Solving cosmology puzzles with holographic cosmology

Horatiu Nastase
IFT-UNESP

Mainz ITP Holography workshop
May 2019

based on: HN and K. Skenderis, 1904.05821, 1905....?, HN 1812.07597

and H. Bernardo and HN, 1812.07586

Summary:

- 0. **Introduction**
- 1. **Holographic cosmology**
- 2. **Hot Big Bang puzzles and their solutions in inflation**
- 3. **Solutions of puzzles in holographic cosmology**
- 4. **Relic and monopole problem and toy model**
- 5. **Cosmological constant problem explained in holographic cosmology**
- 6. **Possible top-down model: "dimensional reduction" of $\mathcal{N}=4$ SYM vs.** $AdS_5 \times S^5$
- 7. **Conclusions.**

0. Introduction

•Inflation is considered almost a "Standard Model" of cosmology, since it agrees with data and solves a set of classic "puzzles" of Hot Big Bang cosmology

•But there is an extension of it into the strong gravity domain, where it can be dealt with holographically (in the AdS/CFT of gauge/gravity duality): holographic cosmology

•Model by P. Mc Fadden and K. Skenderis (2009) offers a phenomenological set-up in this extended paradigm: use 2+1d theories with "generalized conformal structure" and fix parameters from CMBR data.

•Different parametrical fitting than $\Lambda-$CDM with inflation, but fit to CMBR is as good (χ^2 of 0.5 difference, 824.0 vs. 823.4)

•Could be improved by lattice calculation at intermediate coupling (in progress)

•We will show that the classic puzzles of Hot Big Bang cosmology solved by inflation are also solved in holographic cosmology

•For the monopole and relic problem, detailed calculations in a toy model needed

•The cosmological constand problem (high Λ to low Λ) is understood as a natural consequence of RG flow.

•A possible top-down origin, via a "dimensional reduction" of $\mathcal{N} = 4$ SYM vs. $AdS_5 \times S^5$, in "time".

1. Holographic cosmology (McFadden, Skenderis, 2009)

•Wick rotated cosmology (cosmology/domain wall correspondence)

$$\begin{aligned} ds^2 &= +dz^2 + a^2(z)[\delta_{ij} + h_{ij}(z,\vec{x})]dx^i dx^j \,, \\ \Phi(z,\vec{x}) &= \phi(z) + \delta\phi(z,\vec{x})a \,, \end{aligned}$$

with $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$.

•This has a (phenomenological) gravity dual; Wick rotation implies $\bar{q} = -iq$, $\bar{N} = -iN$.

•CMBR observations: power spectra of perturbations γ_{ij} and ζ,

$$\begin{aligned} \Delta_S^2(q) &\equiv \frac{q^3}{2\pi^3}\langle\zeta(q)\zeta(-q)\rangle \\ \Delta_T^2(q) &\equiv \frac{q^3}{2\pi^3}\langle\gamma_{ij}(q)\gamma_{ij}(-q)\rangle . \end{aligned}$$

•A holographic (strong gravity $\to$ perturbative field theory) calculation, either direct, or based on Maldacena's map $Z[\Phi] = \Psi[\Phi]$, extended to this case, gives

$$\Delta_S^2(q) = -\frac{q^3}{16\pi^2 \mathrm{Im} B(-iq)}$$
$$\Delta_T^2(q) = -\frac{2q^3}{\pi^2 \mathrm{Im} A(-iq)}$$

(we used $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$), where

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}$$
$$\Pi_{ijkl} = \pi_{i(k}\pi_{l)j} - \frac{1}{2}\pi_{ij}\pi_{kl}\,, \quad \pi_{ij} = \delta_{ij} - \frac{\bar{q}_i \bar{q}_j}{\bar{q}^2}$$

•Euclidean field theory is super-renormalizable $SU(N)$ gauge theory, with $A_i = A_i^a T_a$, $\phi^M = \phi^{aM} T_a$, $\psi^L = \psi^{aL} T_a$ and "generalized conformal structure" $\to$ dimensions contained in q only, and through $g_{\text{eff}}^2 = \frac{g^2 N}{q}$.

•Action is

$$
\begin{aligned}
S_{\mathrm{QFT}} &= \int d^3x \mathrm{Tr}\left[\frac{1}{2}F_{ij}F^{ij} + \delta_{M_1M_2}D_i\Phi^{M_1}\Phi^{M_2} + 2\delta_{L_1L_2}\bar{\psi}^{L_1}\gamma^i D_i\psi^{L_2}\right.\\
&\left.+\sqrt{2}g_{YM}\mu_{ML_1L_2}\Phi^M\bar{\psi}^{L_1}\psi^{L_2} + \frac{1}{6}g_{YM}^2\lambda_{M_1\ldots M_4}\Phi^{M_1}\ldots\Phi^{M_4}\right]\\
&= \frac{1}{g_{YM}^2}\int d^3x \mathrm{Tr}\left[\frac{1}{2}F_{ij}F^{ij} + \delta_{M_1M_2}D_i\Phi^{M_1}\Phi^{M_2} + 2\delta_{L_1L_2}\bar{\psi}^{L_1}\gamma^i D_i\psi^{L_2}\right.\\
&\left.+\sqrt{2}\mu_{ML_1L_2}\Phi^M\bar{\psi}^{L_1}\psi^{L_2} + \frac{1}{6}\lambda_{M_1\ldots M_4}\Phi^{M_1}\ldots\Phi^{M_4}\right]
\end{aligned}
$$

.

•Then, calculate in field theory

$$
\begin{aligned}
A(q,N) &= q^3N^2f_T(g_{\mathrm{eff}}^2)\ , \quad B(q,N) = \frac{1}{4}q^3N^2f(g_{\mathrm{eff}}^2)\\
f(g_{\mathrm{eff}}^2) &= f_0\left[1 - f_1g_{\mathrm{eff}}^2\ln g_{\mathrm{eff}}^2 + f_2g_{\mathrm{eff}}^2 + \mathcal{O}(g_{\mathrm{eff}}^4)\right]\\
f_T(g_{\mathrm{eff}}^2) &= f_{T0}\left[1 - f_{T1}g_{\mathrm{eff}}^2\ln g_{\mathrm{eff}}^2 + f_{T2}g_{\mathrm{eff}}^2 + \mathcal{O}(g_{\mathrm{eff}}^4)\right]
\end{aligned}
$$

which implies

$$
\Delta_S^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q}\ln\left|\frac{q}{\beta gq_*}\right| + \mathcal{O}\left(\frac{gq_*}{q}\right)^2}, \quad \Delta_T^2(q) = \frac{\Delta_{0T}^2}{1 + \frac{g_Tq_*}{q}\ln\left|\frac{q}{\beta_T gq_*}\right| + \mathcal{O}\left(\frac{g_Tq_*}{q}\right)^2}
$$

•Fit to data is as good as $\Lambda-$CDM with inflation, χ^2 of 824.0 vs. 823.5, and fixes parameters (N, g^2_{eff}, and simplified couplings).

•Find that g^2_{eff} is not perturbative for $l < 30 \Rightarrow$ exclude it from the fit. To put it back: need lattice calculation (in progress). (Afshordi, Coriani, Delle Rose, Gould, Skenderis, 2017)

•Another quantity needed here: global symmetry current correlators, giving

$$\langle j_i^A(q) j_k^B(-q)\rangle = N^2 q \delta^{AB} \pi_{ik} f_J(g^2_{\text{eff}})$$

where again

$$f_J(g^2_{\text{eff}}) = f_{J0}\left[1 - f_{J1} g^2_{\text{eff}} \ln g^2_{\text{eff}} + f_{J2} g^2_{\text{eff}} + \mathcal{O}(g^4_{\text{eff}})\right]$$

2. Hot Big Bang puzzles and their solutions in inflation

1. Smoothness and horizon: observed correlation size $2r_H$/ horizon distance d_H at ls, today:

.

$$N = \frac{2r_H(t_0)}{d_H(t_0)} \simeq 2(1+z_{\rm ls})^{1/2} \simeq 72$$

Inflation: expansion with $a(t) \propto t^n$, $n > 1$ or $e^{Ht} \Rightarrow$ scales expand exponentially and $d_H(t_{ls}) \propto e^{N_e}$, giving

$$\frac{d_H(t_{\rm ls})}{2r_H(t_{\rm ls})} > 1 \Rightarrow e^{N_e} > \frac{a(t_I)H_I}{a_0H_0} \gtrsim e^{56}\frac{\rho_{\rm begRD}^{1/4}}{5\times 10^{13}GeV}$$

2. Flatness problem:

$$\Omega(t) - 1 = \frac{k}{a(t)^2H(t)^2} \propto \left(\frac{t}{a(t)}\right)^2 \propto t^{2(1-p)}$$

needs $p > 1$ or $a(t) \propto e^{Ht}$ (inflation) to decr., then incr.:

$$\Omega_0 - 1 = (\Omega(t_{bi}) - 1)e^{-2N_e}\left(\frac{a(t_I)H_I}{a_0H_0}\right)^2$$

gives same condition as at 1. For $T_I = T_{\rm inflation} \sim 10^{16}GeV$,

$$(\Omega - 1)_I = (\Omega - 1)_{e^+e^-}\left(\frac{a_eH_e}{a_IH_I}\right)^2 = 10^{-16}\left(\frac{T_e}{T_I}\right)^2 \sim 10^{-54}$$

3. Relic and monopole problem

-Monopoles: direct searches: $\exists < 10^{-30}$ monopoles/nucleon $\Rightarrow$ $< 10^{-30}$ monopoles per volume dilution (at phase transition, the Kibble mechanism gives $\sim$ 1 mon./nucleon) $\Rightarrow$ need dilution by $N_e > \ln 10^{10} \simeq 23$ e-folds (for phase transition, before the end of inflation).

-Relics: Not over close the Universe $\Rightarrow < 10^{-11}$ reduction in volume since phase transition (when $\exists \simeq 1$ relic/nucleon)

4. Entropy problem: $S_H(t_{\rm BBN}) \sim 10^{63}$, but at phase transition, ~ 1/horizon. Inflation: large growth of entropy during reheating, and exponential exp. increases entropy in horizon.

5. Perturbations problem: CMBR pert. are *classical*, and were super-horizon in the past. Inflation: scales $\propto e^{Ht}$, but $H \simeq$ const. $\Rightarrow$ scales get out of horizon.

6. Baryon asymmetry problem: $(N_B - N_{\bar{B}})/N_B \sim 10^{-9}$. Its creation needs interactions out of equilibrium. Inflation $\rightarrow$ true (fast expansion) and 10^{-9}: $S_1 \sim 10^9$.

3. Solution of puzzles in holographic cosmology

1. Smoothness and horizon problem

•$\exists$ nongeometric phase, but at the end - geometrical.

•Holographic map nonlocal, even though field theory is causal and local $\rightarrow$ generates apparent nonlocality.

•More precisely, RG flow (UV to IR) dual to *inverse* time evolution: AdS geodesic, joining x and y at spatial distance $L \Rightarrow$ $L = cR^2/r_0$, where $r_0 =$ minimum radial distance in AdS. But $r \rightarrow e^{-t/R}$, so $L = cRe^{-t/R}$, so $k = \frac{H}{c}e^{Ht}$, where k is momentum scale.

•Then, constraint on N_e becomes constraint on amount of RG flow $\Rightarrow$ an amount of 10^{-54} in k^2 (or 63 e-folds) for $T_I \sim 10^{16} GeV$.

2. Flatness problem

•Again RG flow $\leftrightarrow$ inverse time evolution. We want to see then that (grav.) perturbations decrease along the inverse RG flow (from IR to UV).

•For $g^2_{\rm eff} = \frac{g^2 N}{q} \ll 1$ (late times), we find

$$f(g^2_{\rm eff}) = f_0 \left(1 - f_1 g^2_{\rm eff} \ln g^2_{\rm eff} + f_2 g^2_{\rm eff} + \mathcal{O}(g^2_{\rm eff})\right)$$

where $f_1 < 0$ (for best fit, and most of the theor. parameter space) and f_1 dominates over f_2. But since

$$f(g^2_{\rm eff}) \propto q^{2\delta} \sim 1 + 2\delta \ln q \sim 1 - 2\delta \ln g^2_{\rm eff} + ...$$

we have $2\delta \simeq f_1 g^2_{\rm eff} < 0 \Rightarrow T_{ij}$ is marginally relevant.

•CFT terminology, but only generalized conf. structure, yet same results: $\delta < 0 \Rightarrow$ dilution along inverse RG flow.

•Quantitatively, same cond.: at least 10^{-54} of RG flow in k^2 (63 e-folds) for $T_I \sim 10^{16} GeV$.

4. Entropy problem: inflation $\to$ reheating.

•Now $\to \exists$ period corresponding to reheating. But, in field theory: *obvious*: dual field theory has grav. modes + SM modes: transfer of energy from one to the other. Entropy larger in the UV (late times) than IR (initial times) $\to$ # of d.o.f. decreases along RG flow. Large entropy $\to$ large N. $S_1 \sim 10^9$ (UV) to $S_1 \sim 1$ (IR) is a constraint.

5. Perturbations problem

•Also easier: classical $\langle h_{ij}h_{kl}\rangle$ perturbations in CMBR are dual to quantum $\langle T_{ij}T_{kl}\rangle \to$ usual QFT perturbations. But now, no assumptions (like QFT in curved space and Bunch-Davies vacuum) $\to$ initial conditions: vacuum is unique perturbative QFT vacuum.

6. Baryon asymmetry problem. Same solution. But now: reactions out of thermal equilibrium: no thermal equilibrium along the RG flow. Nr. of d.o.f. changes rapidly

4. Relic and monopole problem, and toy model

•$\nexists$ geometry. But monopole defined by topology: abstractly.

•Monopole in the bulk $\rightarrow$ vortex (top. and magn. charge) on the boundary. AdS/CFT: True case: "'t Hooft monopole" $\rightarrow$ "true vortex", but approx. case: "Dirac monopole" $\rightarrow$ "Dirac vortex".

•Constraint: dilution of monopole current $\tilde{j}_i^a$ perturbations in the bulk $\rightarrow$ in inverse RG flow, of 10^{-10} in linear size. $\Rightarrow$ need $\delta(\tilde{j}_i^a) < 0$. For relics, coupling to T_{ij}, need dilution of T_{ij} pert. along the RG flow of 10^{-4} $\rightarrow$ same, and less stringent, as for flatness problem.

•But: A_μ^a (gauge) in bulk $\rightarrow j_i^a$ (global) in QFT. Moreover, magnetic $\tilde{j}_i^a$ replaced by electric j_i^a. Since QFT is phenomenological, no definite $j_i^a \rightarrow$ need toy model.

•Toy model: $SU(N)$ gauge symm., $SO(3)$ global, allowing for vortex solutions. A_μ and 6 complex scalars ϕ_i^a, $i = 1,2$ and $a = 1,2,3$ for $\underline{3}$ of $SO(3)$, all in $SU(N)$. Potential (scalar self-int.)

$$V = \lambda \mathrm{Tr}\, |\vec{\phi}_1 \times \vec{\phi}_2|^2$$

Then the Euclidean action is

$$S = \int d^3x \mathrm{Tr} \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1,2} |D_\mu \vec{\phi}_i|^2 + \lambda |\vec{\phi}_1 \times \vec{\phi}_2|^2 \right]$$

and the $SO(3)$ global currents are

$$j_\mu^a = \sum_{i=1,2} i\epsilon^{abc} \phi_i^{b,*} D_\mu \phi_j^c + h.c$$

where $D_\mu^{AB} = \partial_\mu \delta^{AB} - ig(T_C)^{AB} A_\mu^C$.

•Two loop calculation in dim. reg.: $\exists$ divergences, but removed $\to$ only p dependence in finite piece. Find (one-loop plus 2-loop):

$$\cdot \quad \langle j^a_\mu(p) j^b_\nu(-p)\rangle = N^2 \frac{p}{4}\delta^{ab}\left[\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - 4\cdot 16 \frac{g^2 N}{p} J_0 \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \text{finite}\right]$$

where $J_0 \simeq -\frac{1}{32\pi^2}\frac{1}{\epsilon}$+finite. But: generlized conf. structure $\to$

$$\langle j^a_\mu(p) j^b_\nu(-p)\rangle = \frac{N^2 p}{4}\pi_{\mu\nu}[1+c g^2_{\text{eff}} \ln g^2_{\text{eff}} + \ldots] = \frac{N^2 p}{4}\pi_{\mu\nu}[1 - c g^2_{\text{eff}} \ln p + \ldots]$$

•But definining anomalous dimension as before,

$$\langle j^a_\mu(p) j^b_\nu(-p)\rangle \propto N^2 \pi_{\mu\nu} p^{1+2\delta} \simeq N^2 p \pi_{\mu\nu}[1 + 2\delta \ln p + \ldots]$$

gives $2\delta = -c g^2_{\text{eff}}$. Finally, we obtain

$$\delta_j = \frac{2}{\pi^2} g^2_{\text{eff}} > 0$$

so j^a_i is irrelevant: grows in the UV.

•But: need *vortex* current. In Abelian-Higgs model,

$$j^{\mu}_{\text{vortex}} = \frac{1}{K}\epsilon^{\mu\nu\rho}\partial_\nu j_\rho$$

.

Then the correlators are related as

$$\langle j_\mu(p) j_\nu(-p)\rangle = f\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \Rightarrow \langle j^{\mu}_{\text{vortex}}(p) j^{\nu}_{\text{vortex}}(-p)\rangle = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{p^2}{K^2}f$$

•But, more precisely (Witten; Herzog, Kovtun, Sachdev, Son) conformal structure in 2+1d $\Rightarrow$ (t replaced by K^{ab} in the nonabelian case)

$$\langle j_i(p) j_j(-p) = \left(p^2\delta_{ij} - p_i p_j\right)\frac{t}{2\pi\sqrt{k^2}} + \epsilon_{ijk}p_k\frac{w}{2\pi}$$

•Then implies for the magnetic current

$$\langle \tilde{j}_i(p)\tilde{j}_j(-p)\rangle = \frac{p^2\delta_{ij} - p_i p_j}{2\pi\sqrt{p^2}}\frac{t}{t^2+w^2} - \frac{\epsilon_{ijk}p_k}{2\pi}\frac{w}{t^2+w^2}$$

•For $w = 0 \Rightarrow t \to 1/t$ in Abelian case and $K_{ab} \to (K^{-1})_{ab}$ in the nonabelian case.

•In both cases, S duality $\to$ Maxwell duality in bulk. Acts the same for us.

•Conf. structure or generalized conf. structure $\to$ same form of correlators.

•Then, inversion $\Rightarrow$ $1 + 2\delta \ln p \to \simeq 1 - 2\delta \ln p$, so $\delta(\tilde{j}) = -\delta(j)$. Then $\delta(\tilde{j}) < 0$ and $\tilde{j}$ is relevant, as we wanted.

•Must $\exists$ vortex. Here: Abelian Dirac vortex. $\exists U(1) \subset SO(3)$ with

$$j_\mu = i \sum_{i=1,2} \vec{\phi}_i D_\mu \vec{\phi}_i + h.c.$$

under which $\phi_1^a \to e^{i\alpha}\phi_1^a$, $\phi_2^a \to e^{i\alpha}\phi_2^a$.

•Then, $\exists$ vortex ansatz tht keeps $V = 0$,

$$\phi_1^a = \phi_1(r) f^a e^{i\alpha}\ , \ \ \phi_2^a = \phi_2(r) f^a e^{i\alpha}$$

•Sol. of eq. of m. with ansatz $\to$ vortex nr. $\to$ vortex current.

5. Cosmological constant problem explained in holographic cosmology

•Λ problem: why is Λ so small today (yet $\Lambda \sim H^2 \mathcal{M}_{\rm Pl}^2$ in inflation)? QFT + gravity = problem. About 10^{-120} problem.

•Can map it holographically to solved problem in QFT? Yes.

•Inverse RG flow will dilute Λ (for time evolution).

•In $\mathcal{N} = 4$ SYM vs. $AdS_5 \times S^5$:

$$\frac{1}{\lambda} = \frac{1}{g_{YM}^2 N} = \frac{\alpha'^2}{R^4} \sim \alpha'^2 \mathcal{R}^2 \Rightarrow \alpha' \mathcal{R} \sim \frac{1}{\sqrt{\lambda}}$$

•But since $\frac{2-d}{2}\mathcal{R} = d\Lambda + 8\pi G_N T$ (E.eqs), we get

$$\frac{\Lambda}{M_{\rm Pl}^2} \lesssim \frac{\mathcal{R}}{M_{\rm Pl}^2} \sim \frac{1}{\sqrt{\lambda}}$$

•But: late times $\leftrightarrow$ UV: $q \to \infty \Rightarrow g^2_{\rm eff} \to 0$, so we need: $\mathcal{R}/M^2_{\rm Pl} \sim (g_{\rm eff})^p$, with $p > 0$, unlike $p = -1/2$ before ($g^2_{\rm eff} = g^2 N/q = \lambda/q$. Then we would get

$$\frac{\Lambda}{M^2_{\rm Pl}} \lesssim \left(\frac{g^2 N}{q}\right)^p$$

.

which means natural flowing of Λ from IR to UV is due to dimensional RG flow. **Principle**: quantum Λ in 3+1d FLRW is related to 2+1d QFT scale: low $\Lambda \leftrightarrow$ high q.

•**Example 1**: Holographic dual of Dp-branes. $g^2_{\rm eff} = g^2_{\rm YM} N U^{p-3}$, where $U = r/\alpha' = q$. String frame solution is

$$\begin{aligned}
e^\phi &\sim \left(\frac{g^2_{YM} N}{U}\right)^{5/4} \frac{1}{N} \\
\frac{ds^2}{\alpha'} &\simeq U^2 \sqrt{\frac{U^{3-p}}{g^2_{YM} N d_p}} dx^2_{||} + \sqrt{\frac{g^2_{YM} N d_p}{U^{3-p}}} \frac{dU^2}{U^2} + \sqrt{\frac{g^2_{YM} N d_p}{U^{3-p}}} d\Omega^2_{8-p} \\
&= \frac{U^2}{R^2} dx^2_{||} + R^2 \frac{dU^2}{U^2} + R^2 d\Omega^2_{8-p} \Rightarrow \\
\alpha' \mathcal{R} &\sim \frac{1}{\sqrt{\lambda_{\rm eff}}} \sim \sqrt{\frac{U^{3-p}}{g^2_{YM} N}} \to \sqrt{\frac{U}{g^2_{YM} N}}
\end{aligned}$$

•Validity of sugra: $g^2_{\text{eff}} \ll 1$ and $N \gg 1 \Rightarrow$ for $p = 2$ (our case)

$$g^2_{YM} N^{1/5} \ll U \ll g^2_{YM} N$$

so pert. SYM is valid at large U (and sugra is not).

•String frame has problems. But in *Einstein frame*,

$$\mathcal{R}_E \sim e^{\phi/2}\mathcal{R}_S + \ldots \sim \frac{\lambda_{\text{eff}}^{1/8}}{\sqrt{N}} = \frac{1}{\sqrt{N}}\left(\frac{g^2 N}{U}\right)^{1/8} \Rightarrow \frac{\Lambda_E}{M^2_{\text{Pl}}} \lesssim \frac{\mathcal{R}_E}{M^2_{\text{Pl}}} \propto \left(\frac{g^2 N}{U}\right)^{1/8}$$

so $p = 1/8$. Moreover, after KK reduction on the sphere, we get $a(t) \propto t^7 \Rightarrow$ good FLRW cosmology.

•Caveat: sugra strongly coupled at $t \to \infty \Rightarrow$ need to transition to new FLRW phase (via "reheating").

•**Example 2**: Compactified NS5-branes (on S^3 with a twist)

•Holographic theory: Maldacena-HN solution: In UV, $g_{s,eff} = e^{\phi} \sim e^{-\rho} \to 0$, as well as

$$\alpha' \mathcal{R}_E \sim \sqrt{g_s} \ll 1 \text{ and } \alpha' \mathcal{R}_S \sim \frac{1}{NR^2(\rho)} \sim \frac{1}{N\rho} \to 0$$

Then also

$$\frac{\mathcal{R}_E}{M_{\mathsf{Pl}}^2} \sim \frac{e^{\phi/2}}{M_{\mathsf{Pl}}^2} \to 0$$

so we have

$$\frac{\mathcal{R}}{M_{\mathsf{Pl}}^2} \sim (g_{\mathsf{eff}}^2)^p \Rightarrow \frac{\Lambda}{M_{\mathsf{Pl}}^2} \lesssim \left(\frac{g^2 N}{q}\right)^p , \;\; p > 0$$

as advertised.

•Caveat: UV is not free SYM: $\exists$ KK modes on S^3.

•**Generic holographic cosmology**

•FLRW cosmology: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$.

•For $a(t) \sim e^{kt} \Rightarrow$ Ricci $R = 6k^2$. For $a(t) \sim t^n \Rightarrow$ small

$$\mathcal{R} = \frac{n(2n-1)}{t^2} \propto \frac{1}{[a(t)]^{2/n}} \propto \frac{1}{q^{2r}} \Rightarrow \frac{\Lambda}{M_{\mathrm{Pl}}^2} \lesssim \frac{C}{q^p}$$

where we assumed $t \sim q^r$, $r > 0$, since $p = 2r > 0$ for UV $\leftrightarrow$ large t.

6. Possible top-down model: "dimensional reduction" of $\mathcal{N}=4$ SYM vs. $AdS_5 \times S^5$

•Top-down model (Awad, Das, Nampuri, Narayan, Trivedi, 2008 and Brandenberger, Ferreira, Morrison, Cai, Das, Wang, 2016) modifying $\mathcal{N}=4$ SYM vs. $AdS_5 \times S^5$:

$$ds^2 = \frac{R^2}{z^2}[dz^2 + (-dT^2 + a^2(T)d\vec{x}^2)] + R^2 d\Omega_5^2$$

and $\phi = \phi(T)$. $\exists$ unique solution of e.o.m.,

$$a(T) \propto T^{1/3}\ , \quad e^{\phi(T)} = \left(\frac{T}{R}\right)^{2/\sqrt{3}}$$

•In conformal time, $ds_4^2 = a^2(t)[-dt^2 + d\vec{x}^2]$, $a \sim t^{1/2}$ and we get

$$e^{\phi(t)} = \left(\frac{t}{R}\right)^{\sqrt{3}}$$

which means an $\mathcal{N}=4$ SYM with time-dependent coupling

$$g_{YM}(t) = g_{YM,0}\left(\frac{|t|}{R}\right)^{\sqrt{3}}$$

•But, conformal transf. *on the boundary* by $a^2(t)$ can be removed by a coordinate transf. in the bulk. It is ($\rho = z^2$)

$$
\begin{aligned}
t &= t' + \frac{1}{4t'}\rho' + \frac{1}{16t'^3}\rho'^2 \Rightarrow \\
\phi'(t') &= \phi(t) = \phi\left(t' + \frac{\rho'}{4t'} + \frac{\rho'^2}{16t'^3}\right) = \sqrt{3}\,\ln\left[t' + \frac{\rho'}{4t'} + \frac{\rho'^2}{16t'^3}\right]
\end{aligned}
$$

giving near the boundary at $\rho = 0$

$$
\phi'(t') \simeq \sqrt{3}\left[\ln t' + \frac{\rho'}{4t'^2} + \frac{\rho'^2}{32t'^4}\right]
$$

which means that we have $\mathcal{N} = 4$ SYM with $g^2_{\mathrm{YM}}(t)$ AND a time-dependent VEV

$$
\langle \mathrm{Tr}\,[F^2_{\mu\nu}] \rangle \propto \frac{1}{32t'^4} \neq 0
$$

•To connect with phenomenological holographic cosmology, need a "dimensional reduction" in t. Indeed, there FLRW time $t \overset{\text{Wick}}{\leftrightarrow}$ radial $r \leftrightarrow$ energy scale.

•But: $\Psi[h_{ij}]$ evolved with $H \leftrightarrow$ RG flow of correlators from $Z[h_{ij}]$. Wick rotation: time evolution $\leftrightarrow$ radial evolution.

•Maldacena map: $\Psi[h_{ij}] = Z[h_{ij}]$ is for path integral over fields in past time, with boundary condition h_{ij} at time t.

•Now: $\exists$ radial r and time t, so we generalize:

$$\Psi[h_{ij}]_{t,r} = Z[h_{ij}]_{t,q}$$

•Boundary condition both at time t and at radial scale r. Here $r \leftrightarrow q$: energy scale of QFT. Path integral over times $\leq t$.

•Holographic map is the same $\Rightarrow$ same $\langle T_{ij} T_{kl} \rangle \Rightarrow$ same $\Delta_S(q)$ and $\Delta_T(q)$.

•Field theory: Z: path integral over time until t also. Then

$$e^{-S} = e^{-\int dt \frac{1}{g^2_{YM}(t)} \int d^3x \mathcal{L}_{\mathrm{SYM}}}$$

is dominated by low t (low $g_{\mathrm{YM}}(t)$).

•Then, "dimensional reduction" in t, and

$$\int dt \frac{1}{g^2_{YM}(t)} \sim \frac{1}{g^2_{YM,0}} \int_{t_{\mathrm{Pl}}}^{t_X} \frac{dt}{(t/R)^{\sqrt{3}}} = \frac{R}{g^2_{YM,0}} (t/R)^{1-\sqrt{3}} \Big|_{t_{\mathrm{Pl}}}^{t_X} \equiv \frac{RK}{g^2_{YM,0}} \equiv \frac{1}{g^2_{3d}}$$

•The effective 3d coupling is

$$g^2_{\mathrm{eff}} \equiv \frac{g^2_{3d} N}{\bar{q}} = \frac{g^2_{YM,0} N}{K(R\bar{q})}$$

•Obtain specific 3d QFT $\rightarrow$ but it is excluded from the best fit to CMBR.

•Perhaps need larger coupling (g^2_{eff} is not < 1) $\rightarrow$ then need lattice gauge theory to test it.

•Or maybe use another gravity dual pair and "dimensionally reduce". (here $a(t)$ is "stiff matter").

7. Conclusions

.

- Holographic cosmology fits CMBR as well as Λ−CDM plus inflation.

- The Hot Big Bang cosmology puzzles are solved, just as inflation does, with some being explained more naturally.

- The cosmological constant $\Lambda_{\text{infl.}} \to \Lambda_{\text{now}}$ is explained as inverse RG evolution in scale q by $\frac{\Lambda}{M_{\text{Pl}}^2} \lesssim \left(\frac{g^2 N}{q}\right)^p$, $p > 0$.

- Generalizing the Maldacena map $\Psi[h_{ij}] = Z[h_{ij}]$ to $\Psi[h_{ij}]_{t,r} = Z[h_{ij}]_{t,q}$ and "dimensionally reducing" in t, we can get top-down models. The simplest contradicts CMBR observations.

- Holographic cosmology is a larger paradigm that includes inflation (for perturbative gravity), and has new corners (perturbative gauge theory) that are just as good as inflation.