5d partition functions with A twist

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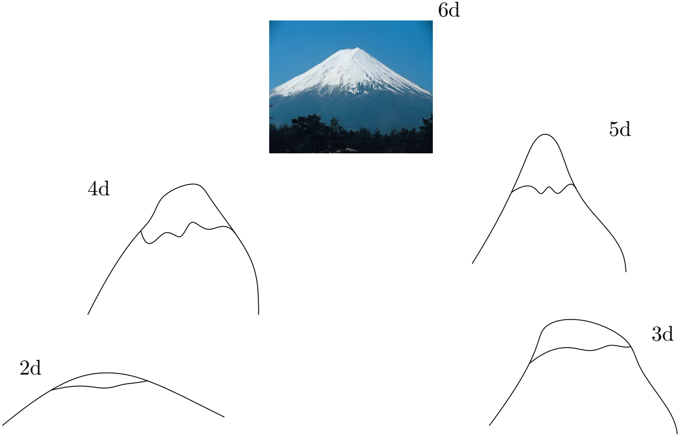
Landscape of SCFTs

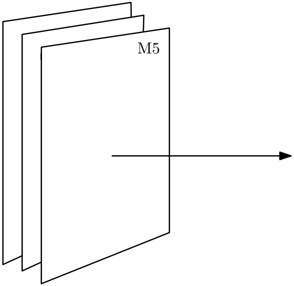
String theory predicts the existence of interacting local SCFTs in $d = 5, 6$



Arise from worldvolume of D- and M-branes. AdS/CFT $\Rightarrow$ conformal

6d

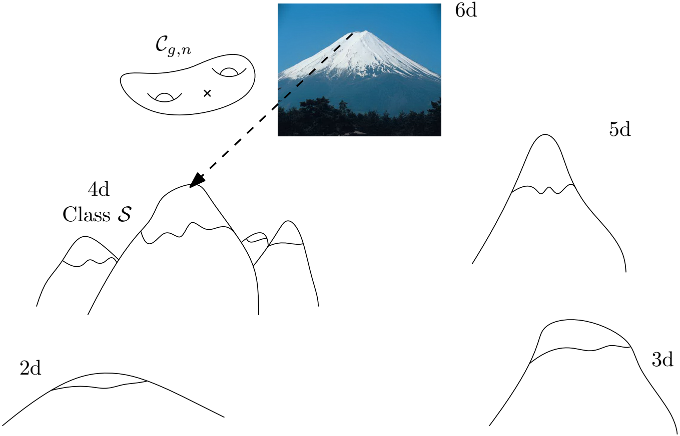
Most famous example: maximal 6d $(2,0)$ theory



Still remains rather M-ysterious. Mileage from:

$$\underbrace{\text{Anomalies}}_{SO(6)\times SO(5)_R} \qquad \underbrace{\text{Holography}}_{\text{AdS}_7\times S^4}$$

Compactification: 6d → 4d [Gaiotto]

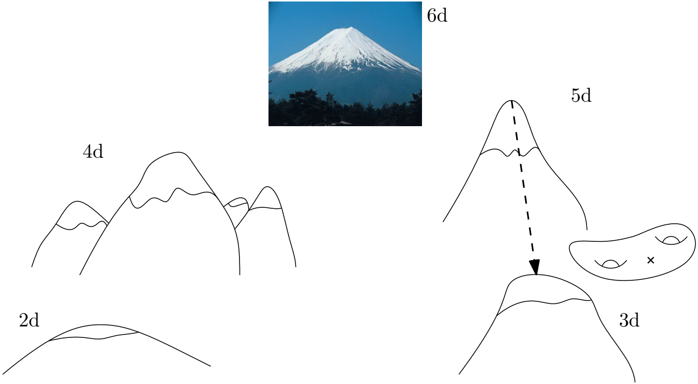


Vast space of new 4d $\mathcal{N} = 2$ and $\mathcal{N} = 1$ SCFTs from 6d

$$\mathcal{T}^{(6d)}, \mathcal{C}_{g,n}; \mathfrak{n} \qquad \Rightarrow \qquad \mathcal{T}^{(4d)}_{\mathcal{C}_{g,n};\mathfrak{n}}$$

4d anomalies and holography known [Maldacena-Núñez,Gaiotto-Maldacena,BBBW]

What about 5d compactifications?



Landscape of 3d SCFTs, $\mathcal{T}^{(3d)}_{\mathcal{C}_{g,n};\mathfrak{n}}$, from 5d? No anomalies. Compute

$$Z_{M_3}[\mathcal{T}^{(3d)}_{\mathcal{C}_{g,n};\mathfrak{n}}]\ ?$$

Much harder, but worth it

Outline

I) 5d $\mathcal{N} = 1$ gauge theories

II) Partition function on $M_3 \times \Sigma_{\mathfrak{g}}$

III) Two applications

I) 5d $\mathcal{N} = 1$ gauge theories

5d $\mathcal{N} = 1$ gauge theories (8 SUSYs)

Multiplets of 5d $\mathcal{N} = 1$ SUSY:

$$\text{vector: } V = (A_\mu, \sigma, \lambda^I, D^{IJ}) \qquad \text{hyper: } \Phi = (q^I, \psi)$$

Global symmetry:

$$SO(6) \times SU(2)_R \times G_F$$

We can write Lagrangians, e.g.,

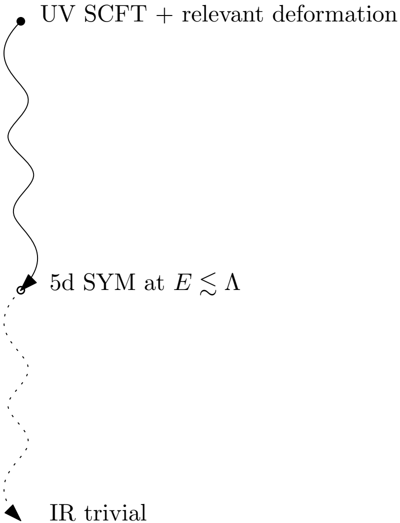
$$S_V = \frac{1}{g_5^2}\text{Tr}\int d^5x\left(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + i\lambda_I \not{D}\lambda^I - D^\mu\sigma D_\mu\sigma - \lambda_I[\sigma, \lambda^I] - \frac{1}{2}D^{IJ}D_{IJ}\right)$$

- Non-renormalizable: $g_5^2 \sim mass^{-1}$ (always effective theories)
- Coupling decreases at large distances:

$$g_5 \to 0 \qquad \text{as } E \to 0$$

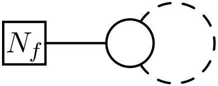
RG flow

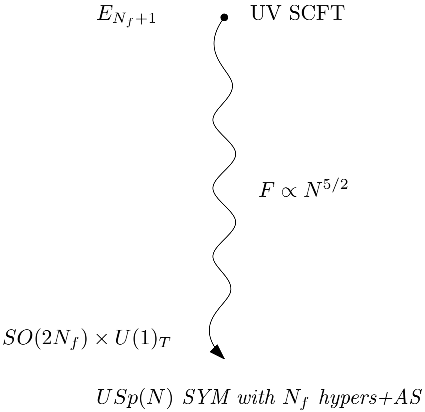
Unlike $d \leq 4$, in $d = 5$ strongly-coupled SCFT at UV:



Example 1: Seiberg theories

A simple example of UV completion as 5d SCFT **[Seiberg]**

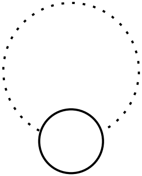


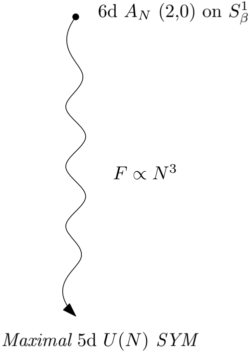


- For $N_f < 8$, gauge theory is the IR description of an SCFT with enhanced E_{N_f+1} (checked on, e.g., $S^1 \times S^4$ **[Kim-Kim-Lee]**)
- Number of d.o.f. grows to $N^{5/2}$

Example 2: Maximal theory

Some 5d theories admit UV completions as 6d theories! **[Douglas, Lambert et al.]**

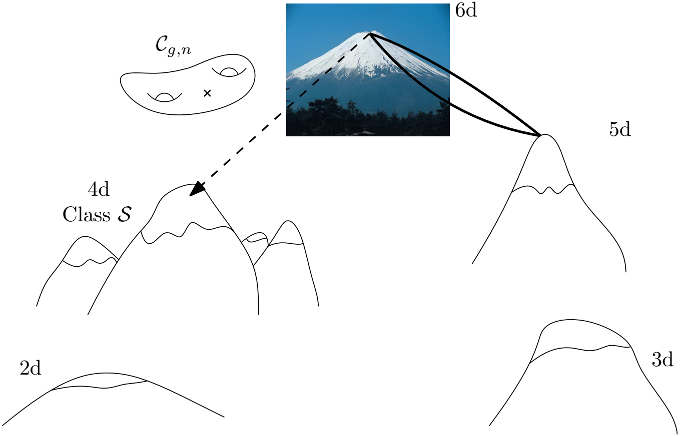




Size of "emergent" S^1_β:

$$\beta = \frac{g_5^2}{2\pi} \qquad \text{grows at large coupling}$$

"5d is the best d"



A (5d) Lagrangian method for (4d) non-Lagrangian theories

Basic idea:

$$M_3 \times \mathcal{C}_{g,n} \times S^1_\beta$$

$$\swarrow \qquad\qquad \searrow$$

$$M_3 \times S^1_\beta \qquad\qquad\qquad M_3 \times \mathcal{C}_{g,n}$$

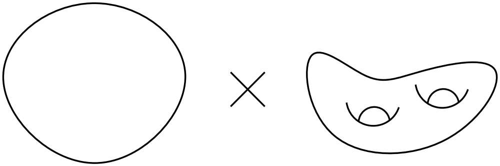
Partition function invariant under RG $\Rightarrow$

$$\underbrace{Z_{M_3 \times S^1_\beta}[\text{Class } \mathcal{S}]}_{Non-Lagrangian} \quad = \quad \underbrace{Z_{M_3 \times \mathcal{C}_{g,n}}[\text{5d SYM}]}_{Lagrangian}$$

Match can be seen as:

- Precision test of *5d maximal SYM* $\Leftrightarrow$ *6d* $(2,0)$ conjecture
- Computational tool for non-Lagrangian theories

II) 5d partition function on



M_3 $\mathcal{C}_{g,n}$

(In this talk we will take $M_3 = S_b^3$ and $\mathcal{C}_{g,n} = \Sigma_g$ for simplicity)

Supersymmetric localization

Basic localization argument **[Witten, Pestun]**

$$Z_{M_d} = \underbrace{\int [D\Phi] e^{-S[\Phi]}}_{\mathcal{Q}\text{ preserved}} \quad \to \quad Z_{M_d}(t) = \int [D\Phi] e^{-(S[\Phi]+t\mathcal{Q}V[\Phi])}$$

If $\mathcal{Q}V[\Phi]$ positive semidefinite and $\mathcal{Q}$-invariant, $\frac{d}{dt}Z_{M_d}(t) = 0$. Taking $t \to \infty$

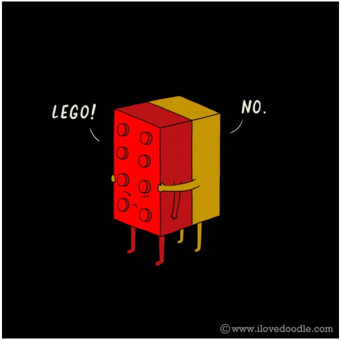
$$Z_{M_d} = \int d\Phi_0 \, e^{-S[\Phi_0]} \, Z^{1-loop}(\Phi_0) Z^{inst}(\Phi_0) \qquad \text{Exact!}$$

1) Couple to supergravity, take rigid limit **[Festuccia-Seiberg]**
2) Solve for background
3) Compute 1-loop det's and instantons in background

Systematic but convoluted and every new manifold requires starting from scratch

The LEGO approach to localization

Another approach is to “glue” together basic building blocks

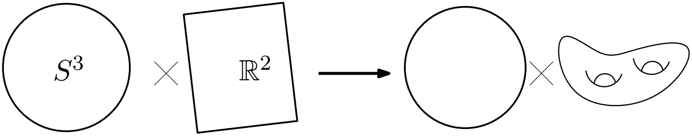


– Works for many $\mathcal{M}_d$ (not only direct products)

–Provides understanding; holom. factorization, dualities, etc.

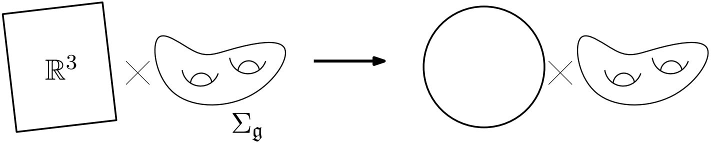
Three perspectives

1) Reduction on S^3: A-model on $\Sigma_\mathfrak{g}$



$$Z_{S^3\times\Sigma_\mathfrak{g}} = Z_{A-model}$$

2) Reduction on $\Sigma_\mathfrak{g}$: Direct sum of 3d theories



$$Z_{S^3\times\Sigma_\mathfrak{g}} = \bigoplus_{\mathfrak{m}\in\Lambda_G} \left(Z_{S^3}\right)^{\mathfrak{m}}$$

3) Uplift from 4d: Nekrasov partition function: $Z_{S^3\times\Sigma_\mathfrak{g}} = \sum_n Z_n^{Nekrasov}$

The final answer

By either method, one finds [MC-Jain-Willett]

$$Z_{S^3_b \times \Sigma_{\mathfrak{g}}}(\nu)_{\mathfrak{n}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u},\nu)^{\mathfrak{n}_i} \mathcal{H}(\hat{u},\nu)^{\mathfrak{g}-1}$$

with $\mathfrak{n}$ = flavor flux on $\Sigma_{\mathfrak{g}}$, ν = fugacity,

$$\Pi_i \equiv e^{2\pi i \partial_{\nu_i} \mathcal{W}}, \quad \Pi_a \equiv e^{2\pi i \partial_{u_a} \mathcal{W}}, \quad \mathcal{H} = e^{-2\pi i \Omega} \det_{ab} \partial_{u_a} \partial_{u_b} \mathcal{W}$$

with $\mathcal{W} = \mathcal{W}^{pert} + \mathcal{W}^{inst}$,

$$\mathcal{W}^{pert}(\mu,\nu) = \sum_{\rho \in R} g_b(\rho(u)+\nu) - \sum_{\alpha \in Ad(G)} g_b(\alpha(u)-1)$$

$$\Omega^{pert} = \sum_{\rho \in R} (r-1)\, l_b(\rho(u)+\nu)$$

$$g_b(u) = \frac{1}{2\pi i} \int du \log s_b(-iQu), \quad l_b(u) = \frac{1}{2\pi i} \log s_b(-iQu)$$

s_b is the double sine function appearing in Z_{S^3}.

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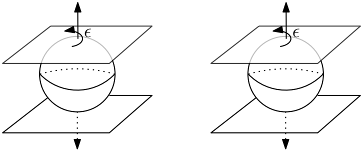
Relation to 5d Nekrasov partition function

In 3rd approach, start with

$$\text{5d } \Omega\text{-backg:} \quad \lim_{\epsilon_2 \to 0} Z_{\mathbb{R}_{\epsilon_1} \times \mathbb{R}_{\epsilon_2} \times S^1} = e^{\frac{1}{\epsilon_2} \mathcal{W}_{NS}^{(5d)}(u,\epsilon_1) + \Omega_{NS}^{(5d)}(u,\epsilon_1) + \mathcal{O}(\epsilon_2)}$$

Then, consider $S^2 \times S^2 \times S^1$:



$$\text{4 fixed points} \quad \Rightarrow \quad Z_{S^2 \times S^2 \times S^1} = \prod_{l=1}^{4} Z^{(l)}_{\mathbb{R}_{\epsilon_1^{(l)}} \times \mathbb{R}_{\epsilon_2^{(l)}} \times S^1}$$

Fiber the S^1 over one S^2. Then, 2d A-model with

$$\mathcal{W} = \mathcal{W}_{NS}^{(5d)}, \qquad \Omega = \Omega_{NS}^{(5d)}$$

Identification $\epsilon_1 = \frac{1}{2}(b - b^{-1})$. For $b = 1 \Rightarrow$ SW prepotential $\mathcal{F}(u)$

Summary

Final result:

$$Z_{S_b^3 \times \Sigma_{\mathfrak{g}}}(\nu)_{\mathfrak{n}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u}, \nu)^{\mathfrak{n}_i} \mathcal{H}(\hat{u}, \nu)^{\mathfrak{g}-1}$$

- **Exact**. Includes all instanton contributions
- Given by **2d TQFT**
- Determined by **5d Nekrasov**:

$$\mathcal{W} = \mathcal{W}_{NS}^{(5d)}\,, \qquad \Omega = \Omega_{NS}^{(5d)}$$

III) Two Applications

Applications

1) Maximal SYM: Superconformal index of 4d class $\mathcal{S}$

2) Seiberg theory and holography

I will mostly focus on 2)

1) Maximal $U(N)$ 5d SYM

First, solve Bethe equations

$$\Pi_a(u) = e^{2\pi i \partial_{u_a}(\mathcal{W}^V(u) + \mathcal{W}^H_{Ad}(u,\nu_{Ad}))} = 1$$

Difficult but simplified for $\nu_{Ad} = \frac{i}{2}(b - b^{-1})$ (Schur limit) $\Rightarrow$

$$Z^{\mathcal{N}=2\ \mathrm{SYM}}_{S^3_b \times \Sigma_\mathfrak{g}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u},\nu_{Ad})^{\mathfrak{n}_i} \mathcal{H}(\hat{u},\nu_{Ad})^{\mathfrak{g}-1} = e^{-\beta E_C} \mathcal{I}(p,q)$$

with

$$\mathcal{I}(p,q) = \left(\frac{p^{\frac{1}{12}N(N^2-1)} V(p)}{(p;p)^{N-1}}\right)^{-\ell_1} \left(\frac{q^{\frac{1}{12}N(N^2-1)} V(q)}{(q;q)^{N-1}}\right)^{-\ell_2} \times \sum_{\lambda \in \Lambda^+_{cr}} \dim_p(R_\lambda)^{-\ell_1} \dim_q(R_\lambda)^{-\ell_2}$$

where $\ell_1 = \mathfrak{g} - 1 + \mathfrak{n}$, $\ell_2 = \mathfrak{g} - 1 - \mathfrak{n}$, identifications

$$p = e^{2\pi b \gamma^{-1}}, \qquad q = e^{2\pi b^{-1} \gamma^{-1}}, \qquad \gamma = -\frac{2\pi Q}{{g_5}^2}$$

$V(p)$ polynomial and $\dim_p(R_\lambda)$ the "quantum dimension" of a representation with highest weight λ. Matches [Rastelli et al.][Beem-Gadde]

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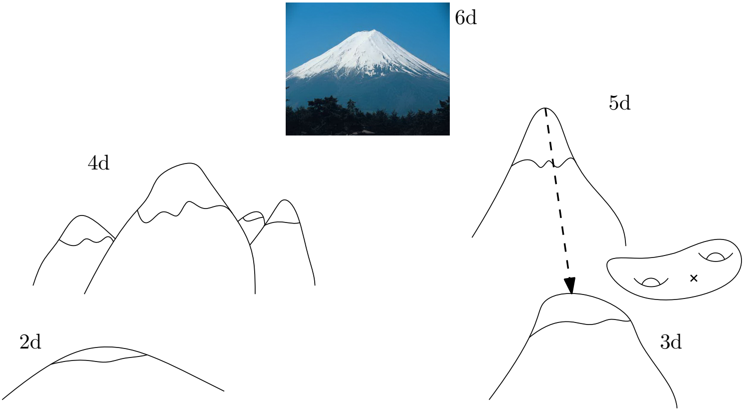
Comments

- Perfect match with previous results [Rastelli et al.][Beem-Gadde] in the Schur limit
- Index known only for this and other special limits
- In principle, 5d partition function gives full answer (might be hard to explicitly solve the Bethe equations in general)

2) Seiberg theory and holography

The Seiberg theory has an actual 5d UV completion



Landscape of new 3d SCFTs?

New SCFTs in 3d

Compactification from 5d provides a map:

$$\mathcal{T}^{(5d)}, \Sigma_{\mathfrak{g}}, \mathfrak{n} \qquad \Rightarrow \qquad \mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}$$

Assuming RG flow ends in a nontrivial fixed point:

Infinite family of 3d SCFTs labeled by $\mathfrak{g}, \mathfrak{n}$

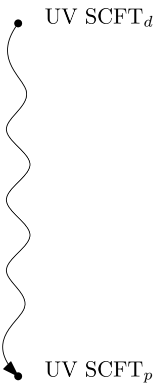
whose partition function on S^3 is

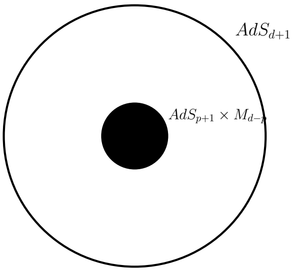
$$Z_{S^3}[\mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}] = Z_{S^3 \times \Sigma_{\mathfrak{g}}}[\mathcal{T}^{(5d)}]_{\mathfrak{n}}$$

How do we know RG flows end in interacting SCFTs?

Holography of RG flows across dimensions

According to AdS/CFT **[Maldacena-Núñez]**





Extremal black p-branes in asymptotically locally AdS are flows across dimensions!

Universal RG flow

Extremal 2-brane solution in $F(4)$ gauged sugra [Núñez et al., Naka]

$$ds_6^2 = e^{2f(r)}ds^2(\mathbb{R}^4)+e^{2g(r)}ds^2(\Sigma_{\mathfrak{g}>1})\,, \quad dA^I = \delta^{I3}\text{dvol}(\Sigma_{\mathfrak{g}})\,, \quad \phi = \phi(r)\,.$$

Indeed interpolates between AdS_6 and AdS_4 and on-shell action [Bobev-MC]

$$F^{sugra}_{\text{AdS}_4} = -\frac{8}{9}(\mathfrak{g}-1)F^{sugra}_{\text{AdS}_6}\,, \qquad \mathfrak{g} > 1\,.$$

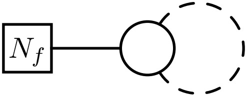
From AdS/CFT, non-trivial prediction for matrix models:

$$Z_{S^3}[\mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}=0}] = \Big(Z_{S^5}[\mathcal{T}^{(5d)}]\Big)^{-\frac{8}{9}(\mathfrak{g}-1)}$$

–Establishes existence of 3d fixed points (large N)

–Relation is *universal* (valid for any 5d $\mathcal{N}=1$ theory on $\Sigma_{\mathfrak{g}}$)

We can test this for the Seiberg theory:



Bethe potential:

$$\mathcal{W}^{pert} = -\sum_{i<j}\left[g_b(1 \pm (u_i - u_j)) + g_b(1 \pm (u_i + u_j))\right] - \sum_i g_b(1 + 2u_i)$$
$$+ \sum_{i<j}\left[g_b(\nu_{AS} \pm (u_i - u_j)) + g_b(\nu_{AS} \pm (u_i + u_j))\right]$$
$$+ \sum_{I=1}^{N_f}\sum_i g_b(\nu_I + u_i)$$

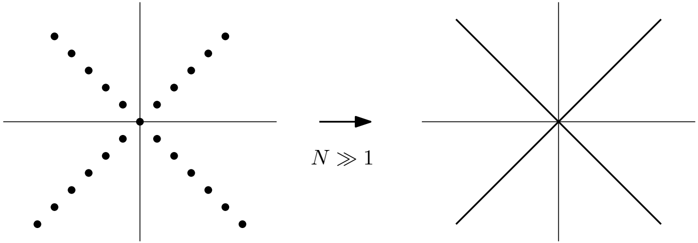
Perturbative accuracy sufficient at large N

Large N method

Solve Bethe equations:

$$e^{2\pi i \partial_{u_i} \mathcal{W}^{pert}} = 1$$

at large N by continuum method **[Herzog-Klebanov-Pufu-Tesileanu]**



Then

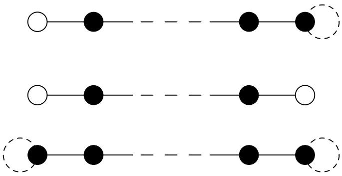
$$F_{S^3 \times \Sigma_{\mathfrak{g}}} \simeq -\frac{8}{9}(\mathfrak{g}-1) \underbrace{\left(-\frac{9\sqrt{2}\pi N^{5/2}}{5\sqrt{8-N_f}} \right)}_{F_{S^5}\ \text{[Jafferis–Pufu]}}$$

Exactly reproduces holographic prediction

Generalizations

5d quiver gauge theories for $Sp(N)$ and $U(N)$ nodes with gravity duals

[Bergman,Rodriguez-Gomez]



Generic flavor fluxes $\mathfrak{n}$ turned on. Precise match with holographic flows

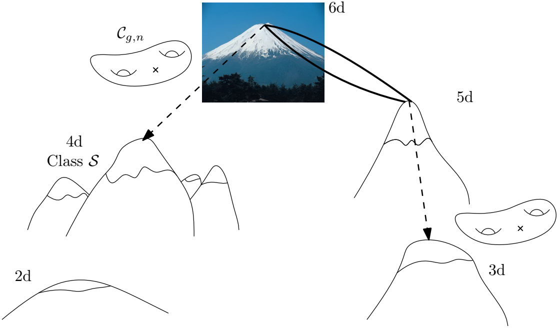
[Bah-Passias-Weck] [Hosseini, Hristov, Passias, Zaffaroni]

Conclusion:

- Holographic duals confirm 3d fixed points (large N)
- Large class of novel 3d SCFTs: $\mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}$

Summary

5d is a fruitful vantage point:



- New tool for superconformal index of class $\mathcal{S}$
- Lagrangian methods for non-Lagrangian theories
- New 3d SCFTs with $Z_{S^3} = Z^{(2d)}_{\text{TQFT}} \overset{univ}{=} Z_{S^5}^{-\frac{8}{9}(\mathfrak{g}-1)}$

Outlook

- Include punctures on $\Sigma_{\mathfrak{g}}$
- General expression for $Z_{S^1 \times S^3}[\text{class } \mathcal{S}]$
- $Z_{M_4}[\text{class } \mathcal{S}]$?

- Purely three-dimensional description of SCFTs?
- Two-dimensional integrable model?
- 3d/2d correspondence à la AGT?

Thank you