Sommerfeld enhancement at finite T

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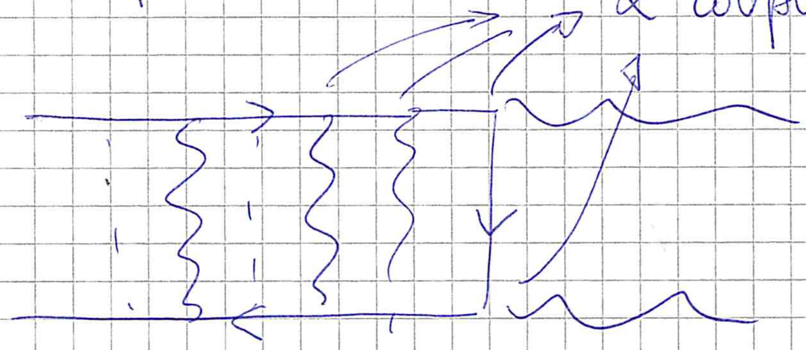
History: QM effects already discussed by Sommerfeld in 1939 for QED & later by Sakharov in 1948 for non-abelian case

It was realised that it was important also for DM phenomenology first by the japanese group of Hisano, Matsumoto & Nojiri in 2003-2005 ... etc...

What is the Sommerfeld enhancement?

→ IR / long range force effect in NR scattering

e.g. annihilation / coannihilation of two charged particles



α coupling Scales in the PBM:

M mass of initial particles

m_V "photon" mass

$\beta = v/c$ particle's velocity

↳ Coulomb force acts also at large distance modifying the wave-functions of the initial state!
The effect is enhanced by resumming all ladder diagrams

The effect is large if the annihilating particles are non-relativistic with $\beta = v/c \ll 1$

but at the same time velocity large enough to be able to neglect the gauge boson mass:

$M\beta^2 \gg \alpha m_V \longrightarrow$ Coulomp potential

Energy of NR particles

How is the effect computed at $T=0$?

$\longrightarrow$ solve the Schrödinger equation for the two particles state in the Coulomb potential

in principle the effect exists $\forall\ \ell$ values of angular momentum, in practise the S-wave with $\ell=0$ is affected most (no velocity suppression!)

$\beta = 0$ Schrödinger eq.:

$$\boxed{\frac{1}{M}\frac{d^2\psi(r)}{dr^2} - V(r)\psi(r) = -M\beta^2\psi(r)}$$

where $V(r) = -\frac{\alpha}{r}e^{-m_V r}$ Coulomb/Yukawa potential

solve with boundary conditions

$$\frac{d\psi}{dr} = iM\beta\,\psi(r) \quad \text{at } r \to \infty$$

"incoming wave"

$p = M\beta$

Usually the solution has to be obtained numerically, but for $m_V = 0$ there is an analytic solution:

$$\sigma_{S.E.}(v) = S(v)\,\sigma_0(v)$$

where $\sigma_0(v)$ = Born cross-section

$S(v) = |\psi(0)|^2$ "enhancement" of wave-function at zero

For $m_V = 0$ one has simply for QED

$$S(\beta) = \frac{\pi\alpha}{\beta}\frac{1}{1-e^{-\pi\alpha/\beta}}$$

$$S(\beta) \xrightarrow[\beta\to 0]{} \frac{\pi\alpha}{\beta}$$ $1/v$ enhancement

$$S(\beta) \xrightarrow[\alpha\to 0]{} \frac{\pi\alpha}{\beta}\frac{1}{1-1+\frac{\pi\alpha}{\beta}+O(\alpha^2/\beta^2)} = 1$$

no enhancement

Generically for non-abelian groups

$$\frac{\pi\alpha}{\beta} \longrightarrow C_F(N)\frac{\pi\alpha_N}{\beta}$$

with $C_F(N)$ group constant depending on initial configuration. C_F can be both positive/negative

$C_F > 0 \rightarrow$ attractive potential & enhancement

$C_F < 0 \rightarrow$ repulsive potential & suppression

For other analytic results, see

Blum, Sato & Slatyer 1603.01383v2

(Hulthén potential $V(r) = -\dfrac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}$

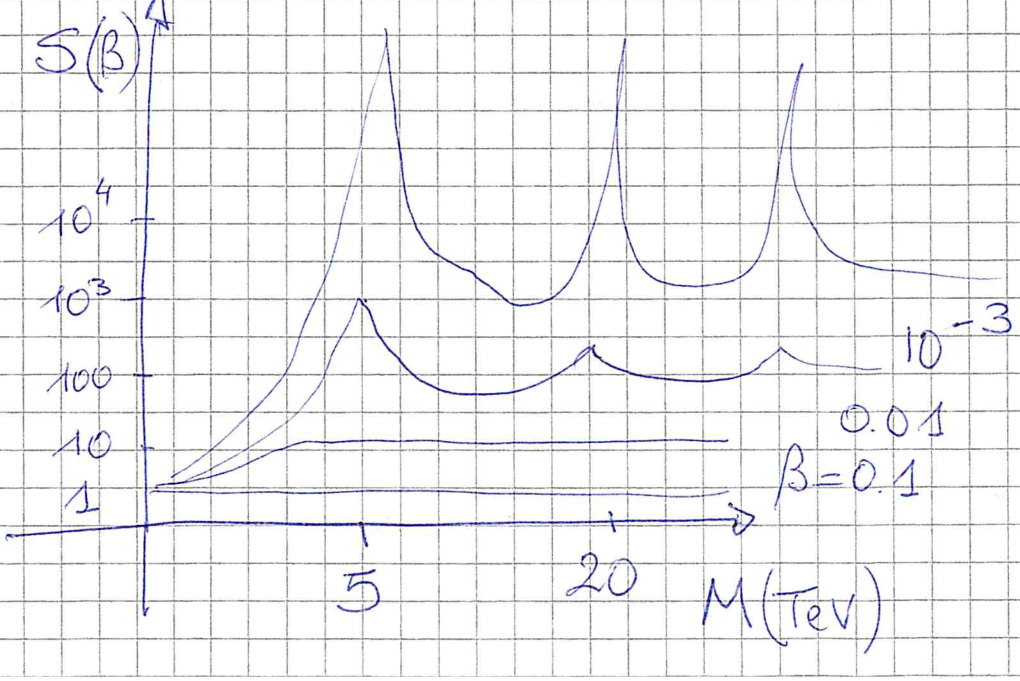
to maintain unitarity bound, which may fail if bound states exists with $\sigma_0 \propto 1/v$ (Bethe) need to include in V also a contact term

$$V_{TOT} = V(\vec{x}) + u\, \delta^3(\vec{x})$$

with $\mathrm{Im}(u) = -\dfrac{\sigma v_0}{2}$)

For $\beta \ll \sqrt{\dfrac{\alpha m_V}{M}}$ then the kinetic energy is negligible and only the potential remains

$\Longrightarrow$ bound states as in hydrogen atom for integer $\left(\dfrac{\alpha M}{4 m_V}\right)^2 \longrightarrow$ resonances!



Lattanzi & Silk

arxiv 0812.0360

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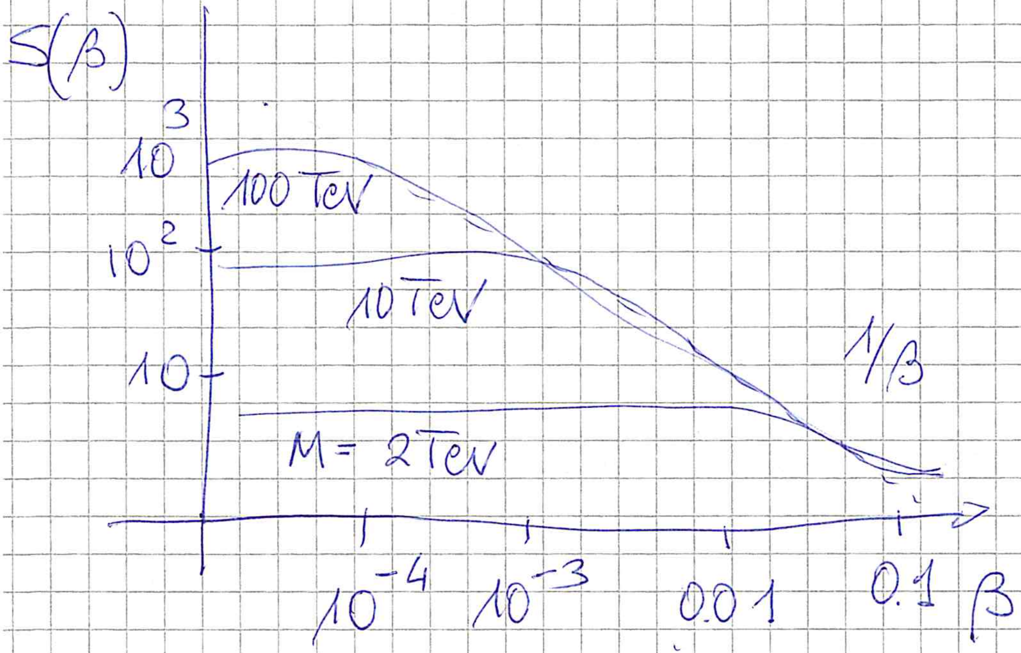

Usually for WIMP decoupling: consider

$$\sigma(\beta) = S(\beta)\,\sigma_0(\beta)$$

& computes it's thermal average as usual

$$\langle \sigma v \rangle_T = \frac{M^4}{4T^4 K_2^2\left(\frac{T}{M}\right)} \int_{\frac{2M}{T}}^{\infty} dz\, z^2 \left(1 - \frac{4M^2}{z^2}\right) \sigma\, K_1(z)$$

since the ~~ex~~ velocity distribution peaks at $v/c \simeq \sqrt{\frac{T}{M}}$ the Sommerfeld effect becomes relevant for temperatures such that

$\beta^2 \gg \frac{\alpha m_V}{M}$ i.e. $T \gg \alpha m_V$

& still $\beta \ll 1$ i.e. $T \lesssim \alpha^2 M$

Lately it has also been found that even bound-states can be formed for sufficiently strong interaction (e.g. colored states)

Those always annihilate & reduce the number density of DM

$\Longrightarrow$ coupled system of Boltzmann equations including both scattering & bound states

Kept in equilibrium by soft emissions

$$\text{DM DM} \Longleftrightarrow \text{Bound State} + \gamma$$

Of course bound states are energetically favored for small T due to Boltzmann factor

$$\frac{n_{BS}}{2n_{DM}} \sim e^{+\frac{\Delta M}{T}} \qquad \Delta M = \text{binding energy}$$

But the presence of a thermal bath can modify the rates:

$\longrightarrow$ affect the Coulomb potential e.g. by Debey screening via $\omega_D = \alpha T$

$\longrightarrow$ destroy bound states by absorption of thermal radiation

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New formalism to include both the Sommerfeld enhancement & bound states at finite temperature: (4)

Tobias Binder, L.C. Kyohei Mukaida

arxiv 1808.06472

PRD 98 (2018) 115023

Based on real time thermal field theory on the Keldysh contour.

Consider the NR effective action for the DM particles, i.e.

$$S_{NR}[\eta, \xi] = \int \eta^{+}(x)\left[i\partial_t + \frac{\nabla^2}{2M}\right]\eta(x) +$$

$$+\int \xi^{+}(x)\left[i\partial_t - \frac{\nabla^2}{2M}\right]\xi(x) + \int i\,\frac{g_X^2}{2}\,J(x)\,D(x,y)\,J(y)$$

$$+\int i\,O_S^{+}(x)\,\Gamma_S(x,y)\,O_S(y)$$

η / ξ particle / antiparticle DM

$D(x,y) \longrightarrow$ gauge boson propagator

$J = \eta^{+}\eta + \xi^{+}\xi$ U(1) DM current (NR)

$O_S = \xi^{+}\eta$ $\qquad \Gamma_S \sim \frac{\pi\alpha_X^2}{M^2}$ annihilation rate

Thermal bath includes: $U(1)$ gauge boson & light fermions ψ

Massive mediator in thermal bath in the HTL approximation, i.e. external energy & momentum smaller than loop momentum, possibly of order $k \sim T$. This is relevant for the soft static propagator: $|\vec{p}| < T$

$$\lim_{\omega \to 0} D^{++}(\omega, \vec{p}) = \frac{i}{\vec{p}^2 + m_V^2 + m_D^2} + \pi \frac{T}{|\vec{p}|} \frac{m_D^2}{(\vec{p}^2 + m_V^2 + m_D^2)^2}$$

"screened" potential — imaginary part

where $m_D^2 = \frac{g_\psi^2 T^2}{3}$

i.e. thermal correction of the mass important for $m_D \gtrsim m_V$ i.e. $T \gtrsim \sqrt{3}\, \frac{m_V}{g_\psi}$

compared to $T=0$ approach for $T > \alpha_\psi m_V$

so in the range $\alpha_\psi m_V \leq T \leq \sqrt{3}\, \frac{m_V}{g_\psi}$

the usual approach with $T \simeq 0$ probably OK

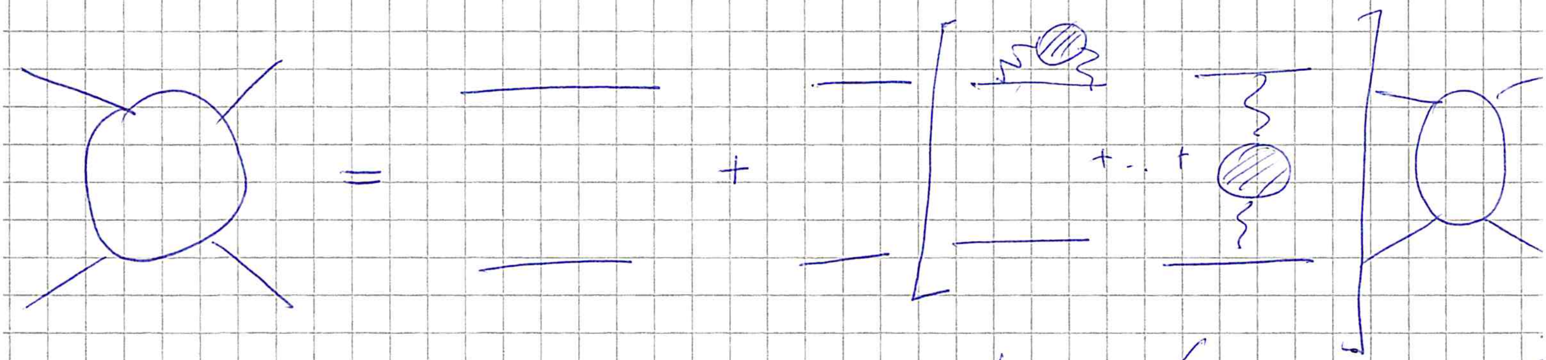
$\Rightarrow$ modification of the Schrödinger equation:

$$V_{eff}(\vec{r}) = -i g_X^2 \int_{-\infty}^{+\infty} d^3q \left(1 - e^{i\vec{q}\cdot\vec{r}}\right) D^{++}(0, \vec{q})$$

$$= -\alpha_X m_D - \frac{\alpha_X}{r} e^{-m_D r} - i \alpha_X T \phi(m_D r)$$

To describe 2-particle states, relevant for scattering & bound states, need the e.o.m. for the 4-point function

$\Longrightarrow$ Two-time Bethe-Salpeter equation.



Interactions of thermal bath with DM

Screening of Coulomb force by thermal bath!

Particular resummation scheme making sure that the resummed Green's function satisfy basic equilibrium relations, i.e. KMS, & has correct vacuum limit & linear response limit.

No vertex corrections included, only self-energy!

From these equation in the dilute limit one can solve for $G^{++--}_{\eta\xi}$ 4-point function that is related to the previous Schrödinger equation

$\longrightarrow$ extract the effective potential