The holographic dual of the Ω-background

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Based on..

- 1903.05095 with Nikolay Bobev and Friðrik Freyr Gautason

Important background literature

- Topological field theory - *[Witten'88]*
- Ω-deformation - *[Nekrasov'02], [Nekrasov, Okounkov'03]*
- Susy field theories on curved space - *[Festuccia, Seiberg'11], [Klare, Tomasiello, Zaffaroni'12]*
- 5d $\mathcal{N} = 4^+$ supergravity - *[Romans'86]*

Motivation

Exact holography?

- The Ω-deformation allows exact evaluation of the partition function on $\mathbb{R}^4$, $Z_{Nekrasov}$.
- $Z_{Nekrasov}$ a basic building block for many other localization results, fundamental importance.

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- *Here:* just the beginning - find the simplest bulk dual!

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- Vector multiplets: (A_μ, ϕ, ψ) with arbitrary gauge groups G
- Hypermultiplets: (q^u, ξ) in some representation of G

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- Supercharges $Q, Q^+_{\mu\nu}, G^\mu$ (scalar, selfdual two-form, vector) on $\mathbb{R}^4$
- Donaldson-Witten topological theory on any $\mathcal{M}_4$ - twist the $SU(2)_{left}$ with the $SU(2)_R$, only Q preserved - works for an arbitrary $\mathcal{N} = 2$ theory

Intro: the Ω-deformation

- Pick any 4d $\mathcal{N} = 2$ susy QFT on $\mathbb{R}^4$, consider supercharge

$$\tilde{Q} = Q + E_a \Omega^a_{\mu\nu} x^\mu G^\nu \ , \qquad (1)$$

  $\Omega^a_{\mu\nu} x^\mu$ the Killing vectors of $SO(4)$ rotations.

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- Choose a symplectic form ω on $\mathbb{R}^4$,

$$\omega \equiv \mathrm{d}x^1 \wedge \mathrm{d}x^2 + \mathrm{d}x^3 \wedge \mathrm{d}x^4 \ , \tag{2}$$

  implying a complex structure $z_1 = x^1 + ix^2, z_2 = x^3 + ix^4$,

$$H \equiv \epsilon_1 |z_1|^2 + \epsilon_2 |z_2|^2 \tag{3}$$

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$$Z_N(a, \epsilon_{1,2}) \equiv \left\langle \exp \int_{\mathbb{R}^4} \omega \wedge Tr(\phi F + \psi\psi) - H\ Tr(F \wedge F) \right\rangle_a \quad (4)$$

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- in an expansion around small $\epsilon_{1,2}$,

$$\log Z_N(a, \epsilon_{1,2}) = \frac{1}{\epsilon_1 \epsilon_2} \mathcal{F}_0 + \frac{(\epsilon_1 + \epsilon_2)}{\epsilon_1 \epsilon_2} \mathcal{H}_{1/2} + \mathcal{F}_1 + \frac{(\epsilon_1 + \epsilon_2)^2}{\epsilon_1 \epsilon_2} \mathcal{G}_1 + ... \quad (5)$$

Intro: gluing copies of $Z_N(a, \epsilon_{1,2})$

- For an $\mathcal{M}_4$ with a $U(1)$-isometry one can localize $Z_{\mathcal{M}_4}$ to the fixed points of the $U(1)$ on $\mathcal{M}_4$, where we locally recover the Ω-deformation on $\mathbb{R}^4$.

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- $\rightarrow$ Construct $Z_{\mathcal{M}_4}$ with a number of $Z_N(a, \epsilon_{1,2})$ for each fixed point, correctly identifying $\epsilon_{1,2}$ at every point and integrating over the Coulomb branch parameter a - "gluing procedure".
- $\rightarrow$ Use $\mathcal{F}_0(a), \mathcal{H}_{1/2}(a), \mathcal{F}_1(a), \mathcal{G}_1(a)$ to construct large class of susy partition functions of a given $\mathcal{N} = 2$ theory.

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- Similarly deform Q to $\tilde{Q}$ on manifolds other than $\mathbb{R}^4$ - NO twist. Even richer story for twists of $\mathcal{N} = 4$ SYM...
- *Here:* stay at the superconformal point, $a = 0$ - full Coulomb branch not visible $\Rightarrow$ only one of $\epsilon_{1,2}$ accessible.

Ω-background

- Instead of modifying the path-integral, consider a background of $4d\,\mathcal{N}=2$ sugra, in particular here off-shell superconformal gravity.
- Explicit background constructed in *[Hama, Hosomichi'13]* from deconstructing the ellipsoid, in agreement with the classification of *[Klare, Zaffaroni'13]*.

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- SUGRA Weyl multiplet: metric $g^{(4)}_{\mu\nu}$, two real 2-forms $T^{\pm}_{\mu\nu}$, $SO(1,1)+SU(2)$ gauge fields A^0_μ, A^{ij}_μ, a real scalar $\tilde{d}$ + fermions.

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- Ω-background: vanishing fermions, flat $\mathbb{R}^4$ metric, $A^0 = A^{ij} = \tilde{d} = T^+ = 0$,

$$T^- = \mathrm{d}b = -2\beta\,\omega\ , \quad b = 2\beta(x_{[2}\mathrm{d}x_{1]} + x_{[4}\mathrm{d}x_{3]})\ . \tag{6}$$

with $\beta = \epsilon_1 + \epsilon_2$ *[Hama, Hosomichi'13], [Klare, Zaffaroni'13]*.

Conformal Killing spinors in the Ω-background

- 12 supercharges preserved,

$$\zeta^+ = \zeta_0^+\,, \quad \zeta^- = \zeta_0^- - \frac{i}{2} b_m \gamma^m \zeta_0^+ + x^m \gamma_m \eta_0^+\,,$$
$$\eta^\pm = \frac{1}{4}\gamma^m \partial_m \zeta^\mp\,, \Rightarrow \eta^+ = \eta_0^+\,, \quad \eta^- = 0 \tag{7}$$

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- not all S-supersymmetries are broken!
- for comparison, on usual $\mathbb{R}^4$ with $T^- = 0$: 8 Q's + 8 S's,

$$\zeta^\pm = \zeta_0^\pm + x^m \gamma_m \eta_0^\mp \,, \quad \eta^\pm = \eta_0^\pm \tag{8}$$

Deformation of the Lagrangian

- Due to the background 2-form T^-, additional couplings to the flat space $\mathcal{N} = 2$ theory dictated by superconformal gravity coupled to vectormultiplets (A_μ, ϕ, ψ) and hypermultiplets (q^u, ξ):

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- $Tr(F \wedge F)$ term absent in the superconformal gravity formalism!
- $\Rightarrow$ We *cannot* switch on freely both deformation parameters $\epsilon_{1,2}$ of Nekrasov using only the background Weyl multiplet.

The bulk: 5d $\mathcal{N} = 4^+$ supergravity

- 5d sugra with no extra matter, minimal physical theory (various string theory embeddings).
- Metric $g^{(5)}_{\mu\nu}$, scalar $X = e^{-\lambda/\sqrt{6}}$, $SO(1,1)_R$ symmetry $f = \mathrm{d}a$, $SU(2)_R$ symmetry $F^i = \mathrm{d}A^i + g\varepsilon^i{}_{jk}A^j \wedge A^k$, two 2-forms $B^{\pm}$ charged under the $SO(1,1)_R$, $H^{\pm} = \mathrm{d}B^{\pm} \mp g\ a \wedge B^{\pm}$, odd-dim. (anti-)selfdual.

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- Bosonic Lagrangian

$$\begin{aligned}\mathcal{L} =& \sqrt{g^{(5)}}\Big[R - \frac{1}{2}|\mathrm{d}\lambda|^2 + 2X^4|f|^2 + g^2(X^2 + 2X^{-1}) \\ & - X^{-2}\left(\mathrm{tr}|F|^2 + B^+ \cdot B^-\right)\Big] + \text{CS terms}\ ,\end{aligned} \tag{10}$$

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- Maximally supersymmetric vacuum (16 supercharges) AdS_5: $\lambda = 0 \Rightarrow X = 1$, $f = F^i = B^\pm = 0$.

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- Very easy! No flow away from AdS$_5$ because of remaining conformal S supercharges.

The bulk: Killing spinors for Ω-AdS$_5$

- Use spinors of definite chirality and $SO(1,1)$ eigenvalues, $\gamma_{1234}\varepsilon_0^{\pm} = \pm\varepsilon_0^{\pm}$ and $\hat{\sigma}_3\varepsilon_{0\pm} = \pm\varepsilon_{0\pm}$.

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- Explicit Killing spinors for Ω-AdS$_5$ (12 independent supercharges)

$$\begin{aligned}\epsilon = z^{-1/2}\varepsilon_0^-{}_+ + (z^{1/2} + z^{-1/2}x^m\gamma_m)\varepsilon_0^+{}_+ \\ + z^{-1/2}\left(1 - \frac{i}{2}\,b_m\gamma^m\hat{\sigma}_+\right)\varepsilon_0^+{}_- \,.\end{aligned} \tag{12}$$

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- $\varepsilon^-{}_- = 0$ but $\varepsilon^+{}_+ \neq 0 \to$ four S's preserved.
- Moving away from the conformal vacuum $=$ giving vev v to the scalar $X^3 = 1 + vz^2 \Rightarrow$ a susy flow breaking conformality, $\varepsilon^+{}_+ = 0$ with an IR singularity as $z \to \infty$.

Uplift to IIB supergravity

- Simple form in string frame:

$$\mathrm{d}s^2 = \sqrt{\Delta}\left(\mathrm{d}s_5^2 + L^2 X \mathrm{d}\theta^2\right) + \frac{L^2}{\sqrt{\Delta}}\left(\frac{1}{X}\cos^2\theta\,\mathrm{d}\Omega_3^2 - X^2\sin^2\theta\,\mathrm{d}\phi^2\right),$$

$$F_5 = -\frac{iL^4}{g_s}(1+\star_{10})\mathrm{d}\left(\frac{\Delta X^2}{z^4}\right)\wedge vol_4\,, \quad \Delta = X\cos^2\theta + X^{-2}\sin^2\theta$$

$$B_2 = i g_s C_2 = \frac{L^2}{4} e^{-\phi}\sin\theta\; B^-\,, \quad C_0 = 0\,, \quad e^{\Phi} = g_s\,.$$

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- The "internal" space becomes a deformation of 5d de Sitter $\rightarrow$ Hull's type IIB* supergravity (e.g. standard IIB sugra with purely imaginary RR fields).

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- Generalize the holographic construction of Ω-like backgrounds to other dimensions.
- Investigate holographic duals to the more general Nekrasov-Okounkov twist of $\mathcal{N} = 2$ theories, as well as the 3 special twists of $\mathcal{N} = 4$ SYM.