Observables in perturbative quantum gravity and perturbative quantum cosmology

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The pAQFT perspective

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- The aim of this program is to study some aspects of observables in QG that are accessible to perturbative methods and to learn more about the algebraic structure they define.
- The ultimate goal is to break away from the classical picture and have an intrinsically quantum formulation.

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- Strict locality is in conflict with diffeomorphism invariance (at least for non-compact M). Main proposals for non-local diff invariant observables: relational observables, dressed observables (analogy to QED and Wilson loops).
- A weaker notion: require all the functional derivatives $\frac{\delta^n F}{\delta g^n}(g_0)[h]$ to be local. This is sufficient for perturbative renormalization in the sense of Epstein-Glaser.

Relational observables I

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- We realize the choice of a coordinate system by constructing four scalars X_g^μ, $\mu = 0, \ldots, 3$ which will parametrize points of spacetime. The fields X_g^μ should transform under diffeomorphisms χ as

$$X^\mu_{\chi^* g} = X^\mu_g \circ \chi \, ,$$

Relational observables II

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- Take another local field $A[g](x)$ (e.g. a metric scalar). Then
$$\mathcal{A}_g := A_g \circ \alpha_g$$
is invariant under diffeos.

Relational observables III

Physical interpretation

Fields X_g^μ are configuration-dependent coordinates such that $[A[g] \circ X_g^{-1}](Y)$ corresponds to the value of the quantity $A[g]$ provided that the quantity X_g has the value $X_g = Y$.

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- By considering $\mathcal{A}[g] = A_g \circ X_g^{-1} \circ X_{g_0}$ we identify this observable with a functional

$$F_{\mathcal{A}}(g) = \int \mathcal{A}[g](x) f(x) = \int A[g](X_g^{-1}(Y)) f(X_{g_0}^{-1}(Y)) \,,$$

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  for a test density f.
- If X_g^μ and $A[g]$ are all local fields themselves, then $F_{\mathcal{A}}$ is non-local with local derivatives.

Examples:

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- See also papers by Fröb et. al. [1703.01158], [1801.02632].

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  - Interaction introduced in the causal approach to renormalization due to Epstein and Glaser ([Epstein-Glaser 73]),
  - Generalization to gauge theories using homological algebra ([Hollands 08, Fredenhagen-KR 11]).

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- Typically $\mathcal{E}(M)$ is a space of smooth sections of some vector bundle $E \xrightarrow{\pi} M$ over M. For the scalar field: $\mathcal{E}(M) \equiv \mathcal{C}^\infty(M, \mathbb{R})$. For perturbative gravity $\mathcal{E}(M) = \Gamma((T^*M)^{\otimes 2})$.

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- The choice of action functional S specifies the dynamics. We use a modification of the Lagrangian formalism (fully covariant).

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- On $\mathcal{F}(M)$ we introduce first classical dynamics by means of a Poisson structure (Peierls bracket): $\{F, G\} = \left\langle \frac{\delta F}{\delta\varphi}, \Delta \frac{\delta G}{\delta\varphi} \right\rangle$, where $\Delta = \Delta^{\mathrm{R}} - \Delta^{\mathrm{A}}$ (Green functions for the linearized action).

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- Use the deformation quantization to construct the non-commutative algebra $\mathfrak{A}(M) = (\mathcal{F}(M)[[\hbar]], \star)$, such that

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- We work all the time on the same vector space of functionals, but we equip it with different algebraic structures (Poisson bracket, $\star$-product).

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- Later on we will see that physical quantities do not depend on this split (background independence).

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$$\mathrm{supp}(\Delta^{\mathrm{R}}) \subset \{(x,y) \in M^2 | y \in J_-(x)\},$$
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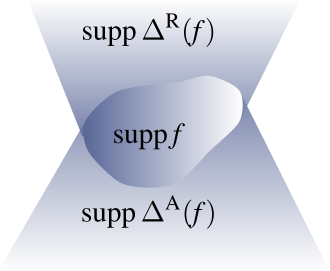

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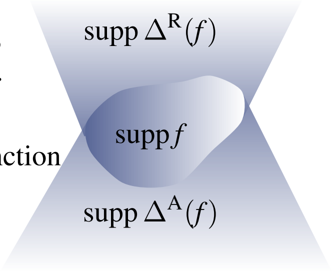

- Their difference is the Pauli-Jordan function

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- The free QFT is defined as $\mathfrak{A}_0(M) \doteq (\mathcal{F}(M)[[\hbar]], \star, *)$, where $F^*(\varphi) \doteq \overline{F(\varphi)}$ and $\mathcal{F}(M)$ is an appropriate functional space (some WF set conditions on $F^{(n)}(\varphi)$s induced by W).

Example: free scalar field

- **Smeared fields**: Let $\mathcal{D}(M) = \mathcal{C}^\infty_c(M,\mathbb{R})$ and $f, f' \in \mathcal{D}(M)$.

$$\Phi(f)[\varphi] \doteq \int f(x)\varphi(x)d\mu_g(x), \quad \Phi(f')[\varphi] \doteq \int f'(x)\varphi(x)d\mu_g(x)$$

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- Formally, we can consider $\Phi_x\doteq\Phi(\delta_x)$, where δ_x is the Dirac delta supported at some $x\in M$.
- for $M=\mathbb{M}$ (Minkowski spacetime):
  $[\Phi_{(0,\mathbf{x})},\Phi_{(0,\mathbf{y})}]_\star=\Delta(0,\mathbf{x};0,\mathbf{y})=0.$
  $[\Phi_{(0,\mathbf{x})},\dot{\Phi}_{(0,\mathbf{y})}]_\star=\partial_{y^0}\Delta(0,\mathbf{x};0,\mathbf{y})=i\hbar\delta(\mathbf{x}-\mathbf{y})$, where dot denotes the time derivative.

Building models in pAQFT: Interaction

- To introduce the interaction, we construct, for a given interaction term $V \in \mathcal{F}_(M)$, the formal S-matrix

$$\mathcal{S}(\lambda V) \doteq \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i\lambda}{\hbar}\right)^n V \cdot_{\mathcal{T}} \ldots \cdot_{\mathcal{T}} V \,,$$

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- Let $F \in \mathcal{F}(M)$ be a functional with local derivatives. It can be non-local, e.g. $F = \int \mathcal{A}_g(x) f(x)$.
- We define the interacting field corresponding to F by

$$F_{\text{int}} = -i\hbar \frac{d}{dt} \left(\mathcal{S}(V)^{-1} \star \mathcal{S}(V + tF)\right)\Big|_{t=0} \,,$$

  where the inverse of $\mathcal{S}$ is the $\star$-inverse.

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  1. **Causal factorization**: $\mathcal{S}(F+G) = \mathcal{S}(F) \star \mathcal{S}(G)$, if $\mathrm{supp}(F)$ is later than $\mathrm{supp}(G)$.

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- Gauge invariance is guaranteed by additional renormalization conditions (Ward identities).

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  where the inverse of $\mathcal{S}$ is the $\star$-inverse.
- Given s, the free classical BV operator, the interacting quantum BV operator $\hat{s}$ is defined by

$$s(F_{\text{int}}) = (\hat{s}F)_{\text{int}} .$$

  The cohomology of $\hat{s}$ characterizes the space of gauge invariant quantum observables.

Correlation functions

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- Interacting correlation functions are obtained as:

$$(\Phi_{\text{int}}(f_1) \star \dots \star \Phi_{\text{int}}(f_n))(0)\,,$$

similarly for other observables in the theory.

What about gravity?

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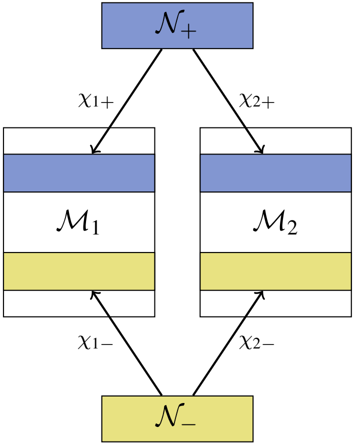
- Formally, this product depends only on L, not on the splitting (Hawkins, KR [1612.09157]). Some obstructions could appear when renormalization is performed.

Relative Cauchy evolution

- Let $\mathcal{N}_+$ and $\mathcal{N}_-$ be two spacetimes that embed into two other spacetimes $\mathcal{M}_1$ and $\mathcal{M}_2$ around Cauchy surfaces, via admissible embeddings $\chi_{k,\pm}, k = 1, 2$.



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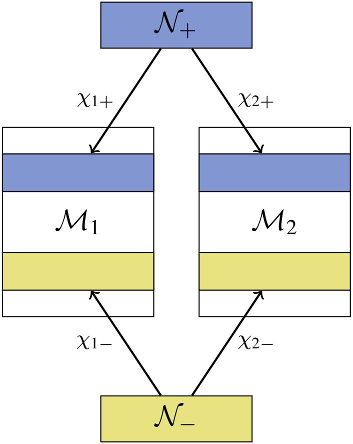
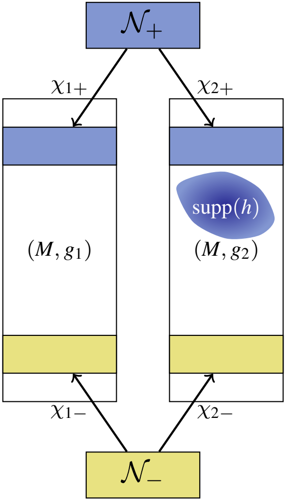
- Then $\beta = \mathfrak{A}\chi_{1+} \circ (\mathfrak{A}\chi_{2+})^{-1} \circ \mathfrak{A}\chi_{2-} \circ (\mathfrak{A}\chi_{1-})^{-1}$ is an automorphism of $\mathfrak{A}(\mathcal{M}_1)$. This is the consequence of the Time-slice axiom of LCQFT.



Background independence

- Let $\mathcal{M}_1 = (M, g_1)$ and $\mathcal{M}_2 = (M, g_2)$, where g_1 and g_2 differ by a (compactly supported) symmetric tensor h (see the diagram).



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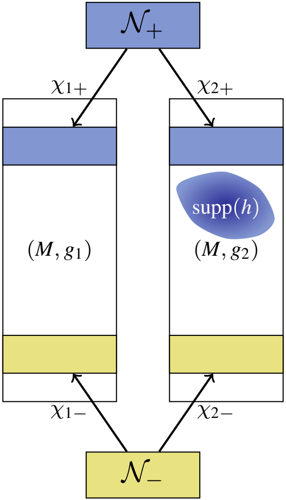

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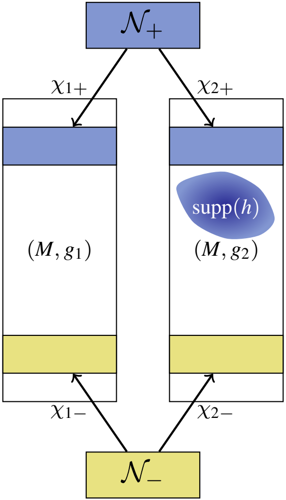
- The infinitesimal background independence is the condition $\Theta_{\mu\nu} = 0$.



Background independence II

- We have shown that

$$(\Theta^{\mu\nu}(F))_{\rm int} \stackrel{o.s.}{=} -\frac{i}{\hbar}[T^{\mu\nu}_{\rm int}, F_{\rm int}]_\star \,,$$

where $T^{\mu\nu}_{\rm int}$ is the interacting stress-energy tensor of L.

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- Next we used the renormalization freedom to ensure that $T^{\mu\nu}_{\text{int}} = 0$ holds, so the interacting theory is background independent.

Conclusions

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- We have shown that our theory is background independent, i.e. independent of the split into free and interacting part.

Thank you very much for your attention!