Glauber Operators in SCET

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Factor workshop, Aug, 2019

based on 1601.04695
with Ira Rothstein

Goals:

- EFT description of IR. $\sum_i$ EFT fields$_i$ $\Leftrightarrow$ QCD IR (Mom. regions via Landau Eqtns, definition of how fields avoid overlaps (subtractions))

  Manifest p.c at start, $\lambda \ll 1$ homogeneous in λ

- Operator description of G, symmetry & universality

Appl.
- Hard & Fwd Scattering in same framework, $\overline{MS}$ style RGEs for BFKL
- Calculate / Parameterize ~~FAct~~
- new approach to demonstrate Fact
- study Glauber effects at subleading power
- calculate $i\pi$ terms in amplitudes (easy)

Outline:
- Intro
- $\mathcal{L}_G^{(0)}$ construction & properties
- Glauber Loops
- Application to Regge & BFKL logs
- Factor & ~~Factor~~

SCET vs. CSS

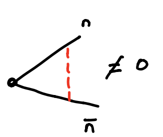
- CSS keeps S & G together, expand later (important for parts of Fact Pf, convergent contour integrals due to presence of $k^+k^- - \vec{k}_\perp^2$)

  deform contour out of G region

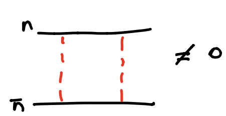
- SCET expands at start, needs additional regulator

  calculate G terms explicitly, Soft graphs have Glauber subtractions

  later decide if they can be absorbed in S (then ignoring G-subtractions)



$\neq 0$



$\neq 0$

non-zero Glauber contrib. in SCET

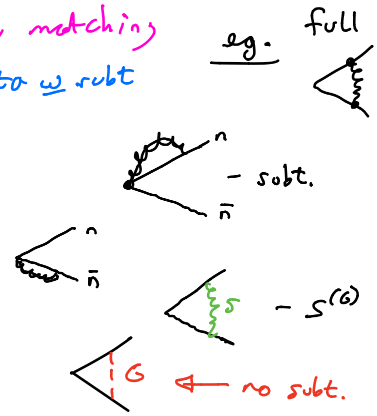
whereas both zero in CSS via contour deformation

SCET$_{II}$ $\quad k_s \sim \lambda$ $\qquad$ (λ vs. λ^2 determined by observables one studies)

hard $\quad p_H^2 \sim Q^2$ offshell $\quad$ int. out. $\qquad$ C from matching

approx to formulate Feyn. Rules, int. over all momenta w/ subt

eg.



scaling

		$(p^+, p^-, p_\perp)$
Coleman-Norton-Landau	n:	$Q(\lambda^2, 1, \lambda)$
	$\bar{n}$:	$Q(1, \lambda^2, \lambda)$
	s:	$Q(\lambda, \lambda, \lambda)$
offshell	G:	$Q(\lambda^a, \lambda^b, \lambda)$

$\xi_n \sim \lambda \,,\; A_n \sim (\lambda^2, 1, \lambda)$

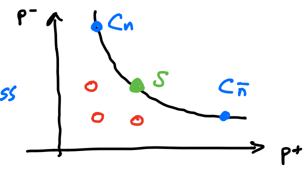
$\psi_s \sim \lambda^{3/2} ,\; A_s \sim \lambda$

$\lambda^{a+b} \ll \lambda^2 \sim p_G^2$

$$\mathcal{L}^{(0)}_{SCET_{II}} = \sum_n \mathcal{L}_n^{(0)} + \mathcal{L}_s^{(0)} + \mathcal{L}_{Hard}^{(0)} + \mathcal{L}_G^{(0)}$$

eg. $C\, (\bar{\xi}_n W_n)\, S_n^\dagger S_{\bar{n}}\, (W_{\bar{n}}^\dagger \xi_{\bar{n}})$

Note: o some $G(\lambda^2)$ offshellness

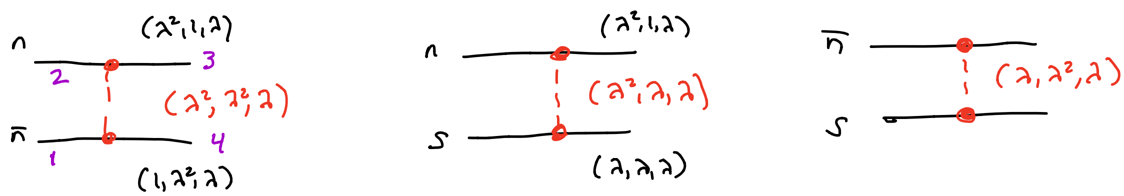


- Standard Factorization violated unless $\cancel{\mathcal{L}_G^{(0)}}$

- Glauber region has $\qquad p^+ p^- \ll \vec{p}_\perp^{\,2} \sim \lambda^2$

  $\frac{1}{\vec{p}_\perp^{\,2}}$ potentials, instantaneous in z & t

eg.



int. out when constructing $\rightarrow \mathcal{L}_G^{(0)}$

G. leaves collinear particles collinear (not Wilson line like s)
(What J.C. referred in his talk as violation of naive soft p.c.)
Depend on 3 components of G momenta, $\perp$ & + or −

- mediates Forward scattering $\quad s \gg -t$

  Forward: $\bar{n}\cdot p_2 = \bar{n}\cdot p_3 \,,\quad n\cdot p_1 = n\cdot p_4$

Match from QCD, integrating Hard & Glauber out:

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{j_n B} + \sum_{n,n'} \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{BC} \frac{1}{P_\perp^2} O_{n'}^{jC}$$

(2-rapidities) (3-rapidities)

↑ more sectors by T-products ↗

discuss highlighted things

$\boxed{\text{Sideboard}}$

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{j_n B} + \sum_{n,n'} \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{BC} \frac{1}{P_\perp^2} O_{n'}^{jC}$$

(2 rapidities) (3 rapidities)

$$O_n^{qB} = \bar{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n , \qquad O_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2}\cdot(i\partial_n - i\overleftarrow{\partial}_n) \mathcal{B}_{n\perp}^{D\mu}$$

similar $O_{\bar{n}}$'s

$$O_s^{q_n B} = 8\pi\alpha_s \, \bar{\psi}_s^n T^B \frac{n\!\!\!/}{2} \psi_s^n , \qquad O_s^{g_n B} = 8\pi\alpha_s \frac{i}{2} f^{BCD} \mathcal{B}_{s\perp\mu}^{nC} \frac{n}{2}\cdot(i\partial_s - i\overleftarrow{\partial}_s) \mathcal{B}_{s\perp}^{nD\mu}$$

$$O_s^{BC} = 8\pi\alpha_s \Big\{ P_\perp^\mu S_n^T S_{\bar{n}} P_{\perp\mu} - P_{\perp\mu} g \widetilde{\mathcal{B}}_{s\perp}^{n\mu} S_n^T S_{\bar{n}} - S_n^T S_{\bar{n}} g \widetilde{\mathcal{B}}_{s\perp}^{\bar{n}\mu} P_{\perp\mu} - g \widetilde{\mathcal{B}}_{s\perp}^{n\mu} S_n^T S_{\bar{n}} g \widetilde{\mathcal{B}}_{s\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} S_n^T i g \widetilde{G}_s^{\mu\nu} S_{\bar{n}} \Big\}^{BC}$$

where

$$\chi_n = W_n^\dagger \xi_n \qquad W_n = W_n[\bar{n}\cdot A_n] , \qquad g\mathcal{B}_{n\perp}^\mu = [W_n^\dagger i D_{n\perp}^\mu W_n]$$

$$\psi_s^n = S_n^\dagger \psi_s \qquad S_n = S_n[n\cdot A_s] , \qquad g\mathcal{B}_{s\perp}^{n\mu} = [S_n^\dagger i D_{s\perp}^\mu S_n]$$

$$\widetilde{\mathcal{B}}_{s\perp}^{n\,AB} = -i f^{ABC} \mathcal{B}_{s\perp}^{nC}$$

$$\widetilde{G}_s^{\mu\nu\,AB} = -i f^{ABC} G_s^{\mu\nu C}$$

S_n fundamental Wilson line $\qquad \mathcal{S}_n$ adjoint Wilson line

Present but Suppressed above: rapidity regulator $|k_z|^{-\eta}$, multipole expansion

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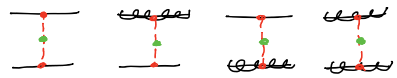
Comments * – SCET p.c. theorem used in construction

– universal for $i, j = q, g$ 

– no hard coefficient or loop corrections to $\mathcal{L}_G^{(0)}$

– only pairs of collinear directions in $\mathcal{L}_G^{(0)}$, rest are T–products

– breaks factorization: $\mathcal{L}_G^{(0)}(\{\xi_{n_i}, A_{n_i}\}, q_s, A_s)$ gives coupling at $\mathcal{O}(\lambda^0)$ btwn n_i, n_j & s

– encodes known examples of fact. violation:

collinear Wilson line universality (back-to-back hadrons John C. et al)

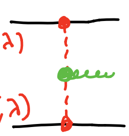
collinear amplitudes in space-like limits (German Rodrigo et al.)

regge amplitude corrections ($\frac{\quad}{iii}$ Leonardo Vernazza et al.)

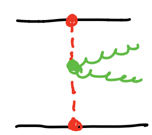
Unrelated: measurement factoring, div. convolutions (defining ren. m.elts.)

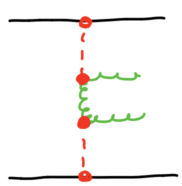
– only $\mathcal{L}_G^{(0)}$ dangerous for FACT, not subleading $\mathcal{L}_G^{(n\geq 1)}$ alone (more from Vita next week)

– O_s^{BC} pure glue ●, ●~~~ One-gluon Feyn. Rule is

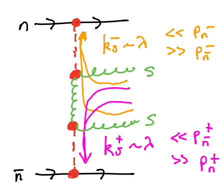
$(\lambda^2,\lambda,\lambda)$  Lipatov Vertex → gluon reggeization, BFKL

$(\lambda,\lambda^2,\lambda)$

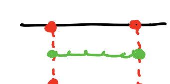
also higher vertices 

soft's from T-product 

(Keep this brief:



*– Glauber Loops give $i\pi$'s

– Note: Distinction btwn fields/propagators & loop Momenta 
(eg. definite propagators in regions but 2 different routings $G_{n\bar n}$ & S, G_{ns} & $G_{\bar n s}$)

*– Glauber is connected to Soft & Collinear Wilson Line directions

$$\frac{1}{n\cdot k \pm i0} \to \Delta\text{sign} \sim i\pi\,\delta(n\cdot k) \to \text{Glauber region}$$

$\mathcal{O}(\lambda)$ or $\mathcal{O}(1)$ $\qquad$ $\mathcal{O}(\lambda^2)$ or $\mathcal{O}(\lambda)$ $\qquad$ (not soft or collinear)

Power Counting (use sideboard!)

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=q,g} \underset{\lambda^2}{O_n^{iB}} \underset{\lambda^{-2}}{\frac{1}{P_\perp^2}} \underset{\lambda^3}{O_s^{j_n B}} = \lambda^3 \;+\; \sum_{n,n'} \sum_{i,j=q,g} \underset{\lambda^2}{O_n^{iB}} \underset{\lambda^{-2}}{\frac{1}{P_\perp^2}} \underset{\lambda^2}{O_s^{BC}} \underset{\lambda^{-2}}{\frac{1}{P_\perp^2}} \underset{\lambda^2}{O_{n'}^{jC}} = \lambda^2$$

$$O_n^{qB} = \overset{\lambda}{\bar\chi_n} T^B \frac{\bar{n}\!\!\!/}{2} \overset{\lambda}{\chi_n}\,, \qquad O_n^{gB} = \frac{i}{2} f^{BCD} \overset{\lambda}{\mathcal{B}^C_{n\perp\mu}} \overset{\lambda^0}{\frac{\bar n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)} \overset{\lambda}{\mathcal{B}^{D\mu}_{n\perp}}$$

similar for $O_{\bar n}$'s

$$O_s^{q_n B} = 8\pi\alpha_s \underset{\lambda^{3/2}}{\bar\psi_s^n} T^B \frac{n\!\!\!/}{2} \underset{\lambda^{3/2}}{\psi_s^n} = \lambda^3\,, \qquad O_s^{g_n B} = 8\pi\alpha_s \frac{i}{2} f^{BCD} \underset{\lambda}{\mathcal{B}^{nC}_{s\perp\mu}} \underset{\lambda}{\frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)} \underset{\lambda}{\mathcal{B}^{nD\mu}_{s\perp}} = \lambda^3$$

$$O_s^{BC} = 8\pi\alpha_s \Big\{ \underset{\lambda}{P_\perp^\mu} S_n^T S_{\bar n} \underset{\lambda}{\mathcal{P}_{\perp\mu}} = \lambda^2 \;+\; \cdots \Big\}$$

Power counting for any graph (without state scaling)

$\sim \lambda^\delta$ any α_s^k, any power   gauge invariant

$$\delta = \delta^{\text{topo}} + \sum_k \underbrace{(k-2)\, V_k^{n\bar n}}_{\text{3 rapidity}} + \underbrace{(k-3)\left(V_k^{ns} + V_k^{\bar n s}\right)}_{\text{2-rapidity}} + (k-4)\left(V_k^n + V_k^{\bar n} + V_k^s\right) + (k-8)\, V_k^{us}$$

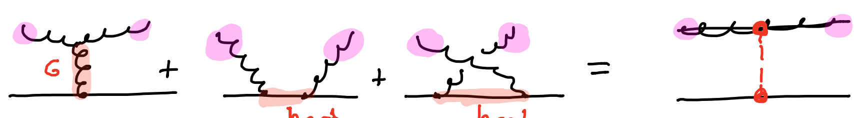
[$\delta^{\text{topo}} = 6 - N^n - N^{\bar n} - N^{ns} - N^{\bar n s} + 2u$ is topological factor having to do with connectedness of various sectors]

(several examples in paper)

N^n = # disconnected n-collinear subgraphs if $\bar n$ & s lines are erased.

Aside (time permitting, (skipped))

$\mathcal{B}^\mu_{n\perp}$ matching



QCD

$$B^\mu_{n\perp} = A^\mu_{n\perp} - \frac{k_\perp^\mu}{\bar n\cdot k}\, \bar n\cdot A_n(k) + \cdots$$

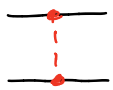
eqtns. of motion

$p\cdot A(p) = 0$ remove $n\cdot A_n$

$p^2 = 0$

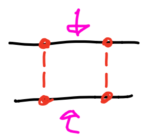
important to get $\bar n\cdot A_n$ terms correctly

Glauber Loops give $i\pi$'s



$$\frac{\nu^{\eta}|2k^z|^{-\eta}}{\vec{k}_\perp^2} \qquad \eta \to 0$$

"rapidity regulator"
no longer instantaneous in z
Note: non-analytic!

- 

effectively eikonal

$$\int \frac{d^dk \;\; |2k^z|^{-2\eta}}{\vec{k}_\perp^2(\vec{k}_\perp-\vec{q}_\perp)^2\left(k^+-\Delta_1(k_\perp)+i0\right)\left(-k^--\Delta_2(k_\perp)+i0\right)}$$

$$= \left(\frac{-i}{4\pi}\right)\int\frac{d^{d-2}k_\perp}{\vec{k}_\perp^2(\vec{k}_\perp-\vec{q}_\perp)^2}\left[-i\pi + \mathcal{O}(\eta)\right]$$

k^0 by contours

Note: $k^\pm$ contours don't converge at ∞

indep of Δ_i
"effectively eikonal"

Fourier Transform: each $|2k^z|^{-\eta} \;\to\; \frac{\eta}{2}|z|^{-1+\eta}\,K_\eta$

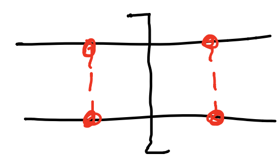
nice physical picture

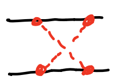
$$\frac{1}{k^z+\Delta+i0} \to (-i)\,\Theta(z)\,e^{iz\Delta}$$

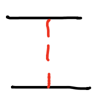
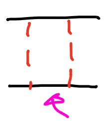
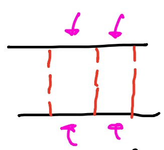
$$(-i)\left(\frac{\eta}{2}\right)^2\int dz_1\,dz_2\;\Theta(z_2-z_1)\;|z_1 z_2|^{-1+\eta}\;e^{i\Delta(z_2-z_1)} = \frac{(-i)}{2}+\dots$$

ordered collapse to $z_i \to 0$!

only encodes cut needed for unitarity



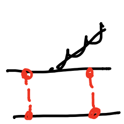
 $= 0$ trivially extends to all crossed Glaubers

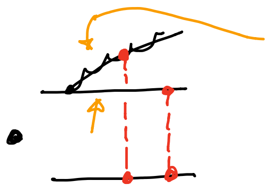
-  +  +  $+ \cdots \;\propto\; \mathrm{FT}\left[\tilde{G}(b_\perp)-1\right]$

$$\tilde{G}(b_\perp) = e^{i\phi(b_\perp)}, \qquad \phi(b_\perp) = -g^2\,T_1^A T_2^A\int\frac{d^{d-2}q_\perp}{\vec{q}_\perp^{\,2}}\,e^{i\vec{q}_\perp\cdot\vec{b}_\perp}$$

eikonal scattering

effectively eikonal for log-div. cases (needed rap. reg.)

-  $= 0$ (interrupted collapse)

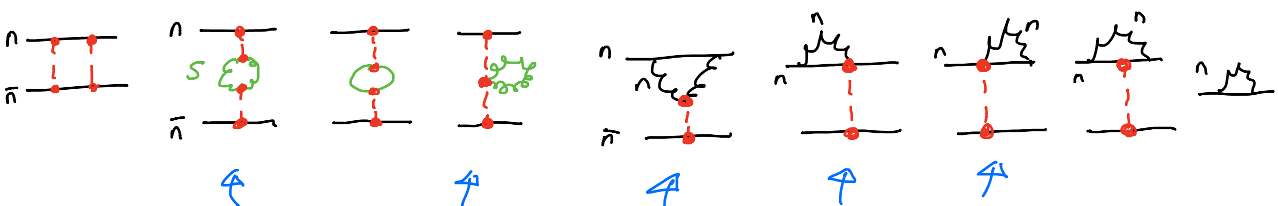
•  not eikonal, k^+ poles on both sides

eff. eikonal with regulator

$\boxed{\text{Resummation}}$

eg. Fwd. One loop Graphs

exactly reproduce leading power QCD amplitude

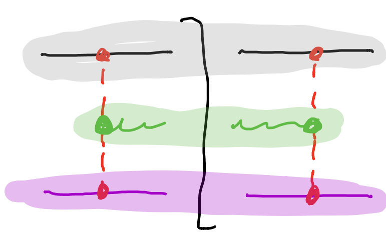


rapidity divergent + $\bar{n}$ analogs

$\ln\left(\frac{\nu^2}{-t}\right)$ $\qquad$ $-\ln\left(\frac{\nu^2}{s}\right)$

$\nu \frac{d}{d\nu}\left(\sum_j O_n^{jA}\right) = \gamma_{n\nu}\left(\sum_j O_n^{jA}\right)$ RGE gives $\left(\frac{s}{-t}\right)^{-\gamma_{n\nu}}$ Regge exponent for gluon

• sum $\ln\frac{s}{t}$, $\ln x$ in DIS, Fwd. Scattering, DY, ...

eg.  + virtual graphs + higher orders in $S, n, \bar{n}$

$$= \int d^2 q_\perp\, d^2 q_\perp' \; \underbrace{C_n(q_\perp, p^-, \nu)}_{n\text{-collinear}} \; \underbrace{S(q_\perp, q_\perp', \nu)}_{\text{soft}} \; \underbrace{C_{\bar n}(q_\perp', p^+, \nu)}_{\bar n\text{-collinear}}$$

$\sum_X |\langle p | O_n | X\rangle|^2$ $\qquad$ $\sum_{X_s} |\langle 0 | O_{s(1,1)} | X_s\rangle|^2$ $\qquad \cdots$

$$\nu \frac{\partial}{\partial \nu} S(q_\perp, q_\perp', \nu) = \int d^2 k_\perp \; \underline{\gamma^{BFKL}(q_\perp, k_\perp)} \; S(k_\perp, q_\perp', \nu)$$

Operator based in SCET $\longrightarrow$ recycle, exploit universality

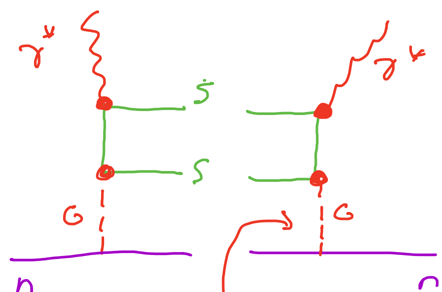
Glauber useful for calculations (not just to prove it cancels)

skip

eg. $\boxed{\text{DIS}}$ with D. Neill, A. Pathak, S. Marzani, I. Rothstein

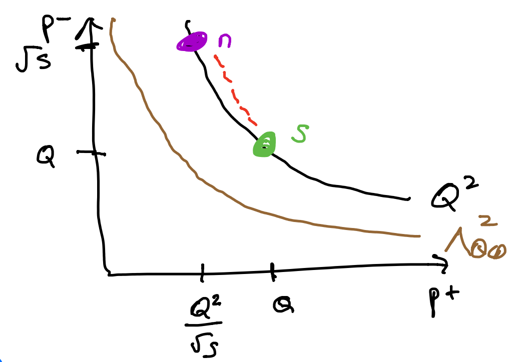
$s \gg Q^2$

$x \ll 1$



perturbative Glauber!

$t \sim Q^2$



gauge inv.

$$\frac{d\sigma}{dQ^2 dx} \propto \int d^2 q_\perp' \; S(q_\perp, q_\perp', x E^-, \nu) \; C_n(q_\perp', E^-, \nu)$$

match onto PDF: $\quad C_n = H_n(\mu) \otimes f_n(\mu)$

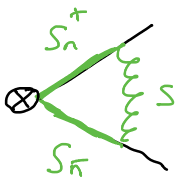
RGE in ν $\quad$ sums $\ln x \;\to\;$ BFKL

RGE in μ $\quad$ sums $\ln \frac{Q}{\Lambda_{QCD}} \;\to\;$ DGLAP

$\boxed{\text{Properties rel. for Fact}}$ • like the Cheshire Cat, (Lewis Carol, Alice in Wonderland)

Why we (usually) don't see Glauber in Hard Matching Calcs.

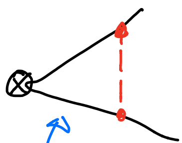
eg.



naive $\tilde{S} = \int \dfrac{d^dk \qquad |k_z|^{-\eta}}{(k^2 - m_{IR}^2 + i0)(n\cdot k + i0)(\bar{n}\cdot k - i0)}$

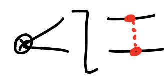
$$= (\cdots) + i\pi\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\right)$$

true $S = \tilde{S} - S^{(G)} = (\cdots)$ only, real



collinear, but effectively eikonal

$G = S^{(G)}$, its G that has cut :



long distance propagation

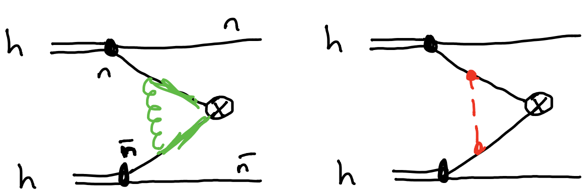
$$S + G = \left(\tilde{S} - S^{(G)}\right) + G = \tilde{S}$$

(A) only G carries info about soft Wilson line directions,
$S = \tilde{S} - S^{(G)}$ <u>does</u> <u>not</u> <u>care</u>

(B) can absorb <u>this</u> <u>Glauber</u> into soft Wilson Lines
if they are taken with proper physical directions,
& just use $\tilde{S}$. Glauber tells us direction, sees long dist. propagation $t \sim \lambda^{-2}$

(done for any SCET matching calculation) (This corresponds to continuation from G to S region in CSS.)

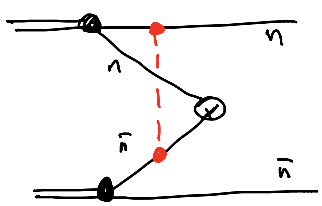
Same in <u>Active - Active</u> Graphs



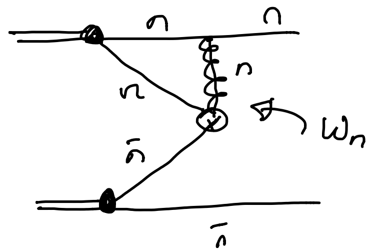
$\Rightarrow$ I define "Active" lines as those that effectively eikonalize due to ~~log~~ div. scaling in regulated integrals

Rest are then "Spectator"

Active-Spectator



G



$C_n = \tilde{C}_n - C_n^{(G)}$

$G = C_n^{(G)}$ here

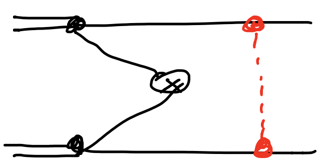
$C_n + G = \tilde{C}_n$

analogous to above but now for collinear

(A) no Wilson line direction dependence in C_n, all in G

(B) can absorb this Glauber into collinear Wilson line, so direction is determined by G graph

Spectator-Spectator



no soft or collinear analogs — at leading power!

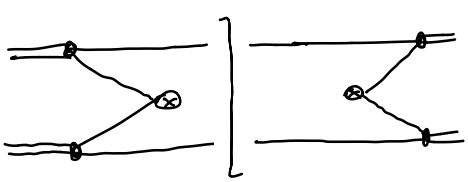
Cancel when we take $|A|^2$

[& integrate over $\Delta p_\perp$ of spectators in general]

Sum of G's here gives phase

Useful to distinguish initial & final state G's (ordered perturbation theory)

Discussed how proof of factorization works in SCET: work in progress with I. Rothstein



- GOPT
- final & initial
- soft complication