Subleading Microstate Counting of AdS_4 Black Hole Entropy

Leo Pando Zayas
University of Michigan and ICTP

Holography, Generalized Geometry and Duality Workshop
Mainz, Germany, May 16, 2019

arXiv:1905.01559, D. Gang, N. Kim and LPZ
arXiv:1808.10445, J. Liu, LPZ and S. Zhou
PRL 120, 221602 (2018), J. Liu, LPZ, V. Rathee and W. Zhao
JHEP 1801 (2018) 026, J. Liu, LPZ, V. Rathee and W. Zhao

Motivation

$$S = \frac{k_B c^3}{\hbar} \frac{A}{4G_N}$$

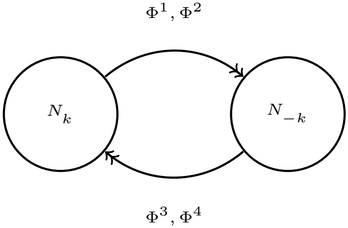
- A confluence of thermodynamical, relativistic, gravitational, and quantum aspects. Hydrogen atom of QG. [Strominger-Vafa].
- An explicit example in $\mathrm{AdS}_4/\mathrm{CFT}_3$: The large-$N$ limit of the topologically twisted index of ABJM correctly reproduces the leading term in the entropy of magnetically charged black holes in asymptotically AdS_4 spacetimes [Benini-Hristov-Zaffaroni].
- Extended also to: dyonic black holes, black holes with hyperbolic horizons, black holes in massive IIA theory and M5-branes.
- Agreement has been shown beyond the large N limit by matching the coefficient of $\log N$ [Liu-PZ-Rathee-Zhao], [Gang-Kim-PZ] (Beyond Bohr energies).

Outline

- The Topologically Twisted Index of ABJM Theory beyond large N
- Magnetically Charged Asymptotically AdS_4 Black Holes
- Logarithmic Corrections in Quantum Supergravity
- Suleading Microstate Counting for the Entropy of Wrapped M5 branes
- Conclusions

ABJM Theory

- AB[1]JM: A 3d Chern-Simons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge group with opposite levels.
- Matter sector: Four complex scalar fields $\Phi_I, (I = 1, 2, 3, 4)$ in the bifundamental representation $(\mathbf{N}, \bar{\mathbf{N}})$ and fermionic partners.
- SCFT $\mathcal{N} = 6$ supersymmetry generically but for $k = 1, 2$, the symmetry is enhanced to $\mathcal{N} = 8$.
- See Pilch's talk for a thorough introduction to ABJM.



[1]Red acknowledging authors present in the audience.

The Topologically Twisted Index of ABJM Theory

- The topologically twisted index for three dimensional $\mathcal{N}=2$ field theories was defined in [Benini-Zaffaroni] (Honda '15, Closset '15) by evaluating the supersymmetric partition function on $S^1 \times S^2$ with a topological twist on S^2.
- Hamiltonian: The supersymmetric partition function of the twisted theory, $Z(n_a, \Delta_a) = \mathrm{Tr}\,(-1)^F e^{-\beta H} e^{iJ_a \Delta_a}$. It depends on the fluxes, n_a, through H and on the chemical potentials Δ_a.
- The topologically twisted index for $\mathcal{N} \geq 2$ supersymmetric theories on $S^2 \times S^1$ can be computed via supersymmetric localization [Crichigno].
- The supersymmetric localization computation of the topologically twisted index can be extended to theories defined on $\Sigma_g \times S^1$.

General form of the Index

- Background:

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2) + \beta^2 dt^2, \quad A^R = \frac{1}{2}\cos\theta d\phi.$$

- The index can be expressed as a contour integral:

$$Z(n_a, y_a) = \sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{int}(x, \mathfrak{m}; n_a, y_a).$$

- Z_{int} meromorphic form, Cartan-valued complex variables $x = e^{i(A_t + i\beta\sigma)} = e^{iu}$, lattice of magnetic gauge fluxes $\Gamma_{\mathfrak{h}}$.
- Flavor magnetic fluxes n_a and fugacities $y_a = e^{i(A_t^a + i\beta\sigma^a)}$.
- Localization: $Z_{int} = Z_{\text{class}} Z_{\text{one-loop}}$.
- E.G.: $Z^{CS}_{class} = x^{k\mathfrak{m}}$, $Z^{gauge}_{1-loop} = \prod_{\alpha\in G}(1 - x^\alpha)(idu)^r$, r – rank of the gauge group, α – roots of G and $u = A_t + i\beta\sigma$.

- The topologically twisted index for ABMJ theory:

$$Z(y_a,n_a) = \prod_{a=1}^{4} y_a^{-\frac{1}{2}N^2 n_a} \sum_{I\in BAE} \frac{1}{\det \mathbb{B}} \times \frac{\prod_{i=1}^{N} x_i^N \tilde{x}_i^N \prod_{i\neq j}\left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)}{\prod_{i,j=1}^{N}\prod_{a=1,2}(\tilde{x}_j - y_a x_i)^{1-n_a}\prod_{a=3,4}(x_i - y_a\tilde{x}_j)^{1-n_a}}.$$

- Contour integral $\rightarrow$ Evaluation (Poles): $e^{iB_i} = e^{i\tilde{B}_i} = 1$

$$e^{iB_i} = x_i^k \prod_{j=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})},$$

$$e^{i\tilde{B}_j} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})}.$$

- The $2N\times 2N$ matrix $\mathbb{B}$ is the Jacobian relating the $\{x_i, \tilde{x}_j\}$ variables to the $\{e^{iB_i}, e^{i\tilde{B}_j}\}$ variables

Algorithmic Summary:

- Given the chemical potentials Δ_a according to $y_a = e^{i\Delta_a}$, and variables $x_i = e^{iu_i}$, $\tilde{x}_j = e^{i\tilde{u}_j}$, the equations (poles):

$$0 = ku_i - i\sum_{j=1}^{N}\left[\sum_{a=3,4}\log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2}\log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right)\right] - 2\pi n_i,$$
$$0 = k\tilde{u}_j - i\sum_{i=1}^{N}\left[\sum_{a=3,4}\log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2}\log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right)\right] - 2\pi \tilde{n}_j.$$

- The topologically twisted index: (i) solve these equations for $\{u_i, \tilde{u}_j\}$; (ii) insert the solutions into the expression for Z.

The large-N limit

- In the large-N limit, the eigenvalue distribution becomes continuous, and the set $\{t_i\}$ may be described by an eigenvalue density $\rho(t)$.

$$u_i = iN^{1/2}\,t_i + \pi - \tfrac{1}{2}\delta v(t_i), \qquad \tilde{u}_i = iN^{1/2}\,t_i + \pi + \tfrac{1}{2}\delta v(t_i),$$

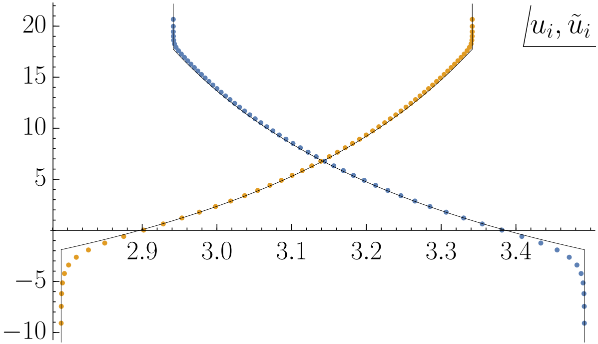


Figure: Eigenvalues for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$.

- Description of the eigenvalue distribution.

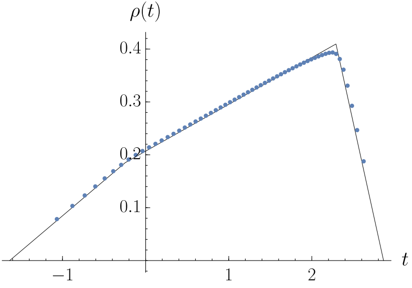


Figure: The eigenvalue density $\rho(t)$ and the function $\delta v(t)$ for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$, compared with the leading order expression.

$$\mathrm{Re}\log Z = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a}$$

Beyond Large N: Numerical Fits

Δ_1	Δ_2	Δ_3	f_1	f_2	f_3
$\pi/2$	$\pi/2$	$\pi/2$	3.0545	-0.4999	-3.0466
$\pi/4$	$\pi/2$	$\pi/4$	$4.2215 - 0.0491n_1$ $-0.1473n_2 - 0.0491n_3$	$-0.4996 + 0.0000n_1$ $+0.0000n_2 + 0.0000n_3$	$-4.1710 - 0.2943n_1$ $+0.0645n_2 - 0.2943n_3$
0.3	0.4	0.5	$7.9855 - 0.2597n_1$ $-0.5833n_2 - 0.6411n_3$	$-0.4994 - 0.0061n_1$ $-0.0020n_2 - 0.0007n_3$	$-9.8404 - 0.9312n_1$ $-0.0293n_2 + 0.3739n_3$
0.4	0.5	0.7	$6.6696 - 0.1904n_1$ $-0.4166n_2 - 0.4915n_3$	$-0.4986 - 0.0016n_1$ $-0.0008n_2 - 0.0001n_3$	$-7.5313 - 0.6893n_1$ $-0.1581n_2 + 0.2767n_3$

- Numerical fit for:

$$\mathrm{Re}\log Z = \mathrm{Re}\log Z_0 + f_1 N^{1/2} + f_2 \log N + f_3 + \cdots$$

- The values of N used in the fit range from 50 to $N_{\max}$ where $N_{\max} = 290, 150, 190, 120$ for the four cases, respectively.
- The index is independent of the magnetic fluxes in the special case $\Delta_a = \{\pi/2, \pi/2, \pi/2, \pi/2\}$

- In the large-N limit, the $k=1$ index takes the form

$$F = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a} + N^{1/2} f_1(\Delta_a, n_a) - \frac{1}{2}\log N + f_3(\Delta_a, n_a) + \mathcal{O}(N^{-1/2}),$$

where $F = \text{Re}\log Z$.

- The leading $\mathcal{O}(N^{3/2})$ term [BHZ], and exactly reproduces the Bekenstein-Hawking entropy of a family of extremal AdS_4 magnetic black holes admitting an explicit embedding into 11d supergravity, once extremized with respect to the flavor and R-symmetries.
- The $-\frac{1}{2}\log N$ term [Liu-PZ-Rathee-Zhao].

Topologically twisted index on Riemann surfaces

- The topologically twisted index can be defined on Riemann surfaces with arbitrary genus. There is a simple relation between the index on $\Sigma_g \times S^1$ and that on $S^2 \times S^1$:
  $F_{S^2\times S^1}(n_a, \Delta_a) = (1-g) F_{\Sigma_g \times S^1}(\frac{n_a}{1-g}, \Delta_a)$.
- Since the coefficient of the logarithmic term in $F_{S^2\times S^1}$ does not depend on n_a we simply have

$$F_{\Sigma_g \times S^1}(n_a, \Delta_a) = \cdots - \frac{1-g}{2} \log N + \cdots .$$

- The $-\frac{1-g}{2} \log N$ from quantum supergravity [Liu-PZ-Rathee-Zhao].

AdS_4/CFT_3

- Holographically, ABJM describes a stack of N M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity, whose low energy dynamics is effectively described by 11 dimensional supergravity.
- The index is computed for ABJM theory with a topological twist, equivalently, fluxes on S^2. On the gravity side it corresponds to microstate counting of magnetically charged asymptotically AdS_4 black holes.

Supergravity solution

- A solution of four dimensional $\mathcal{N}=2$ gauged sugra with prepotential $F=-2i\sqrt{X^0X^1X^2X^3}$ coming from M theory on $AdS_4\times S^7$ with $U(1)^4 \in SO(8)$.
- Background metric :

$$ds^2=-e^{\mathcal{K}(X)}\left(g\,r-\frac{c}{2g\,r}\right)^2dt^2+e^{-\mathcal{K}(X)}\frac{dr^2}{\left(g\,r-\frac{c}{2g\,r}\right)^2}+2e^{-\mathcal{K}(X)}r^2d\Omega_2^2.$$

- Magnetic charges

$$F^a_{\theta\phi}=-\frac{n_a}{\sqrt{2}}\sin\theta,\qquad F^1_{tr}=0.$$

Bekenstein-Hawking entropy and Index

- The Bekenstein-Hawking entropy:

$$
\begin{aligned}
S(n_a) &= \frac{1}{4G_N}A = \frac{2\pi}{G_N}e^{-\mathcal{K}(X_h)}r_h^2 = \frac{2\pi}{G_N}(F_2+\sqrt{\Theta})^{1/2} \\
F_2 &= \frac{1}{2}\sum_{a<b}n_a n_b - \frac{1}{4}\sum_a n_a^2, \quad \Theta = (F_2)^2 - 4n_1n_2n_3n_4.
\end{aligned}
$$

- Extremize the index $Z(n_a, y_a)$ with respect to y_a coincides with the entropy $\ln \mathrm{Re} Z(n_a, \tilde{y}_a) = S_{BH}$.
- Goal: Compute one-loop corrections around this sugra background in 11 Sugra and compare with field theory.

Logarithmic terms in one-loop effective actions

- One-loop effective action is equivalent to computations of determinants.
- For a given kinetic operator A one naturally defines the logarithm of its determinant as

$$\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\sum_n{}'\,\ln \kappa_n$$

  where prime denotes that the sum is over non-vanishing eigenvalues, κ_n, of A.
- It is further convenient to define the heat Kernerl of the operator A as

$$K(\tau) = e^{-\tau A} = \sum_n e^{-\kappa_n \tau}\mid \phi_n\rangle\langle\phi_n\mid .$$

Logarithmic terms in one-loop effective actions

- The heat kernel contains information about both, the non-zero modes and the zero modes.
- Let n_A^0 be the number of zero modes of the operator A.

$$-\frac{1}{2}\,\ln \det{}' A = \frac{1}{2}\int_\epsilon^\infty \frac{d\tau}{\tau}\left(\mathrm{Tr}K(\tau) - n_A^0\right)$$

where ϵ is a UV cutoff.

- At small τ, the Seeley-DeWitt expansion for the heat kernel is appropriate:

$$\mathrm{Tr}K(\tau) = \frac{1}{(4\pi)^{d/2}}\sum_{n=0}^{\infty}\tau^{n-d/2}\int d^dx\,\sqrt{g}\,a_n(x,x).$$

Logarithmic terms in one-loop effective actions

- Since, non-zero eigenvalues of a standard Laplace operator A scale as L^{-2}, it is natural to redefine $\bar{\tau} = \tau/L^2$.

$$-\frac{1}{2}\,\ln \det{}' A = \frac{1}{2}\int_{\bar{\epsilon}}^{\infty}\frac{d\bar{\tau}}{\bar{\tau}}\left(\sum_{n=0}^{\infty}\frac{1}{(4\pi)^{d/2}}\,\bar{\tau}^{\,n-d/2}\,L^{2n-d}\int d^dx\,\sqrt{g}\,a_n(x,x) - n_A^0\right).$$

- The logarithmic contribution to $\ln \det{}' A$ comes from the term $n = d/2$,

$$-\frac{1}{2}\,\ln \det{}' A = \left(\frac{1}{(4\pi)^{d/2}}\int d^dx\,\sqrt{g}\,a_{d/2}(x,x) - n_A^0\right)\log L + \ldots.$$

- On very general grounds (diffeomorphism), the coefficient $a_{d/2}$ vanishes in odd-dimensional spacetimes,.

Quantum Supergravity: Key Facts

- The coefficient of the logarithmic term is well-defined.
- In odd-dimensional spaces the coefficient of the $\log$ can only come from zero modes or boundary modes.
- Corrections to entropy from one-loop part of the partition function:

$$S_1 = \lim_{\beta\to\infty}(1-\beta\partial_\beta)\left(\sum_D (-1)^D(\tfrac{1}{2}\log\det{}'D) + \Delta F_0\right),$$

- D stands for kinetic operators corresponding to various fluctuating fields and $(-1)^D = -1$ for bosons and 1 for fermions.
- The zero modes are accounted for separately by

$$\Delta F_0 = \log\int [d\phi]|_{D\phi=0},$$

where $\exp(-\int d^dx\sqrt{g}\phi D\phi) = 1$.

Quantum Supergravity: Key Facts

- The structure of the logarithmic term in 11d Sugra:

$$\log Z[\beta, \dots] = \sum_{\{D\}} (-1)^D (\beta_D - 1) n_D^0 \log L + \Delta F_{\text{Ghost}}.$$

- Subtract the zero modes (-1) and add them appropriately due to integration over zero modes (β_D).
- The ghost contributions are treated separately.
- An IR window into the UV physics: Requires only information about the massless spectrum and confronts the UV theory!

Zero modes and L^2 cohomology

- For zero modes of A_p in compact space, one requires $\langle dA_p, dA_p\rangle = 0$. This amounts to requiring A_p to be closed. But A_p and $A_p + d\alpha_{p-1}$ are gauge equivalent, and the redundant contributions in the path integral are canceled by the Faddeev-Popov procedure (Closed modulo exact forms). The number of the zero modes is the dimension of the p-th de-Rham cohomology.
- Problem in non-compact spaces: The gauge transformation $d\alpha_{p-1}$ can be normalizable and included in the physical spectrum yet the gauge parameter α_{p-1} can be non-normalizable $p-1$. Faddeev-Popov procedure can only cancel gauge transformations with normalizable α_{p-1}.
- A physical spectrum with some pure gauge modes with non-normalizable gauge parameter is ubiquitous in one-loop gravity computations in AdS [Sen].
- Mathematically, one considers L^2 cohomology, $H^p_{L^2}(M, \mathbb{R})$.

Zero modes

- The Euler characteristics contains relevant information about the number of zero modes

$$\chi(M) = \sum_p (-1)^p \mathrm{dim}\mathcal{H}^p(M,\mathbb{R}).$$

- There is an appropriate modification of the Gauss-Bonnet theorem in the presence of a boundary:

$$\chi = \frac{1}{32\pi^2}\left(\int E_4 - 2 \int \epsilon_{abcd}\, \theta^a_b\, \mathcal{R}^c_d + \frac{4}{3} \int \epsilon_{abcd}\, \theta^a_b\, \theta^c_e\, \theta^e_d \right)$$
$$= 2.$$

- Euler density: $E_4 = \frac{1}{64}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$.
- Generalize to black hole with horizon of Σ_g: $\chi = 2(1-g)$.

Zero modes

- In the non-extremal case the topology of the black hole is homotopic to its horizon Σ_g due to the contractible (t, r) directions.
- The Euler characteristic of the non-extremal black hole is simply $\chi_{\text{BH}} = 2(1-g)$.
- It also indicates that all but the second relative de-Rham cohomology vanish

$$\dim^R \mathcal{H}^2_{L^2}(M, \mathbb{R}) = \int^{\text{Reg}} \text{Pf}(R) = \chi_{\text{BH}} = 2(1-g).$$

- The no- extremal black hole background has only two-form zero modes and their regularized number is:

$$n_2^0 = 2(1-g).$$

- Where are the 2-forms in 11d Sugra?

Quantizing a p-form

- The general action for quantizing a p-form A_p requires a set of $(p-j+1)$-form ghost fields, with $j = 2, 3, \ldots, p+1$.
- The ghost is Grassmann even if j is odd and Grassmann odd if j is even [Siegel '80]

$$\Delta F_{\text{Ghost}} = \sum_j (-1)^j (\beta_{A_{p-j}} - j - 1) n^0_{A_{p-j}} \log L.$$

Quantizing C_3 and zero modes

- The quantization of the three-form $C_{\mu\nu\rho}$ introduces 2 two-form ghosts that are Grassmann odd, 3 one-form ghosts that are Grassmann even and 4 scalar ghosts that are Grassmann odd[Siegel '80].
- Note that $C_{\mu\nu\rho}$ itself can't decompose as a massless two-form in the black hole background and a massless one-form in the compact dimension since S^7 does not admit any non-trivial 1-cycles.
- The only two-form comes from the two-form ghosts when quantizing $C_{\mu\nu\rho}$

Quantizing C_3 and zero modes

- Ghost contribution to one-loop effective action:

$$\Delta F = \Delta F_{\text{Ghost2form}}.$$

- The logarithmic term in the one-loop contribution to the entropy is

$$(2-\beta_2) n_2^0 \log L,$$

- Recall β_2 comes from integrating the zero modes in the path integral, and the minus sign takes care of the Grassmann odd nature of A_2.

Zero modes: Two form zero modes

- The properly normalized measure is $\int d[A_{\mu\nu}] \exp(-L^7 \int d^{11}x \sqrt{g^{(0)}} g^{(0)\mu\nu} g^{(0)\rho\sigma} A_{\mu\rho} A_{\nu\sigma}) = 1$, where we single out the L dependence of the metric, $g^{(0)}_{\mu\nu} = \frac{1}{L^2} g_{\mu\nu}$. Thus the normalized measure is $\prod_x d(L^{\frac{7}{2}} A_{\mu\nu})$. For each zero mode, there is a $L^{\frac{7}{2}}$ factor. Thus in the logarithmic determinant, one has $\beta_2 = \frac{7}{2}$.
- Recall that the $\log L$ correction to the partition function is $((2-\beta_2) n_2^0 \log L$; using that $\beta_2 = 7/2$ and $n_2^0 = 2(1-g))$ leads to:

$$\log Z[\beta, \dots] = -3(1-g) \log L + \cdots \, .$$

Final Result

- The coefficient of the logarithmic term does not depend on β

$$
\begin{aligned}
S_{\text{1-loop}} &= (1-\beta\partial_\beta)(-3(1-g)\log L) + \cdots \\
&= -3(1-g)\log L + \cdots \, .
\end{aligned}
$$

- As this is β independent, it is also valid in the extremal limit, $\beta \to \infty$.
- The AdS/CFT dictionary establishes that $L \sim N^{1/6}$ leading to a logarithmic correction to the extremal black hole entropy of the form

$$
S = \cdots - \frac{1-g}{2}\log N + \cdots \, ,
$$

- Perfectly agrees with the microscopic result!!!

Wrapped M5 branes in AdS/CFT

- Se talks by Bah, Crichigno for treatments of M5 branes and localization, respectively.

$\text{AdS}_4/\text{CFT}_3$	from M2-branes	from M5-branes
M-theory set-up	N M2-branes probing Cone(Y_7)	N M5-branes wrapped on M_3
Dual Field theory	Known only for special examples of Y_7	Systematic algorithm applicable to general M_3
Gravity dual	$AdS_4 \times Y_7$	Warped $AdS_4 \times M_3 \times \tilde{S}^4$
Symmetry	Isometry of Y_7 ($\supset U(1)_R$)	Only $U(1)_R$
L^2/G_4	$\frac{N^{3/2}\pi^2}{\sqrt{27/8\text{vol}(Y_7)}}$	$\frac{2N^3\text{vol}(M_3)}{3\pi^2}$
L/L_p	$\propto N^{1/6}$	$\propto N^{1/3}$

Field theory topologically twisted partition function

- The twisted partition functions for general $\mathcal{N}=2$ theory can be written as [Closset et al. '17, '18].

$$\mathcal{Z}_{p,g}^{\nu_R} = \sum_{\alpha} (\mathcal{H}_{\nu_R}^{\alpha})^{g-1} (\mathcal{F}_{\nu_R}^{\alpha})^{p} \,, \tag{1}$$

- where α labels vacua of the 3d $\mathcal{N}=2$ on $\mathbb{R}^2 \times S^1$, called Bethe-vacua, and $\mathcal{H}$ and $\mathcal{F}$ are called 'handle-gluing' and 'fibering' operators, respectively.

The 3d-3d Correspondence

3D $\mathcal{T}_N[M_3]$ theory on $\mathbb{R}^2 \times S^1$	$PSL(N,\mathbb{C})$ CS theory on M_3
Bethe vacuum α	Irreducible flat connection $\mathcal{A}^\alpha$
Fibering operator $\mathcal{F}^\alpha_{\nu_R=\frac{1}{2}}$	$\exp(-\frac{1}{2\pi i}S_0^\alpha) = \exp(\frac{1}{4\pi i}CS[\mathcal{A}^\alpha; M_3])$
Handle gluing operator $\mathcal{H}^\alpha_{\nu_R=\frac{1}{2}}$	$N\exp(-2S_1^\alpha) = N \times \mathbf{Tor}^{(\alpha)}_{M_3}(\tau_{\text{adj}}, N)$

- $\mathbf{Tor}^{(\alpha)}_{M_3}(\tau, N)$ is analytic torsion (Ray-Singer torsion) for an associated vector bundle in a representation $\tau \in \mathrm{Hom}[PSL(N,\mathbb{C}) \to GL(V_\tau)]$ twisted by a flat connection $\mathcal{A}^\alpha$.
- The dictionary for the handle gluing operator works only for M_3 with vanishing $H_1(M_3, \mathbb{Z}_N)$.

Field Theory Answer from CS invariants

$$
\begin{aligned}
&|\mathcal{Z}^{\nu_R=\frac{1}{2}}_{g,p=0}(\mathcal{T}_N[M_3])| \\
&\xrightarrow{N\to\infty} 2\cos\left((1-g)\theta_{N,M_3}\right) \\
&\qquad \times \exp\left((g-1)\left(\frac{\mathrm{vol}(M_3)}{3\pi}(N^3-N)\right.\right. \\
&-a(M_3)(N-1)-b(M_3)+\log N-c(M_3;N)\Big)\Bigg) \\
&\qquad \times\left(1+e^{-\left(\dots\right)}\right).
\end{aligned}
\tag{2}
$$

- The $1/N$ expansion terminates at $o(N^0)$.
- Analytic result, no numerics involved.
- Logarithmic correction to the $\log\mathcal{Z}$ is $(g-1)\log N$.

- Pernici-Sezgin, Acharya-Gautlett-Kim, Gauntlett-Kim-Waldram, Donos-Gauntlett-Kim-Varela.
- A universal black hole solution, similar to the M2 embedding of [ABCMZ].

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{L^2}{4}F^2\right) . \tag{3}$$

$$\frac{ds^2}{L^2} = -(\rho - \frac{1}{2\rho})^2 dt^2 + \frac{1}{(\rho-\frac{1}{2\rho})^2}d\rho^2 + \rho^2 ds^2(\Sigma_g) ,$$
$$F = \frac{1}{L^2}(\text{volume form on } \Sigma_g) . \tag{4}$$

$$S_{\rm BH} = \frac{A_{\rm horizon}}{4G_4} = \frac{2\pi(g-1)L^2}{4G_4}$$
$$\boxed{S = \frac{(g-1)\text{vol}(M_3)N^3}{3\pi}\checkmark} \tag{5}$$

Logarithmic correction: one-loop quantum gravity

$$\log Z_{1-loop} = (2-\beta_2)n_2^0 \log L = (2-7/2)2(1-g)\log L = (g-1)\log N \checkmark$$

A challenge (prediction) for field theory for $b_1(M_3) \neq 0$:

$$\begin{aligned} \log Z\Big|_{C_3} &= (-1)^1(\beta_{C_3}-1)n_{C_3}^{(0)}\log L \\ &= -\left(\frac{5}{2}-1\right)2(1-g)b_1\log L \\ &= 3(g-1)b_1\log L \\ &= (g-1)b_1\log N, \end{aligned} \tag{6}$$

The full answer:

$$\log Z_{1-loop} = (g-1)(1+b_1)\log N$$

Conclusions and Open Problems

- The degrees of freedom do not live locally at the horizon. Corrections to the Quantum Entropy Formula, extra hair in AdS [Sen] [Hristov-Lodato-Reys]. Reconciling the near-horizon and the asymptotic region in AdS at the quantum level.
- 't Hooft limit:$\lambda = N/k$ held fixed as $N \to \infty$ [PZ-Yu] ✓.
  A window into Mock modularity? [Drukker-Mariño-Putrov]
  $F_{S^3}(\lambda, N) = \sum_{g=0}^{\infty} \left(\frac{2\pi\, i\, \lambda}{N}\right)^{2g-2} F_g(\lambda)$
- Electrically charged, rotating AdS_5 black hole entropy explained from the the $\mathcal{N}=4$ SYM index on $S^1 \times S^3$: Kim, Martelli, Benini, Honda. [Logarithmic police!]
- A *precise setup* to attack important questions of black hole physics in the AdS/CFT: Information loss paradox.