Holographic & geometric aspects of electromagnetic duality in supergravity

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Holography, Generalised Geometry and Duality
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Outlook

- Electric-magnetic duality in N=8 supergravity
- M-theory
- Massive Type IIA
- Type IIB

- Electric-magnetic duality in N=8 supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

metric	8 gravitini	28 vectors	56 dilatini	70 scalars
$(s=2)$	$(s=3/2)$	$(s=1)$	$(s=1/2)$	$(s=0)$

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N=8$ supergravity with $G=U(1)^{28}$ **[$E_{7(7)}$ symmetry]**

[Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

- Reduction of M-theory on a *sphere* S^7 down to 4D produces $N=8$ supergravity with $G=SO(8)$ [de Wit, Nicolai '82]

- Reduction of M-theory on S^1 (Type IIA) and subsequently on S^6 down to 4D produces $N=8$ supergravity with $G=ISO(7)=SO(7)\ltimes\mathbb{R}^7$ [Hull '84]

- Reduction of Type IIB on S^5 and subsequently on S^1 down to 4D produces $N=8$ supergravity with $G=[\,SO(1,1)\times SO(6)\,]\ltimes\mathbb{R}^{12}$ [Inverso, Samtleben, Trigiante '16]

✱ These gauged supergravities believed to be unique for 30 years…

Electric-magnetic deformations

• Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $AdS_5 \times S^5$ (D3-brane ~ N = 4 SYM in 4d) [Maldacena '97]

M-theory : $AdS_4 \times S^7$ (M2-brane ~ ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

• N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$\boxed{D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})}$$

g = 4D gauge coupling
c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

• There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous parameter [similar for $SO(p,q)_c$]

2) Family of $CSO(p,q,r)_c$ theories : $c = 0$ or 1 is an (on/off) parameter

[Dall'Agata, Inverso, Marrani '14]

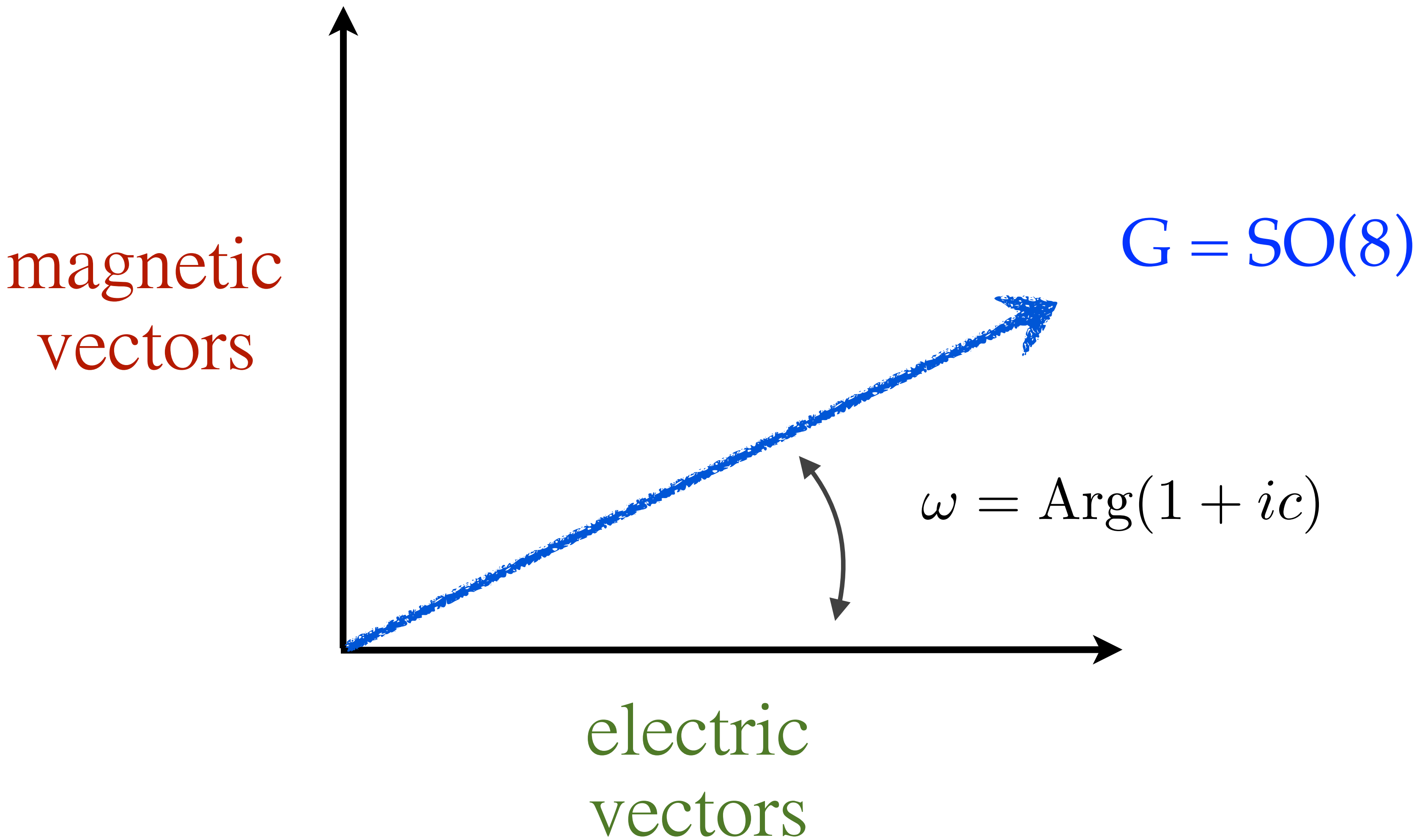
The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

- M-theory

SO(8)$_c$ theories : physical meaning in 4D



$$D = \partial - g\,(A^{\mathrm{elec}} - c\,\tilde{A}_{\mathrm{mag}})$$

SO(8)$_c$ theories : physical meaning in 11D …

?

Obstruction for SO(8)$_c$, *cf*. [de Wit, Nicolai '13]

[Lee, Strickland-Constable, Waldram '15]

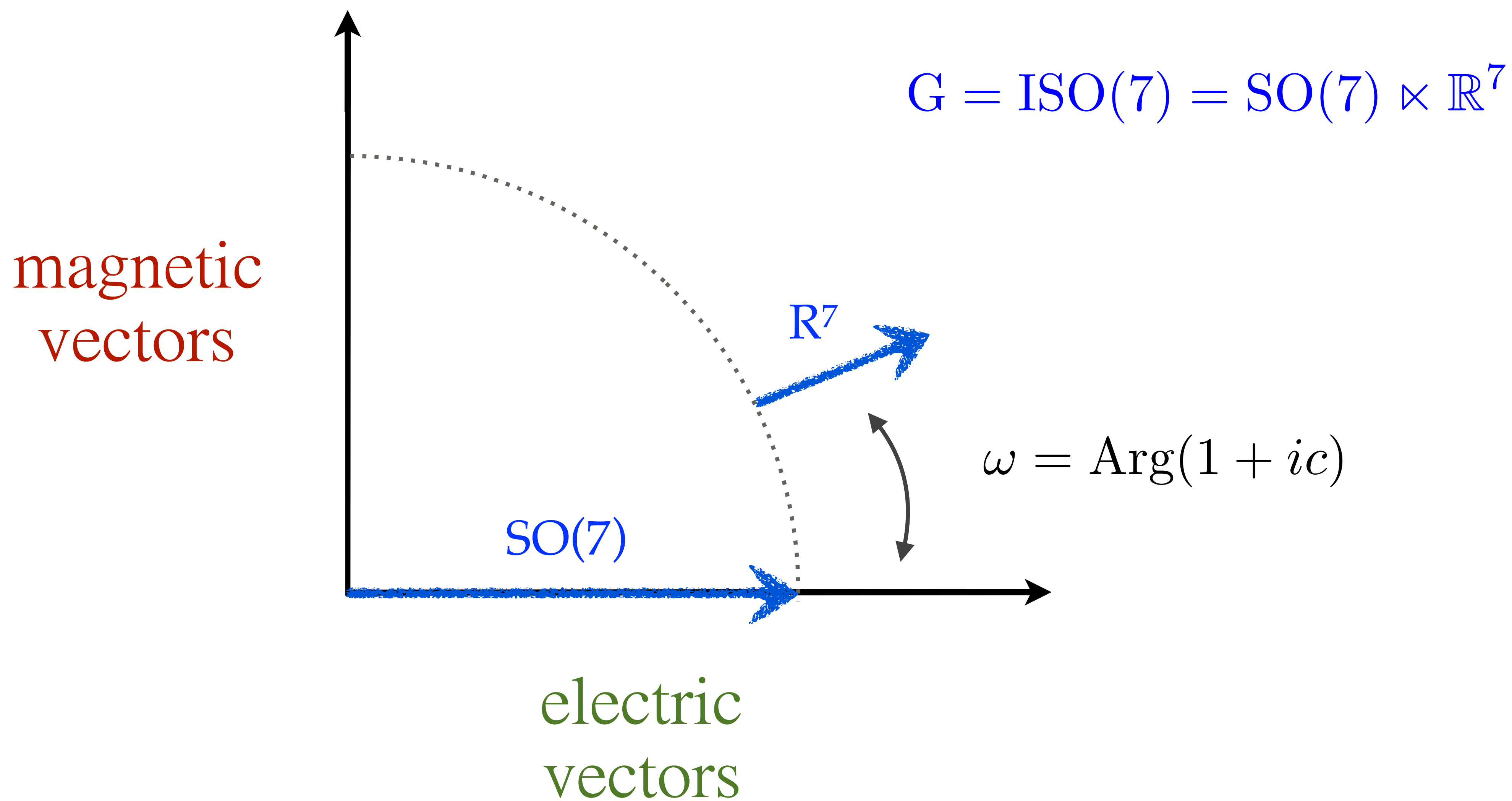
$SO(8)_c$ theories : holographic AdS_4/CFT_3 meaning …

?

Massive Type IIA

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

Why ISO(7)$_c$ works ?



$$\boxed{D = \partial - g\, A^{\text{elec}}_{\text{SO}(7)} - g\,\big(A^{\text{elec}}_{\mathbb{R}^7} - c\,\tilde{A}_{\mathbb{R}^7\,\text{mag}}\big)}$$

4D : ISO(7)$_c$ Lagrangian

$\mathbb{M} = 1, \ldots, 56$
$\Lambda = 1, \ldots, 28$
$I = 1, \ldots, 7$

$$
\begin{aligned}
\mathcal{L}_{\text{bos}} \; = \; & (R - V)\,\text{vol}_4 - \tfrac{1}{48} D\mathcal{M}_{\mathbb{MN}} \wedge * D\mathcal{M}^{\mathbb{MN}} + \tfrac{1}{2}\,\mathcal{I}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)} \wedge * \mathcal{H}^{\Sigma}_{(2)} - \tfrac{1}{2}\,\mathcal{R}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)} \wedge \mathcal{H}^{\Sigma}_{(2)} \\
& + g\,c\,\Big[\mathcal{B}^I \wedge \big(\tilde{\mathcal{H}}_{(2)I} - \tfrac{g}{2}\delta_{IJ}\mathcal{B}^J\big) - \tfrac{1}{4}\,\tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J \wedge \big(d\mathcal{A}^{IJ} + \tfrac{g}{2}\,\delta_{KL}\,\mathcal{A}^{IK} \wedge \mathcal{A}^{JL}\big)\Big]
\end{aligned}
$$

✦ Setting $c = 0$, all the magnetic pieces in the Lagrangian disappear.

∗ *Ingredients :*

- Electric vectors $(21+7)$: $\mathcal{A}^{IJ} = \mathcal{A}^{[IJ]}$ [SO(7)] and $\mathcal{A}^I$ [$\mathbb{R}^7$] with $\mathcal{H}^{\Lambda}_{(2)} = (\,\mathcal{H}^{IJ}_{(2)}\,,\,\mathcal{H}^{I}_{(2)}\,)$
- Auxiliary magnetic vectors (7) : $\tilde{\mathcal{A}}_I$ [$\mathbb{R}^7$] with $\tilde{\mathcal{H}}_{(2)I}$ field strength
- $E_7/SU(8)$ scalars : $\mathcal{M}_{\mathbb{MN}}$
- Auxiliary two-forms (7) : $\mathcal{B}^I$ [$\mathbb{R}^7$]
- Topological term : $g\,c\,[\,\ldots\,]$
- Scalar potential : $V(\mathcal{M}) = \dfrac{g^2}{672} X_{\mathbb{MN}}{}^{\mathbb{R}} X_{\mathbb{PQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MP}} \big(\mathcal{M}^{\mathbb{NQ}}\mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{Q}}_{\mathbb{R}}\,\delta^{\mathbb{N}}_{\mathbb{S}}\big)$

AdS_4 solutions

[AG, Varela '15]

$\mathcal{N}$	G_0	$c^{-1/3}\,\chi$	$c^{-1/3}\,e^{-\varphi}$	$c^{-1/3}\,\rho$	$c^{-1/3}\,e^{-\phi}$	$\frac{1}{4}\,g^{-2}\,c^{1/3}\,V_0$	M^2L^2
$\mathcal{N}=1$	G_2	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2}\,3^{1/2}}{2^{7/3}}$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2}\,3^{1/2}}{2^{7/3}}$	$-\frac{2^{22/3}\,3^{1/2}}{5^{5/2}}$	$4\pm\sqrt{6}\,,\ -\frac{1}{6}(11\pm\sqrt{6})$
$\mathcal{N}=2$	$U(3)$	$-\frac{1}{2}$	$\frac{3^{1/2}}{2}$	0	$\frac{1}{2^{1/2}}$	$-3^{3/2}$	$3\pm\sqrt{17}\,,\ 2\,,\ 2$
$\mathcal{N}=1$	$SU(3)$	$\frac{1}{2^2}$	$\frac{3^{1/2}\,5^{1/2}}{2^2}$	$-\frac{3^{1/2}}{2^2}$	$\frac{5^{1/2}}{2^2}$	$-\frac{2^6\,3^{3/2}}{5^{5/2}}$	$4\pm\sqrt{6}\,,\ 4\pm\sqrt{6}$
$\mathcal{N}=0$	$SO(6)_+$	0	$2^{1/6}$	0	$\frac{1}{2^{5/6}}$	$-3\,2^{5/6}$	$6\,,\ 6\,,\ -\frac{3}{4}\,,\ 0$
$\mathcal{N}=0$	$SO(7)_+$	0	$\frac{1}{5^{1/6}}$	0	$\frac{1}{5^{1/6}}$	$-\frac{3\,5^{7/6}}{2^2}$	$6\,,\ -\frac{12}{5}\,,\ -\frac{6}{5}\,,\ -\frac{6}{5}$
$\mathcal{N}=0$	G_2	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$-\frac{2^{10/3}}{3^{1/2}}$	$6\,,\ 6\,,\ -1\,,\ -1$
$\mathcal{N}=0$	$SU(3)$	0.455	0.838	0.335	0.601	-5.864	$6.214\,,\ 5.925\,,\ 1.145\,,\ -1.284$
$\mathcal{N}=0$	$SU(3)$	0.270	0.733	0.491	0.662	-5.853	$6.230\,,\ 5.905\,,\ 1.130\,,\ -1.264$

✦ N = 2 solution will play a central role in holography !!

10D : ISO(7)$_c$ into type IIA supergravity

[AG, Varela '15]

$$d\hat{s}_{10}^2 = \Delta^{-1}\, ds_4^2 + g_{mn}\, Dy^m\, Dy^n \; ,$$

$$\begin{aligned}\hat{A}_{(3)} &= \mu_I \mu_J \left(\mathcal{C}^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \tfrac{1}{6}\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \tfrac{1}{6}\mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K\right) \\ &\quad + g^{-1} \left(\mathcal{B}_J{}^I + \tfrac{1}{2}\mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2}\mathcal{A}^I \wedge \tilde{\mathcal{A}}_J\right) \wedge \mu_I D\mu^J + \tfrac{1}{2} g^{-2}\, \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ &\quad - \tfrac{1}{2}\, \mu_I\, B_{mn}\, \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \tfrac{1}{6} A_{mnp}\, Dy^m \wedge Dy^n \wedge Dy^p \; ,\end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I \left(\mathcal{B}^I + \tfrac{1}{2}\mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J\right) - g^{-1}\, \tilde{\mathcal{A}}_I \wedge D\mu^I + \tfrac{1}{2} B_{mn}\, Dy^m \wedge Dy^n \; ,$$

$$\hat{A}_{(1)} = -\mu_I\, \mathcal{A}^I + A_m\, Dy^m \; .$$

where we have defined : $\quad Dy^m \equiv dy^m + \frac{1}{2}\, g\, K^m_{IJ}\, \mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g\, \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$\begin{aligned} g^{mn} &= \tfrac{1}{4}\, g^2\, \Delta\, \mathcal{M}^{IJ\,KL}\, K^m_{IJ}\, K^n_{KL} \quad , \quad & B_{mn} &= -\tfrac{1}{2}\, \Delta\, g_{mp}\, K^p_{IJ}\; \partial_n \mu^K\, \mathcal{M}^{IJ}{}_{K8} \; , \\ A_m &= \tfrac{1}{2}\, g\, \Delta\, g_{mn}\, K^n_{IJ}\; \mu_K\, \mathcal{M}^{IJ\,K8} \quad , \quad & A_{mnp} &= \tfrac{1}{8}\, g\, \Delta\, g_{mq}\, K^q_{IJ}\, K^{KL}_{np}\, \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} \; . \end{aligned}$$

N=2 solution of massive type IIA

- N=2 & U(3) AdS_4 point of the $\mathrm{ISO}(7)_c$ theory

$$d\hat{s}^2_{10} = L^2 \frac{\big(3+\cos 2\alpha\big)^{\frac{1}{2}}}{\big(5+\cos 2\alpha\big)^{-\frac{1}{8}}} \Big[ds^2(\mathrm{AdS}_4) + \frac{3}{2}\, d\alpha^2 + \frac{6\sin^2\alpha}{3+\cos 2\alpha}\, ds^2(\mathbb{CP}^2) + \frac{9\sin^2\alpha}{5+\cos 2\alpha}\, \boldsymbol{\eta}^2 \Big] ,$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{\big(5+\cos 2\alpha\big)^{3/4}}{3+\cos 2\alpha} \qquad , \qquad \hat{H}_{(3)} = 24\sqrt{2}\, L^2\, e^{\frac{1}{2}\phi_0} \frac{\sin^3\alpha}{\big(3+\cos 2\alpha\big)^2}\, \boldsymbol{J}\wedge d\alpha ,$$

$$L^{-1}\, e^{\frac{3}{4}\phi_0}\, \hat{F}_{(2)} = -4\sqrt{6}\, \frac{\sin^2\alpha\cos\alpha}{\big(3+\cos 2\alpha\big)\big(5+\cos 2\alpha\big)}\, \boldsymbol{J} - 3\sqrt{6}\, \frac{\big(3-\cos 2\alpha\big)}{\big(5+\cos 2\alpha\big)^2}\, \sin\alpha\ d\alpha\wedge\boldsymbol{\eta} ,$$

$$\begin{aligned} L^{-3}\, e^{\frac{1}{4}\phi_0}\, \hat{F}_{(4)} &= 6\,\mathrm{vol}_4 \\ &\quad + 12\sqrt{3}\, \frac{7+3\cos 2\alpha}{\big(3+\cos 2\alpha\big)^2}\, \sin^4\alpha\ \mathrm{vol}_{\mathbb{CP}^2} + 18\sqrt{3}\, \frac{\big(9+\cos 2\alpha\big)\sin^3\alpha\cos\alpha}{\big(3+\cos 2\alpha\big)\big(5+\cos 2\alpha\big)}\, \boldsymbol{J}\wedge d\alpha\wedge\boldsymbol{\eta} , \end{aligned}$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{5}{8}}\, 3^{-1}\, g^{-2}\, c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}}\, c^{-\frac{5}{6}}$

✦ The angle $0 \leq \alpha \leq \pi$ locally foliates S_6 with S_5 regarded as Hopf fibrations over $\mathbb{CP}^2$

3D : CFT$_3$ dual & matching of free energies

[Schwarz '04]
[Gaiotto, Tomassiello '09]

- 3d SYM + (N=2) Chern-Simons with simple group SU(N) , level k , three adjoint matter and a cubic superpotential W = Tr(X[Y,Z])

- The 3d free energy F = -Log(Z), where Z is the partition function of the CFT on a Euclidean S$_3$, can be computed via localisation over supersymmetric configurations $N \gg k$

$$F = \frac{3^{13/6}\pi}{40}\left(\frac{32}{27}\right)^{2/3} k^{1/3} N^{5/3}$$

[Pestun '07] [Kapustin, Willett, Yaakov '09]
[Jafferis '10] [Jafferis, Klebanov, Pufu, Safdi '11]
[Closset, Dumitrescu, Festuccia, Komargodski '12 '13]

- The gravitational free energy can be computed from the warp factor in the N=2 massive IIA solution. Using the charge quantisation condition $N = -(2\pi\ell_s)^{-5} \int_{S^6} e^{\frac{1}{2}\phi} \hat{*} \hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6} \hat{F}_{(0)} \hat{B}^3_{(2)}$ for the D2-brane, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} k^{1/3} N^{5/3} \qquad \text{provided} \qquad \boxed{g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)}$$

[Emparan, Johnson, Myers '99]

Holographic description of RG flows

[Boonstra, Skenderis, Townsend ’98]

- RG flows are described holographically as non-AdS_4 solutions in gravity

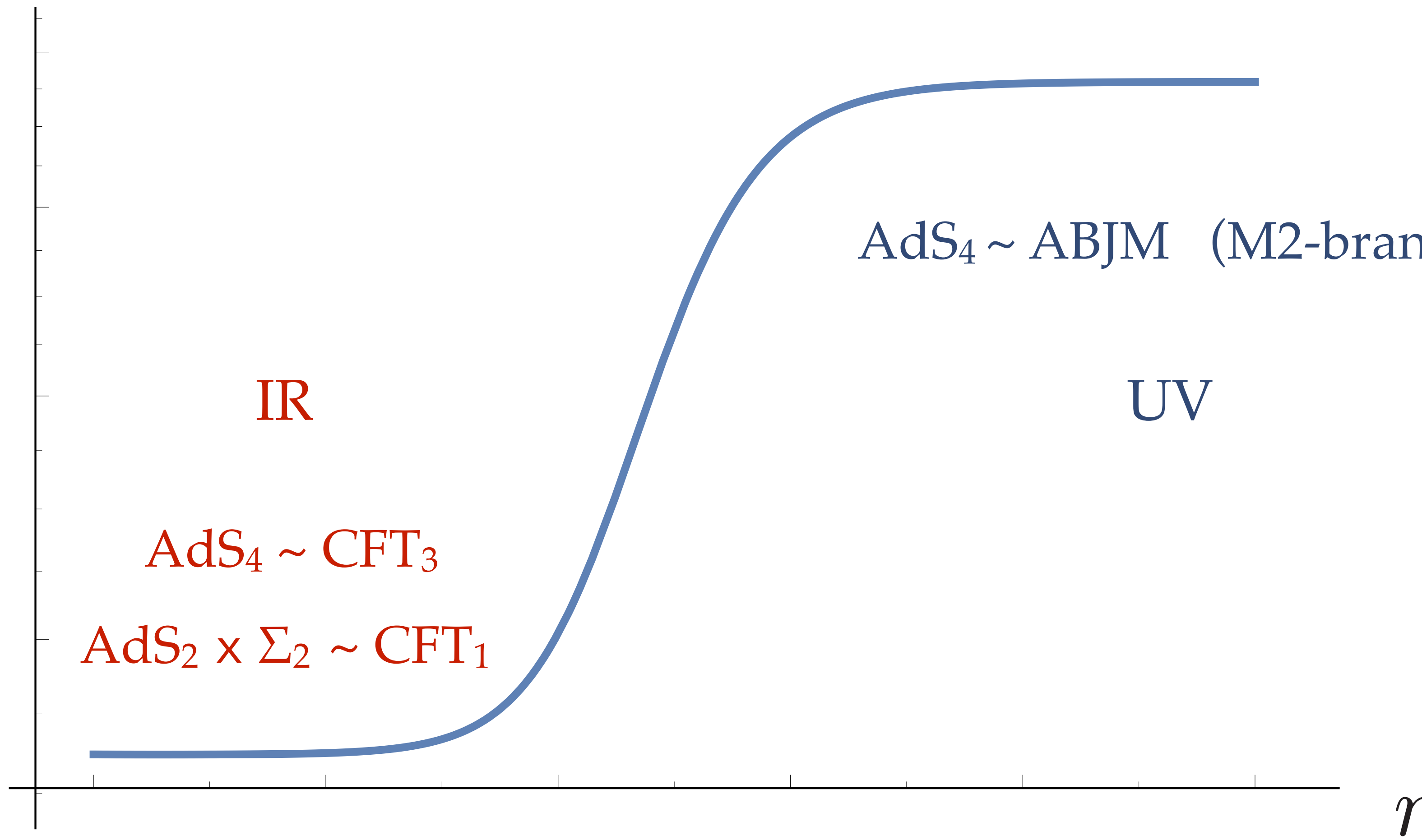
- RG flows on M2-brane :   SO(8)-gauged sugra from M-theory on S^7

[Ahn, Paeng ’00] [Ahn, Itoh ’01]
[Bobev, Halmagyi, Pilch, Warner ’09]

[Cacciatori, Klemm ’09]
[Halmagyi, Petrini, Zaffaroni ’13]
[Chimento, Klemm, Petri ’15]
[Benini, Hristov, Zaffaroni ’15 ’16]

AdS_4 in IR : domain-wall

$AdS_2 \times \Sigma_2$ in IR : black hole



- RG flows on D3-brane :   SO(6)-gauged sugra from type IIB on S^5 and N=4 SYM in 4D

[Freedman, Gubser, Pilch, Warner ’99]
[Pilch, Warner ’00] [Benini, Bobev ’12,’13]

Holographic RG flows on the D2-brane

- D2-brane :

$$d\hat{s}_{10}^2 = e^{\frac{3}{4}\phi}\left(-e^{2U}dt^2 + e^{-2U}dr^2 + e^{2(\psi-U)}ds^2_{\Sigma_2}\right) + g^{-2}e^{-\frac{1}{4}\phi}ds^2_{S^6}$$

$$e^{\hat{\Phi}} = e^{\frac{5}{2}\phi}$$

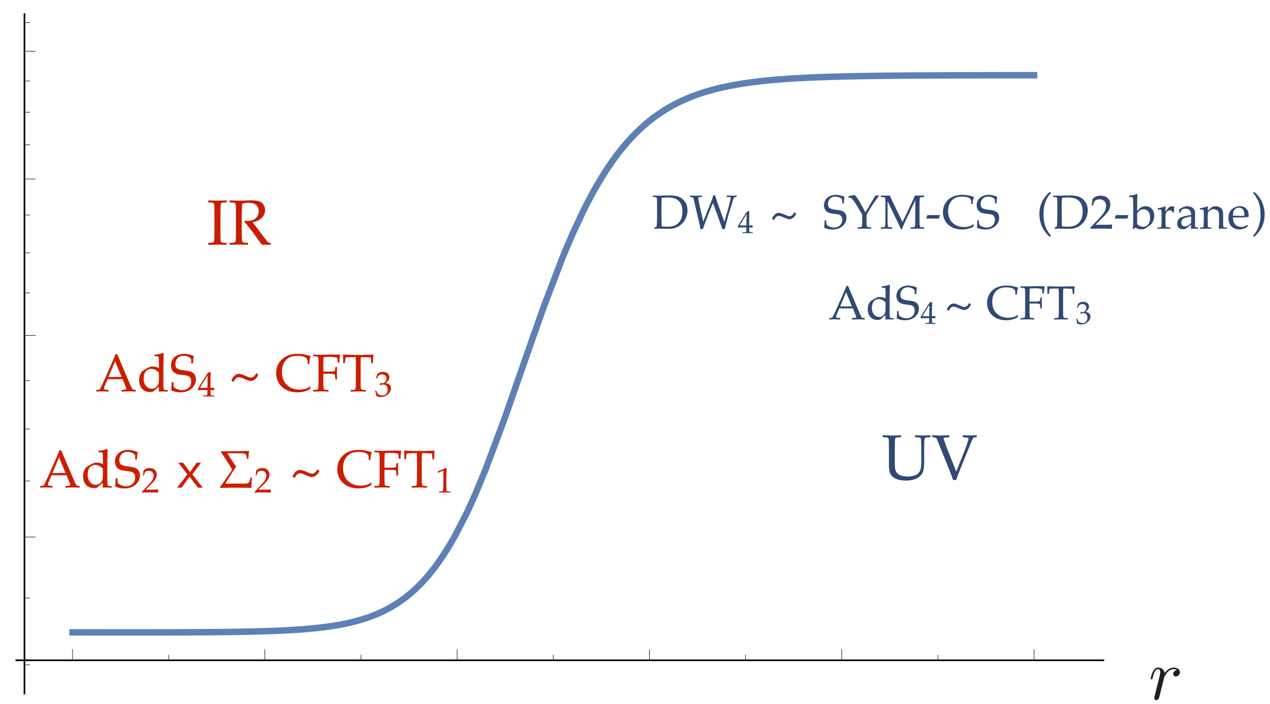
$$\hat{F}_{(4)} = 5\,g\,e^{\phi}\,e^{2(\psi-U)}\,dt\wedge dr\wedge d\Sigma_2$$

with $\quad e^{2U}\sim r^{\frac{7}{4}}\,, \qquad e^{2(\psi-U)}\sim r^{\frac{7}{4}} \quad$ and $\quad e^{\phi}\sim r^{-\frac{1}{4}}$ $\Rightarrow$ DW_4 domain-wall (SYM-CS)

- RG flows on D2-brane : $ISO(7)_c$-gauged sugra from mIIA on S^6

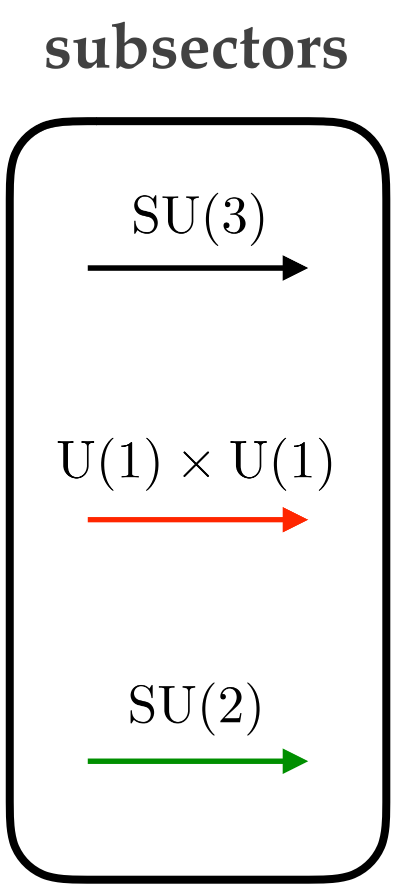
AdS_4 in IR : domain-wall

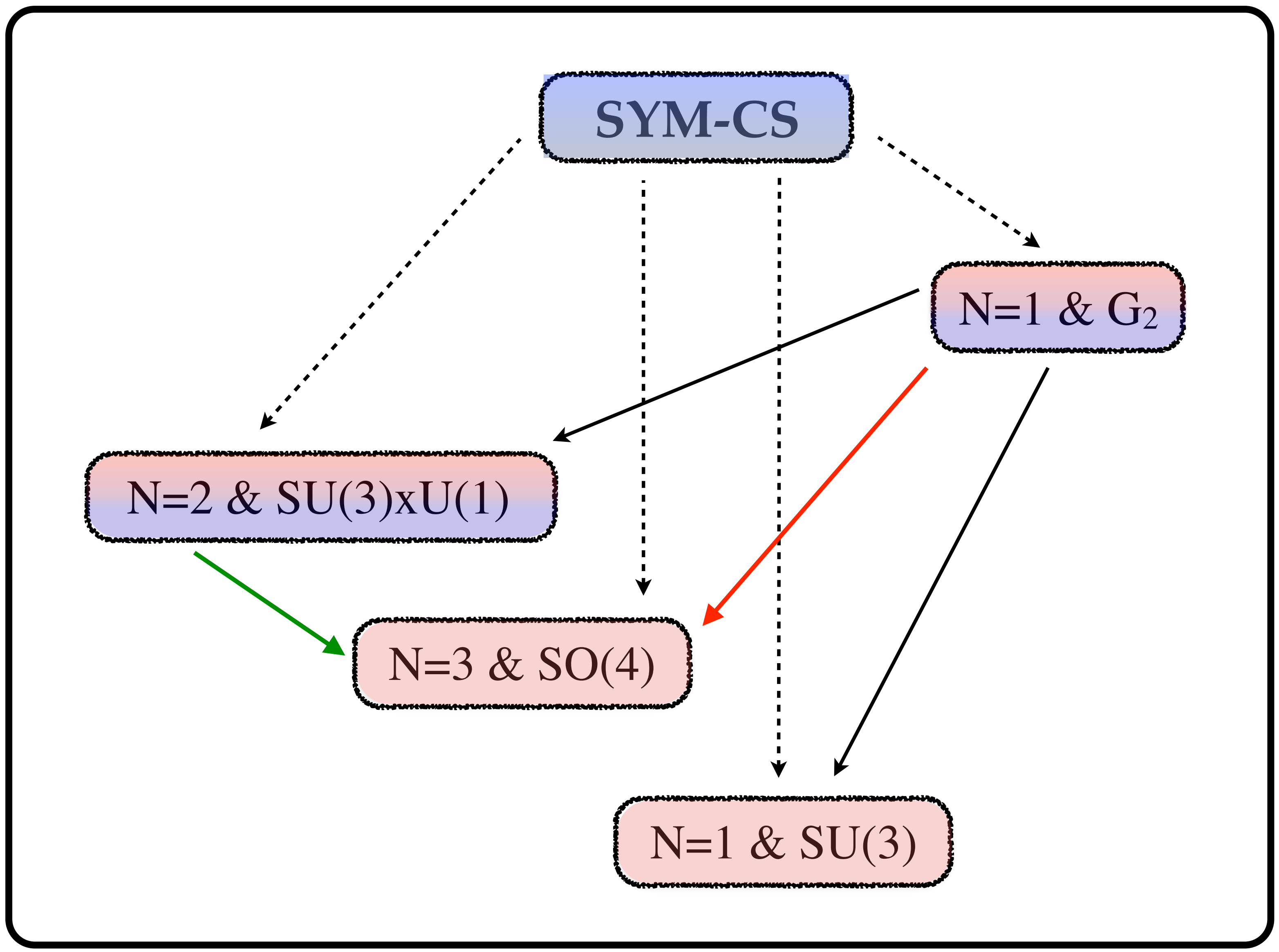
$AdS_2 \times \Sigma_2$ in IR : black hole



Domain-walls

[AG, Tarrío, Varela '16 + in progress]





- RG flows from SYM-CS (dotted lines) and between CFT's (solid lines) dual to BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity

Black holes (I)

$\Lambda = 0, 1$

- SU(3)-invariant sector: N=2 model with 1 vector + 1 hyper (universal)

- Black hole Anstaz :

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2(\psi(r)-U(r))}\left(d\theta^2 + \left(\frac{\sin\sqrt{\kappa}\,\theta}{\sqrt{\kappa}}\right)^2 d\phi^2\right)$$

$$\mathcal{A}^\Lambda = \mathcal{A}_t{}^\Lambda(r)\,dt - p^\Lambda\,\frac{\cos\sqrt{\kappa}\,\theta}{\kappa}\,d\phi$$
$$\tilde{\mathcal{A}}_\Lambda = \tilde{\mathcal{A}}_{t\,\Lambda}(r)dt - e_\Lambda\,\frac{\cos\sqrt{\kappa}\,\theta}{\kappa}\,d\phi$$

$$\mathcal{B}^0 = b_0(r)\,\frac{\sin\sqrt{\kappa}\,\theta}{\sqrt{\kappa}}\,d\theta\wedge d\phi$$

- Attractor equations :

$$\mathcal{Q} = \kappa\,L^2_{\Sigma_2}\,\Omega\,\mathcal{M}\,\mathcal{Q}^x\,\mathcal{P}^x - 4\,\mathrm{Im}(\bar{\mathcal{Z}}\,\mathcal{V})\ ,$$
$$\frac{L^2_{\Sigma_2}}{L_{\mathrm{AdS}_2}} = -2\,\mathcal{Z}\,e^{-i\beta}\ ,$$
$$\langle\mathcal{K}^u,\mathcal{V}\rangle = 0\ ,$$

[Dall'Agata, Gnecchi '10]
[Klemm, Petri, Rabbiosi '16]

- Unique AdS_2 x H^2 :

(N=2 & U(3) AdS_4 vev's)

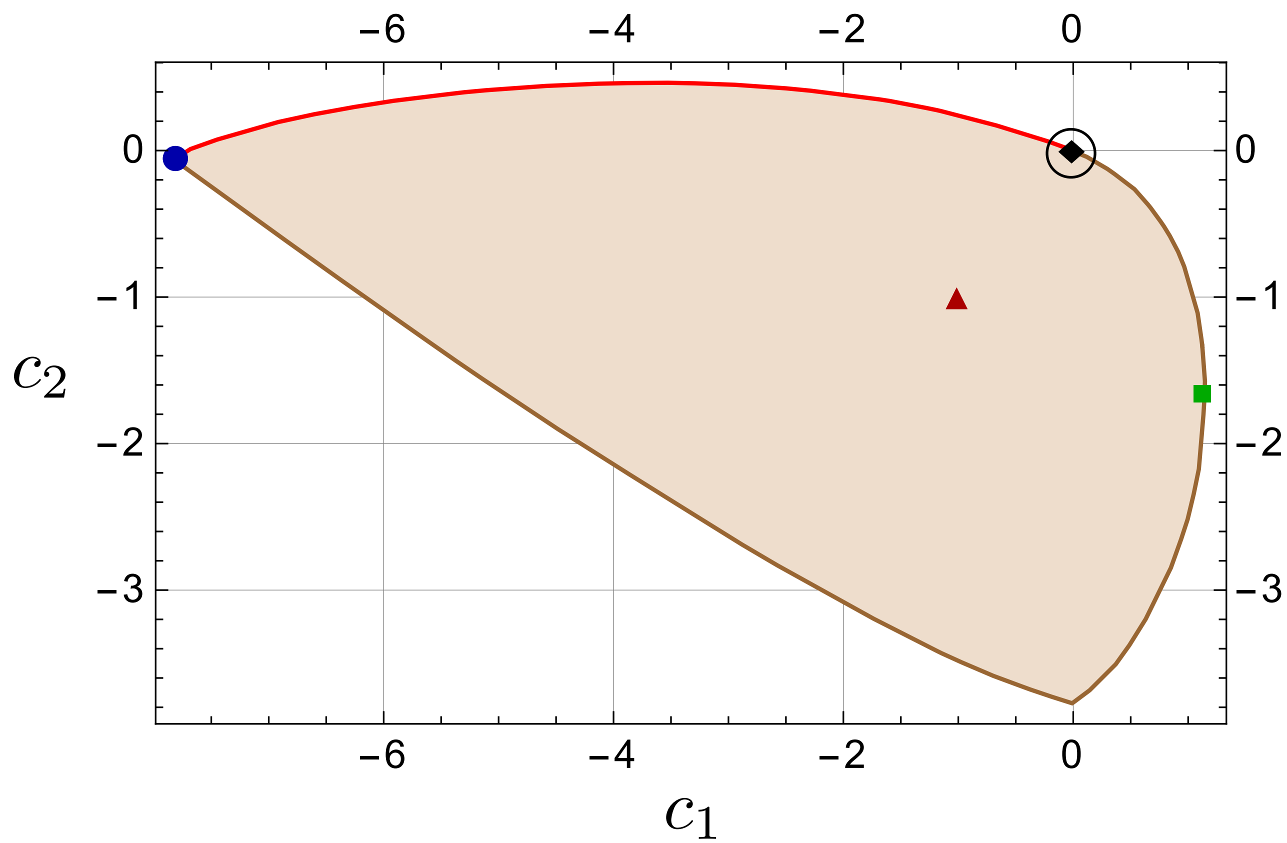
[AG, Tarrío '17]

$$e^{\varphi_h} = \frac{2}{\sqrt{3}}\left(\frac{g}{m}\right)^{\frac{1}{3}}\ ,\quad \chi_h = -\frac{1}{2}\left(\frac{g}{m}\right)^{-\frac{1}{3}}\ ,\quad e^{\phi_h} = \sqrt{2}\left(\frac{g}{m}\right)^{\frac{1}{3}}\ ,\quad a_h = \zeta_h = \tilde{\zeta}_h = 0\ ,$$
$$p^0 + \frac{1}{2}\,m\,b_0^h = \pm\frac{1}{6}\,m^{\frac{2}{3}}\,g^{-\frac{5}{3}}\ ,\quad e_0 + \frac{1}{2}\,g\,b_0^h = \pm\frac{1}{6}\,m^{-\frac{1}{3}}\,g^{-\frac{2}{3}}\ ,$$
$$p^1 = \mp\frac{1}{3}\,g^{-1}\ ,\quad e_1 = \pm\frac{1}{2}\,m^{\frac{1}{3}}\,g^{-\frac{4}{3}}\ ,$$
$$L^2_{\mathrm{AdS}_2} = \frac{1}{4\sqrt{3}}\,m^{\frac{1}{3}}\,g^{-\frac{7}{3}}\ ,\quad L^2_{\mathrm{H}^2} = \frac{1}{2\sqrt{3}}\,m^{\frac{1}{3}}\,g^{-\frac{7}{3}}\ .$$

Black holes (II)

[Dall'Agata, Gnecchi '10]
[Klemm, Petri, Rabbiosi '16]

- Two irrelevant modes (c_1, c_2) when perturbing around the $AdS_2 \times H^2$ solution in the IR



▲ : $AdS_2 \times H^2$ to DW_4

◆ : $AdS_2 \times H^2$ to AdS_4

● : $AdS_2 \times H^2$ to Lifshitz (z=2)

■ : $AdS_2 \times H^2$ to conf-Lifshitz

[AG, Tarrío '17]

- RG flows across dimension from SYM-CS **or** CFT_3 **or** non-relativistic to CFT_1

- Universal (constant scalars) RG flow (◆) CFT_3 to CFT_1

[Caldarelli, Klemm '98]
[Benini, Hristov, Zaffaroni '16]
[Azzurli, Bobev, Crichigno, Min, Zaffaroni '17]

- $AdS_2 \times \Sigma_g$ horizons for mIIA on $H^{(p,q)}$: STU-models with 3 vectors + 1 hyper [AG '17]

[Hosseini, Hristov, Passias '17] [Benini, Khachatryan, Milan '17]

Type IIB

[in progress]

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

Dyonically-gauged [SO(1,1) × SO(6)] ⋉ R^12 supergravity

- Higher-dimensional origin as Type IIB on $S^1 \times S^5$

[Inverso, Samtleben, Trigiante '16]

- New AdS4 vacuum with N=4 & SO(4) symmetry

- Holographic expectation: N=4 interface SYM theory with SO(4) symmetry & Janus solutions

[Bak, Gutperle, Hirano '03 (**N = 0**)]
[Clark, Freedman, Karch, Schnabl '04]

[D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]
[Assel, Tomasiello '18 (**N = 3 , 4**)]

- Classification of (original) interface SYM theories

N=4 & SO(4) N=2 & SU(2) × U(1) N=1 & SU(3) N=0 & SU(4)

[D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

Question : *Simple analytic* holographic duals for the N = 0, 1, 2 interface SYM theories with SU(4), SU(3) and SU(2)×U(1) internal symmetry?

A truncation : $G_0 = SU(3)$ invariant subsector

[Warner '83]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup $G_0 \subset [\,\mathrm{SO}(1,1) \times \mathrm{SO}(6)\,] \ltimes \mathbb{R}^{12}$

- SU(8) R-symmetry branching : gravitini $\mathbf{8} \to \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}$ ➡ N = 2 SUSY

- Scalars fields : $\mathbf{70} \to \mathbf{1}\ (\times 6) + \text{non-singlets}$ ➡ 6 real scalars $(\varphi\,,\, \chi\,,\, \phi\,,\, \sigma\,,\, \zeta\,,\, \tilde{\zeta}\,)$

- Vector fields : $\mathbf{56} \to \mathbf{1}\,(\times 4) + \text{non-singlets}$ ➡ vectors $(\,A^0, A^1\,;\, \tilde{A}_0, \tilde{A}_1\,)$

- N = 2 gauged supergravity with $\mathrm{G} = \mathrm{SO}(1,1)_m \times \mathrm{U}(1)_e$ with 1 vector & 1 hypermultiplet

$$\mathcal{M}_{\text{scalar}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$$

AdS$_4$ vacua

- **N=0 & SU(4) vacuum** [1 free parameter]

$$\chi = \text{free} \quad , \quad e^{-\varphi} = \frac{c}{\sqrt{2}} \, , \qquad e^{2\phi} = \frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = 0$$

… it turns out to be perturbatively unstable

- **N=1 & SU(3) vacuum** [2 free parameters]

$$\chi = 0 \quad , \quad e^{-\varphi} = \frac{\sqrt{5}c}{3} \, , \qquad e^{2\phi} = \frac{6}{5} \frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = \tfrac{2}{3}\sqrt{1-\sigma^2}$$

… the compact U(1)$_e$ symmetry broken by $|\vec{\zeta}|^2 \neq 0$ (charged)

Next step : **Uplift to Type IIB on R × S^5 using E$_{7(7)}$-EFT**

$E_{7(7)}$-EFT

[momentum, winding, …]

- Space-time : external (D=4) + **generalised internal** ($Y^{\mathcal{M}}$ coordinates in **56** of $E_{7(7)}$)

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

[Coimbra, Strickland-Constable, Waldram '11]

- Generalised Lie derivative built from an $E_{7(7)}$-invariant structure Y-tensor

$$\mathbb{L}_{\Lambda} U^{\mathcal{M}} \quad = \quad \Lambda^{\mathcal{N}} \partial_{\mathcal{N}} U^{\mathcal{M}} - U^{\mathcal{N}} \partial_{\mathcal{N}} \Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}} \, \partial_{\mathcal{N}} \Lambda^{\mathcal{P}} \, U^{\mathcal{Q}}$$

[no density term]

Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}} \, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

Two maximal solutions : M-theory (**7** dimensional) & Type IIB (**6** dimensional)

[**massless** theories]

$y^{i=1\dots5}$ (elec) , $\tilde{y}_1$ (mag)

$E_{7(7)}$-EFT

[Hohm & Samtleben '13]

- $E_{7(7)}$-EFT action $\;[\; \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu} \;]$

$$S_{\text{EFT}} = \int d^4x\, d^{56}Y\, e\,\big[\,\hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g)\,\big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\,\partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$\begin{aligned} V_{\text{EFT}}(\mathcal{M}, g) = & -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{N}} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{L}} \mathcal{M}_{\mathcal{N}\mathcal{K}} \\ & -\tfrac{1}{2}\, g^{-1} \partial_{\mathcal{M}} g\, \partial_{\mathcal{N}} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_{\mathcal{M}} g\, g^{-1} \partial_{\mathcal{N}} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} g^{\mu\nu}\, \partial_{\mathcal{N}} g_{\mu\nu} \end{aligned}$$

- Two-derivative potential : **ungauged** N=8 D=4 SUGRA when $\Phi(x, Y) = \Phi(x)$

Generalised Scherk-Schwarz reductions

- SL(8) twist (geometry) :

$$(U^{-1})_A{}^B = \left(\frac{\mathring{\rho}}{\hat{\rho}}\right)^{\frac{1}{2}} \begin{pmatrix} 1 & 0 & 0 & -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 \\ 0 & \delta^{ij} + \hat{K}\, y^i\, y^j & -\lambda\, \hat{\rho}^2 y^i & 0 \\ 0 & -\lambda\, \hat{\rho}^2 y^j\, \hat{K} & \hat{\rho}^4 & 0 \\ -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 & 0 & 0 & \mathring{\rho}^{-4}(1+\tilde{y}_1^2) \end{pmatrix}$$

- EFT fields = Twist × 4D fields :

$$\begin{aligned} g_{\mu\nu}(x,Y) &= \rho^{-2}(Y) g_{\mu\nu}(x) \\ \mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, U_N{}^L(Y)\, M_{KL}(x) \\ \mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}\, A_\mu{}^N(x)\, (U^{-1})_N{}^M(Y) \\ \mathcal{B}_{\mu\nu\,\alpha}(x,Y) &= \rho^{-2}(Y)\, U_\alpha{}^\beta(Y)\, B_{\mu\nu\,\beta}(x) \\ \mathcal{B}_{\mu\nu\,M}(x,Y) &= -2\,\rho^{-2}(Y)\, (U^{-1})_S{}^P(Y)\, \partial_M U_P{}^R(Y)\, (t^\alpha)_R{}^S\, B_{\mu\nu\,\alpha}(x) \end{aligned}$$

- Type IIB fields = EFT fields :

$$\begin{aligned} G^{mn} &= G^{1/2}\, \mathcal{M}^{mn} \,, \\ \mathbb{B}_{mn}{}^\alpha &= G^{1/2}\, G_{mp}\, \epsilon^{\alpha\beta}\, \mathcal{M}^p{}_{n\beta} \,, \\ m_{\alpha\beta} &= \tfrac{1}{6}\, G\, \left(\mathcal{M}^{mn}\, \mathcal{M}_{m\alpha\, n\beta} + \mathcal{M}^m{}_{k\alpha}\, \mathcal{M}^k{}_{m\beta} \right) \,, \\ C_{klmn} &= -\tfrac{1}{4}\, G^{1/2}\, G_{kp}\, \mathcal{M}^p{}_{lmn} + \tfrac{3}{8}\, \epsilon_{\alpha\beta}\, \mathbb{B}_{k[l}{}^\alpha\, \mathbb{B}_{mn]}{}^\beta \end{aligned}$$

Type IIB S-fold backgrounds

$$ds_{10}^2 = \tfrac{1}{2}\sqrt{Y}\, e^{\varphi}\, ds^2_{\mathrm{AdS}_4} + \sqrt{Y}\, e^{-2\varphi}\, d\eta^2 + \frac{1}{\sqrt{Y}}\left[ds^2_{\mathbb{CP}^2} + Y\, \boldsymbol{\eta}^2 \right]$$

$$\mathbb{B}^{\alpha} = -\tfrac{1}{2}\, Y^{-1}\, \epsilon^{\alpha\delta}\, (A^{-t})_{\delta}{}^{\gamma}\, H_{\gamma\beta}\, \boldsymbol{\Omega}^{\beta}$$

$$\widetilde{F}_5 = dC + \tfrac{1}{2}\,\epsilon_{\alpha\beta}\,\mathbb{B}^{\alpha}\wedge\mathbb{H}^{\beta} = \left(4 + \frac{6\,(1-Y)}{Y}\right) Y^{\frac{3}{4}}\,(1+\star)\,\mathrm{vol}_5$$

$$m_{\alpha\beta} = (A^{-t})_{\alpha}{}^{\gamma}\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^{\delta}{}_{\beta}$$

with $\quad Y = 1 + \dfrac{1}{4}\, e^{2\phi}\, |\vec{\zeta}|^2 \quad$ and $\quad (A^{-1})^{\gamma}{}_{\beta} \equiv \begin{pmatrix} \sqrt{1+\tilde{y}_1^2} & -\tilde{y}_1 \\ -\tilde{y}_1 & \sqrt{1+\tilde{y}_1^2} \end{pmatrix} = \begin{pmatrix} \cosh\eta & -\sin\eta \\ -\sin\eta & \cosh\eta \end{pmatrix}$

[(hyperbolic) SO(1,1)-twist over S¹]

N=0 & SU(4)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}} \begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$H_{\gamma\beta} = 0 \qquad Y = 1$$

N=1 & SU(3)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}} \begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$H_{\gamma\beta} \neq 0 \qquad Y = \frac{6}{5}$$

Summary

- Dyonic N = 8 supergravity with ISO(7) gauging connected to massive IIA reductions on S^6
- Dyonic N = 8 supergravity with [SO(1,1) $\times$ SO(6)] $\ltimes$ R^{12} gauging connected to Type IIB reductions on $S^1 \times S^5$.
- N = 2 $AdS_4 \times S^6$ solution of mIIA = Chern-Simons-matter theories with simple gauge group SU(N)
- S-folds of Janus-type $AdS_4 \times R \times S^5$ solutions of type IIB = interface SYM theory?
- Holographic study of RG flows on D2-brane : DW solutions (CFT_3 / CFT_3 & SYM-CS / CFT_3)
  BH solutions (CFT_3 / CFT_1 & SYM-CS / CFT_1)

[Benini, Hristov, Zaffaroni '16]
[Azzurli, Bobev, Crichigno, Min, Zaffaroni '17]
[Hosseini, Hristov, Passias '17] [Benini, Khachatryan, Milan '17] [Liu, Pando-Zayas, Zhou '18]

- Further tests/generalisations on the mIIA duality (semiclassical observables, level-rank duality, …)

[Fluder, Sparks '15] [Passias, Prins, Tomasiello '18]
[Araujo, Nastase '16] [Araujo, Itsios, Nastase, Ó Colgáin '17]

- N=2 & SU(2)$\times$U(1) interface SYM? , $SO(8)_c$ theories? ….

Thank you all !

… and thanks to the organisers

for such a great workshop !