HOLOGRAPHIC ORIGIN of the BEKENSTEIN-HAWKING ENTROPY of 1/16 BPS AdS_5 BLACK HOLES

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1810.11442, 1904.05865 with A. Cabo-Bizet, D. Martelli, S. Murthy

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Black hole microstate counting

- A major achievement of string theory :

provide the microstates that account for the entropy of (supersymmetric) black holes

asymptotically flat   Strominger, Vafa '96 . . .

$$S \propto \sqrt{c}$$

c 2d CFT central charge

related to the Casimir energy by Cardy formula

asymptotically AdS_4 Benini, Hristov, Zaffaroni '15, . . .

Entropy of supersymmetric AdS_4 black holes computed by the Legendre transform of a topologically twisted index

Supersymmetric black holes in AdS_5

Supersymmetric black holes in AdS_5 have been known for 15 years

Gutowski, Reall '04, Chong, Cvetic, Lu, Pope '05, Kunduri, Lucietti, Reall . . .

1/16 BPS, carry angular momentum & electric charge

start from type IIB on $AdS_5 \times S^5$

$\downarrow$ $\searrow$

SO(2,4) x SO(6) symmetry

$\downarrow$ $\downarrow$

break to $R \times U(1)^2 \times U(1)^3$

$\downarrow$ $\downarrow$

E, J_1, J_2 Q_1, Q_2, Q_3

- replace S^5 with more general M_5 ➜ SO(6) broken to just U(1) ➜ $E,\ J_1,\ J_2,\ Q$

Supersymmetric black holes in AdS_5

Bekenstein-Hawking entropy

$$S = \frac{\mathbf{Area}}{4} = \pi\sqrt{3Q^2 - 8c(J_1 + J_2)}$$

$$c = \frac{\pi \ell^3}{8G_N}$$

microscopic origin ??

Supersymmetric black holes in AdS_5

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microscopic origin ??

use AdS_5 / CFT_4

type IIB on AdS_5 x S^5 $\Longleftrightarrow$ $\mathcal{N} = 4$ SYM ,

replace S^5 with more general M_5 $\Longleftrightarrow$ $\mathcal{N} = 1$ $SCFT_4$, e.g. conifold theory

microstates: 1/16 BPS states with assigned angular momenta and charge

Task: count them at large N.

Attempts in the past unsuccessful

Difficulties on field theory side

Why failed?

- 1/16 BPS states not "protected enough"

- natural quantity to consider: superconformal index

Romelsberger '05

Kinney, Maldacena, Minwalla, Raju '05

$$\mathcal{I}(\omega_1,\omega_2) = \underbrace{\mathrm{Tr}\,(-1)^F e^{-\beta\{\mathcal{Q},\bar{\mathcal{Q}}\}}}_{\text{Witten index}}{}^{+\omega_1\underbrace{(J_1+\frac{1}{2}Q)}+\omega_2\underbrace{(J_2+\frac{1}{2}Q)}}$$

Witten index — commute with supercharge $\mathcal{Q}$

ω_1, ω_2 chemical potentials, taken real

At large N, $\mathcal{I}(\omega_1,\omega_2) \sim \mathcal{O}(1)$ ➜ cannot reproduce $\mathcal{O}(N^2)$ entropy

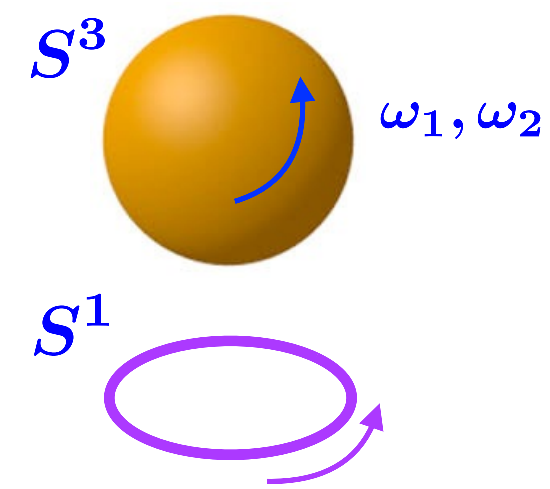
- reason: many cancellations between bosonic and fermionic states

Difficulties on field theory side

Why failed?

- Index also understood as

  partition function on $S^1 \times S^3$, twisted by ω_1, ω_2



Path integral and Index differ by susy Casimir energy

Assel, DC, Martelli '14

$$Z(\omega_1, \omega_2) = \mathrm{e}^{-\mathcal{F}(\omega_1,\omega_2)} \mathcal{I}(\omega_1, \omega_2)$$

At large N , $-\log Z = \mathcal{F} = -\frac{2}{27}\frac{(\omega_1+\omega_2)^3}{\omega_1\omega_2}c \sim \mathcal{O}(N^2)$

anomaly coeff $a = c \sim \mathcal{O}(N^2)$

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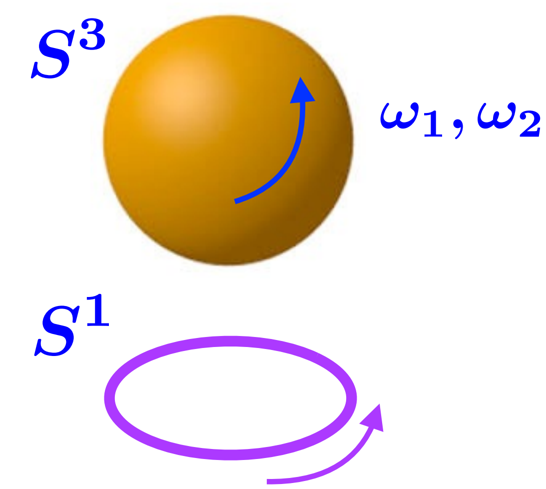

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✤ HOPE?

susy Casimir energy not enough

$\mathcal{F}(\omega_1, \omega_2)$ is a grand-canonical partition function

homogeneous of degree 1 ➜ Legendre transform = 0 ➜ no entropy

Difficulties on gravity side

AdS/CFT (at large N)

$$\mathrm{e}^{-I_{\text{gravity}}[M_5]} = Z[M_4] \qquad M_4 = \partial M_5$$

gravity boundary conditions $\Longleftrightarrow$ QFT background fields

Black hole thermodynamics:

Gibbons, Hawking

relates entropy S and on-shell gravity action

$$I(\beta, \Omega_i, \Phi) = \beta E - S - \beta\Omega_i J_i - \beta\Phi Q$$

Quantum Statistical Relation

$$E = \frac{\partial I}{\partial \beta}\,, \qquad J_i = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_i}\,, \qquad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi} \qquad \beta = T^{-1}$$

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✤ HOPE?

Thermodynamics for supersymmetric black holes is subtle : $\beta \to \infty$

- what are the relevant chemical potentials for $\beta \to \infty$? $\Omega_i \to 1,\ \Phi \to 3/2$ fixed!
- do these match ω_1, ω_2 on the field theory side?

Table of functions

entropy

$$S(J_i, Q) = \pi\sqrt{3Q^2 - 8c(J_1 + J_2)}$$

log of microcanonical partition function

entropy function

Hosseini, Hristov, Zaffaroni '17

$$I(\omega_i, \varphi) = \frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}c \qquad \omega_1 + \omega_2 - 2\varphi = 2\pi i$$

physical interpretation?

supergravity on-shell action (at finite β)

$$I(\beta, \Omega_i, \Phi) = \beta E - S - \beta\Omega_i J_i - \beta\Phi Q$$

$-$log of grand-canonical partition function

SCFT partition function

$$Z(\omega_1, \omega_2) = \mathrm{e}^{-\mathcal{F}(\omega_1,\omega_2)}\mathcal{I}(\omega_1, \omega_2)$$

Casimir energy

index

Legendre transform+ constraint

Legendre transform+ BPS limit

related via AdS/CFT after BPS limit **?**

The non-BPS solution

- Five-dimensional minimal gauged supergravity

sources dual R-current

$$\mathcal{L} = (R + 12) *1 - \frac{2}{3} F \wedge *F + \frac{8}{27} F \wedge F \wedge A$$

The non-BPS solution

- Five-dimensional minimal gauged supergravity

sources dual R-current

$$\mathcal{L} = (R+12) *1 - \frac{2}{3} F \wedge *F + \frac{8}{27} F \wedge F \wedge A$$

- Non-supersymmetric, non-extremal black hole solution

Chong, Cvetic, Lu, Pope

4 parameters $r_+ \, , \, a \, , \, b \, , \, q$

$\updownarrow$

4 independent charges $E \, , J_1 \, , J_2 \, , Q$

$\updownarrow$

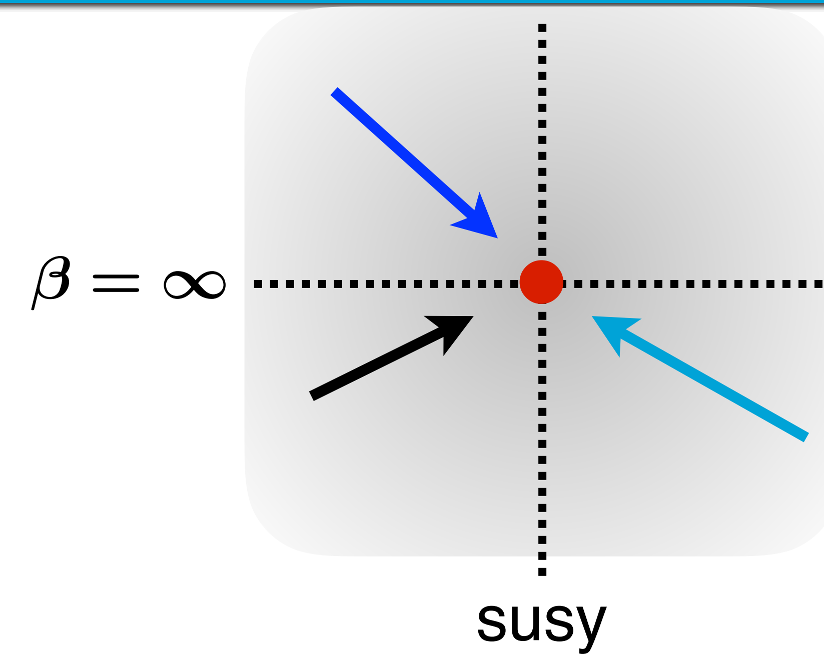
4 independent chemical pot. $\beta \, , \Omega_1 \, , \Omega_2 \, , \Phi$

$$I(\beta, \Omega, \Phi) = \beta E - S - \beta \Omega J - \beta \Phi Q \quad ✔$$

We want to take susy & extremal limit $\beta \to \infty$

The BPS limit

- many possible limits towards susy & extremal BH



- $\beta \to \infty$ implies extremality but not susy

- supersymmetry is :

$$q = -ab + (1 + a + b)\, r_+^2 \pm \underbrace{\sqrt{-r_+^2 (r_+^2 - r_*^2)^2}}$$

reality requires $r_+ = r_*$

$r_* = a + b + ab$

susy & extremal horizon radius

➜ tune two parameters

➜ in the Lorentzian causally meaningful solution, susy implies extremality.

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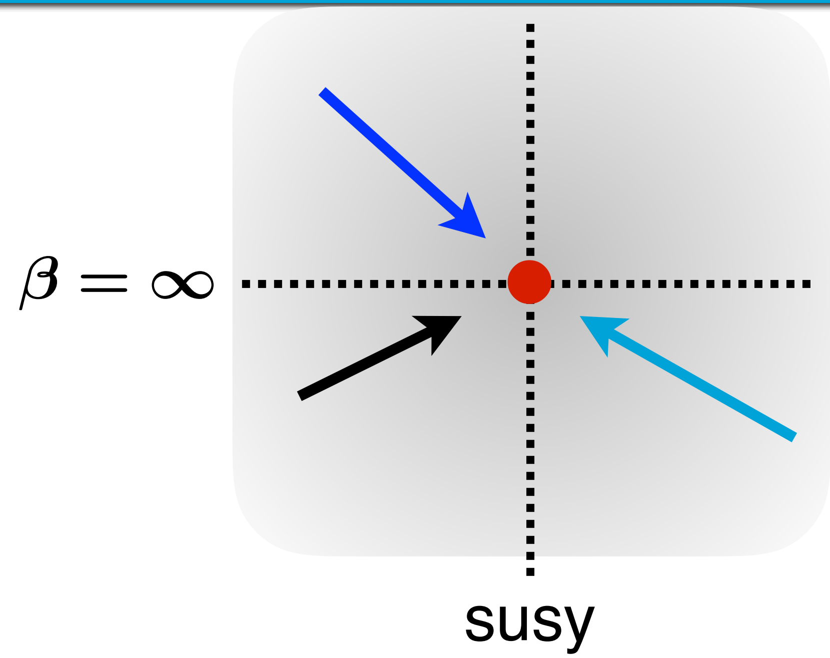

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susy & extremal horizon radius

➜ tune two parameters

➜ in the Lorentzian causally meaningful solution, susy implies extremality.

- we'd like to impose susy and only later $\beta \to \infty$. Makes sense?

The quantum statistical relation holds in a Euclidean section of the solution.

More generally, in the complexified solution.

➜ allow q to be complex ➜ 3-param family of complexified, susy solutions at finite β

BPS limit of BH thermodynamics

$a\,,\; b\,,\; r_+$

$\updownarrow$

$J_1\,,\, J_2\,,\, Q$ $\qquad$ $E = J_1 + J_2 + \frac{3}{2}Q$

follows from superalgebra

$$\{\mathcal{Q}, \overline{\mathcal{Q}}\} = E - J_1 - J_2 - \tfrac{3}{2}Q$$

$\updownarrow$

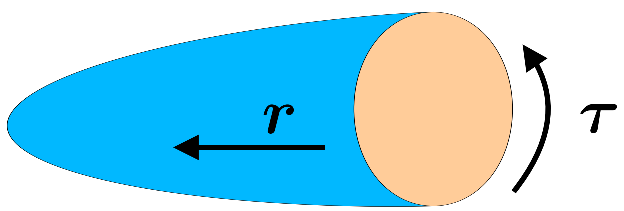
$\beta\,,\, \Omega_1\,,\, \Omega_2\,,\, \Phi$ $\qquad$ $\beta\,(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i$

constraint on chemical potentials

- chemical potentials are complex!

- physical meaning?

  regularity condition ensuring the Killing spinor is antiperiodic along the shrinking thermal circle



crucial that we have not taken $\beta \to \infty$ yet

BPS limit of BH thermodynamics

Define difference between the chemical potentials and their BPS values

$$\omega_1 = \beta(\Omega_1 - 1)\ , \qquad \omega_2 = \beta(\Omega_2 - 1)\ , \qquad \varphi = \beta(\Phi - \tfrac{3}{2})$$ Silva

These are conjugate to J_1 , J_2 , Q if one takes time translations

to be generated by the susy Hamiltonian $\{\mathcal{Q}, \overline{\mathcal{Q}}\} = E - J_1 - J_2 - \frac{3}{2}Q$

(as in the index)

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Silva

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to be generated by the susy Hamiltonian $\{\mathcal{Q}, \overline{\mathcal{Q}}\} = E - J_1 - J_2 - \frac{3}{2}Q$

(as in the index)

The constraint $\beta\,(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i$ becomes:

$$\omega_1 + \omega_2 - 2\varphi = 2\pi i$$

on-shell action $I = \dfrac{16}{27}\dfrac{\varphi^3}{\omega_1\omega_2}\,c$ ➜ matches the entropy function!

- action obtained by background subtraction method (vanishes in AdS)

BPS limit of BH thermodynamics

on-shell action $I(\omega_i, \varphi) = \frac{16}{27} \frac{\varphi^3}{\omega_1 \omega_2} c$

constraint $\omega_1 + \omega_2 - 2\varphi = 2\pi i$

using $E = J_1 + J_2 + \frac{3}{2} Q$ Quantum Statistical Relation becomes :

$$I = -S - \omega_1 J_1 - \omega_2 J_2 - \varphi\, Q$$

Now take extremal limit $r_+ \to r_*$

$\beta \to \infty$ but $\omega_1 , \omega_2 , \varphi$ remain finite ➜ the limit is smooth

➜ these relations define a BPS black hole thermodynamics

BPS limit of BH thermodynamics

Start from $$I(\omega_i, \varphi) = \frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}\,c$$

Entropy $S(J_i, Q)$ as Legendre transform, subject to constraint :

$$S = -I - \omega_1 J_1 - \omega_2 J_2 - \varphi\, Q - \Lambda\,(\omega_1 + \omega_2 - 2\varphi - 2\pi i)$$

Lagrange multiplier

$$-\frac{\partial I}{\partial \omega_i} = J_i + \Lambda\;, \qquad -\frac{\partial I}{\partial \varphi} = Q - 2\Lambda$$

yields: $$S = \pi\sqrt{3Q^2 - 8c(J_1 + J_2)} = \frac{\text{Area}}{4}$$ ✔

- physical derivation of the extremization principle of Hosseini, Hristov, Zaffaroni
- constraint between the charges J_1 , J_2 , Q follows from reality of the entropy

From gravity to field theory

Now that we have gained insight on the gravity side

let’s see how the dual field theory computation is defined.

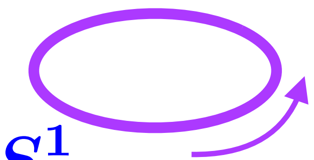
Localization computation

- in the regular Euclidean section, boundary fields are:

$$ds^2 = d\tau^2 + \underbrace{d\theta^2 + \sin^2\theta\,(d\phi_1 - i\,\Omega_1 d\tau)^2 + \cos^2\theta\,(d\phi_2 - i\,\Omega_2 d\tau)^2}_{S^3 \text{ fibered over } S^1}$$

S^3

$$A = i\,\Phi\,d\tau$$



S^1_β

complexify chemical pot, $\beta(1+\Omega_1+\Omega_2-2\Phi) = 2\pi i n\,, \quad n\in\mathbb{Z}$

black hole requires $n=\pm 1$

- for n odd, supercharge is antiperiodic

  ➜ dynamical fields are: periodic bosons, antiperiodic spinors

Localization computation

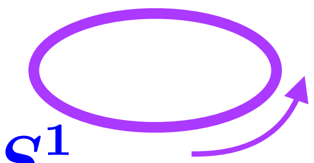
- in the regular Euclidean section, boundary fields are:

S^3

$$ds^2 = d\tau^2 + d\theta^2 + \sin^2\theta\,(d\phi_1 - i\,\Omega_1 d\tau)^2 + \cos^2\theta\,(d\phi_2 - i\,\Omega_2 d\tau)^2$$

S^3 fibered over S^1

$$A = i\,\Phi\,d\tau$$



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black hole requires $n = \pm 1$

- for n odd, supercharge is antiperiodic

➜ dynamical fields are: periodic bosons, antiperiodic spinors

- A localization computation gives the exact partition function:

$$\boxed{Z(\omega_1,\omega_2,\varphi) = \mathrm{e}^{-\mathcal{F}(\omega_1,\omega_2,\varphi)}\,\mathcal{I}(\omega_1,\omega_2,\varphi)}$$

where again $\omega_1 = \beta(\Omega_1 - 1)\,, \qquad \omega_2 = \beta(\Omega_2 - 1)\,, \qquad \varphi = \beta(\Phi - \tfrac{3}{2})$

with $\omega_1 + \omega_2 - 2\varphi = 2\pi i\,n$

The prefactor

localization gives :

$$\boxed{\mathcal{F} = -\frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}\,c}$$

at large N

$$\varphi = \tfrac{1}{2}(\omega_1 + \omega_2 - 2\pi i\, n)$$

- $n = 0 \;\rightarrow\;$ $$\mathcal{F} = -\frac{2}{27}\frac{(\omega_1+\omega_2)^3}{\omega_1\omega_2}\,c$$ susy Casimir energy

- $n = 1 \;\rightarrow\;$ $$\mathcal{F} = -\frac{2}{27}\frac{(\omega_1+\omega_2-2\pi i)^3}{\omega_1\omega_2}\,c$$ $\rightarrow$ matches *minus* the entropy function

$\rightarrow$ Legendre transform of $-\mathcal{F}$ is the Bekenstein-Hawking entropy

$$S = \pi\sqrt{3Q^2 - 8c(J_1+J_2)} = \frac{\text{Area}}{4}$$

The index

$$\boxed{\mathcal{I}(\omega_1, \omega_2, \varphi)}$$

- Translate path integral into Hamiltonian formalism

$$\mathcal{I}(\omega_1, \omega_2, \varphi) = \mathrm{Tr}\, \mathrm{e}^{\pi i(n+1)F}\, \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega_1 J_1+\omega_2 J_2+\varphi\, Q}$$

- Use : $\varphi = \frac{1}{2}(\omega_1 + \omega_2 - 2\pi i\, n)$

$$\mathrm{e}^{\pi i n F} = \mathrm{e}^{-2\pi i n J_1}$$ spin-statistics

- Obtain $\mathcal{I}(\omega_1, \omega_2, \varphi) = \mathrm{Tr}\,(-1)^F\, \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+(\omega_1-2\pi i n)(J_1+\frac{1}{2}Q)+\omega_2(J_2+\frac{1}{2}Q)}$

$$= \mathcal{I}(\omega_1 - 2\pi i\, n\,,\, \omega_2)$$

➜ superconformal index with a shifted chemical potential

Non-trivial as the shift is not an invariance of the index! Introduces extra phases

Cardy limit of the index

- limit of large charges (at finite N) : $\omega_1, \omega_2 \to 0$

$\varphi = \frac{1}{2}(\omega_1 + \omega_2 - 2\pi i\, n)$ remains finite when $n = \pm 1$

Choi, J. Kim, S. Kim, Nahmgoong; Honda; Arabi Ardehali; J. Kim, S. Kim, Song; Amariti, Garozzo, Lo Monaco large-N limit ➜ Benini-Milan

- We find a universal saddle point controlled by anomalies.
  Dominant under some assumptions
  ➜ $\mathcal{N} = 4$ SYM, non-chiral quivers with r<1, orbifold theories

$$\boxed{-\log \mathcal{I} \sim \frac{8\varphi^3}{27\omega_1\omega_2}(5a - 3c) + \frac{8\pi^2\varphi}{3\omega_1\omega_2}(a - c)}$$

generalizes Di Pietro, Komargodski

at large N : $a = c$ ➜ matches entropy function ➜ Bekenstein-Hawking entropy

holds at finite N ➜ prediction for quantum BH entropy

Summary

entropy

$$S(J_i,Q)=\pi\sqrt{3Q^2-8c(J_1+J_2)}$$

Legendre transform

entropy function

$$I(\omega_i,\varphi)=\frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}c \qquad \omega_1+\omega_2-2\varphi=2\pi i$$

BPS limit

supergravity on-shell action (at finite β)

$$I(\beta,\Omega_i,\Phi)=\beta E-S-\beta\Omega_i J_i-\beta\Phi Q$$

SCFT partition function

$$\mathcal{Z}(\omega_1,\omega_2,n)=\mathrm{e}^{-\mathcal{F}(\omega_1,\omega_2,n)}\mathcal{I}(\omega_1,\omega_2,n)$$

variant of Casimir energy

modified index

Open questions

- Why entropy function is encoded both in prefactor and in the index?

  4d Cardy formula relating the degeneracy of states to the vacuum energy?

  (at large N)

  symmetry principle?

- subleading corrections to Bekenstein-Hawking entropy

  localization in supergravity?

- other dimensions, more black hole solutions, … DC, Papini, WIP

- large N limit of the index Benini-Milan

- Sen's approach : zoom near the horizon and use AdS_2 / CFT_1

  should implement the transformation from microcanonical to grand-canonical thanks for your attention !