Holographic Vacua of $\mathcal{N} = 1^*$

Friðrik Freyr Gautason

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based on [1805.03623 + 190x.xxxxx] with Nikolay Bobev, Benjamin Niehoff and Jesse Van Muiden

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KU LEUVEN

Confinement in AdS/CFT

Not many explicit duals to confining $\mathcal{N} = 1$ thories.

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It would be great to have an explicit type IIB background dual to a deformation of $\mathcal{N} = 4$ that exhibits confinement in the IR and is regular (or mildly singular due to explicit branes).

$\mathcal{N} = 1^*$ Gauge theory

$\mathcal{N} = 4$ in $\mathcal{N} = 1$ Language

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$$A_\mu \, , \qquad X_I \, , \qquad \psi_a \, ,$$

where μ is a Lorentz index, $I \in \mathbf{6}$ and $a \in \mathbf{4}$ of $\mathfrak{su}(4) \simeq \mathfrak{so}(6)$. All transform in the adjoint of the gauge group $\mathrm{SU}(N)$

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Only $\mathfrak{u}(1)_r \times \mathfrak{su}(3)_f \subset \mathfrak{su}(4)$ manifest. $i \in \mathbf{3}$ of $\mathfrak{su}(3)$. Kähler and super-potentials for $\mathcal{N} = 4$

$$K = \frac{1}{g_{\mathrm{YM}}^2} \mathrm{Tr} \ \Phi_i^\dagger \Phi_i \ , \qquad W = \frac{1}{g_{\mathrm{YM}}^2} \mathrm{Tr} \ [\Phi_1, \Phi_2]\Phi_3 \ .$$

$\mathcal{N} = 1^*$

Add mass terms

$$\delta W = \frac{1}{g_{\mathrm{YM}}^2} \mathrm{Tr}\left(m_1 \Phi_1^2 + m_2 \Phi_2^2 + m_3 \Phi_3^2\right) .$$

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Three special cases

- $m_1 = m_2 \neq 0\,,\; m_3 = 0 \quad \Rightarrow \quad \mathcal{N} = 2^*\,,$
- $m_1 = m_2 = 0\,,\; m_3 \neq 0 \quad \Rightarrow$ Leigh-Strassler fixed point ,
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Focus on the last case which displays rich vacuum structure.

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Classical vacuum equations

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Most of the time the IR theory will have unbroken $\mathsf{U}(1)$, i.e. free Electro-magnetism. "Coulomb vacuum".

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Each element on the sublattice denotes a dyon that condenses in the vacuum. The dual lattice denotes dyons that are confined. Terminology: all vacua specified by x, y are *massive*

- $D = 1$: Higgs vacuum (classically massive).
- $D = N \;, \; b = 0$: Confining vacuum.
- all other : oblique confining vacuum.

OEIS[A000203]=1, 3, 4, 7, 6, 12, 8, 15, 13, 18, 12, 28, 14, 24, 24, 31, 18, 39, 20, 42, 32, 36, 24, 60, 31, 42, 40, 56, 30, 72, 32, 63, …

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$$\partial V_{\mathrm{D3}} = 0 \quad \Rightarrow \quad [\phi_i, \phi_j] + m \epsilon_{ij}{}^{k} \phi_k = 0 \, .$$

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5D SUPERGRAVITY

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The 5D SUGRA has $\mathfrak{su}(4)$ *gauge* symmetry and among others 42 scalar fields that transform in

$$\mathbf{20}'_0 \oplus \mathbf{10}_{-2} \oplus \overline{\mathbf{10}}_2 \oplus \mathbf{1}_4 \oplus \mathbf{1}_{-4} \ ,$$

under $\mathfrak{su}(4) \times \mathfrak{u}(1)_Y$. The $\mathfrak{u}(1)_Y$ is a remnant of $\mathfrak{sl}(2,\mathbf{R})$ of type IIB.

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The invariant subsector is a $\mathcal{N} = 2$ supergravity with two hypermultiplets. The eight scalars parametrize the scalar manifold

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scalar	$\mathfrak{su}(4)$ rep	dual operator
φ	$\mathbf{1}$	$\mathrm{Tr}\,\lvert F\rvert^2$ and $\mathrm{Tr}\,(F \wedge F)$
ϕ_4	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$\mathrm{Tr}\,(\psi_4 \psi_4)$
ϕ	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$\mathrm{Tr}\,(\psi_i \psi_i)$
α	$\mathbf{20'}$	$\mathrm{Tr}\,(\phi_i \phi_i)$

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Two ways to truncate the theory by imposing discrete symmetries. Both lead to 4-scalar truncations. One of those can be further truncated using residual $\mathfrak{u}(1)$ symmetries.

$$\begin{aligned} \text{self-dual} &: \quad \phi_4\,,\ \phi \quad \rightarrow \quad \text{Re}\,\phi_4\,,\ \text{Re}\,\phi\,, \\ \text{parity-invariant} &: \quad \text{Re}\,\varphi\,,\ \text{Re}\,\phi_4\,,\ \text{Re}\,\phi\,,\ \text{Re}\,\alpha\,. \end{aligned}$$

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First of these was used by GPPZ to find a supersymmetric domain wall solution.

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The second was used to construct a Euclidean domain wall solution on S^4.

Bobev, Elvang, Kol, Olson and Pufu (2016)

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I will focus on the second truncation with metric-scalar sector

$$\mathcal{L} = \frac{1}{4\pi G_N}\sqrt{|g|}\left(\frac{1}{4}R + \frac{1}{2}\mathcal{K}_{i\bar{\jmath}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} - \mathcal{P}\right) ,$$

with

$$\mathcal{K}_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} \mathcal{K} , \qquad \mathcal{K} = -\log\left[(1 - z_1\bar{z}_1)(1 - z_2\bar{z}_2)^3\right] ,$$

$$\mathcal{W} = \frac{3g}{4}(1 + z_1 z_2)\left(1 - z_2^2\right) , \quad \mathcal{P} = \frac{1}{2}\mathrm{e}^{\mathcal{K}}\left[\mathcal{K}^{i\bar{\jmath}} D_i \mathcal{W} D_{\bar{\jmath}}\overline{\mathcal{W}} - \frac{8}{3}\mathcal{W}\overline{\mathcal{W}}\right] .$$

and

$$z_1 = \tanh\frac{1}{2}(3\alpha + \varphi - 3i\phi + i\phi_4) ,$$
$$z_2 = \tanh\frac{1}{2}(\alpha - \varphi - i\phi - i\phi_4) .$$

BPS EQUATIONS ON $\mathbf{R}^{1,3}$ AND S^4

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + \mathcal{R}^2 \mathrm{e}^{2A} \mathrm{d}\Omega_4^2 \,,$$

The BPS equations are

$$(A')^2 = \mathcal{R}^{-2}\mathrm{e}^{-2A} + \frac{4}{9}\mathrm{e}^{\mathcal{K}}\mathcal{W}\widetilde{\mathcal{W}} \,,$$

$$(A' + \mathcal{R}^{-1}\mathrm{e}^{-A})(z^i)' = -\frac{2}{3}\mathrm{e}^{\mathcal{K}}\mathcal{W}\mathcal{K}^{i\tilde{j}}D_{\tilde{j}}\widetilde{\mathcal{W}} \,,$$

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Tildes because in Euclidean signature some scalars have to be analytically continued.

Flat space domain wall obtained by

$$\mathcal{R}^2 \mathrm{d}\Omega_4^2 \to \mathrm{d}s^4_{\mathbf{R}^{1,3}} \, , \quad \tilde{\cdot} \to \bar{\cdot} \, , \quad \mathcal{R} \to \infty \, .$$

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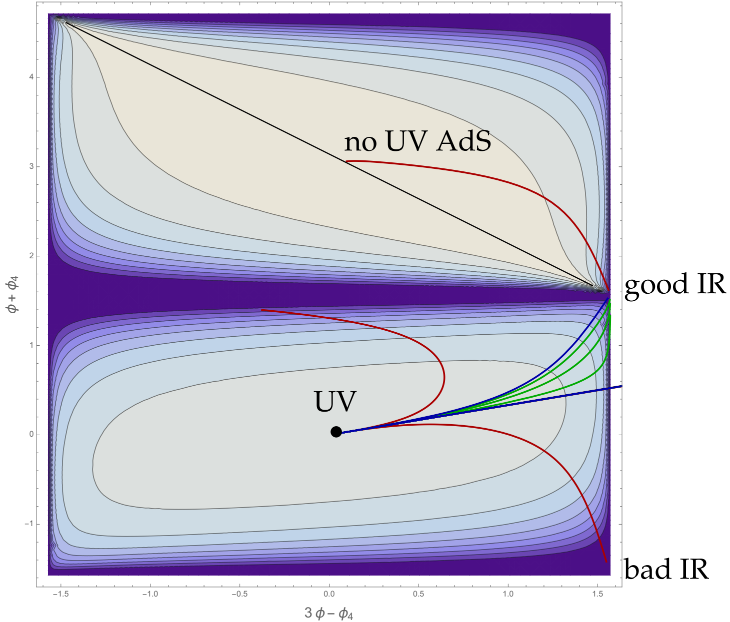

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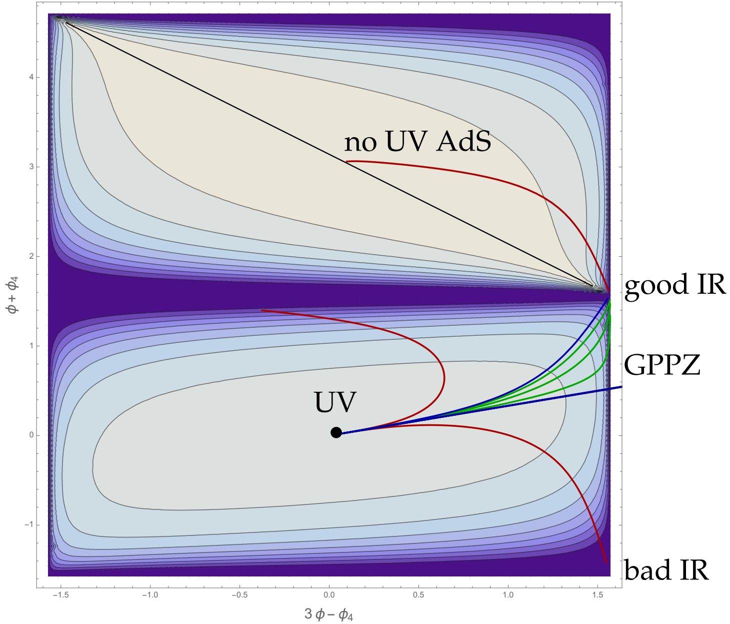

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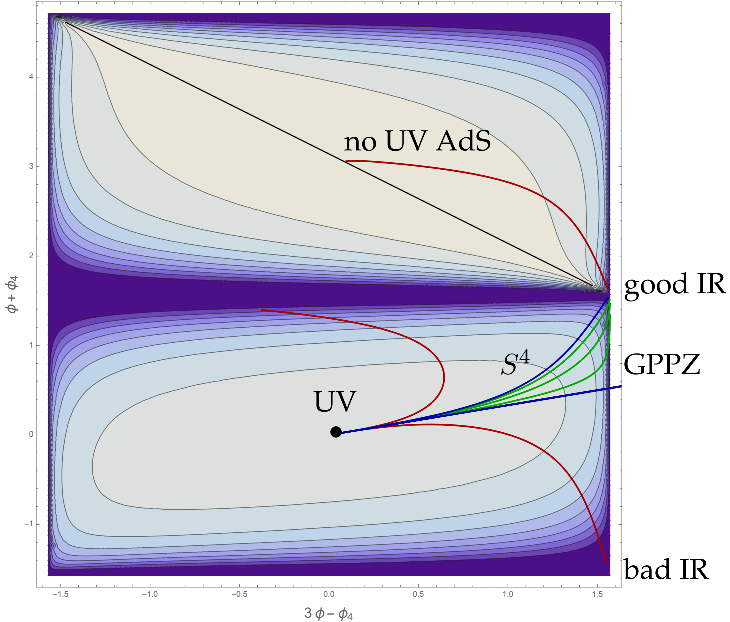
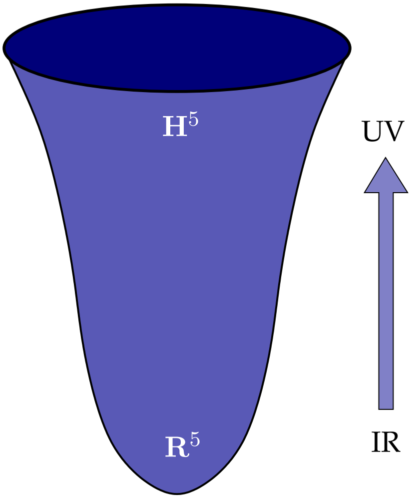


UV-IR matching



10D SOLUTION

To understand the singular flat-space solutions we uplift the GPPZ solutions

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10D SOLUTION

Bobev, FFG, Niehoff and van Muiden (2018)

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\begin{aligned}
t &: \text{ radial of AdS}\,, \\
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$$\begin{aligned}
\mathrm{d}s_{10}^2 &= \frac{4(K_1K_2 - K_3^2)^{1/4}}{g^2\sqrt{g_s}} \left(\frac{\mathrm{d}t^2 + (1-t^2)\left(1-\lambda^2 t^6\right)^{1/3} \mathrm{d}s_4^2}{t^2(1-t)(1-\lambda^2t^6)^{1/2}} + \frac{\left(1-\lambda^2t^6\right)^{1/2}}{K_1K_2 - K_3^2}\, \mathrm{d}\Omega_5^2 \right) \\
\mathrm{d}\Omega_5^2 &= K_4 \mathrm{d}\chi^2 - 4\lambda t^4 (1-t^2)^2 (\cos 2\alpha\, \mathrm{d}\chi - \sin 2\alpha\, \cos 2\chi\, \sigma_3)^2 \\
&\quad + 4\lambda t^6\, \mathrm{d}(\cos 2\alpha\, \cos 2\chi)^2 + \frac{(1-\lambda^2t^8)^2(1-t^2)}{(1-\lambda^2t^6)} (\mathrm{d}\alpha + \sin 2\chi\, \sigma_3)^2 \\
&\quad + \cos^2 2\chi (1+\lambda t^4)^2 (4t^2 \mathrm{d}\alpha^2 + (1-t^2)^2 \sigma_3^2) \\
&\quad + (1-t^2)\left(\sin^2\chi\, K_1 \sigma_1^2 + \sin 2\chi\, K_3 \sigma_1\sigma_2 + \cos^2\chi\, K_2\sigma_2^2 \right) . \\
e^{\Phi} &= \frac{g_s(1+\lambda t^4)}{\sqrt{K_1K_2 - K_3^2}} \Big((1+t^2)(1-\lambda t^4) + 2t^2(1-\lambda t^2)\cos 2\chi\, \cos 2\alpha \Big) ,
\end{aligned}$$

10D SOLUTION

$$C_0 = -\frac{2t^2(1+\lambda t^2)(1-\lambda t^4)\cos 2\chi\ \sin 2\alpha}{g_s(1+\lambda t^4)\big((1+t^2)(1-\lambda t^4)+2t^2(1-\lambda t^2)\cos 2\chi\ \cos 2\alpha\big)}\,,$$

$$\begin{aligned} B_2 + i g_s C_2 = {} & \frac{4}{g^2}\frac{t\mathrm{e}^{-i\alpha}}{K_1K_2-K_3^2} \\ & \times\Big[\big(a_1\mathrm{d}\chi + a_2\sigma_3 - i\left(1-\lambda^2t^8\right)(K_1+K_2)\sin 2\chi\,\mathrm{d}\alpha\big)\wedge\Sigma \\ & -\big(a_3\mathrm{d}\chi + a_4\sigma_3 - i\left(1-\lambda^2t^8\right)(K_1-K_2-2iK_3)\sin 2\chi\,\mathrm{d}\alpha\big)\wedge\overline{\Sigma}\Big]\,, \end{aligned}$$

$$F_5 = -\frac{1}{g^4 g_s}(1+\star_{10})\,\mathrm{d}\left[\frac{\left(1-t^2\right)\left(1-\lambda^2t^8\right)}{t^4\left(1-\lambda^2t^6\right)^{1/3}}\,\mathrm{d}x_0\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3\right]\,.$$

$$K_1 = (1+t^2)(1-\lambda^2t^8)+2t^2\left((1-\lambda^2t^6)+\lambda t^2(1-t^2)\cos(4\alpha)\right)\cos 2\chi\,,$$

$$K_2 = (1+t^2)(1-\lambda^2t^8)-2t^2\left((1-\lambda^2t^6)+\lambda t^2(1-t^2)\cos(4\alpha)\right)\cos 2\chi\,,$$

$$K_3 = 2\lambda t^4(1-t^2)\cos 2\chi\ \sin 4\alpha\,,$$

$$K_4 = (1+t^2)^2(1+\lambda t^4)^2-4t^4(1+\lambda t^2)^2\cos^2 2\chi\,,$$

$$\Sigma = i\sin\chi\,\sigma_1+\cos\chi\,\sigma_2\,,$$

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Solution parametrized by one integration constant $-1 \leq \lambda \leq 1$ that determines the gaugino vev:

$$\langle \psi_4 \psi_4 \rangle \sim N^2 m^3 \lambda \ .$$

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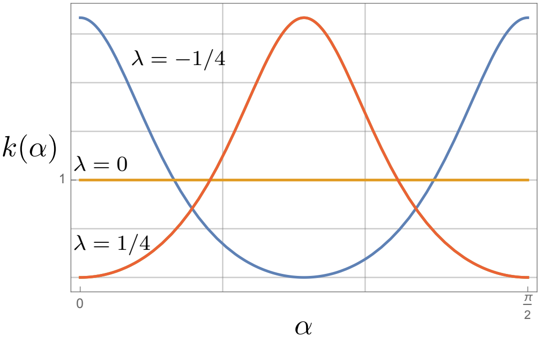
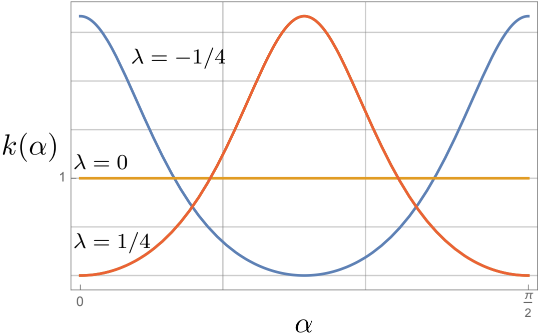
Tension of the (p, q)-fivebranes not constant

$$k(\alpha) = \frac{1-\lambda^2}{1+2\lambda\cos 4\alpha + \lambda^2} \ , \quad \int k(\alpha) \mathrm{d}\alpha = 2\pi \ .$$

(p, q)-FIVEBRANES



(p, q)-Fivebranes



As $|\lambda| \to 1$ we get δ-function peaks at $(1 + \lambda + 4n)\pi/8$, $n = 0, 1, 2, 3.$

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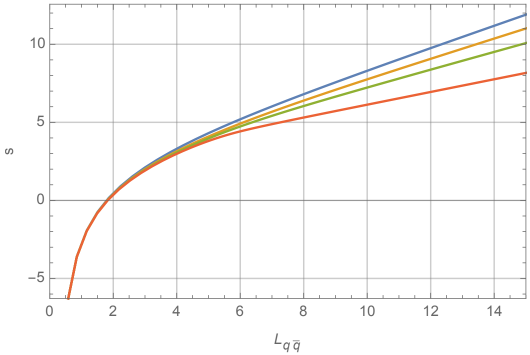

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All line operators show perfect screening.

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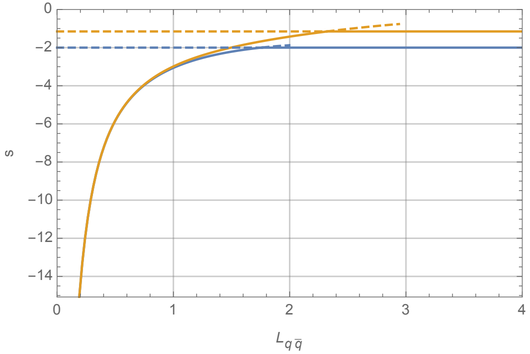

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This suggests that the limit $|\lambda| \to 1$ could be dual to a massive vacuum.

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It would be nice to generalize this story to include $\mathbf{20}'$ vevs. Seems that we must include all eight scalars. Finally we would like to compare the Gaugino vev with computations in the field theory.

Dorey (1999)
Dorey and Kumar (2000)

Thank you