Cosmological Perturbations & Applications to DM & DE Models

1. Introduction

Mainz 1

s-t sketch of SBB — formation of structure problem

s-t sketch of SGC — Newt. pert. regime

Relativistic pert. regime

quantum regime

2. Newtonian Fluctuations

* $\delta\ddot{\rho} \sim G\delta\rho$ (sketch)
* gravity: φ
  matter: ρ, p, v, s } variables
* $$\left.\begin{aligned} &\dot{\rho} + \nabla\cdot(\rho \underline{v}) = 0 \\ &\dot{\underline{v}} + (\underline{v}\cdot\nabla)\,\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0 \\ &\nabla^2\varphi = 4\pi G\rho \\ &\dot{s} + (v\cdot\nabla)\, s = 0 \\ &p = p(\rho, s) \end{aligned}\right\} \text{EOM}$$
* perturbation ansatz $\rho = \rho_0 + \delta\rho$
  procedure
* $$\delta\ddot{\rho} - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = \sigma\nabla^2\delta s$$
  $$\delta\dot{s} = 0$$

  $$\delta p = c_s^2\delta\rho + \sigma\,\delta s$$
  $$c_s^2 = \left.\left(\frac{\delta p}{\delta\rho}\right)\right|_s$$

  Note: $W = \frac{p}{\rho} \neq c_s^2$
* $$k_J^2 = \frac{4\pi G\rho_0}{c_s^2}$$ Jeans length

* Each Fourier mode indep.

  $k > k_J$ oscillations

  $k < k_J$ growth

  $k_J \simeq H$ if $c_s^2 \simeq 1$

  $k_J = \infty$ if $c_s^2 = 0$

  entropy pert. do not grow

  " " seed density flucts.

* Newt. pert. in expanding background

  $\delta_\varepsilon = \frac{\delta\rho}{\rho}$

  $$\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{T}{\rho_0 a^2}\delta S$$

  sub-Jeans $\quad \delta_k(t) \sim a^{-1/2}$ oscill.

  super- " $\quad \delta_k(t) = c_1 t^{2/3} + c_2 t^{-1}$

* sketch (t, x) $\quad t_{eq}, t_{rec}, t_0$

  $$\delta_A(t_{eq}) = \left(\frac{a(t_{rec})}{a(t_{eq})}\right)^{3/2}\delta_B(t_{eq})$$

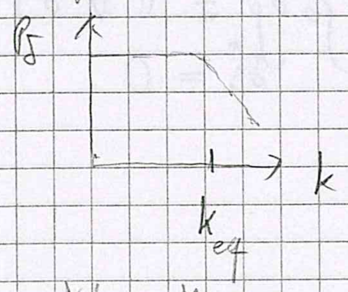
  δ | t sketch

  evidence for CDM

* CDM ≠ coherently oscillating scalar field

  $c_s^2 = 1$

* Application: turnover of matter power spectrum



  Note: Meszaros effect

  δ_ε does not grow in radiation phase

* Application: DE

  Cannot be a fluid with $c_s^2 = w = -1 + \varepsilon$

Reason: ultraviolet UV instability

3. Relativistic Perturbations

* variables: $\delta g_{\mu\nu}$ metric 10 dof
  $\delta\varphi$ matter 1 dof

* $g_{\mu\nu}(\underline{x}, t) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(x, t)$
  $\varphi(\underline{x}, t) = \varphi^{(0)}(t) + \delta\varphi(x, t)$
  $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$

* Classification
  Elimination of coord. artefacts 4 dof

scalar	4 + 1	− 2	2 + 1	− 2	1 dof
vector	4	− 2			2
tensor	2				2

Independent at linear order
single matter does not induce vectors & tensors at linear order

$$ds^2 = a^2[(1+2\phi)d\eta^2 - (1-2\phi)\gamma_{ij}dx^i dx^j]$$
$$\varphi = \varphi^{(0)} + \delta\varphi$$

* $$\phi'' + 2\left(\mathcal{H} - \frac{\varphi_0''}{\varphi_0'}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\varphi_0''}{\varphi_0'}\right)\phi = 0$$

  Compare
  $$\delta_\varepsilon'' + 2H\delta_\varepsilon' - \frac{c_s^2}{a^2}\nabla^2\delta_\varepsilon - 4\pi G\rho\,\delta_\varepsilon = 0$$

  Note: $c_s^2 = 1$ even though $w = 0, \frac{1}{3}$
  $m^2\varphi^2$ $\lambda\varphi^4$

* Application to DM

  DM $\neq$ oscillating scalar field

* Application to DE

  Quintessence s.h. as DE candidate

* Suppression of structure formation in Quintessence for $z < z_c$

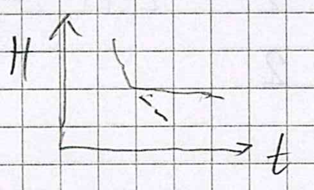
* Application to Hubble tension

  age problem for $\Lambda = 0$ CDM

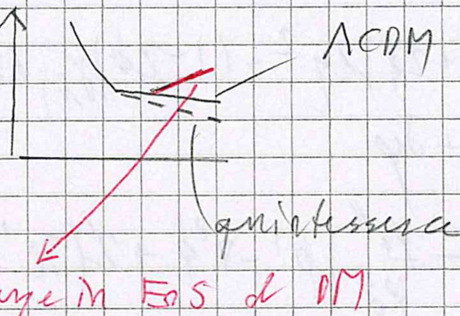
$$H_0 = \frac{2}{3 t_0}$$

add $\Lambda \Rightarrow t_0 > \frac{2}{3} H_0^{-1} \Rightarrow \frac{t_0}{\tau_0} \nearrow$

$\Rightarrow H_0 \searrow$



interpreted in terms of ΛCDM, the model provides smaller H_0



4. Quantum Fluctuations

* $S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right]$

$ds^2 = a^2(\eta) \left[(1+2\phi) d\eta^2 - (1-2\phi) dx^2 \right]$

$\varphi = \varphi_0(\eta) + \delta\varphi(x,\eta)$

* $S^{(2)} = \int d^4x \left[\frac{1}{2} \partial_\mu v \, \partial^\mu v - \frac{z''}{z} v^2 \right]$

$v = a \left(\delta\varphi + \frac{\varphi_0'}{\mathcal{H}} \phi \right)$ $\qquad v = z \zeta$

$z = a \, \varphi_0' / \mathcal{H}$

$v_k'' + \left(k^2 - \frac{z''}{z} \right) v_k = 0$

* $k > \mathcal{H}$ $\quad v_k$ oscillates

$k < \mathcal{H}$ $\quad v_k$ squeezed

$v_k(\eta_i) = \frac{1}{\sqrt{2k}}$ if quantum vacuum IC

* Ex: inflation (sketch)

$$P_\zeta(k,\eta) = k^3 |\zeta_k(\eta)|^2 = k^3 z^{-2}(\eta) |v_k(\eta)|^2$$

$$= k^3 z^{-2}(\eta) \underbrace{|v_k(t_H(k))|^2}_{v_k(t_H)^2 \sim \frac{1}{k}} \underbrace{\left(\frac{z(\eta)}{z(\eta_H(k))} \right)^2}_{\underbrace{\left(\frac{a(\eta)}{a(\eta_H(k))} \right)^2}_{\left(\frac{H}{k} \right)^2}}$$

$$a(t_H(k))^{-1} k^{+1} = H$$

$$= H^2 \left(\frac{a}{z}(\eta) \right)^2$$

* gravit. waves

$h = a u$ $\qquad u$ canonical

$u_k'' + \left(k^2 - \frac{a''}{a} \right) u_k = 0$

$P_h(k,\eta) = k^3 a^{-2}(\eta) |u_k(\eta)|^2 = H^2$

* slight red tilt

* Entropy fluctuations

$$\zeta = \frac{p}{\rho + p} \delta S$$

important: whenever light spectator scalar fields

sketch TD case

* Note: Generic $\nu \sim 1$

not only for inflation

one exception: Ekpyrotic

* Ex Matter Bounce

$$V_k(\eta) = c_1 \eta^2 + c_2 \eta^{-1}$$

↑ dominant

$$P_\zeta(k, \eta) \sim k^3 |V_k(\eta)|^2 a^{-2}$$

$$\sim k^3 |V_k(\eta_H(k))|^2 \left(\frac{\eta_k}{\eta}\right)^2 \sim k^3 k^{-1} k^{-2}$$

$$\eta_H(k) = k^{-1}$$

* Ex SGC