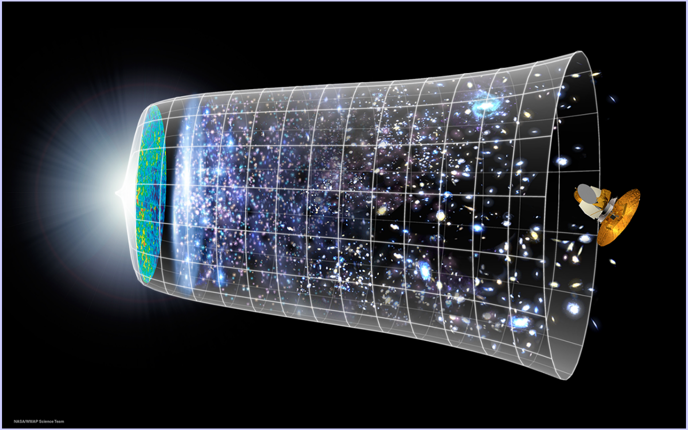
Theory of Cosmological Perturbations and Applications to Early Universe Cosmology

Robert Brandenberger
McGill University

Univ. of New Brunswick, May 28 2019

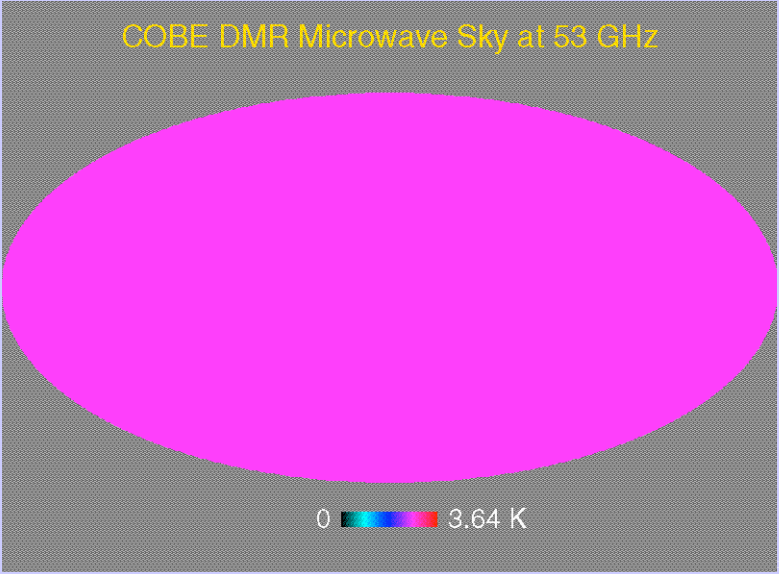
Outline

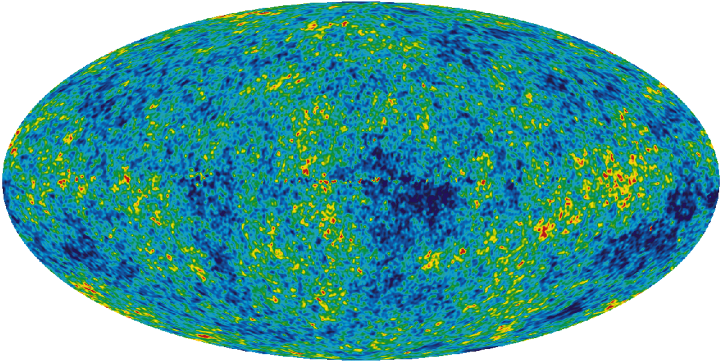
Plan



Credit: NASA/WMAP Science Team

Isotropic CMB Background





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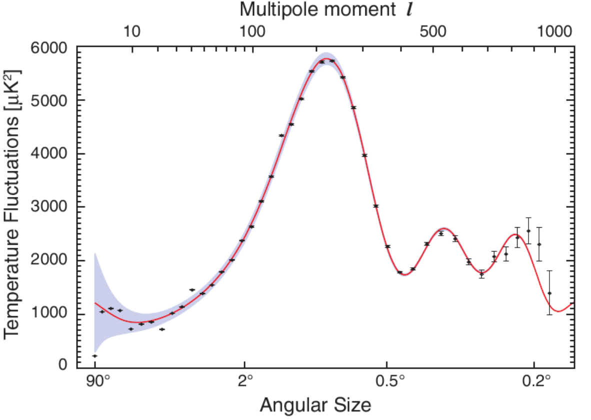
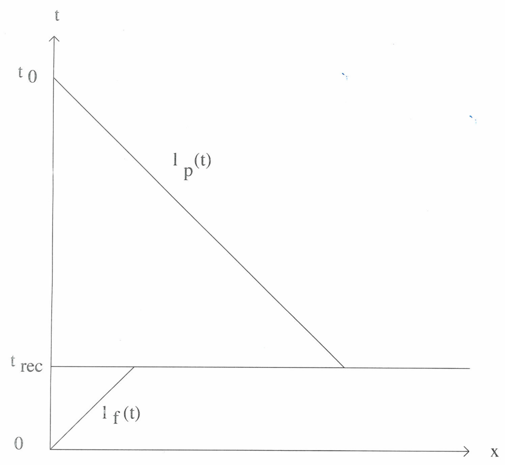
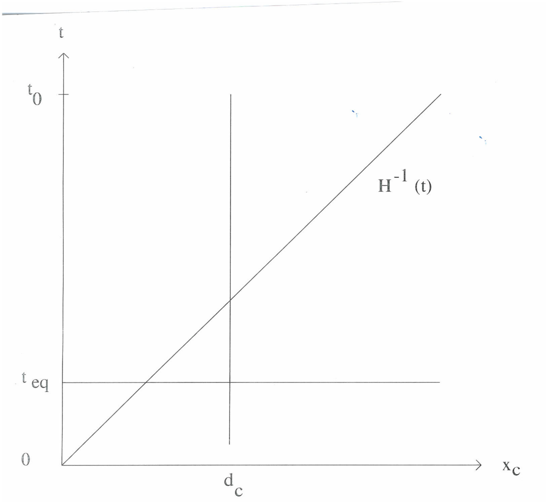


Credit: NASA/WMAP Science Team

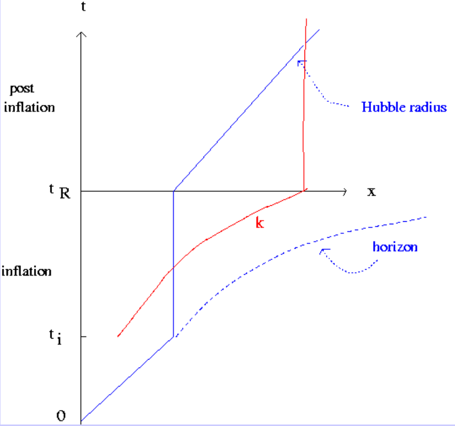
Horizon Problem



Formation of Structure Problem



Space-Time Sketch of Inflationary Cosmology



Plan

Gravitational Instability

Gravitational instability:

$$\ddot{\delta\rho} \sim G\delta\rho \, ,$$

Variables:

φ	:	Newtonian gravitational potential
ρ	:	energy density
p	:	pressure
v	:	fluid velocity
s	:	entropy density

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Basic Equations

Basic equations:

$$
\begin{aligned}
\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho}\nabla p + \nabla \varphi &= 0 \\
\nabla^2 \varphi &= 4\pi G \rho \\
\dot{s} + (\mathbf{v} \cdot \nabla) s &= 0 \\
p &= p(\rho, s) \, .
\end{aligned}
$$

Fluctuations

Perturbation ansatz

$$
\begin{aligned}
\rho &= \rho_0 + \delta\rho \\
\mathbf{v} &= \delta\mathbf{v} \\
p &= p_0 + \delta p \\
\varphi &= \varphi_0 + \delta\varphi \\
s &= s_0 + \delta s \,,
\end{aligned}
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Note: At linear order, each Fourier mode of the fluctuations evolves independently.

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Equation for Linearlized Fluctuations

Linearized equation of motion

$$\begin{aligned} \ddot{\delta\rho} - c_s^2 \nabla^2 \delta\rho - 4\pi G \rho_0 \delta\rho &= \sigma \nabla^2 \delta s \\ \dot{\delta s} &= 0 \, , \end{aligned}$$

$$\delta p = c_s^2 \delta\rho + \sigma \delta S$$

$$c_s^2 = \left(\frac{\delta p}{\delta\rho}\right)_{|s}$$

Jeans scale

$$k_J = \left(\frac{4\pi G \rho_0}{c_s^2}\right)^{1/2} .$$

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Lessons

- Density fluctuations are dynamical.
- Entropy fluctuations do not grow.
- Entropy fluctuations induce density perturbations.
- Growth on super-Jeans scales: $k < k_J$
- Oscillations on sub-Jeans scales: $k > k_J$

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Perturbations about an Expanding Background

$$
\begin{aligned}
ds^2 &= dt^2 - a(t)^2 d\mathbf{x}^2 \\
&= a(\eta)[d\eta^2 - d\mathbf{x}^2]
\end{aligned}
$$

$$\delta_\epsilon \equiv \frac{\delta\rho}{\rho}$$

$$\ddot{\delta}_\epsilon + 2H\dot{\delta}_\epsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\epsilon - 4\pi G\rho_0\delta_\epsilon = \frac{\sigma}{\rho_0 a^2}\delta S \,,$$

Solutions

Super-Jeans growth ($t > t_{eq}$):

$$\delta_k(t) \, = \, c_1 t^{2/3} + c_2 t^{-1} \, ,$$

Sub-Jeans damped oscillations:

$$\delta_k(t) \, \sim \, a^{-1/2}(t) exp(\pm i c_s k \int dt' a^{-1}(t')) \, .$$

Plan

Variables

Metric fluctuations (about a spatially flat FRWL background):

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

- 10 metric degrees of freedom
- Classify fluctuations according to their transformation under spatial rotations: 4 scalar, 4 vector and 2 tensor modes.
- Independent at linear order.
- Remove 4 coordinate (gauge) degrees of freedom(linear space-time reparametrizations).
- 4 gauge modes: 2 scalar and 2 vector

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Scalar Fluctuations = Cosmological Perturbations

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2(\psi\delta_{ij} - E_{,ij}) \end{pmatrix} ,$$

- $\phi(x,\eta), \psi(x,\eta), B(x,\eta), E(x,\eta)$
- Longitudinal gauge: $B = E = 0$.
- Scalar fluctuations are induced by matter.

Vector Perturbations

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 0 & -S_i \\ -S_i & F_{i,j} + F_{j,i} \end{pmatrix} ,$$

$S_i(x,\eta), F_i(x,\eta)$: divergenceless vectors.

- Vector perturbations do not couple to *simple* matter at linear order.
- Vector perturbations decay in an expanding background.

Tensor Fluctuations = Gravitational Waves

$$\delta g_{\mu\nu} = -a^2 \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix},$$

$h_{ij}(x, \eta)$ traceless and divergence-free.

Equations for Cosmological Perturbations

Einstein equations linearized about a FRWL background:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \, .$$

Metric:

$$ds^2 = a^2 \left[(1 + 2\phi) d\eta^2 - (1 - 2\psi)\gamma_{ij} dx^i dx^j \right]$$

Matter (scalar field for simplicity):

$$\varphi(\mathbf{x}, \eta) = \varphi_0(\eta) + \delta\varphi(\mathbf{x}, \eta)$$

$\psi = \phi$ if matter has no anisotropic stress (at linear order).

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$$\phi'' + 2\left(\mathcal{H} - \frac{\varphi_0''}{\varphi_0'}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\varphi_0''}{\varphi_0'}\right)\phi = 0 .$$

Compare with equation for Newtonian fluctuations:

$$\ddot{\delta}_\epsilon + 2H\dot{\delta}_\epsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\epsilon - 4\pi G\rho_0\delta_\epsilon = \frac{\sigma}{\rho_0 a^2}\delta S ,$$

Note: $c_s^2 = 1$ even if $w = 0$ or $w = 1/3$!

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Plan

Starting Point

bf Starting point: Joint action for gravity and matter:

$$S = \int d^4x\sqrt{-g}[-\frac{1}{16\pi G}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)] ,$$

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$
\begin{aligned}
ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\
\varphi &= \varphi_0 + \delta\varphi
\end{aligned}
$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$
\begin{aligned}
S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\
v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\
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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

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Gravitational Waves

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right]$$

- $h_{ij}(\mathbf{x}, t)$ transverse and traceless
- Two polarization states

$$h_{ij}(\mathbf{x}, t) = \sum_{a=1}^{2} h_a(\mathbf{x}, t)\epsilon_{ij}^a$$

- At linear level each polarization mode evolves independently.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) \; = \; a(t) h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k^{''} + \left(k^2 - \frac{a^{''}}{a}\right) u_k \; = \; 0 \, .$$

Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

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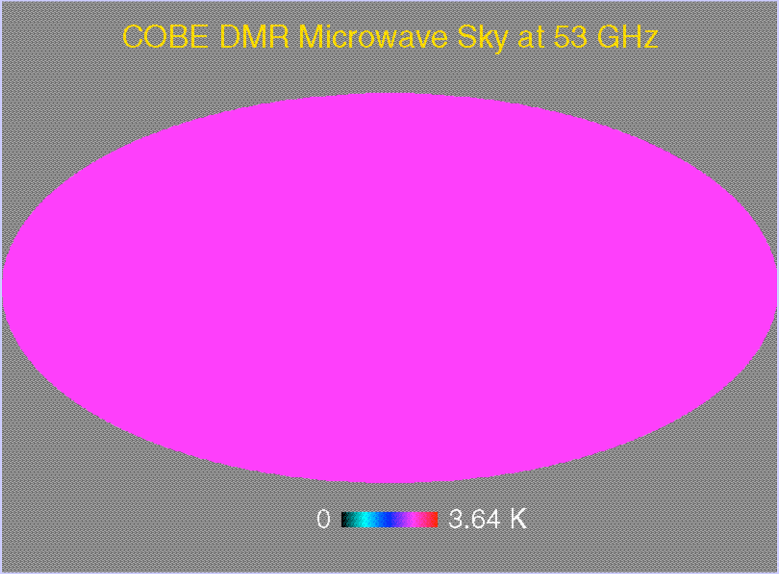

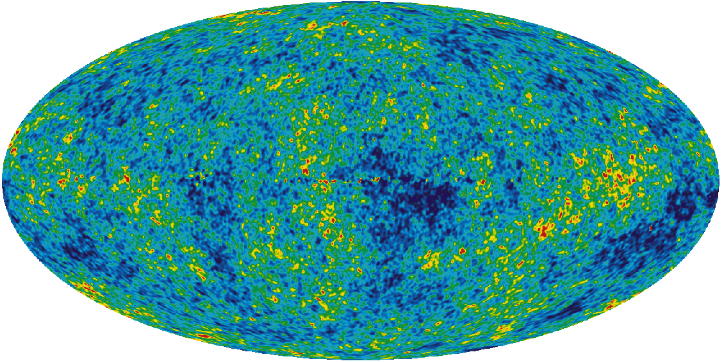

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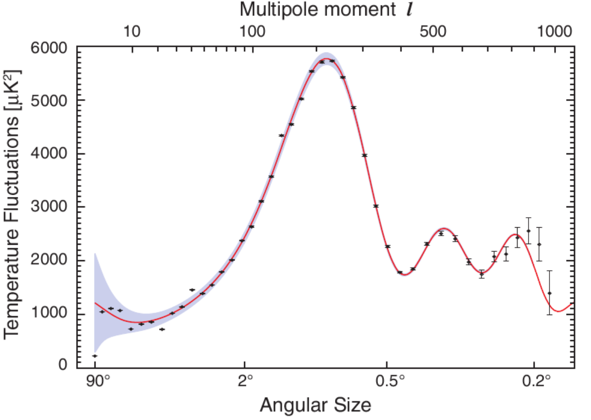

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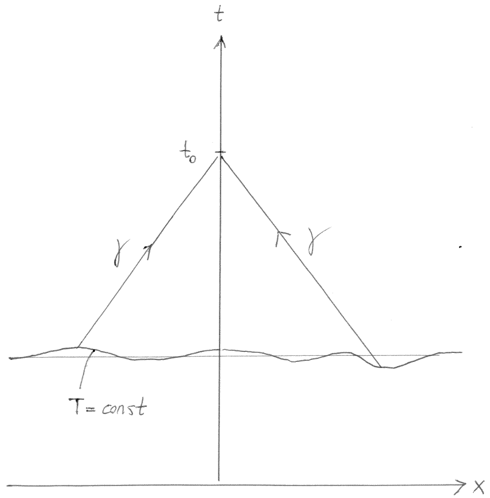
Origin of CMB Anisotropies



Predicting the Data

1970

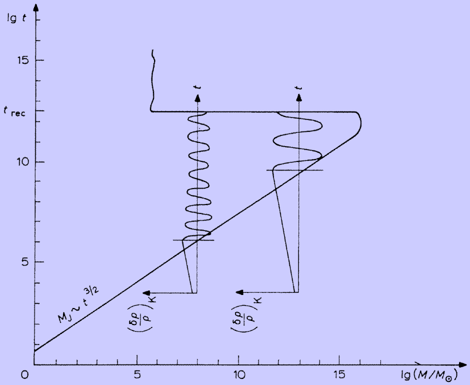


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
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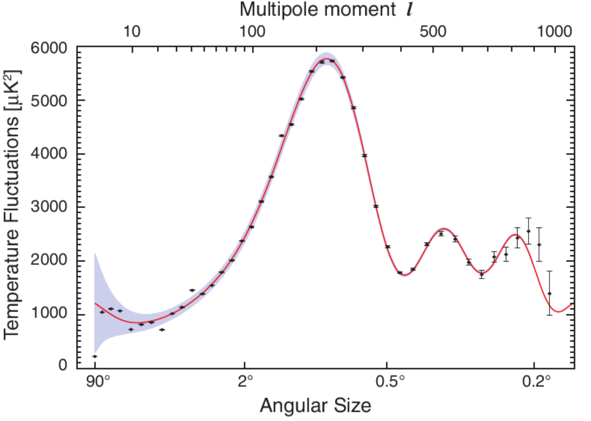
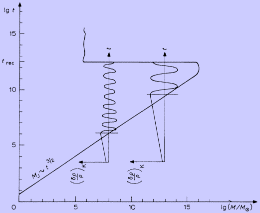


Credit: NASA/WMAP Science Team

Predictions of Sunyaev & Zeldovich, and Peebles & Yu

AtlanticGR
Brandenberger
Introduction
Newtonian
Classical
Quantum
Applications
General
Paradigms
Computations
Recent
Elements
String Gas Cosmology
Swampland
Conclusions

1970 paper



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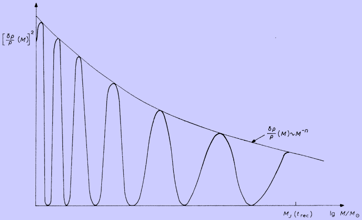

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

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Horizon vs. Hubble radius

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

Horizon: forward light cone, carries causality information

$$l_f(t) = a(t) \int_0^t dt' a(t')^{-1} .$$

Hubble radius: relevant to dynamics of cosmological fluctuations

$$l_H(t) = H^{-1}(t)$$

Horizon vs. Hubble radius

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

Horizon: forward light cone, carries causality information

$$l_f(t) = a(t) \int_0^t dt' a(t')^{-1} .$$

Hubble radius: relevant to dynamics of cosmological fluctuations

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- Solution of the horizon problem: horizon $\gg$ Hubble radius (Criterium 1).
- Solution of the flatness problem.
- Solution of the size and entropy problems.

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Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius (Criterium 2)
- Long propagation on super-Hubble scales (Criterium 3)
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Inflationary Cosmology

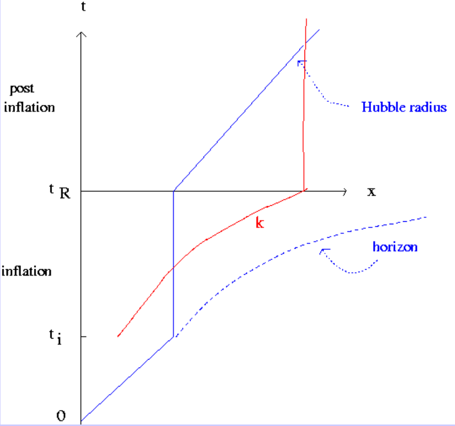
R. Brout, F. Englert and E. Gunzig (1978), A. Starobinsky (1978), K. Sato (1981), A. Guth (1981)

Idea: phase of almost exponential expansion of space $t \in [t_i, t_R]$

Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Space-time sketch of Inflationary Cosmology



Note:

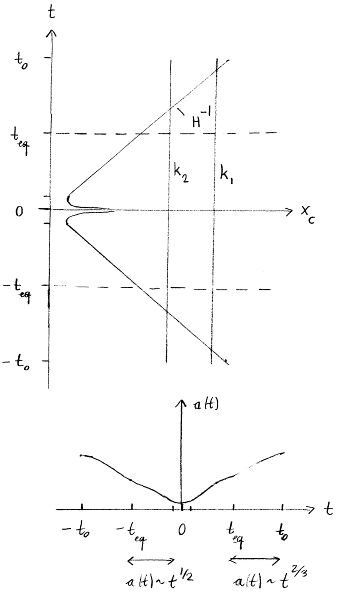
- $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Overview of the Matter Bounce

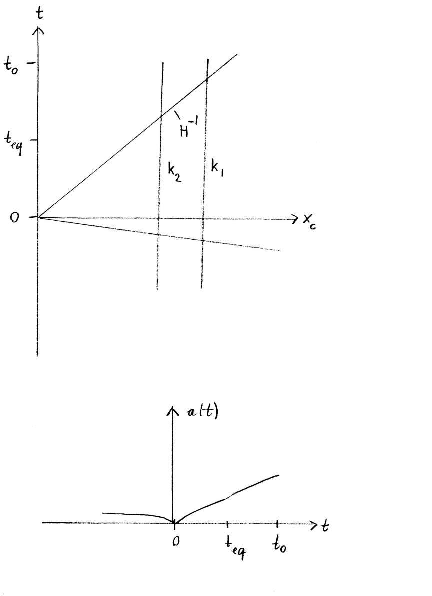
- Begin with a matter phase of contraction during which fluctuations of current cosmological interest exit the Hubble radius.
- Later in the contraction phase the equation of state of matter may be different (e.g. radiation).
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.

Ekpyrotic Bounce Scenario

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok, 2001



Overview of the Ekpyrotic Bounce

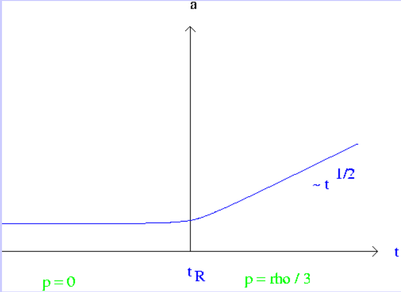
- Assume the existence of matter with an Ekpyrotic equation of state $w = p/\rho \gg 1$.
- Begin with a contracting phase in which Ekpyrotic matter dominates. In this phase, fluctuations of current cosmological interest exit the Hubble radius.
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Original adiabatic fluctuations retain a vacuum spectrum.
- Entropy fluctuations acquire a scale-invariant spectrum on super-Hubble scales.
- Entropy fluctuations seed a scale-invariant spectrum of curvature perturbations (A. Notari and A. Riotto, 2002; F. Finelli, 2002; J-L. Lehners et al, 2007; E. Buchbinder et al, 2007; P. Creminelli and L. Senatore, 2007).

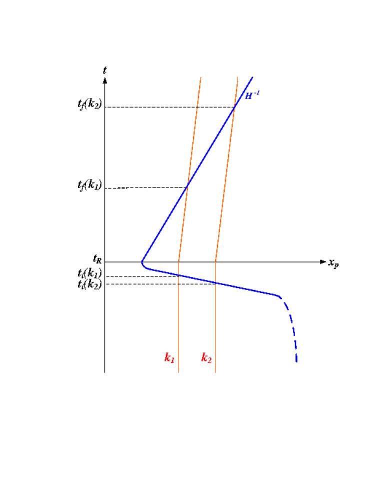
Emergent Universe Scenario

R.B. and C. Vafa, 1989



Space-time sketch of an Emergent Universe

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal fluctuations.

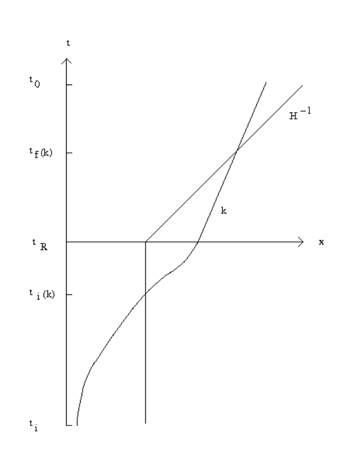
Overview of the Emergent Universe Scenario

- The Universe begins in a quasi-static phase.
- After a phase transition there is a transition to the Hot Big Bang phase of Standard Cosmology.
- Fluctuations originate as thermal perturbations on sub-Hubble scales in the emergent phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales if the thermal fluctuations have holographic scaling.

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.

Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3|\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k,t_i(k))\Big(\frac{a(t)}{a(t_i(k))}\Big)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0$ $\rightarrow$ red spectrum, $n_t < 0$

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Matter Bounce: Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) = k^3|\zeta(k)|^2 \sim k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) = c_1\eta^2 + c_2\eta^{-1},$$

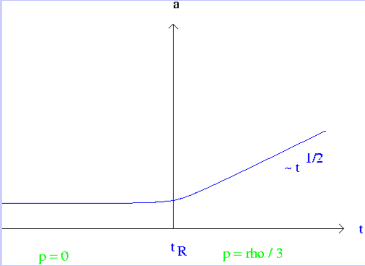
- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_\zeta(k,\eta) &\sim k^3|v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim k^3|v_k(\eta_H(k))|^2\left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \\ &\sim \text{const}, \end{aligned}$$

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Background for Emergent Cosmology



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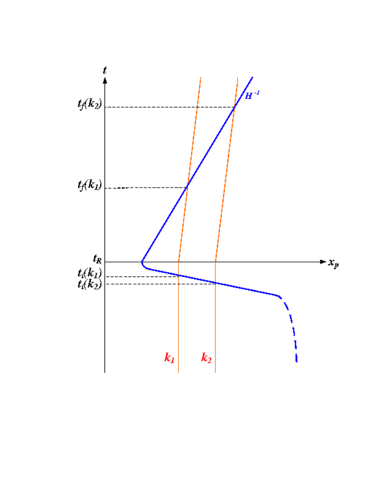
A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal fluctuations.

Method

- Calculate matter correlation functions in the static phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\left((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \,.$$

Key assumption: holographic scaling of thermodynamical quantities: $C_V \sim R^2$

Example: for string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \,.$$

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Power spectrum of cosmological fluctuations

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\begin{aligned}
P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2 k^2 < (\delta M)^2 >_R \\
&= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\
&\sim 8G^2 T
\end{aligned}
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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum

Requirements

- static phase $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$
- Cosmological fluctuations in the IR are described by Einstein gravity.

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$\begin{aligned} P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\ &= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\ &\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \end{aligned}$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)$$

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Consistency Relations

- Two free parameters: $(1 - T/T_H))$ and l_{pl}/l_s
- Four observables: amplitudes and tilts of the scalar and tensor modes.
- $\rightarrow$ two consistency relations
- $r \sim (1 - T/T_H)^2$
- $n_t = 1 - n_s$

Plan

New Degrees of Freedom

Assumption: All spatial dimensions toroidal, radius R.

String states:

- momentum modes: $E_n = n/R$
- winding modes: $E_m = mR$
- oscillatory modes: E independent of R

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New Symmetries: T-Duality

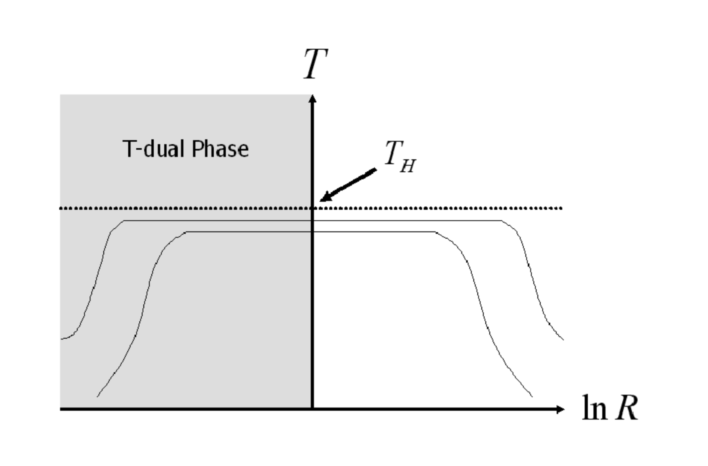
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

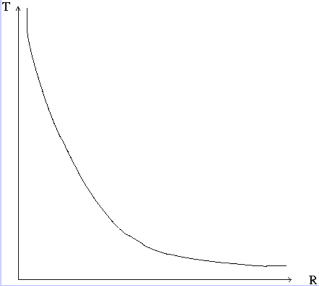
Absence of a Temperature Singularity in String Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology

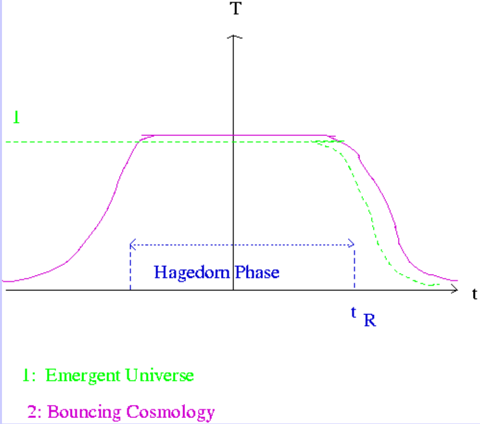


Temperature-size relation in standard cosmology



Dynamics

Assume some action gives us $R(t)$



Position Operators
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Position operators (dual to momenta)

$$|x> = \sum_p \exp(ix \cdot p)|p>$$

Dual position operators (dual to windings)

$$|\tilde{x}> = \sum_w \exp(i\tilde{x} \cdot w)|w>$$

Note:

$$|x> = |x + 2\pi R> , \quad |\tilde{x}> = |\tilde{x} + 2\pi \frac{1}{R}>$$

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Heavy vs. Light Modes

- $R \gg 1$: momentum modes light.
- $R \ll 1$: winding modes light.
- $R \gg 1$: length measured in terms of $|x>$.
- $R \ll 1$: length measured in terms of $|\tilde{x}>$
- $R \sim 1$: both $|x>$ and $|\tilde{x}>$ important.

Conclusion: At string scale densities usual effective field theory (EFT) based on supergravity will break down.

Conclusion: If an effective field theory description is valid, it must be an EFT in 18 spatial dimensions.

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Physical length operator

$$
\begin{aligned}
l_p(R) &= R \quad R \gg 1 \\
l_p(R) &= \frac{1}{R} \quad R \ll 1
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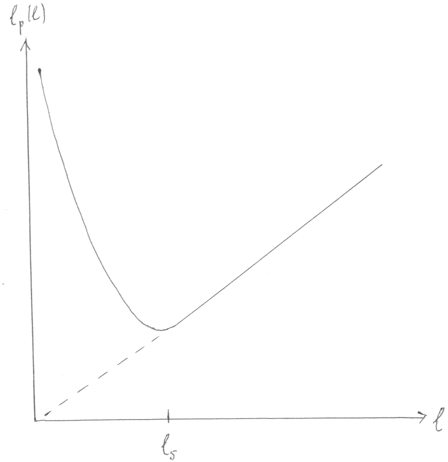
Physical length



String Theory Moduli I

- **Dimension of space-time in superstring theory: 10**
- $\rightarrow$ **6 compact spatial dimensions.**
- Sizes of the extra dimensions: Kahler moduli.
- Shapes of the extra dimensions: Complex Structure moduli.
- These moduli can provide new scalar field degrees of freedom in the **effective 4-d field theory** description.
- **Question**: Can some of these fields play an important role in cosmology?
- The typical range of these fields is of the order of the string scale.

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String Theory Moduli II

- Extra degrees of freedom: branes
- Brane positions in the extra dimensions yield scalar fields.
- Range of these scalar fields of the order of the string scale.
- Axion fields associated with the fluxes on the brane.
- Axion field range of the order of the string scale,

Moduli Stabilization

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [Y-K. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
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- $\rightarrow$ harmonic oscillator potential for θ
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Dilaton stabilization in SGC

- The only remaining modulus is the dilaton.
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum.
- $\rightarrow$ diltaton is stabilized.
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008].
- Gaugino condensation induces (high scale) supersymmetry breaking [S. Mishra, W. Xue, R.B. and U. Yajnik, 2012].

Connection with Cosmology

- Can one of the moduli fields naturally yield inflation?
- Does an alternate cosmological scenario naturally emerge from string theory?

String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings.

Assumption: $g_s \ll 1$.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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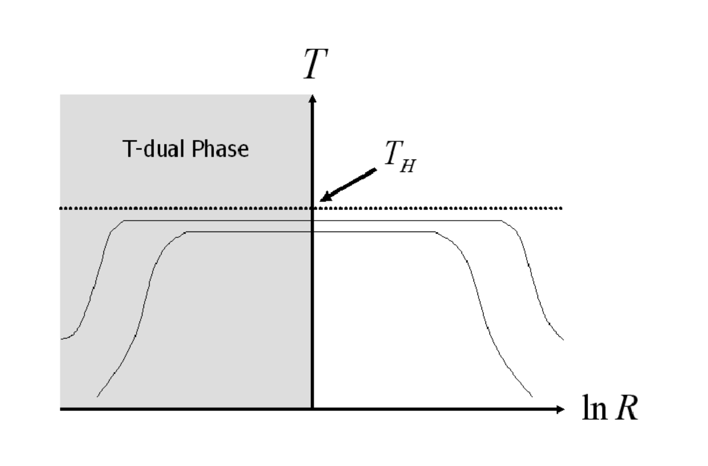
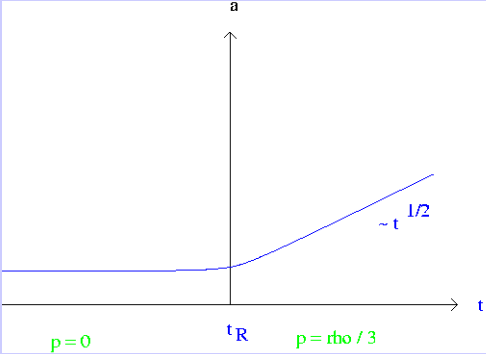
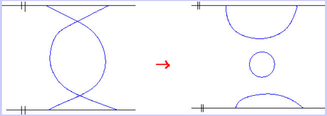


We will thus consider the following background dynamics for the scale factor $a(t)$:



Dynamical Decompactification

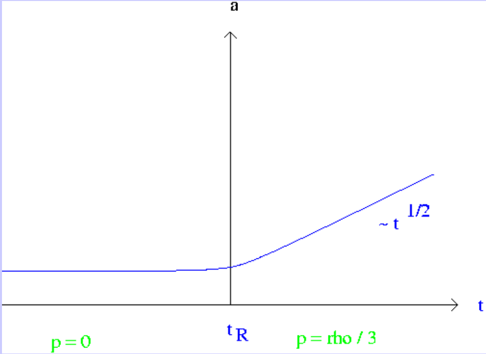
- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

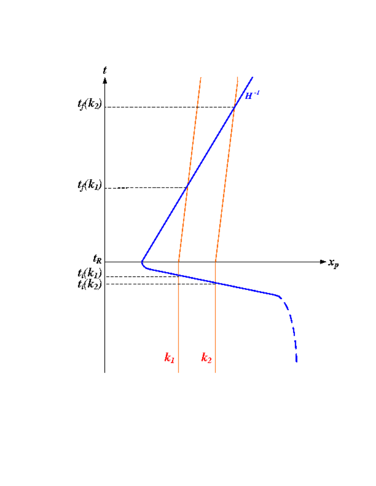
Note: For $R \rightarrow 0$ there is an analogous decompactification mechanism which only allows three dual dimensions to be large.

Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

String Theory and Cosmology

- Can one of the moduli fields naturally yield inflation?
- Can one of the moduli fields yield Dark Energy
- Does an alternate early cosmological scenario naturally emerge from string theory?
- Does an alternate explanation for **dark energy** emerge from string theory?

Consistency Conditions from String Theory

- **Low energy limit** of string theory *should* be describable by an **effective field theory.**
- Not all low energy effective field theories are consistent with string theory.
- There is a huge **landscape** of field theories.
- Most of them are **inconsistent** with string theory - they are in the "swampland".
- Effective field theories consistent with string theories are **islands** in the swampland.

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Swampland Conjectures

H. Ooguri and C. Vafa, hep-th/0605264; G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, arXiv:1806.08362; H. Ooguri, E. Palti, G. Shiu and C. Vafa, arXiv:1810.05506.

Swampland conditions: Scalar fields φ emerging from an **effective field theory** approximation of string theory must satisfy the following conditions:

- The effective field theory is only valid for $\Delta\varphi < dm_{pl}$ (field range condition).
- The potential of φ obeys

$$
\begin{aligned}
|\frac{V'}{V}|m_{pl} &\geq c_1 \quad \text{or} \\
\frac{V''}{V}m_{pl}^2 &\leq -c_2
\end{aligned}
$$

Note: d, c_1, c_2 constants of order 1.

Conditions for Inflation

- Large field range $\delta\varphi > m_{pl}$ required if inflation is to be a local attractor in initial condition space (R.B., arXiv:1601.01918 for a review).
- Slow roll condition 1:

$$\frac{V'}{V} m_{pl} \ll 1 \, .$$

- Slow roll condition 2:

$$\frac{V''}{V} m_{pl}^2 \ll 1 \, .$$

Consequences for Early Time Cosmology I

- **Large field inflation** is in the swamp.
- **Slow roll inflation** is in the swamp.
- **No local or global de Sitter minima.**
- $\rightarrow$ inflation does not naturally fit into string theory [R.B. and C. Vafa, 1989].

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Ekpyrosis: Small field and Large Slope

Consider $\delta t = H^{-1}$:

$$\varphi \sim p^{1/2} \log(p^{-1/2}) m_{pl} \ll 1 \, .$$

Relative slope of the potential:

$$|\frac{V'}{V}| m_{pl} \sim p^{-1/2} \gg 1 \, .$$

Relative curvature of the potential:

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Consequences for Early Time Cosmology II

- Field range condition satisfied in Ekpyrotic scenario.
- Slope and curvature condition on the potential satisfied in Ekpyrotic scenario.

The Ekpyrotic scenario is not in the **swampland**.

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Consequences for Late Time Cosmology

- **Dark Energy** cannot be a **cosmological constant.**
- **Quintessence** as an explanation of **Dark Energy** is constrained and may be ruled out by upcoming observations [L. Heisenberg et al, arXiv:1809.00154].
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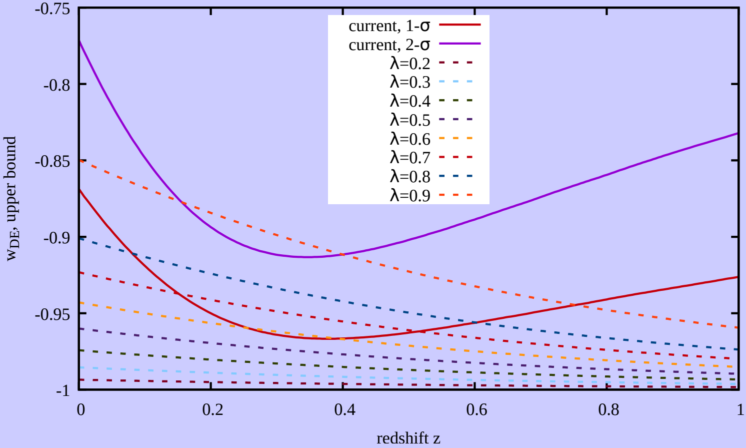
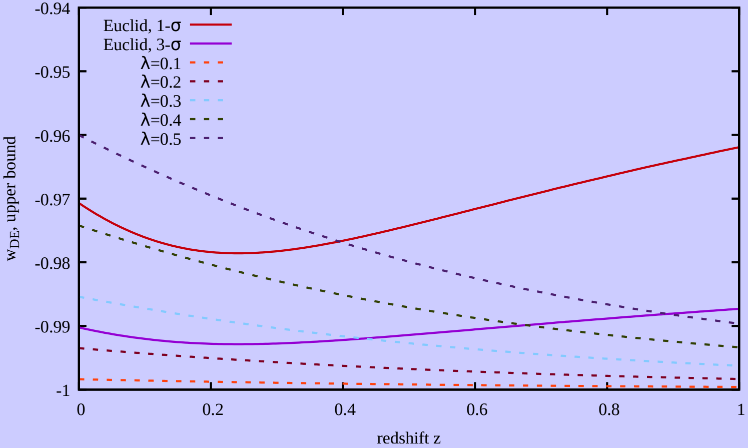
Current Constraints on Quintessence

L. Heisenberg, M. Bartelman, RB and A. Refregier, arXiv:1809.00154.



Euclid Constraints on Quintessence

L. Heisenberg, M. Bartelman, RB and A. Refregier, arXiv:1809.00154.



Criterium 1: Field Range Condition

H. Ooguri and C. Vafa, hep-th/0605264

- Field range condition: A particular effective field theory remains valid only while $\Delta\varphi < 1$ (Planck units).
- **Reason**: Once $\Delta\varphi > 1$ new towers of string states become massless and have to be included in the low energy EFT.
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Criterium 2: No stable de Sitter

G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, arXiv:1806.08362

- Stable de Sitter is inconsistent with string theory
- **Reason:** No de Sitter gound states of supergravity (Gibbons, Maldacena & Nunez).
- At best, string theory effects could provide **metastable** de Sitter or **dynamical quasi de Sitter**, i.e. accelerated phase driven by some effective scalar field.
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Either:

$$|\frac{V'}{V}| > 1$$

or:

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- Slow roll inflation in the swampland.
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Criterium 3: Constraints on the Shape of a Scalar Field Effective Potential

H. Ooguri, E Palti, G. Shiu and C. Vafa, arXiv:1810.05506

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Origin of the Constraint (1)

H. Ooguri, E Palti, G. Shiu and C. Vafa, arXiv:1810.05506

- Assume: uniform rollling of φ.
- Demand: **entropy** increase due to new string states becoming low mass is smaller than the Gibbons-Hawking entropy of an accelerating universe.
- $\rightarrow \ |\frac{V'}{V}| > 1 \, .$
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Origin of the Constraint (2)

- Consider Kahler moduli as candidate for inflation.
- Stringy Kahler moduli stabilization based on string degrees of freedom:
- $\rightarrow \quad V_{eff}(\varphi) = \frac{1}{2} m^2 \varphi^2$ with $|\varphi| < 1$
- $\rightarrow \quad |\frac{V'}{V}| = |\frac{2}{\varphi}| > 1 \, .$

Constraints on Dark Energy

Consequences for Dark Energy:

- Dark Energy cannot be Λ.
- Quintessence as Dark Energy constrained.

Note: In the context of Effective Field Theory it requires extreme fine tuning to obtain the low energy scale of Dark Energy.

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Plan

Conclusions I

- Theory of Cosmological Perturbations is the **cornerstone of early universe cosmology**.
- Fluctuations generated in an early phase of new physics.
- Fluctuations **oscillate** on **sub-Hubble** scales and are **squeezed** on **super-Hubble** scales.
- There are several scenarios of primordial cosmology which are consistent with the current data.
- Inflation is only one of the possible scenarios.

Conclusions II

- **Cosmology of string theory** must take into account the key symmetries of string theory, in particular the T-duality symmetry.
- Standard effective field theory of supergravity will break down in the very early universe.
- Inflation does not naturally emerge from string theory (inflationary models are in the swampland).
- String Gas Cosmology appears to emerge naturally from string theory.
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Conclusions III

- String Gas Cosmology provides an alternative to inflation for producing the spectrum of cosmological perturbations.
- Cosmological evolution is **nonsingular**.
- Our universe emerges from an early Hagedorn phase.
- Thermal string fluctuations in the Hagedorn phase yield an almost scale-invariant spectrum of cosmological fluctuations.
- **Characteristic signal**: blue tilt in the spectrum of gravitational waves.