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EXCELENCIA SEVERO OCHOA

$\boxed{\text{GW FROM PTs}}$ (GW relic SB from) (I.O. PT in the early universe)

GWs in cosmological context:

$$ds^2 = -dt^2 + a^2(t)\,(\delta_{ij} + h_{ij}^{TT})\,dx^i dx^j$$

FRW symmetries: constant time hypersurf. are hom. and isotropic → can distinguish perturb and can decompose it under rotations: S, V, T decoupled at 1st order

$$\ddot{h}_{ij}(t,\underline{k}) + 3H\dot{h}_{ij}(t,\underline{k}) + k^2 h_{ij}(t,\underline{k}) = 0 \quad \text{inflation}$$

$$= 16\pi G \underbrace{\Pi_{ij}(\underline{k},t)}_{\text{tensor an. stress}}$$

Examples in early universe:

$$\left.\begin{aligned} \Pi_{ij} &\sim (\partial_i\phi\,\partial_j\phi)^{TT} \\ &\sim \left(\gamma^2(\rho+p)\,v_i v_j\right)^{TT} \\ &\sim -(E_iE_j + B_iB_j)^{TT} \end{aligned}\right\}$$

I.O. PT everything can be there

why this can be interesting?

typical frequency: $f_* \overset{(\geq)}{=} \dfrac{H_*}{\varepsilon_*}$

happening at time t_*, T_*

$\varepsilon_* < 1$

$(\varepsilon_* = R_* H_*)$

$$f_c = f_* \frac{a_*}{a_0} = \frac{2\cdot 10^{-5}}{\varepsilon_*} \frac{T_*}{\text{TeV}}\ \text{Hz}$$

$H_* \sim \dfrac{T_*^2}{M_{pl}}$ $\qquad$ $\dfrac{T_0}{T_*}$

$\boxed{10^{-3} < \varepsilon_* < 1}$ depending on process

$\boxed{T_* \sim 300\ \text{GeV} \qquad 10^{-4}\ \text{Hz} < f_c < 0.1\ \text{Hz} \qquad \text{LISA !!}}$

LISA $\Rightarrow$ $10^{-5} < f < 0.1$ Hz $\qquad$ GeV $< T_* <$ 1000 TeV

(smallest frequency and smallest ε_*)

EARTH-BASED $\Rightarrow$ 0.1 Hz $< f < 10^3$ Hz $\qquad$ 10^4 GeV $< T_* < 10^{10}$ GeV

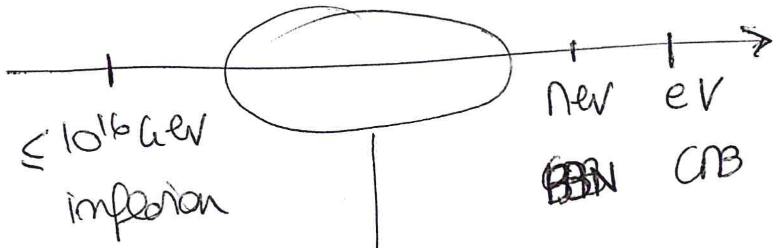
PTA $\Rightarrow$ $f \simeq 10^{-8}$ Hz $\qquad$ 0.1 MeV $< T_* <$ 500 MeV

GW travels freely so bring direct info:

$$\frac{\Gamma(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{pl}} \sim \left(\frac{T}{M_{pl}}\right)^3 < 1$$

$$\left(\Gamma(T) = \sigma n v = (G^2 T^2)(T^3)\, 1\right)$$

Through GWs one can get direct info on interesting epochs, the CMB of the future??



reheating, baryogenesis, EWPT, QCD PT
dark matter ...

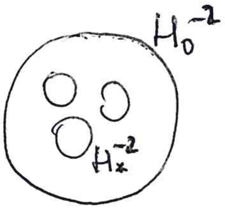
$\Longrightarrow$

$$\boxed{\text{LISA AND I.O. EWPT OR BEYOND}}$$

- new EWPT due to new couplings / fields
- another PT by another field, and/or multi-step PT (even dark sector) extradimensions)

$$V(H,S) = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}a_1|H|^2 S + \frac{1}{2}a_2|H|^2 S^2 +$$
$$+ b_1 S + \frac{1}{2}b_2 S^2 + \frac{1}{3}b_3 S^3 + \frac{1}{4}b_4 S^4$$

what kind of signal does one expect? a SGWB



$$\theta_* = \frac{H_*}{d_A(H_*)} = 10^{-12}\ \text{deg} \quad (10^{-14}\ \text{rad})$$

angle subtending the source region today

EWPT

10^{24} uncorrelated regions

LISA : $\ell \sim \frac{\pi}{\theta} \lesssim 10$

the signal is the superposition of many uncorrelated regions, independent signals

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_c} = \frac{\langle \dot h_{ij} \dot h_{ij} \rangle}{\rho_c G} = \int \frac{df}{f} \frac{d\Omega_{GW}}{d\ln k}$$

$$T_{00} = \frac{\langle \dot h_{ij} \dot h_{ij} \rangle}{G}$$

average over ~~several wavelengths~~

$\lambda < \ell < L_B$

$1/f < \tau < 1/f_B$

to be able to define T_{00}: go beyond linear expansion on mink., necessity to distinguish waves from back

$$\langle \dot h_{ij}(k,t)\, \dot h_{ij}(q,t) \rangle = \delta(k-q)\, \frac{|\dot h^2|}{k^3}$$

space average becomes ensemble average as usually in cosmology

from isotropy of universe

$$\frac{d\rho_{GW}}{d\ln k} = \frac{|\dot h|^2}{G}$$

$$H = \left[0, \frac{(h+v)}{\sqrt{2}}\right]^T \qquad v \sim 246\ \mathrm{GeV}$$

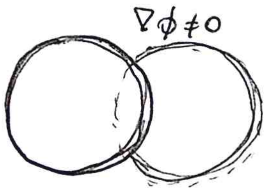
$$V_{\text{tree}}(h) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{c_6}{f^2}|H|^6 + \frac{c_8}{f^4}|H|^8$$

$$= -\frac{1}{2}\mu_H^2 h^2 + \frac{1}{4}\lambda_H h^4 + \frac{c_6}{8f^2} h^6 + \frac{c_8}{16 f^4} h^8$$

$\searrow$ scale of new physics about 1 TeV

+ 1-loop contributions W, Z, top

Suppose this is the settings. Qualitatively:



<u>In vacuum</u>: collision $T_{ij} \propto (\partial_i\phi\, \partial_j\phi)^{TT}$

$\rightarrow$ envelope approx

numerical simulations early '90

(Kamionkowsky) Kosowsky, Turner 92

<u>In fluid, with coupling</u>

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\, \partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\, \partial_\rho\phi\, \partial^\rho\phi + \frac{1}{2} g_{\mu\nu} V_0(\phi)$$

$$T^{f}_{\mu\nu} = \omega\, u_\mu u_\nu - p\, g_{\mu\nu} \qquad \tilde{\eta}\, T_n \quad \boxed{0.01 < \tilde{\eta} < 10}$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu T^{\phi\,\mu\nu} = -\partial_\mu T^{f\,\mu\nu} = \eta\, u^\mu \partial_\mu\phi\, \partial^\nu\phi$$

e.o.m. for Higgs field with interaction with other particles getting a mass, system of Boltzmann eqs., knowing all interactions + particle content... or <u>pheno approach</u>

Ignatius et al 93 $\rightarrow$ Albrecht 82, Kurki-Suonio 87

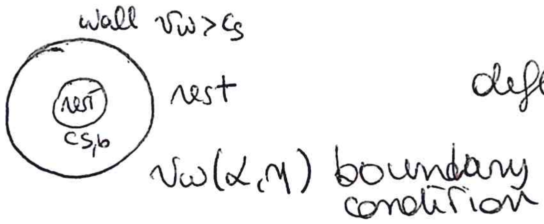
big $\eta \rightarrow$ big coupling $\qquad \eta \propto \frac{\phi^2}{T}$ or constant...

$$\begin{cases} e_s = 3a_s T^4 + V_0(\phi) \\ p_s = a_s T^4 - V_0(\phi) \end{cases}$$

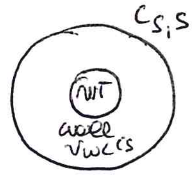
bag e.o.s. or not ...

analyse the system of a fluid on a ~~phase~~ transition front: cons of en. and momentum, stable stationary solutions

detonation



deflagration



collision of these fronts generates GWs

$$\Pi_{ij} \sim (\gamma^2 w \, v_i v_j)^{TT}$$

~~[illegible] people have simulated the system~~

the total GW signal:

$$\Omega_{GW}(f) = \Omega^{\phi}_{GW} + \Omega^{f}_{GW}$$

difficult to analyse!

- analytical attempts using Kolmogorov turbulence

Weir, Hindmarsh Rummukainen from 2013 onwards

- simulations solving the full system of equations

The gross features of the GW signal can be guessed depend only on a small number of parameters :

$$\Omega^*_{GW} = \frac{\rho_{GW}}{\rho_{tot}} \simeq \frac{1}{\rho_{tot}} \frac{\langle \dot{h}^2 \rangle}{G} \simeq \left(\frac{H}{\beta}\right)^2 \left(\frac{\Pi}{\rho_{tot}}\right)^2$$

$$\ddot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}$$

$G = \frac{H^2}{\rho_{tot}}$ Friedmann

(suppose P.T. is fast and neglect H_*)

$\langle \dot{h}^2 \rangle \sim \frac{G^2 \Pi^2}{\beta^2}$ amplitude of shear stresses

characteristic time

INVERSE DURATION OF P.T.

critical bubble action

$$\beta = -\left.\frac{dS}{dt}\right|_{t_*} = \frac{d}{dt} \ln P = \frac{d}{dt} \ln (A e^{-S}) = \frac{\dot{P}}{P}$$

$(T_n \sim T_*)$

everything occurs fast and the P.T. is not so sharp

$\frac{\beta}{H_*} \sim 4 \ln\left(\frac{m_{PL}}{T}\right)$

$P = \frac{\text{nucleation rate}}{\text{Volume}}$

$S(t) \simeq S_* - \beta(t - t_*) + \mathcal{O}(t-t_*)^2$

$$R_* = \frac{\max(v_w, c_s)}{\beta}$$

size of the bubbles at the end of the P.T. $\rightarrow f_*(T_*, \epsilon_*)$

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$$\boxed{\text{detectable ?}}$$

$$\Omega^{\circ}_{GW} = \frac{\rho^{\circ}_{GW}}{\rho^{\circ}_{c}} = \frac{1}{\rho^{\circ}_{c}}\, \rho_{GW} \left(\frac{a_*}{a_0}\right)^4 = \Omega^{\circ}_{rad} \left(\frac{\rho_{GW}}{\rho_{rad}}\right)$$

$$= \underbrace{\Omega^{\circ}_{rad}}_{10^{-5}} \underbrace{\left(\frac{H}{\beta}\right)^2 \left(\frac{\Pi}{\rho_{tot}}\right)^2}_{} \gtrsim 10^{-13} \qquad (\text{LISA})$$

$$\frac{H}{\beta} \cdot \frac{\Pi}{\rho_{tot}} > 10^{-4}$$

$\frac{H}{\beta} \to 10^{-3} < \varepsilon_* < 1$

$\frac{\Pi}{\rho_{tot}} \to$ relevant fraction to get something observable !

- Characteristic frequency → seen ok
- characteristic amplitude → seen okish
- $\boxed{\text{Power spectrum ? Shape ?}}$

$$\frac{d\Omega_{GW}}{d\ln k}$$

$\boxed{\text{GW generation from bulk fluid motions}}$ most interesting case

$$h''_{ij}(\underline{k},\eta) + 2\mathcal{H}\, h'_{ij}(\underline{k},\eta) + k^2 h_{ij}(\underline{k},\eta) = 8\pi G a^2 \underbrace{\Pi_{ij}(\underline{k},\eta)}_{[\eta_{in},\,\eta_{fin}]}$$

$$h_{ij}(\underline{k},\eta) = \int_{\eta_{in}}^{\eta} d\zeta\, \frac{\sin(k(\eta-\zeta))}{\zeta}\, 8\pi G a^2\, \Pi_{ij}(\zeta)$$

+ matching with homogeneous solution to get the amplitude today + $|h'(k,\eta_0)|^2$

In the end:

$$\frac{d\Omega_{GW}}{d\ln k} \sim K_v^2\, (kR_*)^3 \int_{\eta_{in}}^{\eta_{fin}} \frac{d\eta}{\eta} \int_{\eta_{in}}^{\eta_{fin}} \frac{d\zeta}{\zeta} \cos(k(\eta-\zeta))\, \tilde{\Pi}(k,\eta,\zeta)$$

$$K_v = \frac{\langle w\, \gamma^2 v^2 \rangle}{\rho_{tot}} \qquad \tilde{\Pi}_{ij} = \frac{\Pi_{ij}}{\rho_{tot}\, K_v\, R_*^{3/2}}$$

$$\langle \tilde{\Pi}_{ij}(\underline{k},\eta)\, \tilde{\Pi}^*_{ij}(q,\zeta) \rangle = \delta(\underline{k}-q)\, \tilde{\Pi}(k,\eta,\zeta)$$

to derive the P.S. one needs:

1) estimate of the kinetic energy fraction K

→ depends on the energy budget of the P.T.

strength, friction, gradient energy

(how much goes to radiation) $\alpha = \frac{V_0}{3aT^4}$ $\quad$ $K_\phi + K_v + K_{therm} = 1$

$(a = \pi^2/90\, g_*)$

2) knowledge of the unequal time correlator of the shear stresses generated by the fluid

- numerical simulations: appearance of compression waves in the fluid (sound); directly evaluate GW production for fairly weak P.T.s; sound waves remains long after the ϕ is gone (ultimately dissipated by fluid viscosity) ($\alpha \leq 0.1$) ($\langle v_{rms} \rangle \sim O(10^{-3})$) only about 5% of $\rho \times v$ is generated

- turbulence, done analytically:

know $\langle v_i(k)\, v_j(q) \rangle$, assume $\omega = \text{const}$

$$\langle \Pi_{ij}\, \Pi_{lm} \rangle = \omega^2 \langle v_i\, v_j\, v_l\, v_m \rangle$$

+F.T.

suppose gaussian use Wick theorem

+ time behaviour of turbulence (decay, decor.)

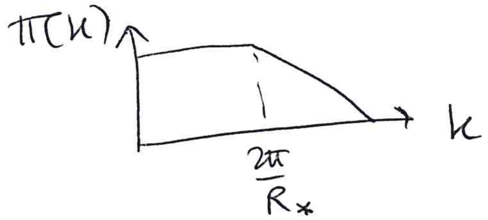
→ get U.T.C

Again, some properties can be guessed

- life time of the shear stresses $\tau_v \gtrless H_*^{-1}$
- decorrelation of the " " $\tau_c < H_*^{-1}$

  $\tau_c < \tau_v$
- space correlation structure: white noise $k < \frac{2\pi}{R_*}$

  (by causality)



$\cancel{\boxed{\tau_v < H_*^{-1}}}$

$\cancel{\frac{d\Omega_{GW}}{d\ln k} \sim k^2 (kR_*)^3 (H_*\tau_v)(H_*\tau_c)\tilde{P}(k)}$

$\cancel{\boxed{\tau_v > H_*^{-1}}}$ ~~" " $(H_*\tau_c)$ " " $\Rightarrow$ enhanced amplitude~~

~~sound phase~~

~~(~ multi...ons)~~

~~$\tau_c \sim R_*/c_s$~~

~~$\tau_v \sim R_*/v_{rms}$ compr.~~

~~Ω_{GW} k^3 H_* k~~

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generic picture expected (work in progress)

ϕ

sound phase

$$\tau_c \sim \frac{R_*}{c_s}$$

turbulence phase

$$\tau_c \sim \frac{R_*}{v_{rms,\perp}} \sim \tau_{eddy}$$

$$Re(R_*) = \frac{v_{rms,\perp} R_*}{\nu} = 1$$

can take several Hubble times up to 100 GeV

if $\tau_{sh} \sim \frac{R_*}{v_{rms,\parallel}} < H_*^{-1}$

$\to \tau_{sh} = \tau_v \to$ turbulence

otherwise continue until dissipated by viscosity ($\nu \Delta v$)

$[\tau_c < \tau_v \to v_{rms,\parallel} < c_s]$

$[v_{rms\,\parallel} > v_{rms,\perp} \Rightarrow \tau_{sh} < \tau_{eddy}]$

Open questions:

- how much $v_{rms\,\perp}$ out of $v_{rms\,\parallel}$?
- do shock really form vorticity ?
- what is vorticity for relativistic flows in relativistic fluids ?

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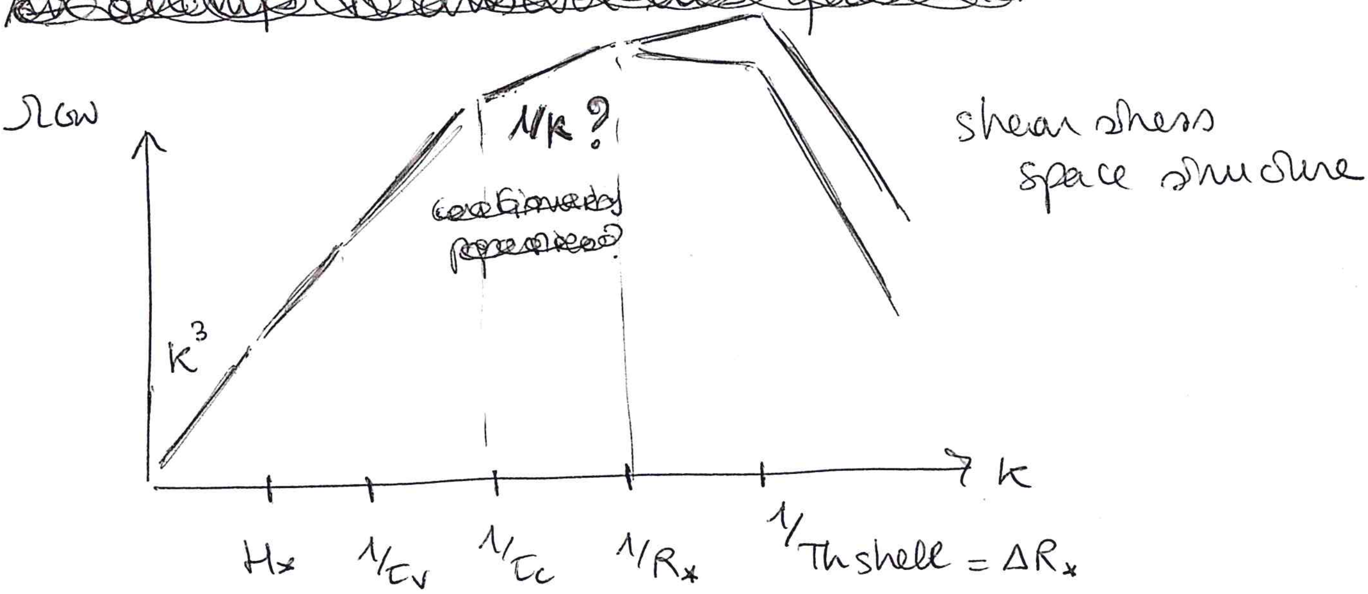

$$\pi(k,\eta,\zeta) = \underbrace{\Pi(k,\eta)}_{\substack{\text{space F.T.}\\ \text{correlation}\\ \text{structure}}} \; \underbrace{\pi(k,\eta-\zeta)}_{\Theta(\tau_c - |\eta-\zeta|)} + \leftrightarrow$$

$$\boxed{\tau_v < H_*^{-1}}$$

$$\frac{d\Omega_{GW}}{d\ln k} \sim k^2 (kR_*)^3 (H_*\tau_v)(H_*\tau_c)\, \tilde{P}(k)$$

$\boxed{\tau_v > H_*^{-1}}$ " " $(H_*\tau_v)$ "

enhancement of amplitude for long-lasting sources