_2 are on shell but in $\pm x$ direction. There's a 90° scattering. But there's no classical process doing that. We can deform integration to where k_1 & k_2 are highly virtual. The inner loop is short distance.

Libby & Sterman part 2: power counting for momenta to determine regions of leading power

Key point #2

E.g. Drell-Yan



Collinear momenta – A & B
Soft momenta S ($|k| \ll Q$)
Hard momenta H.

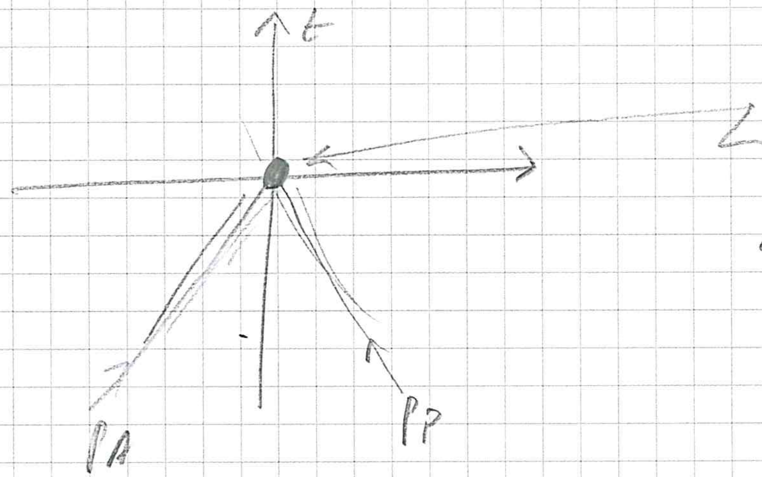
N.B. disconnected hard part with extra jets or... is possible only with nonleading power

From this leading region structure, one can derive standard collinear factorization
(Further ingredients: appropriate approximation, Ward identities, unitary/causality)
It involves
- std power counting
- sum over regions with subtraction
- cancellation of soft, ...
- obtaining of operator defns. of pdfs.

Corresponding space-time view

(natural, but not yet with codified derivation (!))

Key pt #2.1



N.B 1. Time dilated, Lorentz-contracted beam particles (pace Gribov, Strikman et al about contraction).

2. There are 2 transverse dimensions $\perp$ paper & proton size transversely ~ 1 fm.

#3 **Standard power counting** for sizes of momentum components.

Use LF coords. $(+,-,T)$.

Key point #3

Write:		
	collinear to A	$Q(1, \lambda^2, \lambda)$
	collinear to B	$Q(\lambda^2, 1, \lambda)$
	Soft (S)	$Q(\lambda,\lambda,\lambda)$ or $Q(\lambda^2,\lambda^2,\lambda^2)$

Possibible meanings of λ

(a) $\lambda = m/Q$.

(b) λ is "quasi-radial" integration variable.

View (a) is natural. Explain why it's not suitable (as in my QCD book): We need to integrate over all momenta & many other scaling give leading contributions

∴ Go to view (b) & write (for ex.) a collinear-A momentum as

$$k = Q\left(x,\ \lambda \tilde{k}^-,\ \lambda \tilde{k}_T\right)$$

Here x parameterizes position on pinch surface

λ ——————— " ——— "distance" from pinch surface

$\tilde{k}^-$ & $\tilde{k}_T$ are quasi-angular coordinates with normalization condition $|\tilde{k}^-| + |\tilde{k}_T| = 1$ (or similar).

[After talk comment: purist version has single λ collectively for array of loop momenta & a single condition on pseudo-angular coordinates]

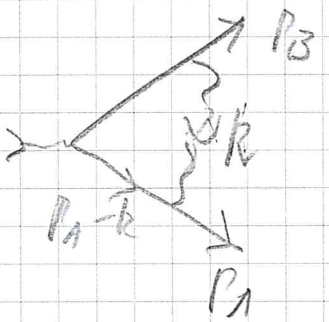
For soft momentum we write

$$k = \lambda_S Q(\tilde{k}^+, \tilde{k}^-, \tilde{k}_T)$$

with say $|\tilde{k}^+| + |\tilde{k}^-| + |\tilde{k}_T| = 1$ [Issue of $\lambda_s = \lambda \vee \lambda^2$ needs separate discussion]

Complication in approximating soft gluon coupling to collinear line:

Eg.



[perhaps inside a bigger graph for a reaction in e^+e^- annihilation]

Line $p_A - k$ has denominator

$$\frac{1}{(p_A-k)^2 - m^2 + i\epsilon} = \frac{1}{p_A^2 - m^2 - 2p_A\cdot k + k^2 + i\epsilon}$$

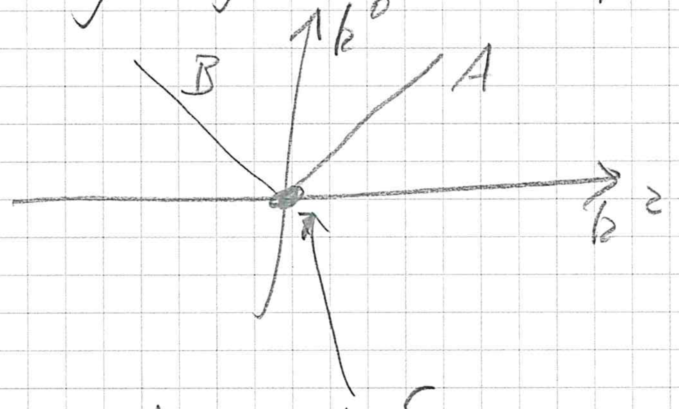
If all $\tilde{k}^\mu$ are comparable, then to leading power we get

$$\frac{1}{p_A^2 - m^2 - 2p_A^+ k^- + i\epsilon}$$

This linearization plus a standard argument with Ward identities & a sum over graphs gives a term in a soft factor constructed from a vacuum matrix element of Wilson lines

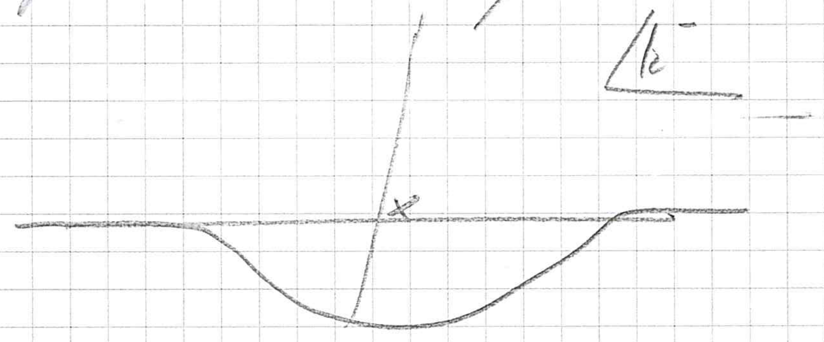
But $-2p_A\cdot k + k^2 = -2(p_A^+\tilde{k}^- + \dots)\lambda_S Q + \lambda_S^2 Q^2(2\tilde{k}^+\tilde{k}^- - \tilde{k}_T^2)$

So in integrating round the soft point in



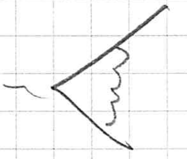
there is a place where $2p_A^+k^-$ is much smaller than λQ^2 & the soft approximation is bad.

But in this graph we can deform k^- to a larger value



Similarly for k^+.

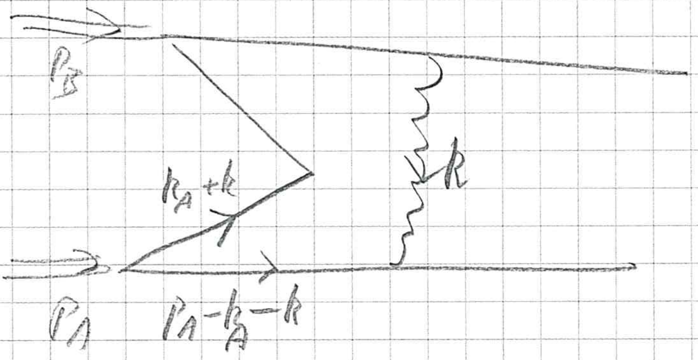
Hence normal soft scaling can be used, for this graph i.e.



#4 Glauber

Key point: classic simple case of failure of std power counting

But in



k^+ & k^- are trapped at small values to gives effective scaling $k \sim (\lambda^2, \lambda^2, \lambda)$.

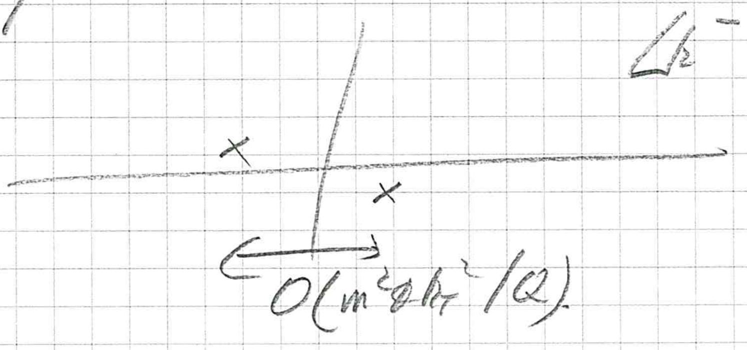
Eg. on A side, good approx. is

$$\int dk^- \frac{1}{(k_A+k)^2 - m^2 + i\epsilon} \; \frac{1}{(p_A-k_A-k)^2 - m^2 + i\epsilon}$$

$$= \int dk^- \frac{1}{2k_A^+k^- + 2k_A^+k_A^- - (k_{AT}+k_T)^2 - m^2 + i\epsilon \ldots} \; \frac{1}{-2k^-(p_A^+-k^+) - \ldots + i\epsilon}$$

(NB for pinch of collinear lines $0 \le k_A^+ \le p_A^+$)

Then k^- singularities are



NB $\frac{1}{k^2+i\epsilon} \simeq \frac{1}{-k_T^2}$, so it gives no relevant pole.

"Involves both "initial-state" & "final-state" lines"

Key point

General case

Qu When does the standard power counting fail?
(see my QCD book for some details)

Ans (a) When integration gets close to another region.
This is taken care of by a proper subtraction procedure in summing graph over regions

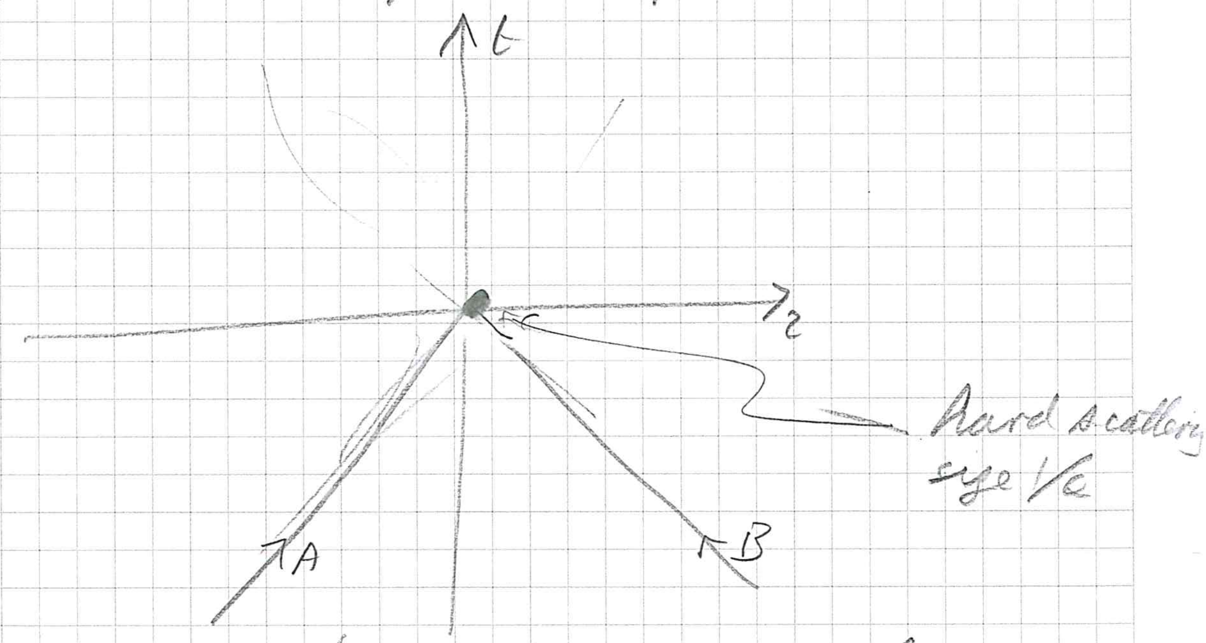
(b) In Glauber-like regions with a trap.

Generalized Libby-Sterman argument can be used to determine the possibilities
— see Sec. 5.11 of my QCD book for a brief sketch of what I mean. It's too brief to satisfy experts.

Cancellation v. non-cancellation of Glauber

In *fully* inclusive DY, a sum of cuts can be applied to cut graphs for cross-section. It can be shown (CSS) that Glauber becomes untrapped after sum over cuts. (Basic argument, pre-QCD!, was given by deTar, S.D.Ellis & Landshoff in 1974.)

An intuitive hint as to the underlying reason is by saying that in a space-time picture:



anything outside the past light-cone of hard scattering does not affect the inclusive x-section.

Don't forget:

2 transverse dimension with proton spectators part spread over 1 fm

Lorentz contraction of valence part of hadron

Implementing this correctly in momentum space (by CSS) used 2 versions of opposite light-front perturbation theory.

Key point: Std. factorization needs to be extended whenever Glauber cancelation fails (as in diffractive Drell-Yan).

Warning: For some interesting cases — e.g. jet production in hadron-hadron collisions — there fail to exist adequate proofs of Glauber cancellation, even though the cancelation is expected

A few comments

(a) A more systematic formulation in coordinate space would be useful

(b) Further investigation of the interface w/ nonperturbative QCD is needed
N.B. proofs of factorization as formulated in Feynman-graph language often rely on big rapidity differences — eg. between a soft & a collinear momentum.