A geometric dual of c-extremization

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Based on work with J. Gauntlett, D. Martelli and J. Sparks

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Holography, Generalized Geometry and Duality

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Motivation

Motivation

2d $\mathcal{N} = (0, 2)$ SCFTs admit a conserved $U(1)_R$ R-symmetry.

Theories (to some extent) characterized by a central charge obeying the c-theorem.

Question? How do you determine the R-symmetry and central charge in the IR from UV data?

Answer: c-extremization [Benini, Bobev].

An extremization principle determines the IR R-symmetry and central charge using UV data.

Close analogy to a-maximization [Intriligator, Wecht] in 4d.

Motivation from 4d

a-max solves a similar problem in 4d.

4d $\mathcal{N} = 1$ SCFTs admit a conserved $U(1)_R$ R-symmetry and have a monotonically decreasing central charge, a.

a-max: maximizing the a-central charge in the space of all R-symmetries gives the exact IR R-symmetry and central charge.

Excellent matching with explicit AdS_5 solutions. Finding explicit metrics is difficult though!

Given topological data (no metric) shown that one can compute the a-central charge holographically [Martelli, Sparks, Yau].

Is there a similar story for c-extremization?

a-maximization: the field theory story

‘t Hooft anomalies completely determine the a and c central charges, [Anselmi, Freedman, Grisaru, Johansen],

$$a = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)\ , \quad c = \frac{3}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

Given an R-symmetry R_0, then $R_t = R_0 + s_I F_I$ is an equally good R-symmetry.

[Intriligator, Wecht] showed that the exact Superconformal R-symmetry maximises a w.r.t. the s_I.

The exact central charge is then a evaluated at the fixed point.

a-maximization: the geometric story

Assume holographic theory $\Rightarrow$ by AdS/CFT there exists a dual AdS_5 solution. Central charge 'a' $\sim$ 1/volume of internal manifold, e.g. S^5.

But what if you don't know the metric?

[Martelli, Sparks, Yau]: given topological data (*no metric necessary*) about the internal manifold there is an extremization problem which agrees with a-max.

Restrict to special theories: toric Sasaki-Einstein. Theories specified by a set of vectors $\{v_a\}$. a-max equivalent to maximizing

$$Z(b) = \frac{1}{24} \sum_a \frac{(v_{a-1}, v_a, v_{a+1})}{(b, v_{a-1}, v_a)(b, v_a, v_{a+1})} ,$$

with respect to the *Reeb vector b*.

Volume $\sim$ central charge is $\mathrm{vol}(Y) = 8\pi^3 Z(b^*)$.

Field theory: wrapped D3 and M2 branes

Wrapped D3 branes

Classic story: stack of N D3-branes probed by a Calabi–Yau three-fold cone. Near-horizon, $\mathrm{AdS}_5 \times Y_5$.

Theory on branes is a 4d quiver theory (when cone resolvable), i.e. $\mathcal{N} = 4$ SYM.

Place theory on a Riemann surface.

Topological twist $\rightarrow$ 2d $\mathcal{N} = (0, 2)$ SCFTs.

Typically no Lagrangian description, at best a UV Lagrangian: believe flows to a superconformal fixed point.

Certain protected BPS quantities at the fixed point can be computed using the UV description.

C-extremization

Central charge and R-symmetry two such quantities: use c-extremization.

Under the RG-flow the R-symmetry can mix with Flavour *and* Baryonic symmetries.

One should construct the most general trial R-symmetry, and extremize

$$c_{trial} = 3\mathrm{Tr}\gamma^3 R^2_{\text{trial}} \ .$$

[Benini, Bobev, (Crichigno)] exact IR R-symmetry is obtained by extremizing the *quadratic* c_{trial}.

Wrapped M2-branes

Play a similar game with wrapped M2-branes.

Stack of N M2-branes probed by a Calabi–Yau four-fold cone. Near horizon, $\mathrm{AdS}_4 \times Y_7$.

Theory on branes is a 3d SCFT, i.e. ABJM.

Place theory on a Riemann surface.

Topological twist $\rightarrow$ 1d $\mathcal{N} = 2$ Superconformal Quantum Mechanics.

View as horizon of AAdS_4 magnetically charged BPS Black holes.

What is the entropy?

$\mathcal{I}$-extremization

How do we compute the entropy?

$\mathcal{I}$-extremization [Benini, Hristov, Zaffaroni] gives an extremization principle for finding the entropy of magnetically charged BHs.

Extremize the logarithm of the topologically twisted index;

$$\mathcal{I} = -\log Z_{S^1 \times \Sigma_g}$$

partition function of 3d theory with twist.

Tested successfully in multiple cases: e.g. [Hosseini, Zaffaroni], [Azzurli, Bobev, Crichigno, Min, Zaffaroni], [Cabo-Bizet, Giraldo-Rivera, Pando Zayas].

Holography: wrapped D3 and M2 branes

AdS solutions

Via AdS/CFT there should be a class of AdS solutions dual to these field theories.

Solutions classified:

- D3-branes: AdS_3 classification of [Kim]

$$\mathrm{d}s^2_{10} = L^2 \mathrm{e}^{-\frac{B}{2}} (\mathrm{d}s^2_{\mathrm{AdS}_3} + \mathrm{d}s^2_7)$$
$$F_5 = -L^4 (\mathrm{dvol}_{\mathrm{AdS}_3} \wedge F + *_7 F)$$

- M2-branes: AdS_2 classification of [Kim, Park]

$$\mathrm{d}s^2_{11} = L^2 \mathrm{e}^{-\frac{2B}{3}} (\mathrm{d}s^2_{\mathrm{AdS}_2} + \mathrm{d}s^2_9)$$
$$G_4 = L^3 \mathrm{dvol}_{\mathrm{AdS}_2} \wedge F$$

An action

Substituting ansäzte into type IIB, respectively 11d EOMs gives EOMs for Y_{2n+1} metric and fields.

Equations of motion can be derived from an action [Gauntlett, Kim]

$$\int_{Y_{2n+1}} \mathrm{e}^{(1-n)B}\left[R_{2n+1} - \frac{2n}{(n-2)^2} + \frac{n(2n-3)}{2}(\mathrm{d}B)^2 + \frac{1}{4}\mathrm{e}^{2B}F^2\right]\mathrm{dvol}_{2n+1}$$

$n = 3$ for IIB, $n = 4$ for 11d sugra.

No supersymmetry imposed yet though!

Imposing supersymmetry

Imposing supersymmetry further constrains Y_{2n+1} geometric structure.

Y_{2n+1} is equipped with a unit norm Killing vector $\xi = \frac{1}{c}\partial_z$; *R-symmetry vector*. Dual one-form $\eta = c(\mathrm{d}z + P)$.

Metric takes the form

$$\mathrm{d}s^2_{2n+1} = c^2(\mathrm{d}z + P)^2 + \mathrm{e}^B \mathrm{d}s^2_{2n} \,,$$

with $\mathrm{d}s^2_{2n}$ a Kähler metric

$$\mathrm{e}^B = \frac{c^2}{2} R_{2n} \,,$$

and $\mathrm{d}P = \rho$, the Ricci-form of the transverse Kähler metric.

$$F = -\frac{1}{c} J + c\, \mathrm{d}\left[\mathrm{e}^{-B}(\mathrm{d}z + P)\right]$$

Restricting the action

Restrict the action to solutions preserving supersymmetry. The action reduces to

$$S_{\mathrm{SUSY}} = \int_{Y_{2n+1}} \eta \wedge \rho \wedge \frac{J^{n-1}}{(n-1)!}$$

Refer to as off-shell.

S_{SUSY} depends only on the Kähler class $[J] \in H_B^{1,1}(\mathcal{F}_\xi)$ and ξ.

We have not imposed E.O.M. for F yet

$$\mathrm{d}\left[\mathrm{e}^{(3-n)B} *_{2n+1} F\right] = 0 \quad \Leftrightarrow \quad \Box R = \frac{1}{2} R^2 - R_{ij} R^{ij}$$

This implies

$$\int_{Y_{2n+1}} \eta \wedge \rho^2 \wedge \frac{J^{n-2}}{(n-2)!} = 0$$

Extremal problem

For $n = 3$ central charge given by [Brown, Henneaux].

$$c = \frac{3L}{2G_3} = \frac{3L^8}{(2\pi)^6 g_s^2 \ell_s^8} S_{\text{SUSY}} \, .$$

For $n = 4$ one compares to Bekenstein-Hawking entropy

$$S_{\text{BH}} = \frac{A}{4} = \frac{1}{4G_2} = \frac{4\pi L^9}{(2\pi)^8 \ell_p^9} S_{\text{SUSY}}$$

Extremizing S_{SUSY} (i.e. imposing EOM) is equivalent to extremizing the central charge/entropy respectively!

Example: $Y_7 = T^2 \times Y_5$.

Specialization

Look at the $n = 3$ case in more detail.

Specialize further to $Y_7 = T^2 \times Y_5$.

For other cases see [Gauntlett, Martelli, Sparks], [Hosseini, Zaffaroni], [Kim, Kim].

Field theory interpretation: reducing D3-brane on $T^2 \to$ 2d $\mathcal{N} = (0, 2)$ SCFT.

Y_7 geometry

R-symmetry vector ξ tangent to Y_5 and transverse Kähler form is $J = A\mathrm{vol}_2 + \omega$.

Flux quantisation: two types of five cycles, Y_5 and $T^2 \times \sigma_a$. Gives flux integers N and M_a.

The central charge is

$$c_{\text{sugra}} = \frac{12(2\pi)^2 N M_1}{\int_{\sigma_1} \eta \wedge \rho}$$

with

$$\frac{\int_{\sigma_i} \eta \wedge \rho}{\int_{\sigma_1} \eta \wedge \rho} = \frac{M_i}{M_1} \ , \ i = 2, .., b_3(Y_5) \ .$$

Toric Formulae and non-convex cones

Using equivariant localization one can evaluate the above formulae.

In the toric case everything simplifies nicely.

$$\int_{S_a} \eta \wedge \rho = 2(2\pi)^2 \frac{(v_{a-1}, v_a, v_{a+1})}{(\vec{b}, v_{a-1}, v_a)(\vec{b}, v_a, v_{a+1})}$$

and

$$\int_{Y_5} \eta \wedge \rho \wedge \rho = \frac{32\pi^3}{b_1} \sum_{a=1}^{d} \frac{(v_{a-1}, v_a, v_{a+1})}{(\vec{b}, v_{a-1}, v_a)(\vec{b}, v_a, v_{a+1})}$$

Example: $Y^{p,q}$ field theory

Field theory reduction of $Y^{p,q}$ on T^2 considered in [Benini, Bobev, Crichigno].

Field theory specified by integers $p > q > 0$.

C-extremization gives

$$c_{\text{c-ext}} = -\frac{6BN^2p^2(p^2 - q^2)}{q^2}$$

with R-charges

$$R[Y] = R[Z] = \frac{q^2 - p^2}{q^2}N\,, \quad R[U_1] = R[U_2] = \frac{p^2}{q^2}N\,.$$

$R[Y], R[Z]$ are negative. Contradiction for a chiral operator.

Conclude superconformal fixed point does not exist.

Example: $\mathcal{Y}^{\mathfrak{p},\mathfrak{q}}$ gravity

Putative dual geometry found in [Donos, Gauntlett, Kim].

Geometry reminiscent of Sasaki-Einstein $Y^{p,q}$ solutions.

Toric data of the solution computed in [Couzens, Martelli, Schäfer-Nameki].

Gravity specified by integers $\mathfrak{q} > \mathfrak{p} > 0$.

Implication that the toric diagram is not convex.

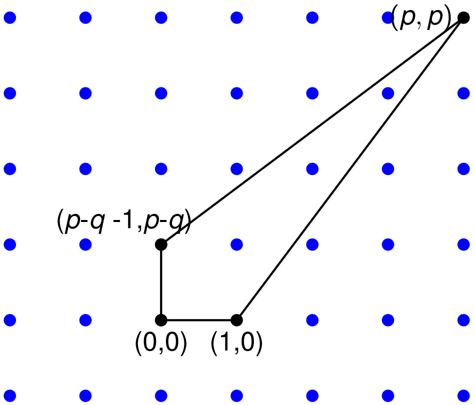
$$c_{\text{grav}} = -\frac{6MN\mathfrak{p}^2(\mathfrak{p}^2 - \mathfrak{q}^2)}{\mathfrak{q}^2}$$

$$R[S_2] = R[S_4] = \frac{\mathfrak{q}^2 - \mathfrak{p}^2}{\mathfrak{q}^2}N\,, \quad R[S_1] = R[S_3] = \frac{\mathfrak{p}^2}{\mathfrak{q}^2}N\,.$$

Everything nice and positive. Formally matches the field theory result.

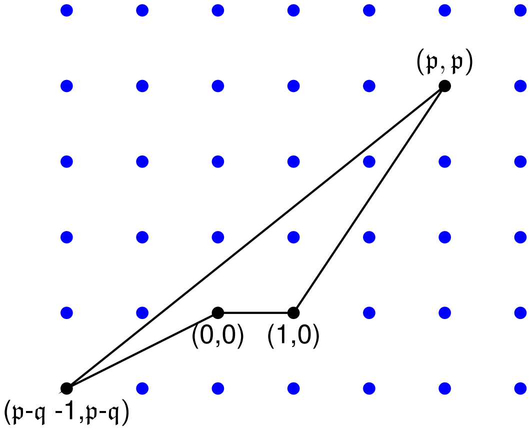
Toric diagram: $Y^{p,q}$

The usual Sasaki-Einstein manifold has a convex toric diagram.



Toric diagram: $\mathfrak{Y}^{\mathfrak{p},\mathfrak{q}}$

Instead the $\mathfrak{Y}^{\mathfrak{p},\mathfrak{q}}$ geometry has a concave toric diagram.



$Y_5 = \mathscr{L}^{\mathrm{a,b,c}}$

We construct a new family of supergravity solutions which are specified by 3 integers; $\mathtt{a}, \mathtt{b}, \mathtt{c}$.

Analogues of $L^{a,b,c}$ solutions and contain $Y^{p,q}$ solutions as a subcase.

Cohomogeneity two ansatz determined by two polynomial functions.

Perfectly regular solutions labeled by integers satisfying

$$-\mathtt{a} + \mathtt{b} = \mathtt{c} + \mathtt{d}\,, \quad \mathtt{b} > \mathtt{a} > 0\,, \quad \mathtt{b} > \mathtt{c} \geq \mathtt{d} > 0$$

$Y_5 = \mathscr{L}^{\mathtt{a},\mathtt{b},\mathtt{c}}$: Continued

Toric data

$$v_1 = (1,1,0)\,, \quad v_2 = (1,-\mathtt{ak},\mathtt{b})\,, \quad v_3 = (1,\mathtt{al},\mathtt{c})\,, \quad v_4 = (1,0,0)$$

Central charge from metric and geometric dual given by

$$c = 6\frac{\mathtt{abcd}}{\mathtt{ab}+\mathtt{cd}}NM$$

and R-charges

$$R[S_1] = R[S_3] = \frac{\mathtt{cd}}{\mathtt{ab}+\mathtt{cd}}N\,, \quad R[S_2] = R[S_4] = \frac{\mathtt{ab}}{\mathtt{ab}+\mathtt{cd}}N$$

$L^{a,b,c}$ on T^2

Metrics look like the Sasaki-Einstein $L^{a,b,c}$ metrics.

Is the field theory the $L^{a,b,c}$ quiver theory reduced on T^2 with Baryonic flux?

Topological twist with

$$T_{\text{top}} = BT_B$$

C-extremization gives

$$c_{\text{c-ext}}(L^{a,b,c}) = 6\frac{abcd}{ab-cd}BN^2$$

and R-charges

$$R_{\text{c-ext}}[Y] = R_{\text{c-ext}}[Z] = \frac{ab}{ab-cd}\ , \quad R_{\text{c-ext}}[U_1] = R_{\text{c-ext}}[U_2] = -\frac{cd}{ab-cd}N$$

$L^{a,b,c}$ continued

This doesn't look quite right though......

There is no choice of B such that the central charges and R-charges are simultaneously positive.

Notice that the results agree if

$$\boldsymbol{a} = -\mathtt{a}$$

though this is formal! Both should be positive.

There are infinitely many new solutions without a correct field theory dual.

What is the fate of the reduction on T^2? Reduction on S^2 behaves similarly as well.

Obstruction for Kähler cones.

There is an obstruction.

Let Y_5, have a cone of Calabi-Yau type. It admits a Kähler cone metric and global holomorphic volume form, then there is no supersymmetric $AdS_3 \times T^2 \times Y_5$ solution with this complex structure.

In the 'toric' case there must be one edge vector such that $(b, u_a) < 0$.

The polytope is now no longer convex.

This is the mathematical statement of the issues encountered above.

Conclusions

Summary

We have found a geometric dual to c-extremization and $\mathcal{I}$-extremization. See also [Gauntlett, Martelli, Sparks], [Hosseini, Zaffaroni], [Kim, Kim].

No metric necessary!

New solution that also exhibits the non-convex property as noted in [CMS] with respect to $Y^{p,q}$.

In fact we have obstruction results, which preclude the existence of Kähler cones over Y_5

Future directions

What are the dual field theories, and what is the fate of the compactified quiver theories?

Results preclude the existence of flow solutions from AdS_5 to AdS_3, what exactly goes wrong?

Work in progress on new examples.

Thank you
For your attention.