Ricci Reheating

- Outline

1. • What do we know about the early universe/reheating?
2. • Overview of traditional Gravitational Reheating + Problems
3. • Ricci Reheating: Viable Gravitational Reheating
4. • Ricci Reheating phenomenology + Higgs Connection

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1. Show slide 2 ("What do we know?")

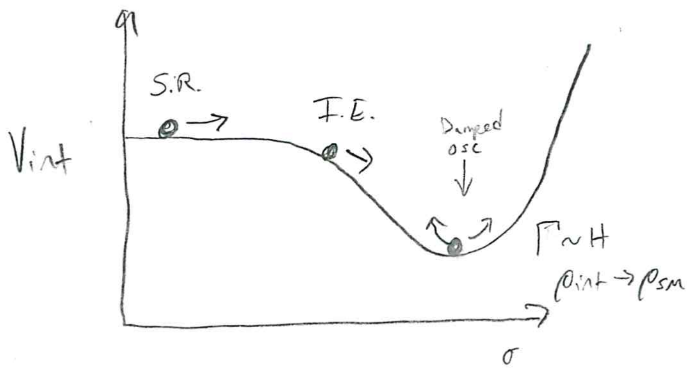
- Lots of evidence for a period of inflation at the energy scale $E_{inf} \sim \sqrt{H_{inf} M_p}$, $H_{inf} \lesssim 10^{14}\,GeV$.

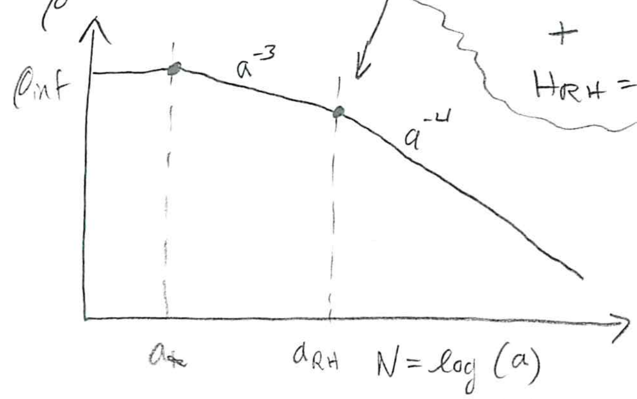
- Need thermal plasma for BBN at $T \sim 1\,MeV$.

- Don't know about the gap $\Delta E \lesssim 10^{19}\,GeV$ in between.

Usual assumed picture is: (Need $\Gamma > H_{BBN} = \frac{T_{BBN}^2}{M_p}$)

$3M_p^2 H_{RH}^2 = \frac{\pi^2}{30} g_* T_{RH}^4$ + $H_{RH} = \Gamma$





- That was the standard picture, which uses direct couplings between the inflaton + SM to reheat.

- But does one need any non-gravitational couplings?

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2 Gravitational Reheating

- Change in spacetime curvature at the end of inflation means there is a time-dep background which can excite non-conformally coupled light fields. (L.H. Ford, PRD 1987)

Example: Scalar Field (Massless) $\boxed{R = -6\frac{a''}{a^3}}$

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}\xi R\phi^2\right]$$

Canonically normalize to get $(\phi \to \varphi/a)$

$$S = \frac{1}{2}\int d\tau\, d^3x\left[(\varphi')^2 - (\nabla\varphi)^2 + a^2\left(\xi - \frac{1}{6}\right)R\,\varphi^2\right]$$

$$\Rightarrow \quad \varphi_k'' + \left[k^2 - a^2\left(\xi - \frac{1}{6}\right)R\right]\varphi_k = 0$$

If we assume that the universe is dominated by one fluid with equation of state $w = P/\rho$, the

$$H^2 = \left(\frac{a'}{a^2}\right)^2 = H_*^2 \left(\frac{a_*}{a}\right)^{3(1+w)}$$

and

$$R = -3(1-3w) H^2$$

$$R = \begin{cases} -12H^2 & (w=-1) \ (\text{de Sitter}) \\ -3H^2 & (w=0, \ \text{Matter dom}) \\ 0 & (w=1/3, \ \text{Rad dom}) \end{cases}$$

$m_{eff}^2 \sim -(\xi - \frac{1}{6}) R$ becomes τ-dep, fields can be excited unless $\xi = \frac{1}{6}$.

Ford calculated

$$\rho_{GPP} \sim (1-6\xi)^2 \, 10^{-2} H^4 \qquad (\xi \sim \tfrac{1}{6})$$

Later, see e.g. (hep-th/9503149)

$$\rho_{GPP} \sim 10^{-2} H^4 \qquad (\xi \sim 1)$$

Summary

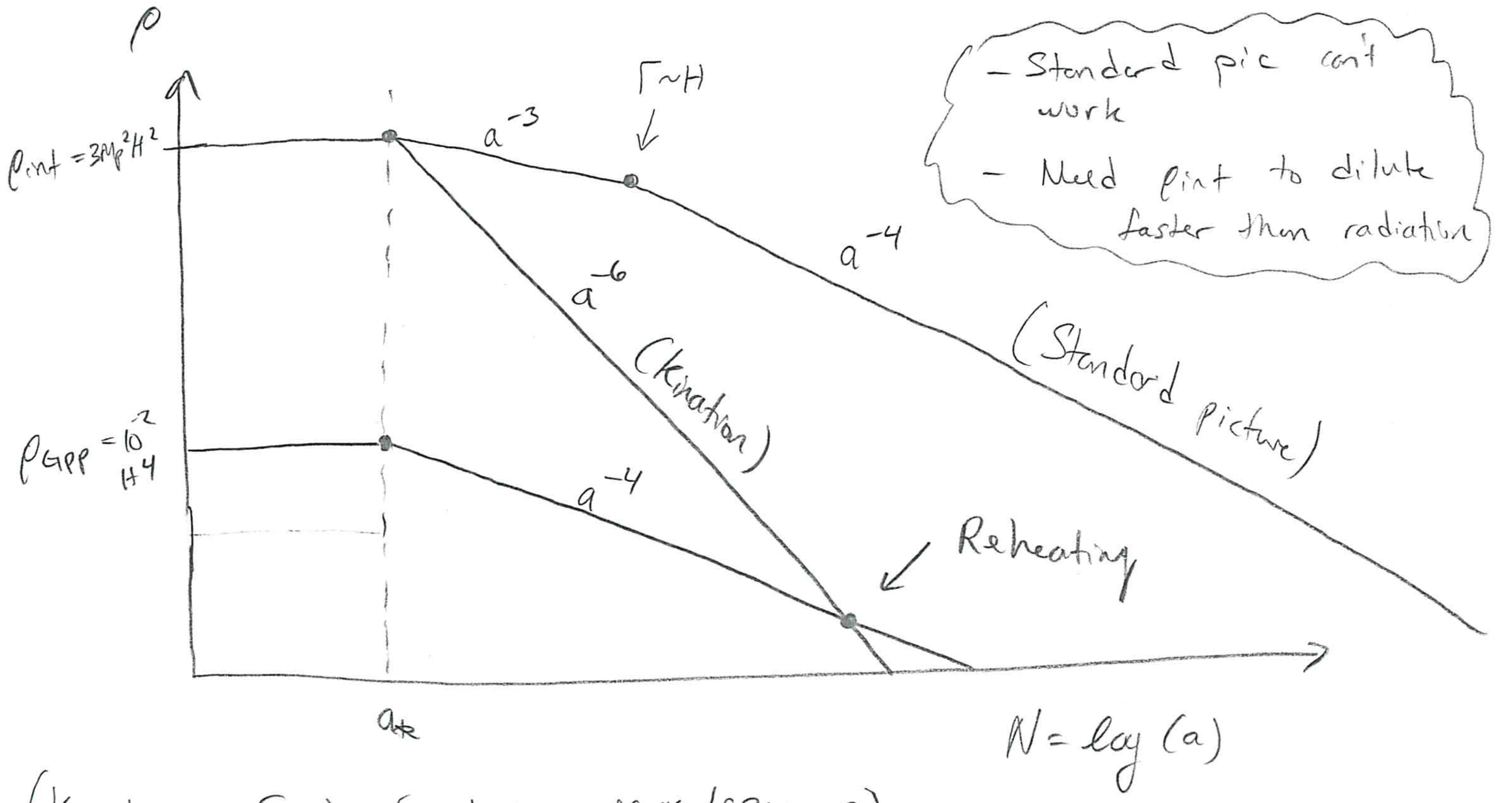
– The excitation of light, non-conformally coupled fields at the end of inflation will yield a gravitationally produced radiation energy density of

$$\rho_{GPP} \sim 10^{-2} H^4$$

Compared to the inflaton energy density $\rho_{inf} \sim 3M_p^2 H^2$

we have

$$\Omega_{GPP} \sim \frac{\rho_{GPP}}{\rho_{inf}} = \frac{10^{-2} H^4}{3M_p^2 H^2} \sim \underbrace{\frac{10^{-2}}{3}\left(\frac{H}{M_p}\right)^2}_{\text{suppression due to grav. coupling}} \lesssim 2.4 \times 10^{-12}$$



(Kination: Ford, Spokoiny gr-qc/9306008)

$$\dot{\rho} + 3H(1+w)\rho = 0 \quad \Rightarrow \quad \rho \sim \left(\frac{a_*}{a}\right)^{3(1+w)}$$

So we require $w > \frac{1}{3}$ to dilute faster than radiation. How to get?

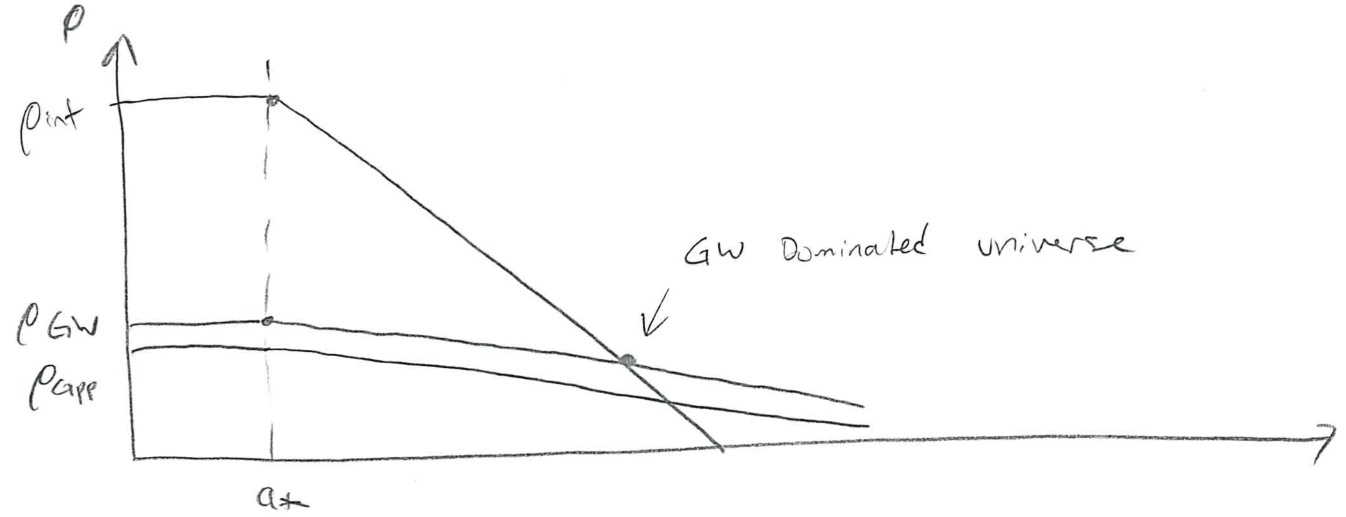
$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \, ,$$

if $\frac{1}{2}\dot{\phi}^2 \gg V$, then $w \to 1$, hence "kination"

Problem with Grav. Reheating

- GW also produced ~~with a densi~~ gravitationally :(

$$\rho_{GW} \sim 2 \times \rho_{GPP}$$



$$\Delta N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \frac{\Omega_{GW}}{\Omega_\gamma} \cong \begin{cases} 9 & \frac{\Omega_{GW}}{\Omega_\gamma} = 2 \\ 0.3 & \frac{\Omega_{GW}}{\Omega_\gamma} = \frac{2}{30} \end{cases}$$

Need to increase $\frac{\rho_{app}}{\rho_{GW}}$ by $O(100)$. One option is to add $N \sim 10-100$ light, non conformally coupled fields. But in SM, only have (potentially) the Higgs. What else?

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(3) Ricci Reheating (ϕ = Reheaton)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \xi R \phi^2 - V(\phi) \right]$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4$$

$m^2 << |\xi R|$

$$V_{eff} = \frac{1}{2} \xi R \phi^2 - V(\phi) = \frac{1}{2}(m^2 - \xi R)\phi^2 + \frac{\lambda}{4}\phi^4$$

$$R = \begin{cases} -12H^2 & (w=-1, \text{ inflation}) \\ -3H^2 & (w=0, \text{ MD}) \\ 0 & (w=1/3, \text{ RD}) \\ 6H^2 & (w=1, \text{ KD}) \end{cases} = \underline{-3(1-3w)H^2}$$

∗ If the universe transitions from $w=-1$ to $w > \frac{1}{3}$, R changes sign and a tachyon is induced for ϕ! (370)

Recall:

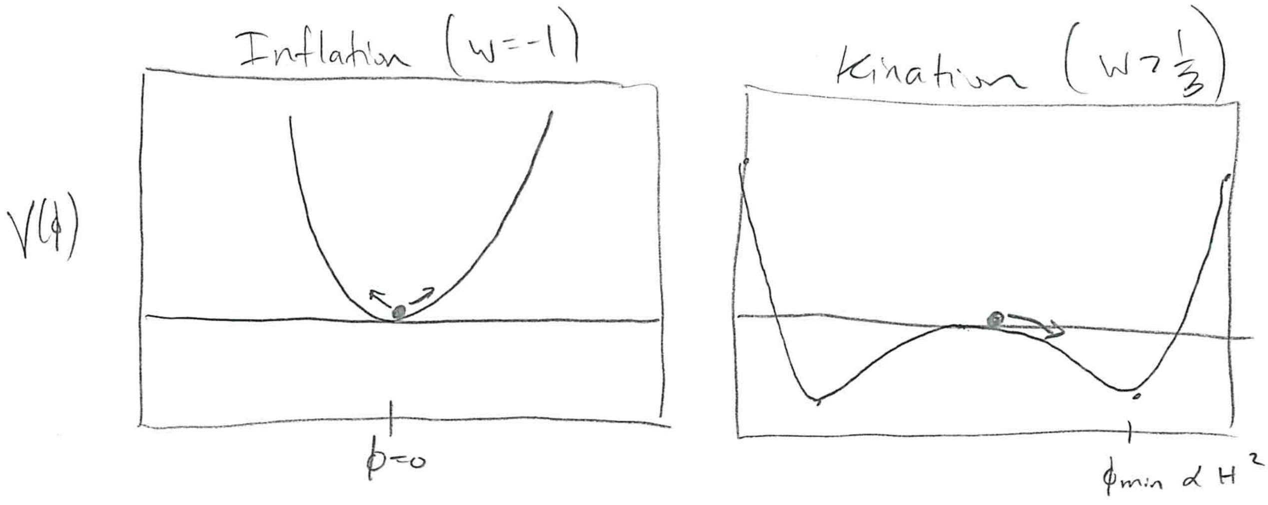
$$m_{eff}^2 = -\xi R = \xi\, 3(1-3w)\, H^2$$

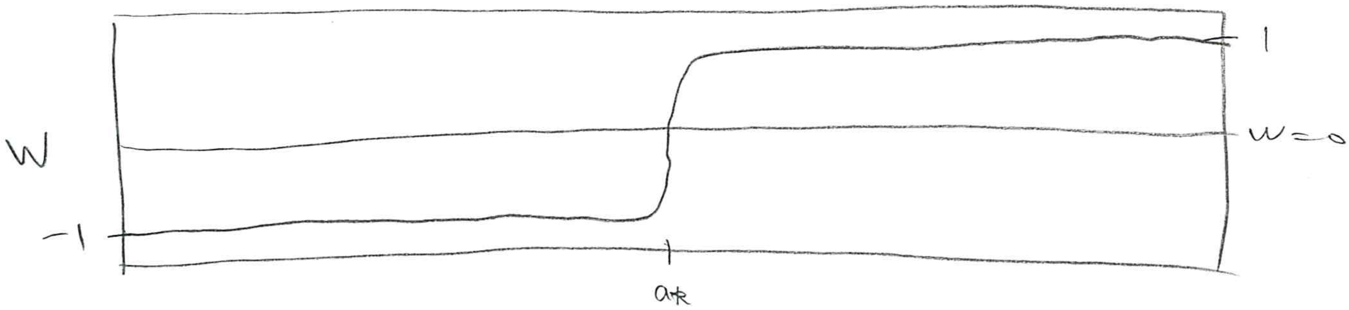
Tachyonic for $\xi > 0$ and $w > \frac{1}{3}$.

During inflation: $m_{eff}^2 = 12 \xi H^2$

After inflation: $m_{eff}^2 = -6\xi H^2$ (for $w=1$)

Sketch





The minima of the effective potential are

$$\phi_{min} = \pm \left(\frac{\xi R}{\lambda}\right)^{\frac{1}{2}} = H \sqrt{\frac{3\xi(3w-1)}{\lambda}} \propto a^{-2} - a^{-3}$$

$$H(t) = H_* \left(\frac{a_*}{a}\right)^{\frac{3(1+w)}{2}}, \quad a \propto t^{\frac{2}{3(1+w)}}, \quad H \propto \frac{1}{t}$$

Depth of the minima:

$$\Delta V_{eff} = -\frac{\xi^2 R^2}{4\lambda} = -\frac{9\xi^2}{4\lambda}(3w-1)^2 H^4$$

Very sensitive to H: $\Delta V_{eff} \propto a^{-8} - a^{-12}$

Field should roll to the minimum quickly to extract energy.

Initial Conditions

We treat the system classically by assuming the superhorizon fluctuations at the end of inflation contribute to the RMS value of the classical field:

$$\phi_*^2 = \langle \hat{\phi}_*^2 \rangle = \int_0^{H_*} \frac{dk}{k} P_\phi(k) \approx \left(\frac{H_*}{2\pi}\right)^2 \frac{1}{3\sqrt{12\xi}} =$$

$$\boxed{\phi_* = \sqrt{\langle \hat{\phi}_*^2 \rangle} = \frac{H_*}{2\pi}\sqrt{\frac{H_*}{3 m_{eff}^*}}}$$

$(m_{eff}^* = \sqrt{12\xi}\, H$

$\rightarrow$ mass suppressed

$$\dot{\phi}_*^2 = \langle \hat{\dot{\phi}}_*^2 \rangle = \int_0^{H_*} \frac{dk}{k} P_{\dot\phi}(k) \approx \frac{H_*^4}{12\pi^2}\sqrt{12\xi}$$

$$\boxed{\dot{\phi}_* = \sqrt{\langle \dot{\phi}_*^2 \rangle} = m_{eff}^* \phi_*}$$

Reheaton Dynamics (Movie here?)

- ϕ knocked off unstable max by quantum fluc.
- Rolls to minimum, gains energy (tachyonic growth)
- Tachyonic growth ends when the field is turned by the quartic.
- Field oscillates back, but quickly see the bare potential since H decreasing
- Field behaves as rad or matter until decay / energy transfer —

Initial Rolling / Tachyonic growth phase

$$\ddot{\phi} + 3H\dot{\phi} - \xi R \phi \simeq 0 \quad , \quad R \propto H^2, \quad H \propto a^{-3(1+w)/2}$$
$$\propto \frac{2}{3t(1+w)}$$

Scaling soln for growth

$$\phi \approx \frac{\phi_*}{2}\left(1 + \frac{\gamma_*}{\gamma}\right)\left(\frac{H_*}{H}\right)^{\gamma} \quad , \quad \gamma = \sqrt{\xi(3w-1)/3}$$
$$\gamma_* = \sqrt{12\xi}/3$$

$\phi \propto H^{-\gamma}$ and $\phi_{min} \propto H$, so the reheaton rolls to the minimum faster than the minimum approaches the origin if $\gamma > 1$. $\Rightarrow \xi \gtrsim 1$

- Can also set $\phi = \phi_{min}$ to get the Hubble rate at the minimum

$$\left(\frac{H_{min}}{H_*}\right)^{1+\gamma} = \frac{1}{36\pi}\sqrt{\frac{\lambda}{\gamma_*}}\,\frac{(\gamma_* + \gamma)}{\gamma^2}$$

For $\xi, w \sim O(1)$ we find

$$H_{min} \sim 0.1\,\lambda^{1/4} H_* \sim 10^{-2} H_* \quad (10^{-5} \lesssim \lambda \lesssim 10^{-2})$$

How long does it take to get to the min?

- B/c H^{-1} is the relevant timescale: $N = \int H\,dt = \log \frac{a}{a_*}$

$$N_{min} = \log\left(\frac{a_{min}}{a_*}\right) = \frac{2}{3(1+w)} \log\left(\frac{H_*}{H_{min}}\right) \sim O(1)$$

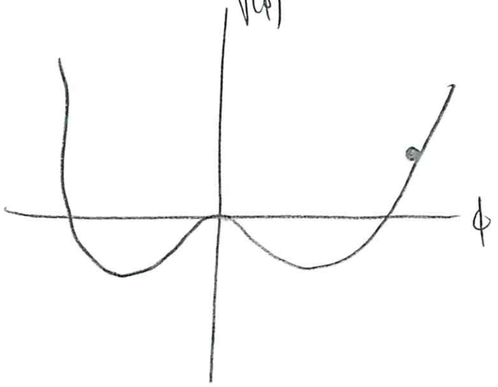
$$\phi_{min}(N_{min}) \cong \frac{\phi_{min}(N=0)}{100}, \quad \Delta V_{eff}(N_{min}) \cong \frac{\Delta V_{eff}(N=0)}{(10^2)^4}$$

→ Quickly aproachs symmetric phase.

Energy extracted

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \xi\left(3H^2\phi^2 + 6H\phi\dot{\phi}\right) \quad \text{(NMC)}$$

At first turning point (quartic ~~dominates~~ important , $\dot{\phi} = 0$)



$$\longrightarrow \quad \rho_\phi = \frac{\lambda}{4}\phi_1^4 + 3\xi H_1^2 \phi_1^2$$

Approximate $N_1 = N_{min}$, $\phi_1 = \phi_{min}$ to obtain

$$\rho_\phi^1 \approx (1+w)(3w-1)\frac{27\xi^2}{\lambda} H_{min}^4 \equiv \Lambda$$

* Since after, quickly approach rad scaling

$$\rho_{eff} \equiv \Lambda e^{-4(N-N_{min})}, \quad \begin{cases} 1 & N < N_{matt} \\ e^{N-N_{matt}} & N \geq N_{matt} \end{cases}$$

* How well does this work? (Show plot)

<u>Reheating</u>

- If ϕ transfers energy before kination ends, then reheating occurs at the end of kination

$$\frac{\pi^2}{30} g_{RH} T_{RH}^4 = \frac{3}{2} M_p^2 H_{KE}^2 \Rightarrow T_{RH} = \left(\frac{45}{\pi^2 g_{RH}}\right)^{\frac{1}{4}} \sqrt{M_p H_{KE}}$$

With $H_{ke} = \sqrt{2}\, H_* \left(\Omega_\phi^*\right)^{\frac{3(1+w)}{2(3w-1)}}$

$$\Omega_\phi^* = \frac{\rho_\phi^{eff}(N=0)}{3M_p^2 H_*^2} = \frac{\Lambda e^{4N_{min}}}{3M_p^2 H_*^2}$$

Otherwise, reheating is determined by microphysics as in the ordinary inflationary case

$$3M_p^2 H^2 \Big|_{H=\Gamma} = \frac{\pi^2}{30}\, g_{RH}\, T_{RH}^4$$

$$\Rightarrow T_{RH} = \left(\frac{90}{\pi^2 g_{RH}}\right)^{\frac{1}{4}} \sqrt{\Gamma M_p}$$

In case 1, for $\xi = w = 1$

$$T_{RH} \approx 10^3\,\mathrm{GeV} \left(\frac{H_*}{10^{11}\,\mathrm{GeV}}\right)^2 \left(\frac{\lambda}{10^{-4}}\right)^{-\frac{1}{5}}$$

(show plot)

4 Ricci Reheating Pheno and Higgs Connection

- Blue-tilting of the primordial GW spectrum
  ↳ Consequence of the kination period

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_{tot}} \propto \frac{a^{-4}}{a^{-3(1+w)}} = a^{3w-1}$$

→ Growing function for $w > \frac{1}{3}$

→ Modes begin evolving when they reenter the horizon after inflation. For a given mode, this happens when

$$K = aH \quad ,$$

$$\frac{K}{K_{ke}} = \frac{aH}{a_{ke} H_{ke}} = \left(\frac{a_{ke}}{a}\right)^{\frac{3w+1}{2}}$$

$$\Rightarrow \Omega_{GW} \propto \left(\frac{k}{K_{ke}}\right)^{\frac{2(3w-1)}{3w+1}} \qquad (K \geq K_{ke})$$

(+) slope for $w > \frac{1}{3}$ ⇒ blue tilt

$$k_\star = a_\star H_\star$$

$$\Omega_{GW}(k) = \Omega_{GW}^{flat} \begin{cases} 1, & k < k_{ke} \\ (k/k_{ke})^{\frac{2(3w-1)}{3w+1}} & k_{ke} \leq k \leq k_\star \\ 0 & k > k_\star \end{cases}$$

$$\Omega_{GW}^{flat} \propto \frac{1}{12\pi^2}\left(\frac{H_\star}{M_p}\right)^2 \Rightarrow \rho_{GW}^{flat} = \frac{1}{4\pi^2} H_\star^4 = 2\times10^{-2} H_\star^4$$

Redshift everything to today → show plot

Make sure Ω_{GW} contribution to N_{eff} isn't too large

$$\Delta N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \frac{\Omega_{GW}^0}{\Omega_\gamma^0} \lesssim 0.3$$

$$\Omega_{GW}^0 = \int \frac{df}{f} \Omega_{GW}^0(f) \rightarrow \text{dominated by endpoint}$$

$$f_{BBN} = H_{BBN}\left(\frac{a_{BBN}}{a_0}\right) = \frac{T_{BBN}^2}{M_p}\frac{T_0}{T_{BBN}} = \frac{T_{BBN}T_0}{M_p} \simeq 10^{-11}\,\text{Hz}$$

$$T_{RH} \lesssim 10^5 \cdot T_{BBN} = 100\,\text{GeV}$$ (since need long kination)

Higgs as the Reheaton

– Higgs quartic runs, so need to use the correct value at the minimum, which we take to be the renormalization scale. $\mu = \phi_{min}$

$$\phi_{min}^2 = \frac{\xi R}{\lambda} = H_{min}^2 \frac{3\xi(3w-1)}{\lambda}$$

For consistency, must solve

$$\boxed{\phi_{min}^2 = \mu^2 = \frac{3\xi(3w-1)}{\lambda(\mu)} H_{min}^2(\mu)}$$

(Solns exist only for $\lambda(\phi_{min}) > 0$)

We use 3-loop SM running for λ (1307.3536)

$\Rightarrow$ Taking H_{R} and/or ξ too large will result in field excursions which probe the unstable region of the Higgs potential.

$$\Rightarrow \boxed{T_{RH} = 10^{-3} - 1\ \text{GeV}}$$