AdS_3 solutions with large, small and smaller superconformal symmetries

Niall T. Macpherson

Holography, Generalised Geometry and Duality
MITP

May 13, 2019

- NTM, arXiv:1812.10172 [hep-th],
- Y. Lozano, NTM, C. Nunez, A. Ramirez, arXiv:19xx.xxxx [hep-th],
- A. Legramandi, NTM, arXiv:19xx.xxxx[hep-th].

Motivation

- By now AdS-CFT generally well established.
- One avatar where CFT side is rather more developed than gravity side is AdS_3-CFT_2.
- Must arrange for large compact internal space when embedded into 10/11d.

$$ds^2 = e^{2A} ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{M}_{7/8})$$

- Systematic way to proceed is to assume extended supersymmetry. Then $\mathrm{M}_{7/8}$ should realise R-symmetry geometrically.
- Large amount of supersymmetry implies tractability, very detailed classifications, nice mathematical structure.
- Excludes many physical phenomena!
- However, with solutions known, can then systematically break some supersymmetry and find many more.

Supersymmetric AdS_3

- A rich variety of superconformal algebras exist for $d = 2$ exhibiting distinct R-symmetries.
- They are all direct sums of left and right algebras, preserving $\mathcal{N} = (n_+, n_-)$.

  - AdS_3 Killing spinors $\quad \nabla_\mu \zeta_\pm = \pm\frac{1}{2}\gamma_\mu\zeta_\pm \quad \Rightarrow \quad n_\pm.$

- Those that can be embedded into 10 and 11d supergravity with an AdS_3 factor are classified [Beck-Gran-Gutowski-Papadopoulos]

AdS_3 KSAs in type II and $d = 11$

$N_\sigma/2$	$\mathfrak{g}_L/\mathfrak{c}$	$\mathfrak{t}_0/\mathfrak{c}$
n	$\mathfrak{osp}(n\|2)$	$\mathfrak{so}(n)$
$2n, n > 1$	$\mathfrak{sl}(n\|2)$	$\mathfrak{u}(n)$
$4n, n > 1$	$\mathfrak{osp}^*(4\|2n)$	$\mathfrak{sp}^*(n) \oplus \mathfrak{sp}^*(1)$
8	$\mathfrak{f}(4)$	$\mathfrak{spin}(7)$
7	$\mathfrak{g}(3)$	$\mathfrak{g}_2$
4	$\mathfrak{D}(2,1,\alpha)$	$\mathfrak{so}(3) \oplus \mathfrak{so}(3)$
4	$\mathfrak{sl}(2\|2)/1_{4\times 4}$	$\mathfrak{so}(3)$

- 16 real supercharges, ie $\mathcal{N} = (8, 0)$ or $\mathcal{N} = (4, 4)$ is maximal for AdS_3 [Haupt-Lautz-Papadopoulos].
- F_4 and $G_{(3)}$ in IIA studied: [Dibitetto-Lo Monaco-Passias-Petri-Tomasiello], see Dibitetto's talk last week.
- Large and small superconfromal symmetries: main focus of this talk.

Small $\mathcal{N} = (4, 0)$ supersymmetry

- Small $\mathcal{N} = (4, 0)$ superconformal algebra contains bosonic sub-algebra
$$\mathfrak{sl}(2) \oplus \mathfrak{su}(2).$$
with SU(2) the R-symmetry, which may be realised geometrically in terms of an 2/3-sphere.
- Such CFT's are characterised by an integer level k and a central charge of the form
$$c = 6k.$$
- Canonical example of geometry realising this algebra is $\mathrm{AdS}_3 \times S^3 \times M_4$ for $M_4 = T^4, K3$. Actually preserves small $\mathcal{N} = (4, 4)$. The dual CFT's are symmetric product orbifolds $Sym^N(M_4)$
- These solutions lie in the more general small $\mathcal{N} = (4, 0)$ class
$$ds^2 = \frac{1}{\sqrt{h_5}}\Big(ds^2(\mathrm{AdS}_3) + ds^2(S^3)\Big) + \sqrt{h_5}ds^2(\mathrm{CY}_2), \qquad \nabla^2_{\mathrm{CY}_2} h_5 = 0,$$
$$F_3 = 2c\mathrm{Vol}(\mathrm{AdS}_3) + 2c\mathrm{Vol}(S^3) \pm c \star_{CY_2} dh_5, \qquad e^{-\Phi} = c\sqrt{h_5}.$$
- Most known geometries are of this type, or are either orbifolds or (non-Abelian) T-duals of this
- a notable exception is the classification of such solutions in IIB with 3-form fluxs set to zero [Couzens-Lawrie-Martelli-Schafer-Nameki-Wong].

Large $\mathcal{N}=(4,0)$ supersymmetry

- The large $\mathcal{N}=(4,0)$ superconfromal algebra $\mathfrak{D}(2,1,\alpha)$, contains bosonic subgroup
$$\mathfrak{sl}(2)\oplus\mathfrak{su}(2)\oplus\mathfrak{su}(2).$$
with SO(4) R-symmetry which can be realised by products of 2 and 3-spheres.
- CFTs with this symmetry are characterised by two integer levels $k_\pm$ in terms of which the central charge and α take the form
$$c=6\frac{k_+k_-}{k_++k_-},\qquad \alpha=\frac{k_+}{k_-}.$$
- Small superconformal algebra recovered in $\alpha\to 0$ limit.
- The canonical example of a geometry realising this symmetry is $\text{AdS}_3\times S^3\times S^3\times S^1$, which preserves $\mathcal{N}=(4,4)$.
- Took a long time to nail down the CFT dual, in large part due to an erroneous calculation of the supergravity BPS spectrum. This now seems to be resolved [Eberhardt-Gaberdiel-Rajesh Gopakumar-Wei] with the corrected spectrum matching the symmetric orbifold CFT $\mathcal{S}_\kappa$ [Gukov-Martinec-Moore-Strominger].
- Large $\mathcal{N}=(4,4)$ classified in M-theory [Bachas-D'Hoker-Estes-Gutperle -Feldman-Krym], compact cases all locally the IIA lift of $\text{AdS}_3\times S^3\times S^3\times\mathbb{R}^2$ (or are they? See later)
- All type II solutions with large $\mathcal{N}=(4,0)$ I was aware of are related to this by orbifoldings or duality - this is no longer the case.

Talk outline

- **Constructing solutions from R-symmetry**

  - *Brief review of methods employed to construct solutions*

- **Type II solutions with large (and small) $\mathcal{N}=(4,0)$ on $\mathbf{AdS}_3\times S^3\times S^3$**

  - *Compact examples with D-branes and O-planes back reacted on $AdS_3\times S^3\times S^3\times S^1$ and $AdS_3\times S^3\times\mathbb{R}^4$*

- **Massive IIA solutions with small $\mathcal{N}=(4,0)$ on $\mathbf{AdS}_3\times S^2\times \mathbf{M}_5$**

  - *Compact examples with branes back reacted on $AdS_3\times S^2\times T^4$*

- **Solutions with smaller superconformal symmetries**

  - *Breaking to $\mathcal{N}=(3,0)$ and $\mathcal{N}=(1,0)$ by fibering S^3 over S^3*

- **Concluding remarks**

Constructing solutions from R-symmetry

Realising an R-symmetry with spinors

- In type II supergravity a warped $\text{AdS}_3\times\text{M}_7$ solution preserving $\mathcal{N}=(n,0)$ supersymmetry admits MW Killing spinor of the form

$$\epsilon_1=\sum_{I=1}^{n} v_+\otimes\zeta^I\otimes\chi_1^I,\qquad \epsilon_2=\sum_{I=1}^{n} v_\pm\otimes\zeta^I\otimes\chi_2^I,$$

  with ζ^I (AdS_3), $\chi^I_{1,2}$ (M_7), $v_\pm$ (Auxiliary) all Majorana.

- Extended superconformal symmetry comes with an R-symmetry G_R (and associated Lie algebra T_{G_R} and Killing vectors $K^I_{G_R}$) under which $\chi^I_{1,2}$ should be charged

$$\mathcal{L}_{K^i_{G_R}}\chi^I_{1,2}=(T^i_{G_R})^I{}_J\chi^J_{1,2}$$

  Providing a map between each of the components of $\chi^I_{1,2}$.

- If one then imposes that all physical fields respect G_R, it is sufficient to solve for any $\mathcal{N}=1$ sub-sector to know $\mathcal{N}=(n,0)$ is preserved.

- We assume that M_7 decomposes as a manifold realising the R-symmetry and some base

$$ds^2(\text{M}_7)=e^{2C}ds^2(\text{M}_{G_R})+ds^2(B)\qquad\Rightarrow\qquad \chi^I_{1,2}=\xi^I_{G_R}\otimes\eta^B_{1,2}$$

- Supersymmetry then implied by reduced conditions on $\eta^B_{1,2}$ only.

- In many cases (and all considered here) M_{G_R} will just involve 2 and 3-spheres, and $\xi^I_{G_R}$ combinations of their Killing spinors.

Solving the $\mathcal{N}=1$ sub-sector

- $\mathcal{N}=1$ AdS$_3$ solutions in type II recently classified [Dibitetto-Lo Monaco-Passias-Petri-Tomasiello]

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(\text{M}_7), \qquad H = c_0 \text{Vol}(\text{AdS}_3) + H_3,$$

$$F = f + e^{3A}\text{Vol}(\text{AdS}_3) \wedge \star_7 \lambda(f)$$

- SUSY imposes either $|\chi_1|^2 \pm |\chi_2|^2 \propto e^{\pm A}$. Assume $|\chi_1|^2 = |\chi_2|^2$, then $c_0 = 0$
- Two 7d spinors give rise to a $G_2 \times G_2$-structure with bi spinor

$$e^{-A}\chi_1 \otimes \chi_2^\dagger = \Psi_+ + i\Psi_-,$$

Supersymmetry equivalent to familiar geometric conditions

$$d_H(e^{2A-\Phi}\Psi_\mp) = 0, \qquad d_H(e^{3A-\Phi}\Psi_\pm) - 2\mu e^{2A-\Phi}\Psi_\mp = \frac{1}{8}e^{3A} \star_7 \lambda(f)$$

(cf. (Mink/AdS)$_4$ classification [Grana-Minasian-Petrini-Tomasiello]) and an additional 7d Mukaui pairing constraint $e^A(f, \Psi_\pm) = \frac{\mu}{2}e^{-\Phi}\text{Vol}(\text{M}_7)$.

- Solving these is sufficient for $\mathcal{N}=(1,0)$, if we want $\mathcal{N}=(n,0)$ we need to impose by hand that the fluxes and other fields depend on invariant forms of G_R only!
- $\Psi_\pm$ will be wedge products of bi-spinors on M_{G_R} and B, former can be factored out to give conditions on B only.
- At this point one has a class of solutions defined by conditions on B that preserves $\mathcal{N}=(n,0)$ and given R-symmetry.

Large (and small) $\mathcal{N} = (4,0)$ on $\mathbf{AdS}_3 \times S^3 \times S^3$ in Type II

NTM

$\mathcal{N}=(4,0)$ on $\mathrm{AdS}_3\times S^3\times S^3$: SO(4) Spinors

- Seek solutions with large $\mathcal{N}=(4,0)$ so $\mathrm{G}_R=\mathrm{SO}(4)_R\simeq \mathrm{SO}(3)\times\mathrm{SO}(3)$. Can be realised with products of 2/3 spheres
- The Killing spinors on S^2 realise SU(2) doublets [NTM-Tomasiello]

$$\nabla_\mu\xi=\frac{i}{2}\gamma_\mu\xi \quad \Rightarrow \quad \mathcal{L}_{K^i}\xi^a=\frac{i}{2}(\sigma_i)^a{}_b\xi^a,\quad \xi^a=\begin{pmatrix}\xi\\ \xi^c\end{pmatrix} \text{ and } \begin{pmatrix}\hat\gamma\xi\\ \hat\gamma\xi^c\end{pmatrix}$$

- S^3 has two types of spinor charged under just one factor of $\mathrm{SU}(2)_+\times\mathrm{SU}(2)_-$ [NTM-Montero-Prins]

$$\nabla_\mu\xi_\pm=\pm\frac{i}{2}\gamma_\mu\xi_\pm \quad \Rightarrow \quad \mathcal{L}_{K^i_\pm}\xi^a_\pm=\pm\frac{i}{2}(\sigma_i)^a{}_b\xi^a_\pm,\quad \mathcal{L}_{K^i_\pm}\xi^a_\mp=0$$

- From this, and $\sigma_i\sigma_j=\delta_{ij}+i\epsilon_{ijk}$ it follows that

$$\eta^I_{\mathrm{SO}(4)}=\mathcal{M}^I_{ab}\xi^a_1\otimes\xi^b_2,\qquad \mathcal{M}^I=(\sigma_2\sigma_i,-i\sigma_2),$$

  transforms under the $\mathrm{SO}(3)_D$ and $\mathrm{SO}(3)_{AD}$ subgroups of $\mathrm{SU}(2)_1\times\mathrm{SU}(2)_2$ as

$$\mathcal{L}_{K^i_D}\eta^I_{\mathrm{SO}(4)}=(T^i_D)^I{}_J\eta^J_{\mathrm{SO}(4)},\qquad \mathcal{L}_{K^i_{AD}}\eta^I_{\mathrm{SO}(4)}=(T_{AD})^I{}_J\eta^J_{\mathrm{SO}(4)}$$

  where $K^i_D=K^i_1+K^i_2$, $K^i_{AD}=K^i_1-K^i_2$ and

$$(T^i_D)_{IJ}=\left(\begin{array}{c|c}\epsilon_{ijk} & \underline{0}\\ \hline \underline{0}^T & 0\end{array}\right),\qquad (T^i_{AD})_{IJ}=\left(\begin{array}{c|c}0_{3\times3} & \underline{c_i}\\ \hline -\underline{c_i}^T & 0\end{array}\right),\qquad (c_i)_j=\delta_{ij}$$

  - Block from makes breaking to subgroups easy later.

- We consider $S^3\times S^3$, which can also realise $\mathcal{N}=(4,4)$ when a solution supports two SO(4) spinors.

$\mathcal{N}=(4,0)$ on $\text{AdS}_3\times S^3\times S^3$: Solving an $\mathcal{N}=1$ sub-sector

- We consider solutions in type II of the form

$$ds^2=e^{2A}ds^2(\text{AdS}_3)+e^{2C_1}ds^2(S^3_1)+e^{2C_2}ds^2(S^3_2)+e^{2k}dr^2,$$

with flux built from dr, $\text{Vol}(S^3_{1,2})$ and functions of r only - $\text{SO}(4)_R\times\text{SO}(4)_F$ isometry.

- Most general Majorana SO(4) spinors are of the form

$$\chi^I_1=e^{\frac{A}{2}}\eta^I_{\text{SO}(4)}\otimes\begin{pmatrix}\sin(\beta_1+\beta_2)\\ i\cos(\beta_1+\beta_2)\end{pmatrix},\qquad \chi^I_2=e^{\frac{A}{2}}\eta^I_{\text{SO}(4)}\otimes\begin{pmatrix}\sin(\beta_1-\beta_2)\\ i\cos(\beta_1-\beta_2)\end{pmatrix},$$

there are two independent copies constructed from the ξ^a_+ and ξ^a_- SU(2) doublets respectively.

- Computing $\chi^1\otimes\chi^{2\dagger}$ for $\mathcal{N}=1$ sub-sector leads to SU(3)-structure

$$\Psi_+=\frac{e^A}{8}\text{Re}\Big[e^{i\beta_2}e^{-iJ}-e^k dr\wedge e^{i\beta_1}\Omega\Big],\quad \Psi_-=\frac{e^A}{8}\text{Im}\Big[-e^{i\beta_2}e^k dr\wedge e^{-iJ}+e^{i\beta_1}\Omega\Big],$$

where SU(3)-structure forms are canonical in terms of complex vielbein

$$E^i=\frac{1}{2}(e^{C_1}K^1_i+ie^{C_2}K^2_i),\qquad dK^a_i=\pm\frac{1}{2}\epsilon_{ijk}K^a_j\wedge K^a_k$$

- Plugging $\Psi_\pm$ into the supersymmetry conditions yields a system of ODE's that can be solved uniquely.

$\mathcal{N} = (4,0)$ on $\text{AdS}_3 \times S^3 \times S^3$: Local Solutions

- There are 4 distinct local solutions, 2 in each of IIA and IIB:
  1. D8/O8 system back reacted on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$, large $\mathcal{N} = (4,0)$.

     -Non trivial F_0, F_4. Generically non compact
  2. D5s back reacted $\text{AdS}_3 \times S^3 \times S^3 \times S^1$, large $\mathcal{N} = (4,0)$.

     - Non trivial F_3. Generically non compact
  3. Solution with large $\mathcal{N} = (4,4)$ and O2 source

     - Non trivial F_4, H, can be lifted to M-theory. Non compact
  4. D5s and/or O5 back reacted on $\text{AdS}_3 \times S^3 \times \mathbb{R}^4$, small $\mathcal{N} = (4,0)$

     - Non Trivial F_3. Compact with tuning.

     - SO(4) is realising $\text{SU}(2)_R$ and SU(2) outer-automorphism of dual CFT_2
- New large $\mathcal{N} = (4,0)$ solutions are all non compact?

  - Recall a local solution is not a global solution.
- **Globally compact solutions with large $\mathcal{N} = (4,0)$ are possible.**

$\mathcal{N} = (4, 0)$ on $\mathrm{AdS}_3 \times S^3 \times S^3$: Compact solutions with D8/O8

- The local solution with D8/O8 back reacted on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ depends on a single $O(1)$ polynomial H_8:

$$ds^2 = \frac{1}{\sqrt{H_8}}\left(L^2 ds^2(\mathrm{AdS}_3) + \frac{L^2}{\cos^2\beta_1} ds^2(S^3_1) + \frac{L^2}{\sin^2\beta_1} ds^2(S^3_2)\right) + \sqrt{H_8} q^2 dr^2,$$

$$F_4 = 2q^2 H_8 \left(L^2 \mathrm{Vol}(\mathrm{AdS}_3) + \frac{L^2}{\cos^2\beta_1}\mathrm{Vol}(S^3_1) + \frac{L^2}{\sin^2\beta_1}\mathrm{Vol}(S^3_2)\right) \wedge dr,$$

$$e^{-\Phi} = q H_8^{\frac{5}{4}}, \quad H_8 = a + F_0 r, \quad \beta_1, a, q, L \ - \text{Constants}$$

- When $a, F_0 > 0$, the interval is bounded from below at $\rho = -\frac{a}{F_0}$ where the solution becomes a D8/O8 stack wrapped on $\mathrm{AdS}_3 \times S^3 \times S^3$. But the interval is not bounded from above!
- The local AdS_7 solutions in massive IIA exhibit analogous non compact behaviour
  [Apruzzi-Fazzi-Rosa-Tomasiello]

  - There local patches glued together with D8 brane defects
  - At the intersection of patches F_0 has a discontinuity
  - But g, e^{Φ}, B continuous
  - Must ensure $d_H F = \delta$ and κ-symmetry solved at defect.

$\mathcal{N} = (4,0)$ on $\mathrm{AdS}_3 \times S^3 \times S^3$: Compact solutions with D8/O8

- Easiest way to build a compact solution is to make F_0 change sign at $r = 0$

$$r < 0: \; H_8 = 1 + |F_0|r, \qquad r > 0: \; H_8 = 1 - |F_0|r$$

- Solution bounded between two D8/O8 at $r = \pm\frac{1}{|F_0|}$

  - D8 defect at $r = 0$ is calibrated and obeys BI provided $N_8 = 4\pi|F_0|$

  - Solution is under parametric control (tune with D2 charge N_2)

  - Possible interpretation: ($\mathrm{O8}_-$, k D8's) and ($\mathrm{O8}_-$, (16-k) D8s)

- Holographic central charge consistent with large $\mathcal{N} = (4,0)$

$$c = \frac{3}{2^4\pi^6}\int_{\mathrm{M}_7} e^{A-2\Phi}\mathrm{Vol}(\mathrm{M}_7) = 6N_2\frac{N_4^1 N_4^2}{N_4^1 + N_4^2} + O(1)$$

- Can insert arbitrary number of defects between the two D8/O8 stacks and construct infinitely many compact solutions with large $\mathcal{N} = (4,0)$!

  - Just need to impose that H_8 is piece-wise linear.

- May be possible to play the same game with other large $\mathcal{N} = (4,0)$ local solutions: Most interesting is the $\mathcal{N} = (4,4)$ solution

  - Can be lifted to M-theory, so would conflict with claim [Bachas-D'Hoker-Estes-Krym] that all compact solutions locally $\mathrm{AdS}_3 \times S^3 \times S^3 \times \mathbb{R}^2$

  - However, the possibility of defects apparently not considered

$\mathcal{N}=(4,0)$ on $\mathrm{AdS}_3\times S^3\times S^3$: Compact solutions with D5s and O5

- The solution with small $\mathcal{N}=(4,0)$ is a deformation of $\mathrm{AdS}_3\times S^3\times\mathbb{R}^4$ depending on a single function h_5 of the form

$$ds^2=L^2\left[\frac{1}{\sqrt{h_5}}\Big(ds^2(\mathrm{AdS}_3)+ds^2(S^3_2)\Big)+\sqrt{h_5}\Big(dr^2+r^2ds^2(S^3_1)\Big)\right],$$

$$F_3=c_1\Big(\mathrm{Vol}(\mathrm{AdS}_3)+\mathrm{Vol}(S^3_2)\Big)+c_2\mathrm{Vol}(S^3_1)\qquad e^{-\Phi}=\frac{c_1}{2L}\sqrt{h_5},$$

$$h_5=a+\frac{c_2}{c_1r^2},\qquad c_1,c_2,a,L\ -\text{constants}$$

- A possible way to interpret this is simply D5s (or an O5 Hole if $c_2/c_1<0$) backreated on $\mathrm{AdS}_3\times S^3\times\mathbb{R}^4$. Then the solution can be realised as a near horizon limit of the brane set up

	0	1	2	3	4	5	6	7	8	9
$D1$	×	×	/	/	/	/				
$D5_1$	×	×	×	×	×	×				
$D5_2$	×	×					×	×	×	×

One goes near D1 and $D5_1$ only - of course this solution is non compact.

- It may be possible to glue local copies of this solution together with defect branes, but there is actually an easier way.

-If one merely assumes $(a<0,\ c_2/c_1>0)$ the interval becomes bounded!

$\mathcal{N}=(4,0)$ on AdS$_3\times S^3\times S^3$: Compact solution with D5s and O5

- The metric is then diffeomorphic to

$$ds^2 = L^2\left[\frac{\cos r}{\sin r}\left(ds^2(\text{AdS}_3)+ds^2(S^3_2)\right)+\frac{c_2\sin r}{c_1\cos r}\left(\sin^2 r dr^2+\cos^2 r ds^2(S^3_1)\right)\right],$$

- Now $r\in[0,\frac{\pi}{2}]$

  - at $r=0$ there is an O5 wrapped on AdS$_3\times S^3_2$

  - at $r=\frac{\pi}{2}$ the metric becomes that of D5s wrapped on AdS$_3$ and either S^3

- The central charge is indeed consistent with small $\mathcal{N}=(4,0)$, and independent of the O5 charge

$$c = 6N_1N_5+O(1)$$

- Solution is under parametric control - Large L (and so D1 charge) ensures small curvature arbitrarily close to singularities.

- Surprisingly there is no conflict with small charge of the O5!

  - Indeed it is not usually possible to simply flip the sign in a D brane warp factor to get a compact solution

  - usually can't make curvature and O-plane charge simultaneously small.

Small $\mathcal{N} = (4,0)$ on $\mathbf{AdS}_3 \times S^2 \times \mathbf{M}_5$ in IIA

Y. Lozano, NTM, C. Nunez, A. Ramirez

Small $\mathcal{N}=(4,0)$ on $\mathrm{AdS}_3\times S^2\times \mathrm{M}_5$: Realising $\mathrm{SU}(2)_R$

- S^2 realises an $\mathrm{SU}(2)_R$ in terms ξ^a, but only 2 independent spinors...
- To realise $\mathcal{N}=(4,0)$ need to introduce $\eta^a=(\eta,\eta^c)^T$ then

$$\chi^I=(\mathcal{M}^I)_{ab}\xi^a\otimes\eta^b \quad\Rightarrow\quad \mathcal{L}_{K^i}\chi^I=\frac{i}{2}(\Sigma_i)^I{}_J\chi^J$$

where $(\mathcal{M}^I)=(\sigma_2\sigma_i,-i\sigma_2)$ and Σ_i 4d rep of SU(2). χ^I Majorana.

- General case contain 4 such SU(2) spinors, rather unwieldy, will focus on

$$ds^2=e^{2A}ds^2(\mathrm{AdS}_3)+e^{2C}ds^2(S^2)+ds^2(\mathrm{M}_{SU(2)})+V^2,\quad H=H_3+e^{2C}H_1\wedge\mathrm{Vol}(S^2)$$

SUSY then follows from the system (RR fluxes also defined)

$$2\mu e^C+\sin\alpha e^A=e^{2C}H_1+\frac{1}{2}e^AV-\frac{1}{4}d(e^{2A}\sin\alpha\cos\alpha)=0$$

$$d(e^{3A-\Phi}\sin\alpha\sin\beta)-2e^{2A-\Phi}\cos\alpha\sin\beta V=d(e^{A-\Phi}\sin\alpha\cos\beta)\wedge V=0,$$

$$d(e^{3A-\Phi}\sin\alpha\Omega)-2e^{2A-\Phi}\cos\alpha V\wedge\Omega=0,$$

$$d(e^{3A-\Phi}\sin\alpha\cos\beta J)-2e^{2A-\Phi}\cos\alpha\cos\beta V\wedge J-e^{3A-\Phi}\sin\alpha\sin\beta H_3=0,$$

$$(\sin\beta e^{2A}d(e^{-2A}J)+\cos\beta H_3)\wedge V=\Omega\wedge H_3=(\sin\beta dJ+\cos\beta H_3)\wedge J=0,$$

- $\beta\neq 0$ related to [Couzens-Lawrie-Martelli-Schafer Nameki-Wong]

  - Case is restrictive. Instead focus of $\beta=0$.

Small $\mathcal{N}=(4,0)$ on $\mathrm{AdS}_3\times S^2\times \mathrm{M}_5$: Conformal CY Class

- System at $\beta=0$ contains the class with NS sector ($u'=\partial_r u$)

$$ds^2=\frac{u}{\sqrt{h_4h_8}}\left(ds^2(\mathrm{AdS}_3)+\frac{1}{4\Delta}ds^2(S^2)\right)+\sqrt{\frac{h_4}{h_8}}ds^2(\mathrm{CY}_2)+\frac{\sqrt{h_4h_8}}{u}dr^2,$$

$$e^{-\Phi}=h_8^{\frac{5}{4}}h_4^{\frac{1}{4}}u^{-\frac{1}{2}}\sqrt{\Delta},\quad B=\frac{1}{2}\left(-r+\frac{uu'}{4h_4h_8\Delta}\right)\mathrm{Vol}(S^2),\qquad \Delta=1+\frac{(u')^2}{4h_4h_8}$$

- Can also turn on H_3 such that $H_3\wedge dr=H_3\wedge J=H_3\wedge\Omega=0$, but I don't.
- H and magnetic components of F_0,F_2,F_4,F_6 non trivial, BI impose

$$h_8'=F_0,\qquad u''=0,\qquad \frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}h_4+h_4''=0$$

  away from localised sources
- When $u=\Delta=1$, have a D8-D4 system wrapped on $\mathrm{AdS}_3\times S^2$ (cf. [Youm])

  - when ∂_r an isometry, T-dual of D5's on $AdS_3\times S^3\times \mathrm{CY}_2$
- Generic system is deformation of the D8-D4 one.

  - Contains NATD.
- General class that NATD of $\mathrm{AdS}_3\times S^3\times T^4$ suggested [Sfetsos-Thompson]

Small $\mathcal{N} = (4,0)$ on $\text{AdS}_3 \times S^2 \times \text{M}_5$: Branes and planes on T^4

- Potentially MANY compact solutions contained here, simplest to assume $\text{CY}_2 = T^4$ (or indeed $K3$)

  - Functions depend on interval only, $(h_4,\ h_8,\ u)$ all $O(1)$ polynomials

  - 6 parameters to tune, leading to rich variety of local solutions

- Two simple compact cases have interval bounded between

  - D8/O8 and smeared D4's or D6s and O6

- Possible to achieve the following physical boundary behaviours

Regular zero	Local Singularity	Smeared on T^4
NATD	D6, D8, O6, O8	D2, D4

- Can't all exist in same local patch, in most cases interval is semi infinite.

- As before one can glue solutions together with defects

  - This time D8-D6 and smeared D4-D2 bound states

- Can use defect branes to make NATD of $\text{AdS}_3 \times S^3 \times T^4$ compact!

  - NATD gives rise to solutions depending on a semi infinite interval. Can often be interpreted as quiver of infinite length.

  - Can try to complete the quiver by inserting a flavour node (eg NATD $\text{AdS}_5 \times S^5$ [Lozano-Nunez]). - Constructions typically pretty complicated.

  - Glueing another local solution onto NATD, comparatively simple.

Solutions with smaller superconformal symmetries

Legramandi-NTM

Realising $\mathcal{N}=(3,0)$ and $\mathcal{N}=(1,0)$ on $\text{AdS}_3\times S^3\times S^3$

- To construct less supersymmetric examples we return to $\text{AdS}_3\times S^3\times S^3$

  - Simplest to take orbifolds of the 3-spheres. Indeed $\mathcal{N}=(3,3)$ and $\mathcal{N}=(1,1)$ orbifolds exist for $\text{AdS}_3\times S^3\times S^3\times S^1$ [Eberhardt-Zadeh]

  - Possible to do the same in presence of D8s/O8 also.

- Using the SO(4) spinors we can be a little more ambitious though, recall

$$\eta^I_{\text{SO}(4)}=\begin{pmatrix}\eta^i\\ \eta^4\end{pmatrix},\qquad \text{SO}(3)_D:\qquad \mathcal{L}_{K^i_D}\eta^j=\epsilon^{ij}{}_k\eta^k,\qquad \mathcal{L}_{K^i_D}\eta^4=0$$

- It's $SO(3)_{AD}$ that provides a map between η^i,η^4, so we break it!

- The breaking can be done at the level of fluxes by allowing them to depend on the invariant forms of $\text{SO}(3)_D$

$$(\text{Vol}(S^3_1),\ \text{Vol}(S^3_2))\longrightarrow(\text{Vol}(S^3_1),\ \text{Vol}(S^3_2),\ K^1_i\wedge K^2_i,\ K^1_i\wedge dK^2_i,\ dK^1_i\wedge K^2_i)$$

- But we can also break $SO(3)_{AD}$ with the metric by fibering S^3_1 over S^3_2

$$ds^2(S^3_1)=\frac{1}{4}\sum_{i=1}^{3}\left(K^1_i\right)^2\to\frac{1}{4}\left(K^1_i+\lambda K^2_i\right)^2$$

  This is similar to how $\mathcal{N}=3$ is realised in the AdS_4 solution of Pang and Rong [De Luca-Lo Monaco-NTM-Tomasiello-Varela]

Realising $\mathcal{N}=(3,0)$ and $\mathcal{N}=(1,0)$ on $\text{AdS}_3\times S^3\times S^3$

- We do both forms of breaking, but in either case, the result is that

  - η^i gives $\mathcal{N}=(3,0)$ and so $\mathfrak{osp}(3|2)$ and $\text{SO}(3)_R\times\text{SO}(4)_F$

  - η^4 gives $\mathcal{N}=(1,0)$ system with $SO(3)\times SO(4)$ flavour symmetry

- We thus seek solutions of the from

$$ds^2=e^{2A}ds^2(\text{AdS}_3)+\frac{e^{2C_1}}{4}\left(K_i^1+\lambda K_i^2\right)^2+\frac{e^{2C_2}}{4}\left(K_i^2\right)^2+e^{2k}dr^2$$

- For the $\mathcal{N}=(3,0)$ must impose that flux depends on invariant forms, for $\mathcal{N}=(1,0)$ this is automatic

  - IIB: F_1,F_3,F_5,H, IIA: F_0,F_2,F_4,H

- Solutions certainly exist, but at this time I only know them some semi analytically

- IIB can realise, D5s, NS5s, O5, (regular?) IIA is richer

Regular zero	Local Singularity	Smeared Singularity
?	D2 , D8, O2, O6, O8, NS5	?

- Of course, similar to the $\mathcal{N}=(4,0)$ case, there is the possibility of supersymmetry enhancements

  - A careful analysis will reveal if this is possible, but we are not that far yet.

Conclusions

- **Have found new classes of AdS_3 solutions with large and small $\mathcal{N} = (4, 0)$ supersymmetry and compact internal space.**

  - Clearly the next step is to try and ascertain what the CFT duals are.

- **Have only studied a small portion of possibilities**

  - Expect classes of $\mathrm{AdS}_3 \times S^2 \times S^3$ and $\mathrm{AdS}_3 \times S^2 \times S^2$ solutions with large $\mathcal{N} = (4, 0)$ and many more with small.

  - Did not consider M-theory at all, or IIB for small $\mathcal{N} = (4, 0)$.

- **Would be interesting to study the less supersymmetric cases in more detail.**

- **Have recently classified AdS_2 solutions in M-theory with magnetic flux and SU(4)-structure** [Hong-NTM-Pando Zayas].

  - would be interesting to employ R-symmetry techniques there

  - New black hole near horizons?

Thank you