Testing General Relativity with Cosmological Observations

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Outline

Introduction

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- Light deflection
- Perihel advance of mercury & many other binary systems
- Shapiro time delay
- $\cdots$
- Gravitational waves

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$$R_{\mu\nu} = 0\,.$$

Can we also test these equations with matter,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad ?$$

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Cosmology is a non-vacuum solution of Einstein's equation:

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \qquad z + 1 = a_0/a(t)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{8\pi G}\right)$$

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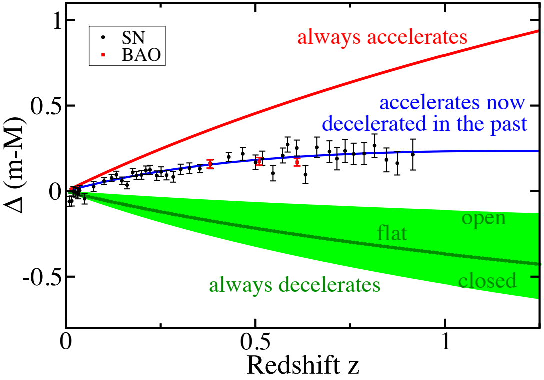
$$F = \frac{L}{4\pi d_L^2}$$

$$d_L(z) = (1+z)\chi_K\left(\int_0^z \frac{dz'}{H(z')}\right), \qquad \chi_K(\lambda) = \frac{\sin(\sqrt{K}\lambda)}{\sqrt{K}}$$



$m - M \propto \log d_L$

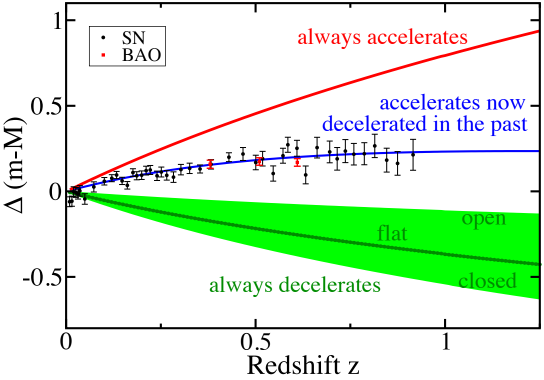
$\Delta(m - M) \propto \log(d_L/d_L^{\text{Milne}})$

$\Omega_\Lambda \simeq 0.7$

$\Omega_m \simeq 0.3$

$\Omega_X = \rho_X/\rho_c$

Compilation by Huterer & Shafer '17.
Binned from 870 SNe Ia (black) and 3 BAO points (from BOSS DR12, red).



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NO !

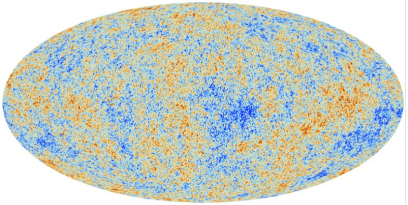
We have 'postulated' the existence of dark matter and dark energy to fit this data.

Introduction

In this talk I shall argue that with the help of clustering observations, i.e. using the fact that the Universe is not perfectly homogeneous and isotropic, we can actually test Einstein's equations to some extent. . .

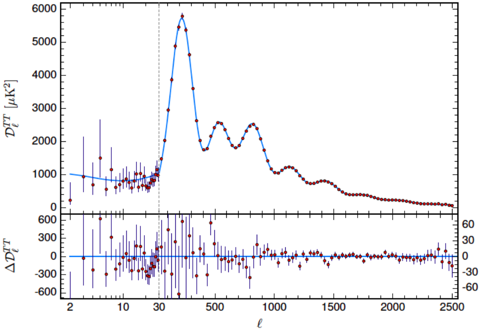
The CMB

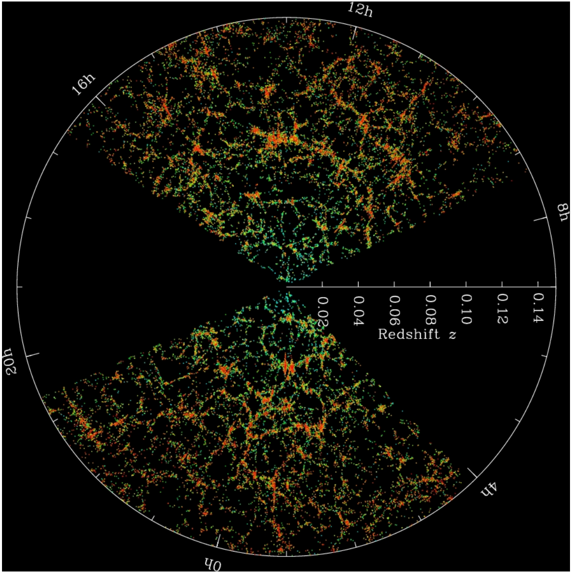
CMB sky as seen by Planck



$D_\ell = \ell(\ell+1)C_\ell/(2\pi)$

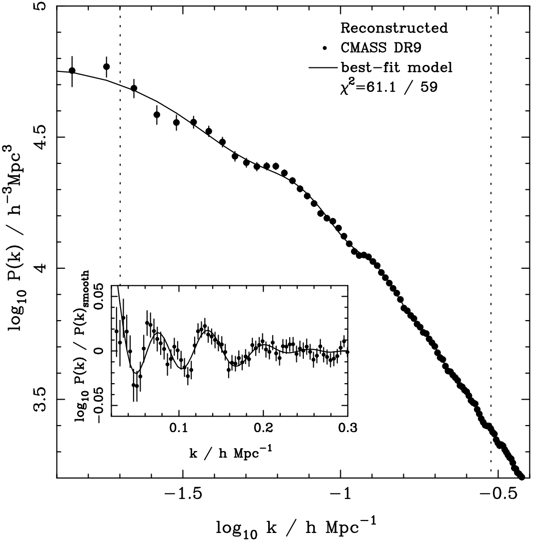
The Planck Collaboration: Planck results 2018 [1807.06209]





M. Blanton and the Sloan Digital Sky Survey Team.

Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)



from Anderson et al. '12

SDSS-III (BOSS) power spectrum.

Galaxy surveys $\simeq$ matter density fluctuations, biasing and redshift space distortions.

But...

- We have to take fully into account that all observations are made on our past lightcone which is itself perturbed.
  We see density fluctuations which are further away from us, further in the past.
  We cannot observe 3 spatial dimensions but 2 spatial and 1 lightlike, more precisely we measure 2 angles and a redshift.

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- But of course much more for future surveys like DESI, Euclid, WFIRST and SKA.

Cosmological distances

In a Friedmann Universe the (comoving) radial distance is

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + \Omega_\Lambda}}$$

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Depending on the observational situation we measure directly $r(z)$ or

$$\begin{aligned} d_A(z) &= \frac{1}{(1+z)}\chi_K(r(z)) \quad \text{the angular diameter distance} \\ d_L(z) &= (1+z)\chi_K(r(z)) \quad \text{the luminosity distance.} \end{aligned}$$

At small redshift all distances are $d(z) = z/H_0 + \mathcal{O}(z^2)$, for $z \ll 1$. At larger redshifts, the distance depends strongly on $\Omega_K,\ \Omega_\Lambda, \cdots$.

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- Whenever we convert a measured redshift and angle into a length scale, we make assumptions about the underlying cosmology.

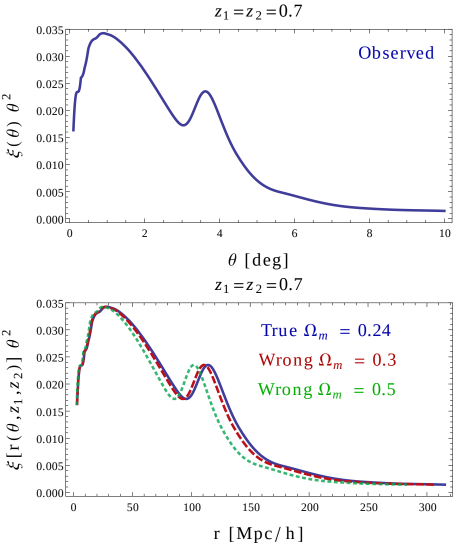
Very large scale galaxy surveys

If we convert the measured correlation function $\xi(\theta, z_1, z_2)$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales.

$$r(z_1, z_2, \theta) \stackrel{(K=0)}{=} \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}.$$

$$r_i = r(z_i) = \int_0^{z_i} \frac{dz}{H(z)}$$

(Figure by F. Montanari)



We now consider fluctuations in the matter distribution and in the geometry first to linear order. (See Yoo et al. '09; Yoo '10, Bonvin & RD '11; Challinor & Lewis '11)

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We can count the galaxies inside a redshift bin and small solid angle, $N(\mathbf{n}, z)$ and measure the fluctuation of this count:

$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)} = \frac{\rho(\mathbf{n}, z)V(\mathbf{n}, z) - \bar{\rho}(z)\bar{V}(z)}{\bar{\rho}(z)\bar{V}(z)}.$$

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$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle \,, \qquad \mathbf{n} \cdot \mathbf{n}' = \cos\theta \,.$$

This quantity is directly measurable.

The total galaxy density fluctuation per redshift bin, per sold angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations from scalar perturbations to 1st order

$$\begin{aligned}\Delta(\mathbf{n},z) \quad = \quad & bD_{cm} - (2-5s)\Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot{\Phi} + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2-5s}{r(z)\mathcal{H}} + 5s\right) & \left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r(z)} dr(\dot{\Phi}+\dot{\Psi})\right) \\ & + \frac{2-5s}{2r(z)}\int_0^{r(z)} dr\left[2 - \frac{r(z)-r}{r}\Delta_\Omega\right](\Phi+\Psi).\end{aligned}$$

(Bonvin & RD '11, Challinor & Lewis '11)

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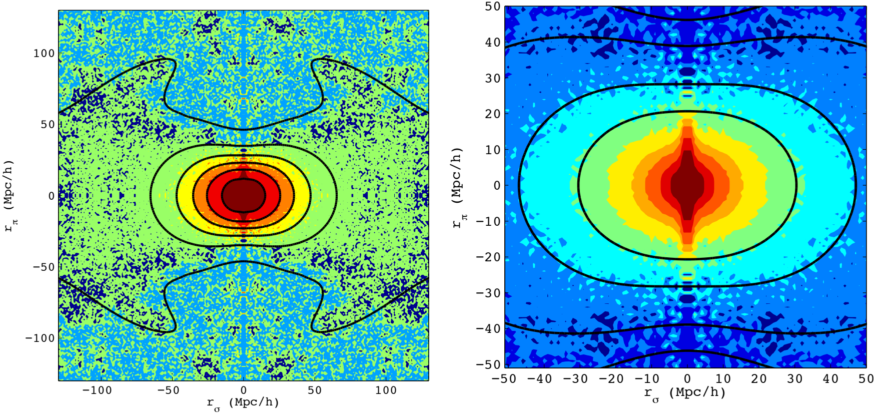
(Bonvin & RD '11, Challinor & Lewis '11)

Redshift space distortions in the BOSS survey

(from Reid et al. '12)



The angular power spectrum of galaxy density fluctuations

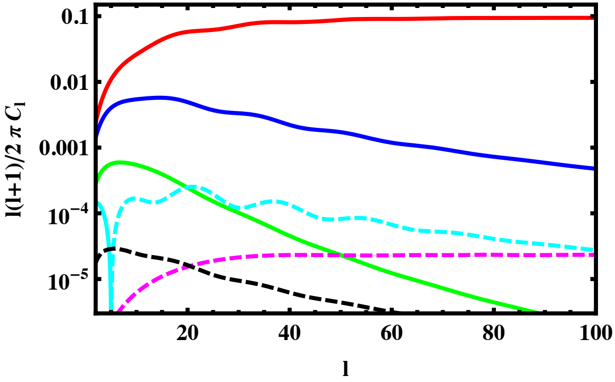
For fixed z, we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \qquad C_\ell(z, z') = \langle a_{\ell m}(z) a^*_{\ell m}(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(\cos\theta)$$

$$\cos\theta = \mathbf{n} \cdot \mathbf{n}'$$

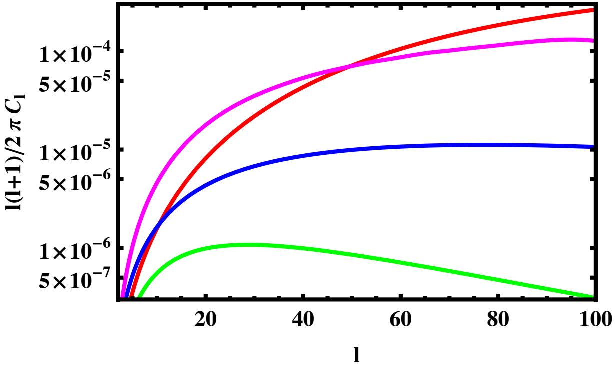
Contributions to the transverse power spectrum at redshift $z = 0.1,\ \Delta z = 0.01$ (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{Doppler}$ (cyan), $C_\ell^{lensing}$ (magenta) C_ℓ^{grav} (black).

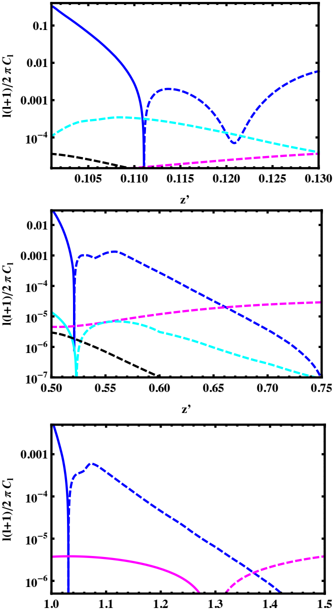
The transversal power spectrum

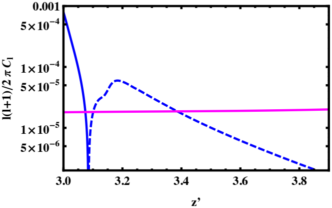
Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$ (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{lensing}$ (magenta).

The radial power spectrum

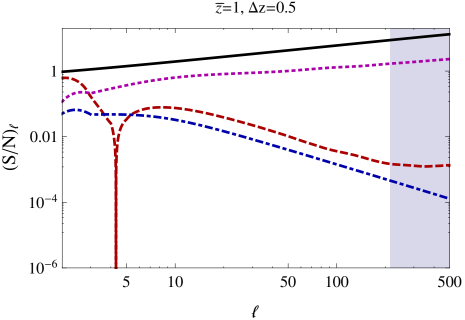




The radial power spectrum $C_\ell(z,z')$
for $\ell = 20$
Left, top to bottom: $z = 0.1,\ 0.5,\ 1,$
top right: $z = 3$

Standard terms (blue), $C_\ell^{lensing}$ (magenta), $C_\ell^{Doppler}$ (cyan), C_ℓ^{grav} (black),
(from Bonvin & RD '11)

The signal to noise of different contributions for an Euclid-like survey:



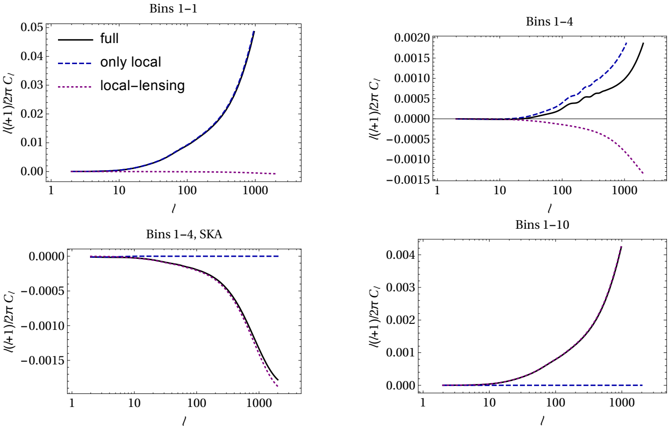
— total, - - - rsd, · · · lensing, · - · - large scale relativistic terms

From Di Dio, Montanari, RD & Lesgourgues (2013).

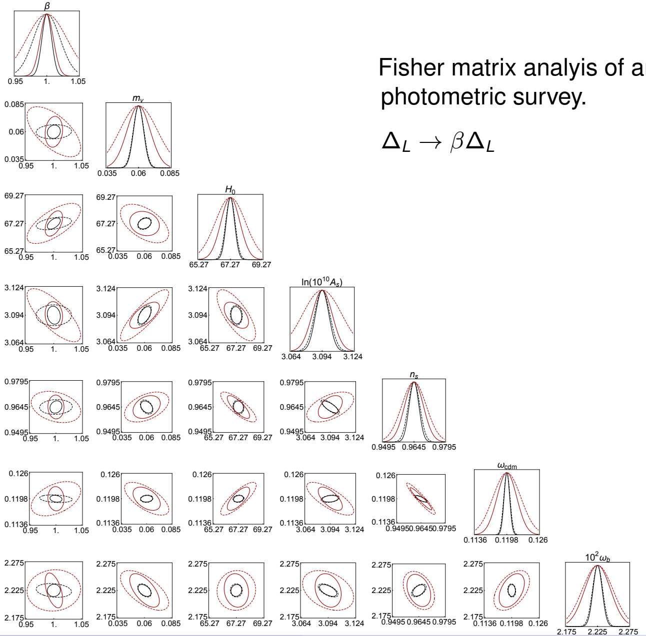
Well separated redshift bins measure mainly the lensing-density correlation:

$$\langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle \simeq \langle \Delta^L(\mathbf{n}, z)\delta(\mathbf{n}', z')\rangle \quad z > z'$$

$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$



(Montanari & RD) [1506.01369]



Fisher matrix analyis of an Euclid-like photometric survey.

$$\Delta_L \to \beta\Delta_L$$

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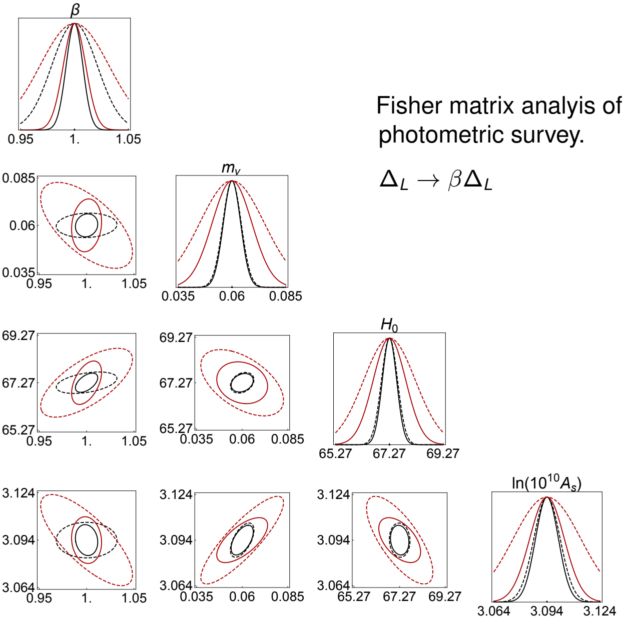
Testing GR with the lensing potential



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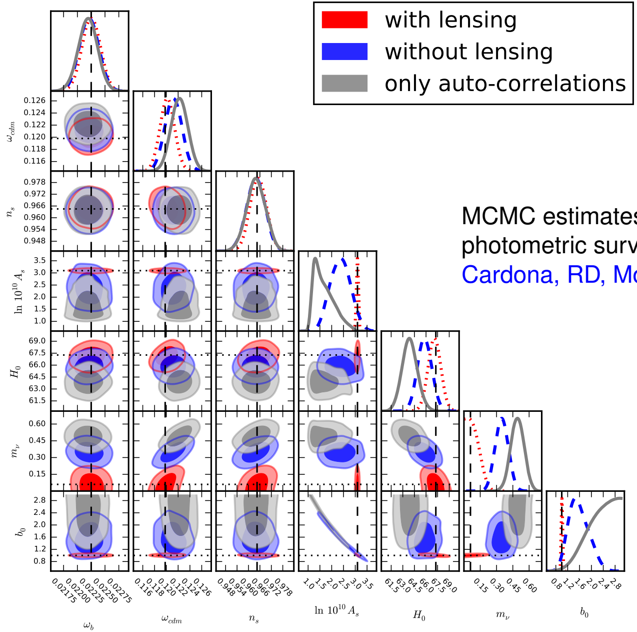
Neglecting the lensing potential biases cosmological parameters



MCMC estimates for an Euclid-like photometric survey
Cardona, RD, Montanari & Kunz, (2016)

E_g statistics

In GR photon propagation, which governs weak lensing is sensitive to the sum of the Bardeen potentials, $\Phi + \Psi$, which density fluctuations generate Φ. In standard GR $\Phi = \Psi$ such that the following combination is independent of both, bias and scale:

$$E_g(k, z) \equiv \frac{H(z)(\Phi + \Psi)}{3H_0^2(1+z)V} = f(z) \simeq [\Omega_m(z)]^{0.55} .$$

(Zhang et al., 2007) This can be converted to (Pullen et al., 2015)

$$E_g(\ell, z) = \Gamma(z) \frac{C_\ell^{\kappa\delta}(z_*, z)}{\beta C_\ell^{\delta\delta}(z, z)}$$

It has, however been pointed out (Moradinezhad Dizgah & RD 2016), that when observing galaxies, we do not directly observe $C_\ell^{\kappa\delta}$ or $C_\ell^{\delta\delta}$ but rather

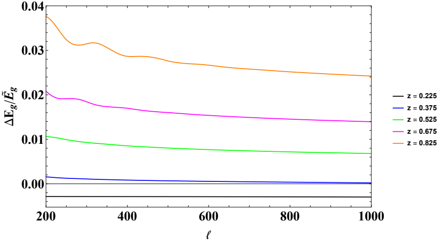
$$\begin{aligned} C_\ell^{\kappa g}(z_1, z_2) &\simeq b(z_2) C_\ell^{\kappa\delta}(z_1, z_2) - (2 - 5s(z_2)) C_\ell^{\kappa\kappa}(z_1, z_2) \\ C_\ell^{gg}(z_1, z_2) &\simeq b(z_1) b(z_2) C_\ell^{\delta\delta}(z_1, z_2) + (2 - 5s(z_1))(2 - 5s(z_2)) C_\ell^{\kappa\kappa}(z_1, z_2) \\ & \quad - b(z_2)(2 - 5s(z_1)) C_\ell^{\kappa\delta}(z_1, z_2) - b(z_1)(2 - 5s(z_2)) C_\ell^{\kappa\delta}(z_2, z_1) \end{aligned}$$

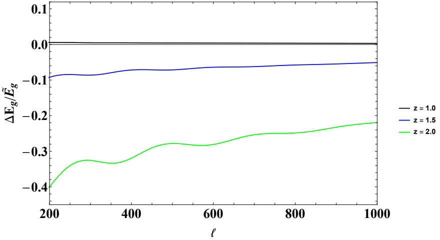
For low redshifts these corrections are not very relevant, but at high redshifts they are.

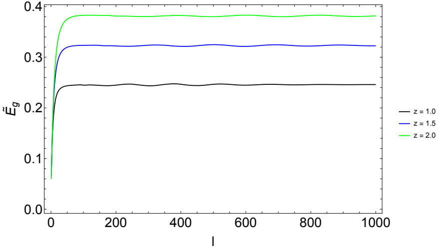
E_g statistics

Euclid like survey

DES-like survey







(From Ghosh & RD [1812.09546])

For intensity mapping $s \equiv 0.4$ and the correction terms vanish (Pourtsidou 2016).

Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These are easier to measure (less noisy) but:
  • they require an fiducial input cosmology converting redshift and angles to length scales,
  $$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z')\cos\theta}\ .$$
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  This complicates especially the determination of error bars in parameter estimation
  • it is not evident how to correctly include lensing.
- Future large & precise 3d galaxy catalogs like **Euclid**, **DESI**, **SKA**, **LSST** etc. will be able to determine directly the measured 3d correlation functions and spectra, $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.

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- These spectra are not only sensitive to the matter distribution (density) but also to the velocity via (redshift space distortions) and to the perturbations of spacetime geometry (lensing) .

Conclusions

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- Especially $C_\ell^{\kappa g}(z,z')$ and $C_\ell^{gg}(z,z')$ if suitably corrected allow for quite model independent tests of GR via e.g. the E_g-statistics.
- The spectra $C_\ell(z,z')$ depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities, bias, etc. Their measurements provide a new route to estimate cosmological parameters and to test general relativity.
