Background Independent Quantum Field Theory

and

Gravitating Vacuum Fluctuations

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The most profound difference between Quantum Gravity and QFT on classical (curved) spacetime is the fundamental requirement of Background Independence:

No particular spacetime is singled out a priori; the internal dynamics of the quantum system ("universe") selects the state it likes to be in and determines the emergent geometric properties, if any.

“The new strategy is to free oneself of the background spacetime that seemed indispensable for formulating and addressing physical questions.

The goal is to lift this anchor and learn to sail the open seas.”

(A. Ashtekar)

A notorious contribution to the Cosmological Constant (Problem):

$$\sum_{\text{modes}} \frac{1}{2}\hbar\omega = \Delta\rho \quad , \quad \Delta\Lambda = 8\pi G \cdot \Delta\rho$$

$$\frac{1}{2}\hbar \int_{p<\mathcal{P}} \frac{d^3p}{(2\pi)^3} \, |\vec{p}| \sim \mathcal{P}^4 \quad ,$$

E.g. cutoff at $\mathcal{P} = m_{Pl}$: $\Delta\Lambda = \mathcal{O}(1) \cdot m_{Pl}^2$

Observations:

$$\Lambda \approx 2.8 \cdot 10^{-122} \, m_{Pl}^2 \quad , \quad \rho_\Lambda = (2.2 \text{ meV})^4$$

requires enormous finetuning $\rightsquigarrow$

small value of Λ is "unnatural" ??

A continuum-based approach to quantum gravity:

The gravitational Effective Average Action

(i) Background Independence via background field technique

(ii) fundamental QFT "=" $\lim_{k\to 0}\big(\text{effective QFT}\big)_k$

$$\varphi = \left(h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu} \text{ , ghosts, matter} \right)$$

$$e^{W_k[J;\bar{g}]} = \int \mathcal{D}\varphi \; \exp\Big\{ -S_{tot}[\varphi;\bar{g}] +$$

$$+ \int J\varphi \quad - \frac{1}{2}\int dx \sqrt{\bar{g}(x)} \; \varphi(x) \underbrace{R_k(-\Box_{\bar{g}})}\varphi(x) \quad \Big\}$$

$$= \begin{cases} 0 & \text{if} \quad (-\Box_{\bar{g}}) \gtrsim k^2 \quad \text{"UV modes"} \\ k^2 & \text{if} \quad (-\Box_{\bar{g}}) \lesssim k^2 \quad \text{"IR modes"} \end{cases}$$

$\Gamma_k[h_{\mu\nu}, \cdots ; \bar{g}]$:= (modif.) Legendre transf. of W_k

$\Box_{\bar{g}}$ - Eigenbasis $\Upsilon = \{\chi_n\}$

$$-\Box_{\bar{g}}\, \chi_n[\bar{g}](x) = \mathcal{E}_n[\bar{g}]\, \chi_n[\bar{g}](x)$$

$\mathcal{E}_n[\bar{g}] \geq k^2$: UV modes

$\mathcal{E}_n[\bar{g}] = k^2$: Cutoff mode χ_{COM} (lowest UV mode)

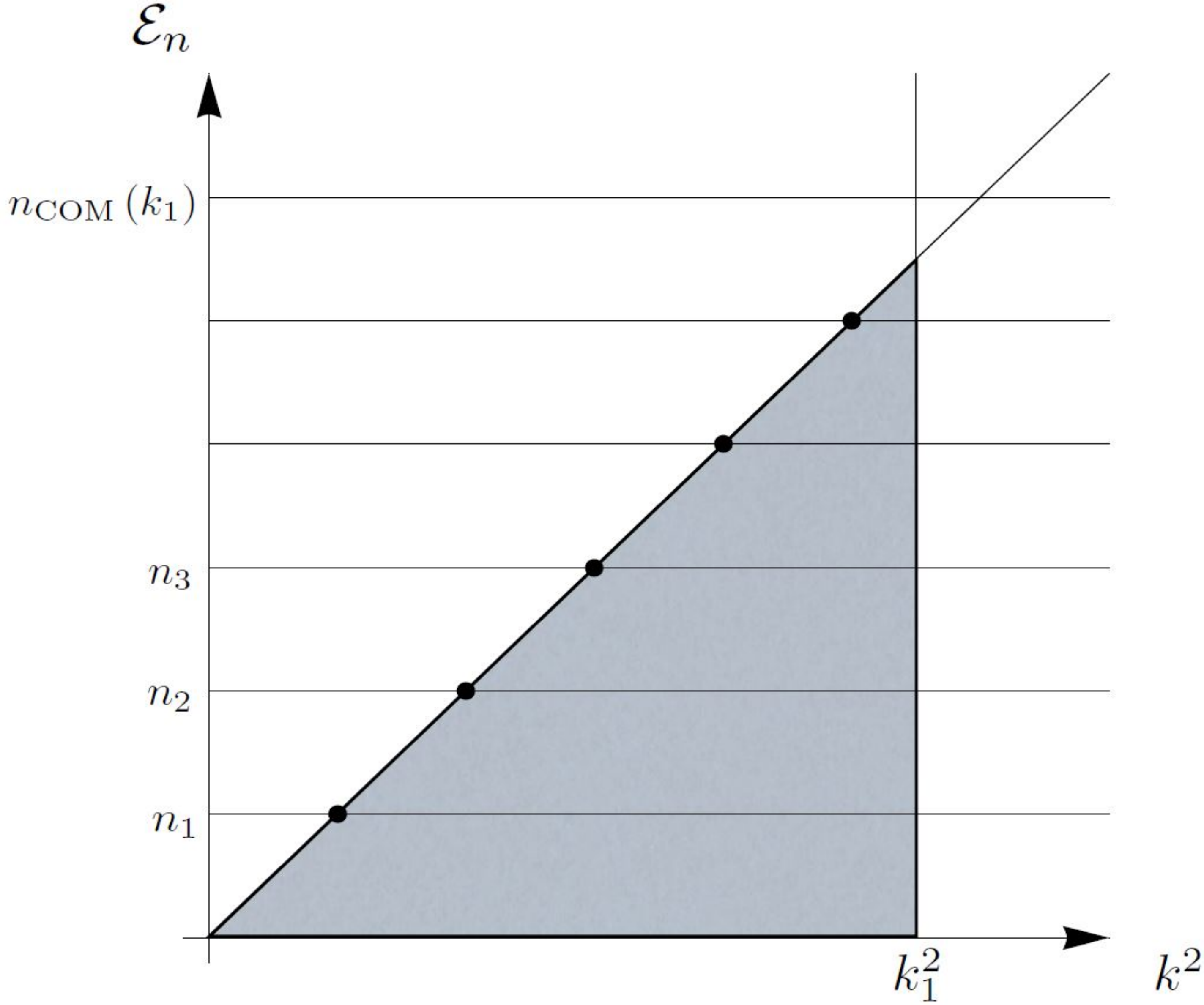
$\mathcal{E}_n[\bar{g}] < k^2$: IR modes

Eigenbasis decomposes:

$$\Upsilon[\bar{g}] = \Upsilon_{\text{IR}}[\bar{g}](k) \cup \Upsilon_{\text{UV}}[\bar{g}](k)$$

The IR - UV separation underlying a given functional $(\varphi, \bar{g}) \longmapsto \Gamma_k[\varphi; \bar{g}]$ depends on its second argument, $\bar{g}_{\mu\nu}$!

$$\Rightarrow \qquad n_{\text{COM}} \equiv n_{\text{COM}}[\bar{g}](k) \quad , \; \cdots$$


$\mathcal{E}_n$
$n_{\mathrm{COM}}(k_1)$
n_3
n_2
n_1
k_1^2
k^2

Assume RG trajectory $k \longmapsto \Gamma_k$ is computed.

$$\left(\frac{\delta}{\delta\varphi}\right)^n \Gamma_k \rightsquigarrow \langle \hat\varphi(x_1) \cdots \hat\varphi(x_n) \rangle_{\bar g}$$

The dynamics of the system itself determines the background geometry it "likes" to be in.

<u>Selfconsistent background metrics</u>:

$$0 \overset{!}{=} h_{\mu\nu} \equiv \langle \hat h_{\mu\nu} \rangle_{\bar g} \equiv \langle \hat g_{\mu\nu} \rangle_{\bar g} - \bar g_{\mu\nu}$$

i.e. $\langle \hat g \rangle_{\bar g} = \bar g$ for $\bar g = \bar g_k^{sc}$

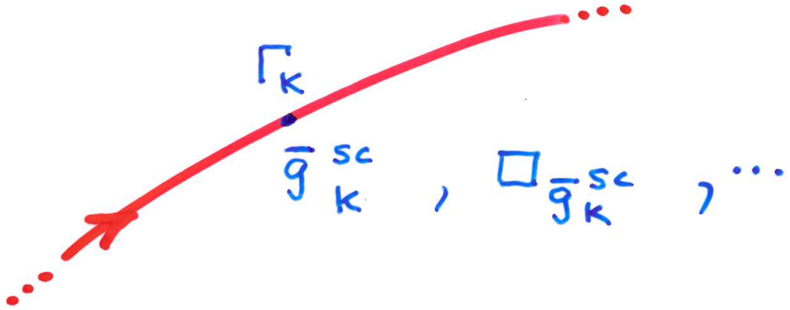
<u>The "tadpole condition"</u>:

$$\frac{\delta}{\delta h(x)} \Gamma_k[h;\bar g] \Big|_{h=0,\ \bar g = \bar g_k^{sc}} = 0$$

!!!

Assume generalized trajectory is computed:

$$k \longmapsto \left(\Gamma_k \,,\, \bar{g}^{sc}_k \right)$$



$\Longrightarrow$ family of Laplacians: $\quad \Box_{\bar{g}^{sc}_k}$

$\Longrightarrow$ family of eigenvalue eqs.:

$$-\Box_{\bar{g}^{sc}_k} \chi_n(x,k) = \mathcal{F}_n(k)\,\chi_n(x,k)$$

The primary Γ_k - trajectory induces

a <u>spectral</u> <u>flow</u> : $\quad k \longmapsto \{ \mathcal{F}_n(k) \}$

Go on-shell setting $\bar{g} = \bar{g}_k^{sc}$, $h = 0$.

$\left(\frac{\delta}{\delta h}\right)^n \Gamma_k [h; \bar{g}_k^{sc}]$ acquires an additional scale dependence via the continually changing, self-determined background geometry.

$\Longrightarrow$ Interpretation of the coarse graining procedure becomes non-trivial and can lead to striking surprises.

__Recall:__

$$\Gamma_k : \underbrace{(h, \bar{g})}_{k\text{-indep.}} \longmapsto \Gamma_k [h; \bar{g}]$$

cutoff in the spectrum of $\square_{\bar{g}} \rightsquigarrow \square_{\bar{g}_k^{sc}}$

Which modes are "cut off" really when the curve parameter k assumes a certain value?

Given a family $k \longmapsto \{\mathcal{F}_n(k),\ \chi_n(\cdot,k)\}$,

compute the cutoff mode from all spectra:

The "quantum number"

$$n_{COM}(k) \equiv n_{COM}[\bar{g}^{sc}_{k}](k)$$

at the point Γ_{k_1} follows from

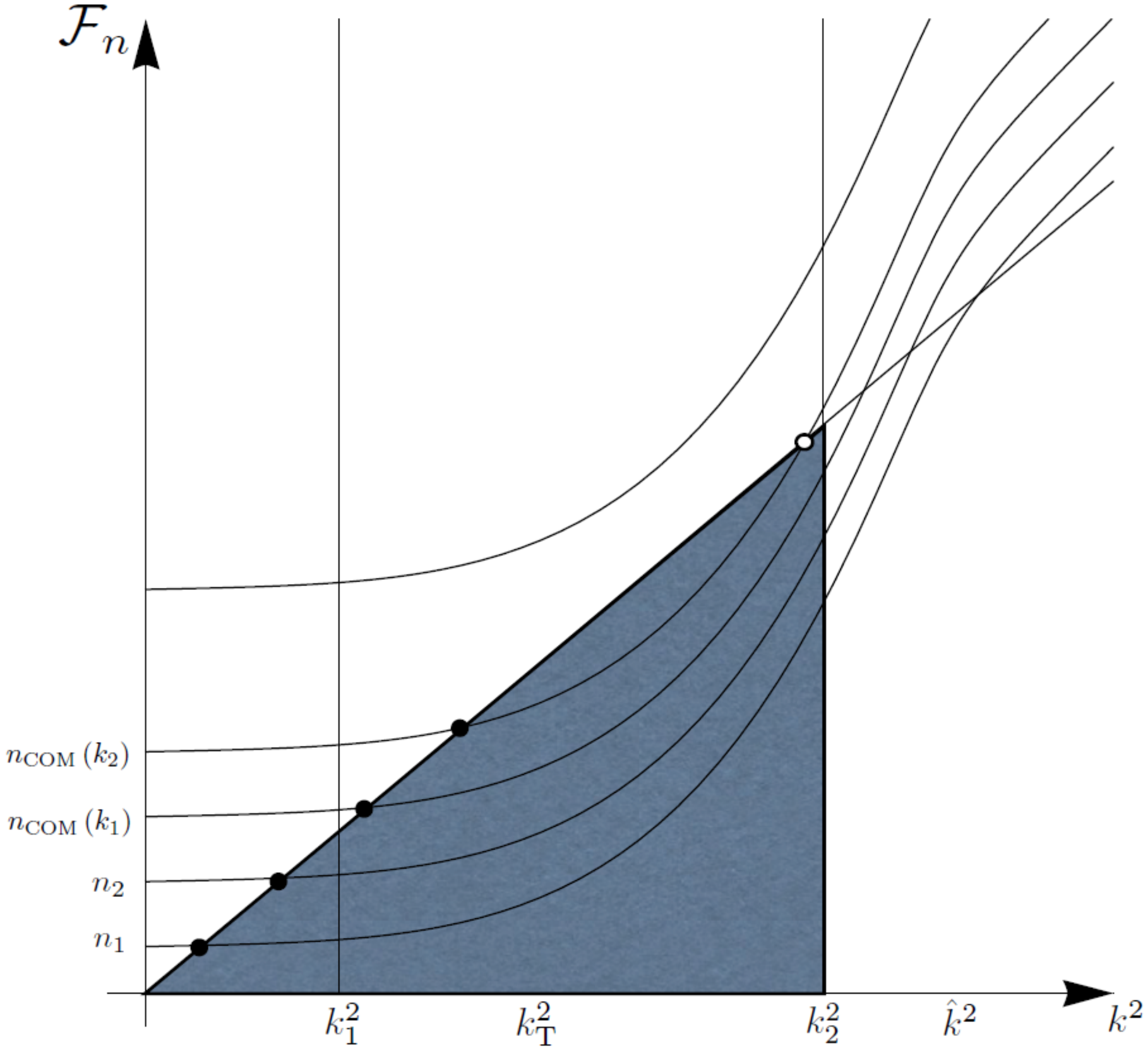
$$\mathcal{F}_{n_{COM}}(k)\Big|_{k=k_1} \overset{!}{=} k_1^2$$

by solving for $n_{COM} \equiv n_{COM}(k_1)$.

$\Upsilon_{UV}(k_1)$: modes with $\mathcal{F}_n(k_1) \geqslant \mathcal{F}_{n_{COM}(k_1)}(k_1)$

$\Upsilon_{IR}(k_1)$: " " $<$

The dof's of $(\text{eff. QFT})_{k_1}$!



$\mathcal{F}_n$
$n_{\mathrm{COM}}(k_2)$
$n_{\mathrm{COM}}(k_1)$
n_2
n_1
k_1^2
k_{T}^2
k_2^2
$\hat{k}^2$
k^2

Einstein-Hilbert truncation

Tadpole condition:

$$G_{\mu\nu} + \Lambda_k g_{\mu\nu} = 0$$

Solutions of the rescaling type:

$$\left(\bar{g}_k^{sc}\right)_{\mu\nu} = \frac{\Lambda_0}{\Lambda_k}\left(\bar{g}_0^{sc}\right)_{\mu\nu}$$

Spectral flow:

$$\mathcal{F}_n(k) = \frac{\Lambda_k}{\Lambda_0}\mathcal{F}_n(0)$$

$$\chi_n(x,k) = \chi_n(x,0)$$

Finding the cutoff modes:

- solve $\mathcal{F}_{n_{\text{COM}}}(0) \stackrel{!}{=} q^2 \rightsquigarrow n_{\text{COM}} = n^0_{\text{COM}}(q)$
- the solution to $\mathcal{F}_{n_{\text{COM}}}(k) \stackrel{!}{=} k^2$:

$$n_{\text{COM}}(k) = n^0_{\text{COM}}\big(q(k)\big) \quad \text{with}$$

$$\boxed{q(k)^2 = \Lambda_0 \frac{k^2}{\Lambda_k} \equiv \frac{\Lambda_0}{\lambda_k}}$$

Example: S^4 spacetimes

Radius of the selfconsistent sphere:

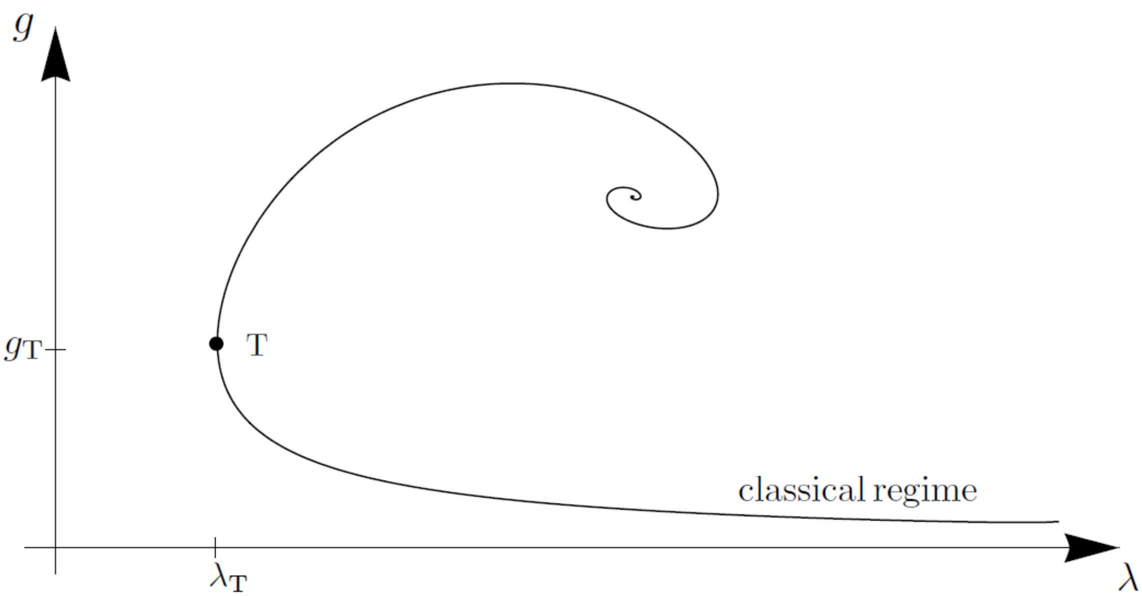
$$r_k = r_0 \left(\frac{\Lambda_0}{\Lambda_k} \right)^{1/2}$$

For tensors of any rank $(n \in \mathbb{N})$:

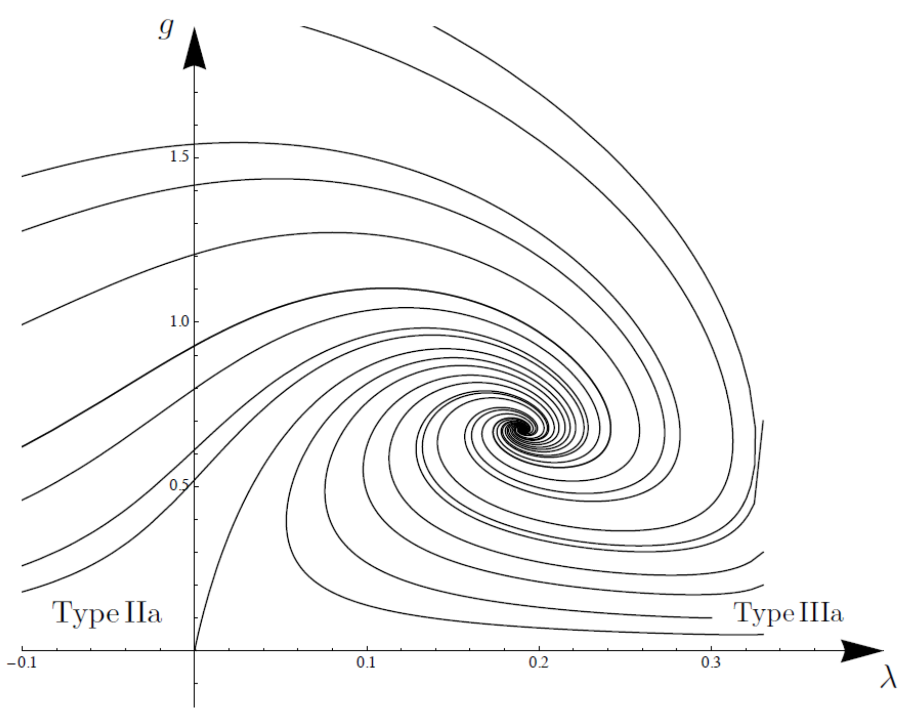
$$\mathcal{F}_n(k) \approx \left(\frac{n}{r_k} \right)^2 \quad \text{if} \quad n \gg 1$$

"Quantum number" of the COM:

$$\boxed{n_{COM}(k) = r_0 \, q(k)}$$

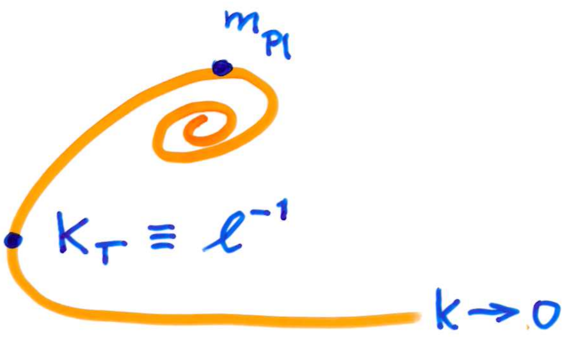

g
g_{T}
T
classical regime
λ_{T}
λ

(b)


g
1.5
1.0
0.5
Type IIa
Type IIIa
-0.1
0.1
0.2
0.3
λ

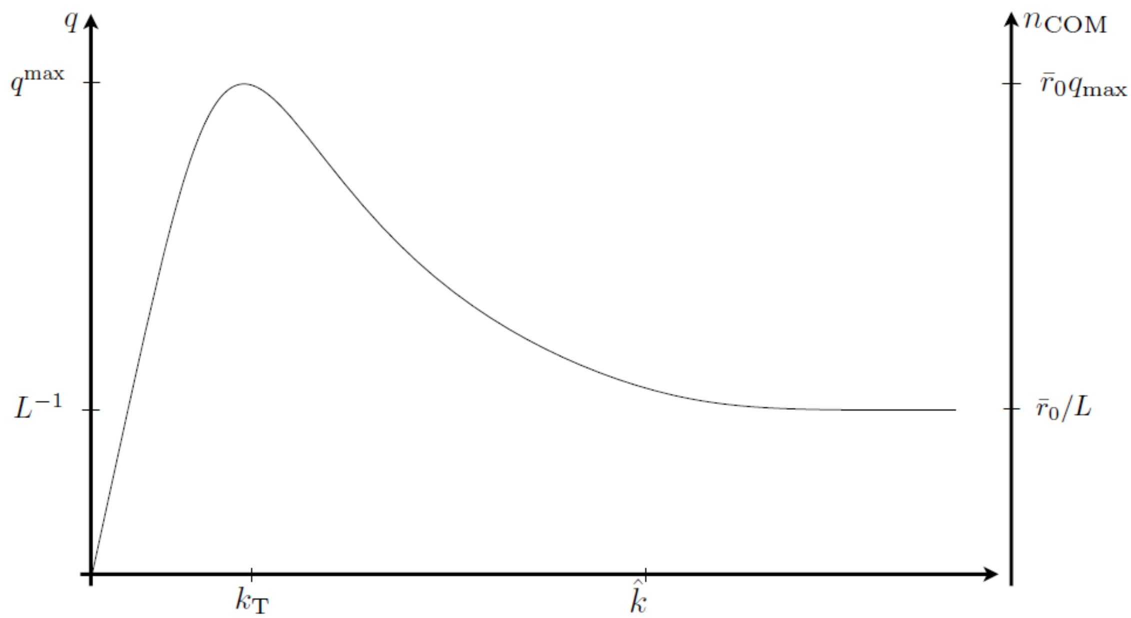
(a)

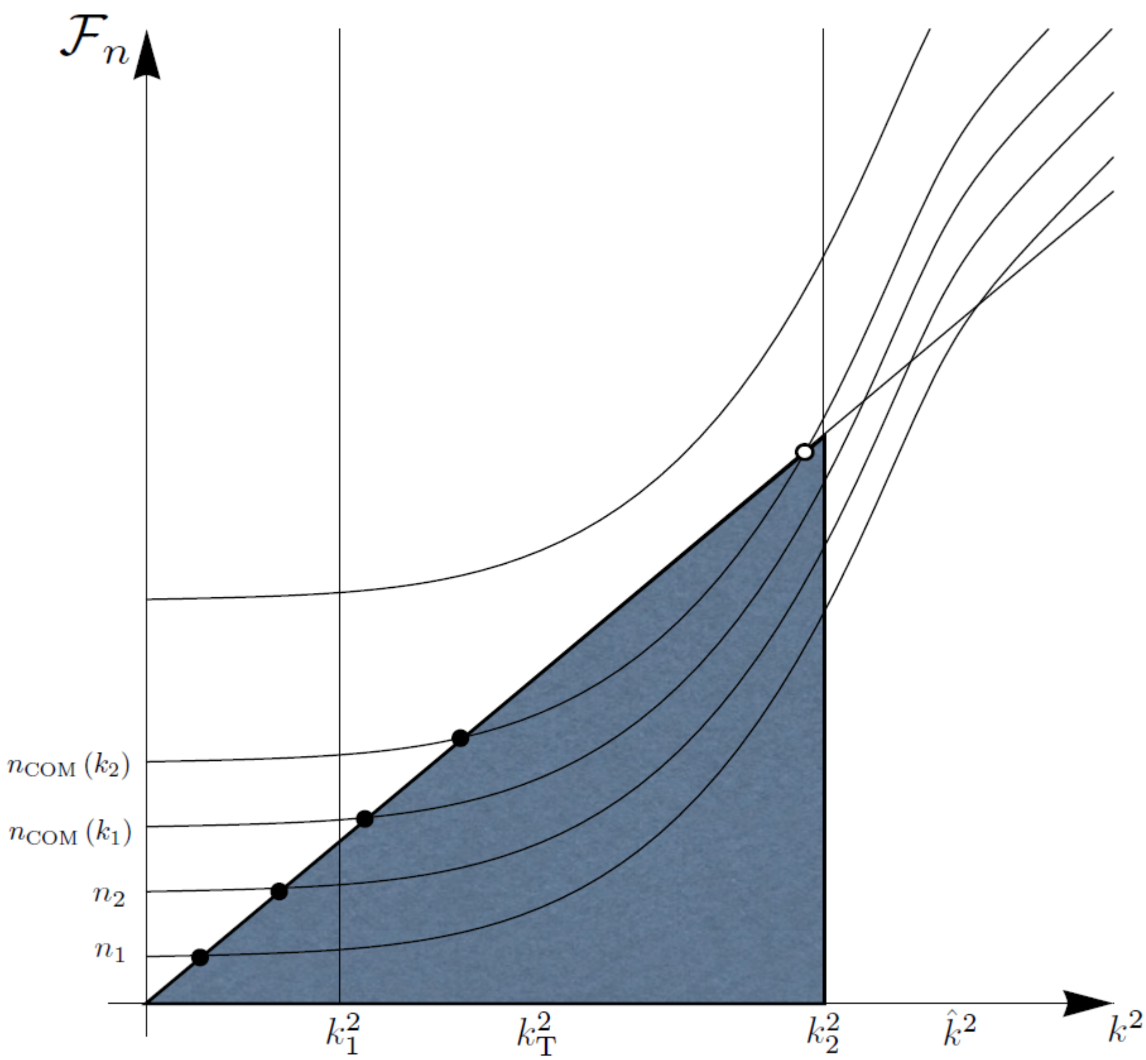
Example: Type III a trajectory



$$\frac{\Lambda_k}{\Lambda_0} = \begin{cases} 1 + \ell^4 k^4 & \text{for } 0 \leq k \lesssim m_{Pl} \\ L^2 k^2 & \text{for } k \gtrsim m_{Pl} \end{cases}$$

$$g(k)^2 = \begin{cases} \dfrac{k^2}{1 + \ell^4 k^4} & \text{semi classical} \\ L^{-2} & \text{fixed point} \end{cases}$$


q
$q^{\max}$
L^{-1}
n_{COM}
$\bar{r}_0 q_{\max}$
$\bar{r}_0/L$
k_{T}
$\hat{k}$



$\mathcal{F}_n$
$n_{\mathrm{COM}}(k_2)$
$n_{\mathrm{COM}}(k_1)$
n_2
n_1
k_1^2
k_{T}^2
k_2^2
$\hat{k}^2$
k^2

Above the turning point, lowering k converts UV-modes to IR modes (rather than vice versa).

At low scales $(\text{eff.QFT})_k$ has more dof's to deal with than at high scales.

Resolution to the "paradox":

When k is increased, the self-selected S^4 shrinks, causing $\mathscr{F}_n(k)$, n fixed, to grow.

The familiar Running Picture:

$\boldsymbol{\Gamma_k}[\boldsymbol{h}, \boldsymbol{matter}; \overline{\boldsymbol{g}}]$ describes the "particle physics" of matter quanta and gravitons propagating on the running $\overline{\boldsymbol{g}}_{\boldsymbol{k}}^{\boldsymbol{sc}}$ geometry. The high-k matter description applies to high-k backgrounds!

The new Rigid Picture:

Try to construct a new action functional Γ_q by eliminating $\overline{\boldsymbol{g}}_{\boldsymbol{k}}^{\boldsymbol{sc}}$ everywhere in favor of the (essentially flat) macroscopic metric $\overline{\boldsymbol{g}}_{\boldsymbol{o}}^{\boldsymbol{sc}}$.
Pretend that only the particle physics runs, while the background metric stays fixed.

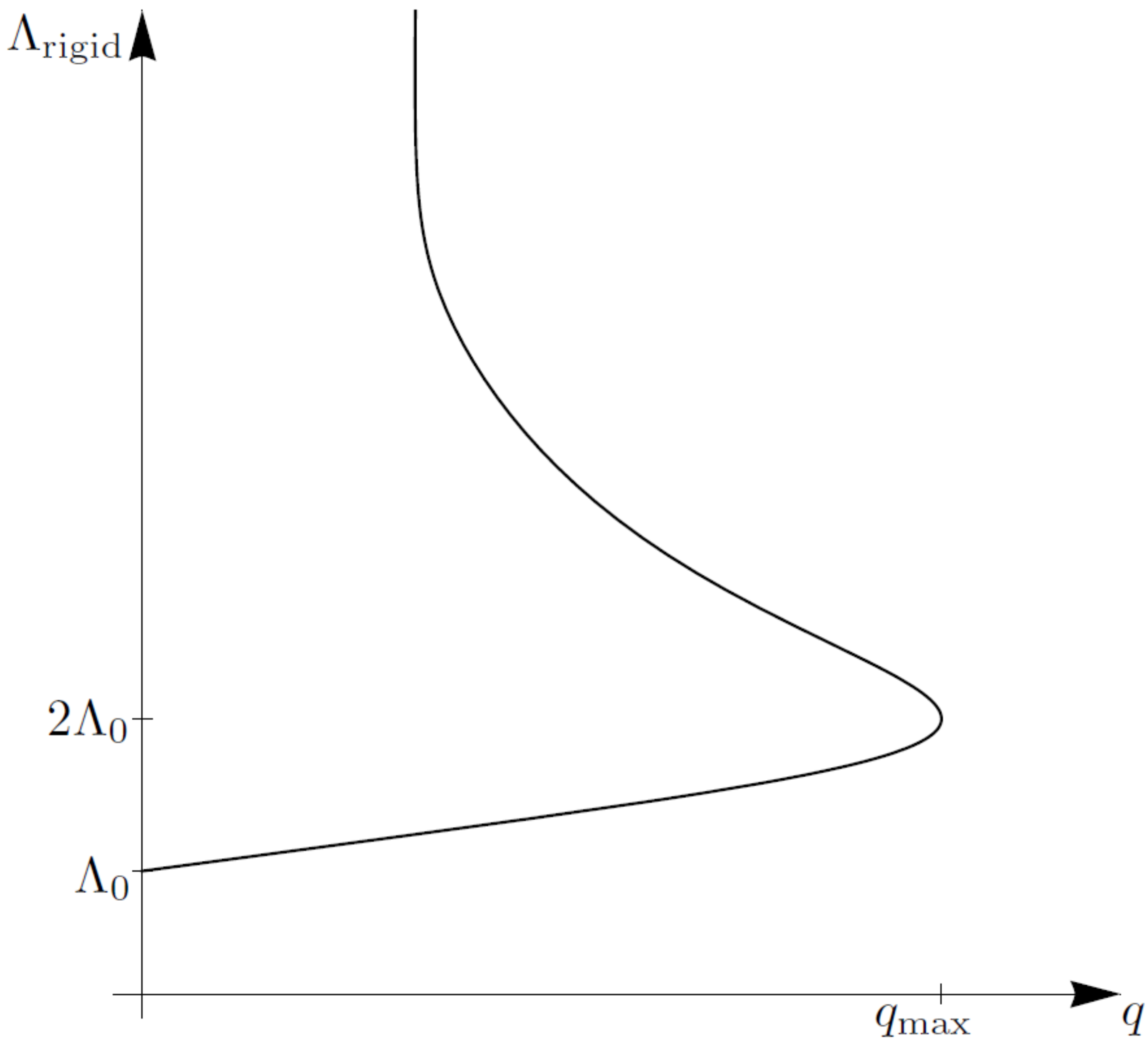
This involves re-interpreting the cutoff scale as an eigenvalue of the Laplacian built from $\overline{\boldsymbol{g}}_{\boldsymbol{o}}^{\boldsymbol{sc}}$ rather than $\overline{\boldsymbol{g}}_{\boldsymbol{k}}^{\boldsymbol{sc}}$.
If the latter is k^2, the former equals q^2.

In the rigid picture, *q* plays the same role *k* plays in the running picture.

The corresponding action functional $\boldsymbol{\Gamma_q = \Gamma_{k(q)}}$ would require solving $q = q(k)$ for $k = k(q)$.

Globally, this is impossible however !

The rigid picture is applicable from *k=0* to k_T only. It breaks down at the turning point which acts as a “scale horizon”.



Λ_{rigid}
$2\Lambda_0$
Λ_0
q_{max}
q

Application to the Cosmological Constant Problem

The familiar summation of zero-point energies quantizes the modes in $\Upsilon_{IR}(\mathscr{P})$ employing the rigid picture.

Within its domain of applicability ***($\mathscr{P} < k_T$)***, the resulting vacuum energy changes Λ by at most a factor of 2.

⟹ Traditional “rigid” calculations obtaining factors like 10^{120} overstretch their actual domain of validity quite considerably.

⟹ It is incorrect to claim that these calculations imply a naturalness problem for a small value of Λ.

The running picture allows for a consistent interpretation of scales above the turning point also.

$\Longrightarrow$ It becomes unavoidable to appreciate the k-dependence of the metric.

$\Longrightarrow$ When Λ_k grows, the running Einstein equation $\boldsymbol{R(\bar{g}_k^{sc}) = 4\Lambda_k}$ allocates the growing curvature not to cosmological scales, but to shorter and continuously decreasing length scales.

Fluctuations at scale k curve spacetime at scale k!

A fatal limitation of the standard approach:
Fluctuations of any scale can generate curvature on the cosmological scale only.

Conclusion

Fundamentally the question about the gravitational manifestation of vacuum fluctuations involves only a very simple dynamics (harmonic oscillator,…) and seems unrelated to any specific UV completion of quantum gravity; nonperturbative effects (Asymptotic Safety,…) play no essential role.

In order to get a qualitatively correct picture, it is crucial though to respect Background Independence and to avoid the misconception of a pre-existing spacetime.