Spin 2 operators in holographic SCFTS in four and five dimensions

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MITP workshop

Spin 2 fluctuations

- Spin 2 fluctuations in spaces with warped AdS factors
- Holographic duals of $d = 4$ $N = 2$ SCFTs
- Universal spin 2 fluctuations in $d = 4$ $N = 2$ SCFT duals Chen,Gutperle, Uhlemann, 1903.07109
- Dual operators and supermultiplets
- Holographic duals of $d = 5$ SCFTs
- Universal spin 2 fluctuations in $d = 5$ SCFT duals Gutperle, Uhlemann, Varela 1805.11914

Spin 2 fluctuations

- KK spectrum of supergravity solutions with AdS_{d+1} factor gives short protected multiplets in dual CFT, e.g type IIB supergravity on $AdS_5 \times S^5$ Kim and van Nieuwenhuizen
- spacetimes with AdS_{d+1} and S_p factors warped over Σ

$$ds^2 = f_1(y) ds^2_{Ads_{d+1}} + f_2(y) ds^2_{S_p} + g_{ab}(y) dy^a dy^b$$

  Janus solutions, duals of Wilson lines in $N = 4$ SYM, duals of $N = 2$ SCFTs (LLM, GM), duals of 5d SCFTs, 6d SCFTs, massive IIA
- Complicated ! Linearization, gauge symmetry, diagonalization of coupled fluctuations, PDE on Σ
- For a special class of fluctuations one can solve the first three problems Csaki et al. hep- th/0001033], Bachas and Estes, arXiv:1103.2800

Spin 2 fluctuations

SYM Operator	desc	SUGRA	dim	spin	Y	$SU(4)_R$	lowest reps
$\mathcal{O}_k \sim \mathrm{tr}X^k$, $k \geq 2$	–	$h^\alpha_\alpha \quad a_{\alpha\beta\gamma\delta}$	k	$(0,0)$	0	$(0,k,0)$	**20',50,105**
$\mathcal{O}_k^{(1)} \sim \mathrm{tr}\lambda X^k$, $k \geq 1$	Q	$\psi_{(\alpha)}$	$k+\frac{3}{2}$	$(\frac{1}{2},0)$	$\frac{1}{2}$	$(1,k,0)$	**20,60,140'**
$\mathcal{O}_k^{(2)} \sim \mathrm{tr}\lambda\lambda X^k$	Q^2	$A_{\alpha\beta}$	$k+3$	$(0,0)$	1	$(2,k,0)$	$\mathbf{10}_c,\mathbf{45}_c,\mathbf{126}_c$
$\mathcal{O}_k^{(3)} \sim \mathrm{tr}\lambda\bar\lambda X^k$	$Q\bar Q$	$h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$	$k+3$	$(\frac{1}{2},\frac{1}{2})$	0	$(1,k,1)$	**15,64,175**
$\mathcal{O}_k^{(4)} \sim \mathrm{tr}F_+X^k$, $k \geq 1$	Q^2	$A_{\mu\nu}$	$k+2$	$(1,0)$	1	$(0,k,0)$	$\mathbf{6}_c,\mathbf{20}_c,\mathbf{50}_c$
$\mathcal{O}_k^{(5)} \sim \mathrm{tr}F_+\bar\lambda X^k$	$Q^2\bar Q$	ψ_μ	$k+\frac{7}{2}$	$(1,\frac{1}{2})$	$\frac{1}{2}$	$(0,k,1)$	$\mathbf{4}^*,\mathbf{20}^*,\mathbf{60}^*$
$\mathcal{O}_k^{(6)} \sim \mathrm{tr}F_+\lambda X^k$	Q^3	"λ"	$k+\frac{7}{2}$	$(\frac{1}{2},0)$	$\frac{3}{2}$	$(1,k,0)$	**4,20,60**
$\mathcal{O}_k^{(7)} \sim \mathrm{tr}\lambda\lambda\bar\lambda X^k$	$Q^2\bar Q$	$\psi_{(\alpha)}$	$k+\frac{9}{2}$	$(0,\frac{1}{2})$	$\frac{1}{2}$	$(2,k,1)$	**36,140,360**
$\mathcal{O}_k^{(8)} \sim \mathrm{tr}F_+^2X^k$	Q^4	B	$k+4$	$(0,0)$	2	$(0,k,0)$	$\mathbf{1}_c,\mathbf{6}_c,\mathbf{20'}_c$
$\mathcal{O}_k^{(9)} \sim \mathrm{tr}F_+F_-X^k$	$Q^2\bar Q^2$	$h'_{\mu\nu}$	$k+4$	$(1,1)$	0	$(0,k,0)$	**1,6,20'**
$\mathcal{O}_k^{(10)} \sim \mathrm{tr}F_+\lambda\bar\lambda X^k$	$Q^3\bar Q$	$A_{\mu\alpha}$	$k+5$	$(\frac{1}{2},\frac{1}{2})$	1	$(1,k,1)$	**15,64,175**
$\mathcal{O}_k^{(11)} \sim \mathrm{tr}F_+\bar\lambda\bar\lambda X^k$	$Q^2\bar Q^2$	$a_{\mu\nu\alpha\beta}$	$k+5$	$(1,0)$	0	$(0,k,2)$	$\mathbf{10}_c,\mathbf{45}_c,\mathbf{126}_c$
$\mathcal{O}_k^{(12)} \sim \mathrm{tr}\lambda\lambda\bar\lambda\bar\lambda X^k$	$Q^2\bar Q^2$	$h_{(\alpha\beta)}$	$k+6$	$(0,0)$	0	$(2,k,2)$	**84,300,2187**
$\mathcal{O}_k^{(13)} \sim \mathrm{tr}F_+^2\bar\lambda X^k$	$Q^4\bar Q$	"λ"	$k+\frac{11}{2}$	$(0,\frac{1}{2})$	$\frac{3}{2}$	$(0,k,1)$	$\mathbf{4}^*,\mathbf{20}^*,\mathbf{60}^*$
$\mathcal{O}_k^{(14)} \sim \mathrm{tr}F_+\lambda\bar\lambda\bar\lambda X^k$	$Q^3\bar Q^2$	$\psi_{(\alpha)}$	$k+\frac{13}{2}$	$(\frac{1}{2},0)$	$\frac{1}{2}$	$(1,k,2)$	$\mathbf{36}^*,\mathbf{140}^*,\mathbf{360}^*$
$\mathcal{O}_k^{(15)} \sim \mathrm{tr}F_+F_-\lambda X^k$	$Q^3\bar Q^2$	ψ_μ	$k+\frac{11}{2}$	$(\frac{1}{2},1)$	$\frac{1}{2}$	$(1,k,0)$	**4,20,60**
$\mathcal{O}_k^{(16)} \sim \mathrm{tr}F_+F_-^2X^k$	$Q^4\bar Q^2$	$A_{\mu\nu}$	$k+6$	$(1,0)$	1	$(0,k,0)$	$\mathbf{1}_c,\mathbf{6}_c,\mathbf{20'}_c$
$\mathcal{O}_k^{(17)} \sim \mathrm{tr}F_+F_-\lambda\bar\lambda X^k$	$Q^3\bar Q^3$	$h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$	$k+7$	$(\frac{1}{2},\frac{1}{2})$	0	$(1,k,1)$	**15,64,175**
$\mathcal{O}_k^{(18)} \sim \mathrm{tr}F_+^2\bar\lambda\bar\lambda X^k$	$Q^4\bar Q^2$	$A_{\alpha\beta}$	$k+7$	$(0,0)$	1	$(0,k,2)$	$\mathbf{10}_c,\mathbf{45}_c,\mathbf{126}_c$
$\mathcal{O}_k^{(19)} \sim \mathrm{tr}F_+^2F_-\bar\lambda X^k$	$Q^4\bar Q^3$	$\psi_{(\alpha)}$	$k+\frac{15}{2}$	$(0,\frac{1}{2})$	$\frac{1}{2}$	$(0,k,1)$	$\mathbf{4}^*,\mathbf{20}^*,\mathbf{60}^*$
$\mathcal{O}_k^{(20)} \sim \mathrm{tr}F_+^2F_-^2X^k$	$Q^4\bar Q^4$	$h^\alpha_\alpha \quad a_{\alpha\beta\gamma\delta}$	$k+8$	$(0,0)$	0	$(0,k,0)$	**1,6,20'**

Table 7: Super-Yang-Mills Operators, Supergravity Fields and $SO(2,4)\times U(1)_Y\times SU(4)_R$ Quantum Numbers. The range of k is $k\geq 0$, unless otherwise specified.

D'Hoker and Freedman, TASI lectures

Spin 2 fluctuations

- KK spectrum of supergravity solutions with AdS_{d+1} factor gives short protected multiplets in dual CFT, e.g type IIB supergravity on $AdS_5 \times S^5$ Kim and van Nieuwenhuizen
- spacetimes with AdS_{d+1} and S_p factors warped over Σ

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Spin 2 fluctuations

- metric fluctuation in AdS_{d+1}

$$ds^2 = f_1 \left(ds^2_{AdS_{d+1}} + h_{\mu\nu} dx^\mu dx^\nu \right) + \hat{g}_{ab} dz^a dz^b$$

- transverse, symmetric, traceless tensor with mass M in AdS_{d+1}

$$h_{\mu\nu}(x,z) = h^{[tt]}_{\mu\nu}(x)\psi(z) \;, \qquad \Box^{(2)}_{AdS_{d+1}} h^{[tt]}_{\mu\nu} = (M^2 - 2) h^{[tt]}_{\mu\nu} \;.$$

- Linearized Einstein equations, decouple and reduce to scalar Laplace equation

$$\frac{1}{\sqrt{-g}} \partial_M \sqrt{-g} g^{MN} \partial_N h_{\mu\nu} = 0$$

- Uses: spin 2 excitations on defects, massive gravity, massive IIA warped compactifications Richard et. al. 1410.4669, Passias and Tomasiello 1604.04286, Pang et. al. 1711.07781, Passias and Richmond 1804.09728

Holographic duals of $d = 4$, $N = 2$ SCFTs

- M-theory: $AdS_5 \times S_1^\beta \times S^2$ warped over 3 dim space x_1, x_2, y Lin, Lunin, Maldacena, hep-th/0409174; Gaiotto and Maldacena 0904.4466

$$ds^2 = 4f_1 ds^2_{AdS_5} + f_2 ds^2_{S^2} + f_4 dy^2 + f_6 \left(dx_i dx^i + \frac{f_3}{f_6} \left(d\beta + A_i dx^i \right)^2 \right)$$

- metric factors

$$f_1 = e^{2\tilde{\lambda}} , \qquad f_2 = y^2 e^{-4\tilde{\lambda}} , \qquad f_3 = 4e^{2\tilde{\lambda}}(1 - y^2 e^{-6\tilde{\lambda}}) ,$$

$$f_4 = \frac{e^{-4\tilde{\lambda}}}{1 - y^2 e^{-6\tilde{\lambda}}} , \qquad f_6 = f_4 e^D , \qquad A_i = \frac{1}{2}\epsilon_{ij}\partial_j D$$

- $\tilde{\lambda}$ is expressed in terms of D

$$e^{-6\tilde{\lambda}} = -\frac{\partial_y D}{y(1 - y\partial_y D)}$$

Holographic duals of $d = 4$ a $N = 2$ SCFTs

- supergravity solution preserving 16 Susys if D satisfies 3dim Toda equation

$$(\partial_1^2 + \partial_2^2)D + \partial_y^2 e^D = 0$$

- Boundary condition $Vol(S^2) \rightarrow 0$ smoothly at $y = 0$ if

$$\partial_y D|_{y=0} = 0 \ , \qquad\qquad D|_{y=0} = \text{finite}$$

- Four cycle $S^2 \times S^1$, S^1 closes off at $y = y_c$

$$e^D|_{y \sim y_c} \sim y - y_c$$

- $M5$ brane sources can be includes as source terms in the Toda equation

$$(\partial_1^2 + \partial_2^2)D + \partial_y^2 e^D = -2\pi \sum_i \delta^{(2)}(x - x^{(i)})\theta(2N_5^{(i)} - y)$$

Holographic duals of $d = 4$ a $N = 2$ SCFTs

- Simple example: M5 brane wrapped on $g > 1$ Riemann surface (no punctures) Maldacena and Nunez, hep-th/0007018

$$e^D = \frac{1}{x_2^2}\left(\frac{1}{4} - y^2\right)$$

x_1, x_2 parameterize H_2, quotient by Γ produces Σ_g. $y \in [0, \frac{1}{2}]$.

- Toda equation is nonlinear PDE, difficult to find solutions systematically. Petropoulos et al. 1308.6583.
- Isometry along x_1 we can reduce M-theory to type IIA by change of variables

$$e^D = \sigma^2 \,, \qquad y = \sigma \partial_\sigma V \,, \qquad x_2 = \partial_\eta V$$

Holographic duals of $d = 4$ a $N = 2$ SCFTs

- metric and dilaton

$$ds^2 = 4f_1 ds^2_{AdS_5} + f_2(d\sigma^2 + d\eta^2) + f_3 ds^2_{S^2} + f_4 d\beta^2 \ , \qquad e^{2\phi} = f_8$$

- Solution is determined by a function $V(\sigma, \eta)$ satisfying the cylindrical Laplace equation

$$\ddot{V} + \sigma^2 V'' = 0 \ , \qquad \dot{V} \equiv \sigma \partial_\sigma V \ , \qquad V' \equiv \partial_\eta V$$

- metric functions

$$f_1 = \left(\frac{2\dot{V} - \ddot{V}}{V''}\right)^{\frac{1}{2}} , \qquad f_3 = f_1 \frac{2V''\dot{V}}{\tilde{\Delta}} , \qquad f_8 = \left(\frac{4(2\dot{V} - \ddot{V})^3}{V''\dot{V}^2\tilde{\Delta}^2}\right)^{\frac{1}{2}} ,$$

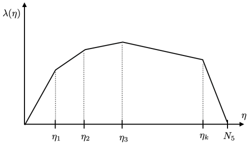
$$f_2 = f_1 \frac{2V''}{\dot{V}} , \qquad f_4 = f_1 \frac{4V''}{2\dot{V} - \ddot{V}}\sigma^2 , \qquad \tilde{\Delta} = (2\dot{V} - \ddot{V})V'' + (\dot{V}')^2$$

Holographic duals of $d = 4$ a $N = 2$ SCFTs

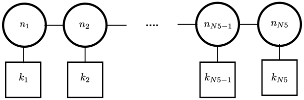
- Boundary condition on V ion $\sigma \in [0, \infty], \eta \in [0, \eta_c]$.

$$\dot{V}|_{\eta=0} = \dot{V}|_{\eta=\eta_c} = 0, \qquad \dot{V}|_{\rho} = \lambda(\eta)$$

- $\lambda(\eta)$ is a line charge density, determining the fluxes



- Generic field theory long linear quiver with bi-fundamental flavors (NS5 branes) and fundamental flavors (D6) branes Aharony et al. 1206.5916, many others.



Universal Spin 2 supergravity modes

- metric fluctuation is AdS_5 in the $N=2$ M-theory background

$$ds^2 = 4f_1\left(ds^2_{AdS_5} + h_{\mu\nu}dx^\mu dx^\nu\right) + \hat{g}_{ab}dz^a dz^b$$

- symmetric traceless tensor of mass M

$$h_{\mu\nu}(x,z) = h^{[tt]}_{\mu\nu}(x)\psi(z)\ , \qquad \Box^{(2)}_{AdS_5} h^{[tt]}_{\mu\nu} = (M^2-2)h^{[tt]}_{\mu\nu}$$

- equation of motion for $h_{\mu\nu}$

$$\frac{1}{\sqrt{-g}}\partial_M\sqrt{-g}\,g^{MN}\partial_N h_{\mu\nu} = 0$$

- equation for ψ

$$\frac{4}{f_1^{3/2}\sqrt{\hat{g}}}\partial_a\left[f_1^{5/2}\sqrt{\hat{g}}\,\hat{g}^{ab}\partial_b\right]\psi = -M^2\psi$$

Universal Spin 2 supergravity modes

- the equation of motion for $h_{\mu\nu}$

$$\left[\frac{4e^{6\tilde{\lambda}}}{y^2}\nabla^2_{S^2} - \frac{4}{y\partial_y e^D}\partial_y y^2 e^D \partial_y - \frac{4y}{\partial_y e^D}\partial_m g_3^{mn}\partial_n + M^2\right]\psi = 0$$

where

$$\partial_m g_3^{mn}\partial_n = \partial^2_{x_1} + \partial^2_{x_2} - 2\left(A_1\partial_1 + A_2\partial_2\right)\partial_\beta + \left(A_1^2 + A_2^2 + \frac{f_6}{f_3}\right)\partial^2_\beta\ . \qquad (1)$$

- Expand in spherical harmonics on S^2 and KK modes on S^1

$$\psi = \sum_{\ell mn}\phi_{\ell mn}(y, x_1, x_2)Y_{\ell m}e^{in\beta}\ , \qquad \nabla^2_{S^2}Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}$$

quantum number l related to $SU(2)_R$ charge and n to $U(1)_R$ charge in $N=2$ SCFT.

Universal Spin 2 supergravity modes

- resulting PDE looks pretty hopeless

$$\Big[- \frac{4y}{\partial_y e^D} \left(\frac{1}{y^2} \partial_y y^2 e^D \partial_y + \partial_1^2 + \partial_2^2 - 2in\,(A_1\partial_1 + A_2\partial_2) \right) + \frac{4n^2 y}{\partial_y e^D} \left(A_1^2 + A_2^2 \right) + n^2 (y \partial_y D) + \frac{4\ell(\ell+1)}{y \partial_y D} + M^2 - 4\ell(\ell+1) - n^2 \Big] \phi_{\ell mn} = 0$$

- The following ansatz solves the PDE

$$\phi^{\mathrm{a}}_{\ell mn} = y^{\ell} e^{\frac{n}{2} D}, \qquad M^2 = -4 + (2 + 2\ell + n)^2 \ . \tag{2}$$

  if D satisfies the Toda equation.
- Universal solutions present in any holographic dual of $d = 4, N = 2$ SCFTs
- There are other solutions, but they are all not regular and non-normalizable.

Dual operator and supermultiplets

- $4d$ $N = 2$ superconformal algebra $SU(2,2|2)$.
- $SU(2)_R \times U(1)_R$ symmetry quantum numbers R, r. Spins $j, \bar{j}$.
- operator in superconformal multiplet

$$[j, \bar{j}]_{\Delta}^{R;r}$$

- Poincare supersymmetries

$$Q : [1,0]_{\frac{1}{2}}^{1;-1}, \qquad \bar{Q} : \ [0,1]_{\frac{1}{2}}^{1;+1}$$

- Identify KK quantum numbers with R-charges

$$R = 2l, \quad r = 2n$$

- Mass $M^2 = -4 + (2 + 2\ell + n)^2$ gives dimension of massive spin 2 operator

$$[2,2]_{\Delta=4+2l+n}^{2l;2n}$$

Dual operator and supermultiplets

- Supermultiplets are left-right combinations of 4 types L, A_1, A_2, B_1.

	$\bar{L}$	$\bar{A}_1$	$\bar{A}_2$	$\bar{B}_1$
L	$[j;\bar{j}]_\Delta^{(R;r)}$ $\Delta > 2+R+\max\{j-\frac{1}{2}r, \bar{j}+\frac{1}{2}r\}$	$[j;\bar{j}\geq 1]_\Delta^{(R;r>j-\bar{j})}$ $\Delta = 2+R+\bar{j}+\frac{1}{2}r$	$[j;\bar{j}=0]_\Delta^{(R;r>j)}$ $\Delta = 2+R+\frac{1}{2}r$	$[j;\bar{j}=0]_\Delta^{(R;r>j+2)}$ $\Delta = R+\frac{1}{2}r$
A_1	$[j\geq 1;\bar{j}]_\Delta^{(R;r<j-\bar{j})}$ $\Delta = 2+R+j-\frac{1}{2}r$	$[j\geq 1;\bar{j}\geq 1]_\Delta^{(R;r=j-\bar{j})}$ $\Delta = 2+R+\frac{1}{2}(j+\bar{j})$	$[j\geq 1;\bar{j}=0]_\Delta^{(R;r=j)}$ $\Delta = 2+R+\frac{1}{2}j$	$[j\geq 1;\bar{j}=0]_\Delta^{(R;r=j+2)}$ $\Delta = 1+R+\frac{1}{2}j$
A_2	$[j=0;\bar{j}]_\Delta^{(R;r<-\bar{j})}$ $\Delta = 2+R-\frac{1}{2}r$	$[j=0;\bar{j}\geq 1]_\Delta^{(R;r=-\bar{j})}$ $\Delta = 2+R+\frac{1}{2}\bar{j}$	$[j=0;\bar{j}=0]_\Delta^{(R;r=0)}$ $\Delta = 2+R$	$[j=0;\bar{j}=0]_\Delta^{(R;r=2)}$ $\Delta = 1+R$
B_1	$[j=0;\bar{j}]_\Delta^{(R;r<-(\bar{j}+2))}$ $\Delta = R-\frac{1}{2}r$	$[j=0;\bar{j}\geq 1]_\Delta^{(R;r=-(\bar{j}+2))}$ $\Delta = 1+R+\frac{1}{2}\bar{j}$	$[j=0;\bar{j}=0]_\Delta^{(R;r=-2)}$ $\Delta = 1+R$	$[j=0;\bar{j}=0]_\Delta^{(R;r=0)}$ $\Delta = R$

Table 15: Consistent two-sided multiplets in four-dimensional $\mathcal{N}=2$ theories.

Cordova, Dumitrescu and Intriligator, 1612.00809

- for $l=0, n=0$ this is the stress tensor which sits in the multiplet $A_2\bar{A}_2$ with primary

$$[0,0]^{0;0}_{\Delta=2}$$

Dual operator and supermultiplets

- Match conformal dimension Δ, spins $(j\bar{j})$ and R charges R, r
- Multiplets from KK supergravity fluctuations are shortened and protected
- B_1 mutliplet only has $j \leq 1$
- A_1 multiplets contain spins $j > 2$. KK Supergravity has no such states
- the $n = 0, l > 0$ spin 2 operator is $Q^2\bar{Q}^2$ descendants $A_2\bar{A}_2$ of primary

$$A_2\bar{A}_2 : \qquad [0,0]^{2\ell,0}_{\Delta=2+2\ell}$$

- For $n > 0, l > 0$ spin 2 operator $Q^2\bar{Q}^2$ descendants in multiplets $A_2\bar{L}$ and $L\bar{A}_2$

$$A_2\bar{L} : \quad [0,0]^{2\ell,-2n}_{2+2\ell+n} , \qquad\qquad L\bar{A}_2 : \qquad [0,0]^{2\ell,2n}_{2+2\ell+n}$$

Summary and open questions

Summary:

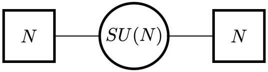
- Found massive spin 2 excitation in holographic $N = 2$ duals, with $SU(2)_R \times U(1)_R$ charges
- Universal, this excitation is present in all holographic duals and does not depend on details of the theory.
- PDE satisfied due Toda equation for function D
- excitation falls into shortened multiplets $A_2\bar{A}_2$ and $A_2\bar{L} + L\bar{A}_2$

Open questions:

- Can we identify this operator on weakly coupled gauge theory side ?
- Can we find the whole multiplet in KK supergravity ?
- Are there other solutions (nonuniversal) ?

Dual operator and supermultiplets

- Simpler field theory (not a long quiver) $N_f = 2N_C, SU(N_C)$



- mesonic operator from fundamental hypers

$$\mathcal{M}^{Ia}{}_{Jb} = \frac{1}{\sqrt{2}} \sum_{i=1}^{N_f} q^{a}_{Ji} \bar{q}^{Ii}_{b}$$

- $SU(2)_R$ singlet and a triplet,

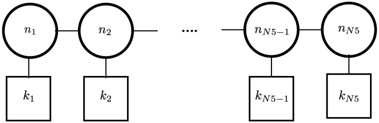
$$\mathcal{M}_{\mathbf{1}} = \mathcal{M}^{I}_{I}\ , \qquad\qquad (\mathcal{M}_{\mathbf{3}})_{I}^{\ J} = \mathcal{M}^{J}_{I} - \frac{1}{2}\delta^{J}_{I}\mathcal{M}^{K}_{K}$$

- ϕ has R-charge -2, candidate for $[2,2]^{2l;2n}_{\Delta=4+2l+n}$

$$\mathrm{tr}\Big(T_{\mu\nu} (\mathcal{M}_3)^{\ell} \phi^{n} \Big)$$

Dual operator and supermultiplets

- For long linear quivers

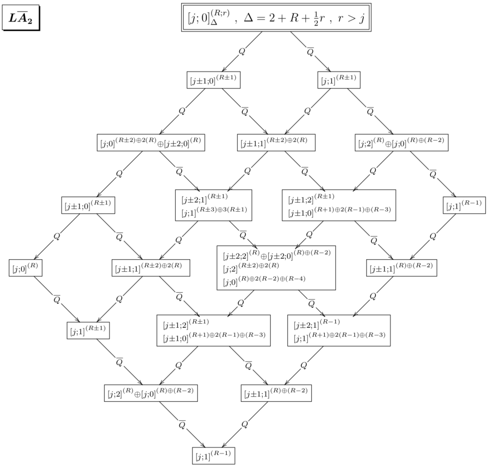


  There are many more operators which carry the R-charges (ϕ_i for gauge factors, $\mathcal{M}$ mesons for hypers)

- F and D term equations relate many of those, can we find a protected one ?
- has to work for any quiver dual to holographic SCFTs (at large N).

Supermultiplet in supergravity

Can we find the whole multiplet in supergravity ?

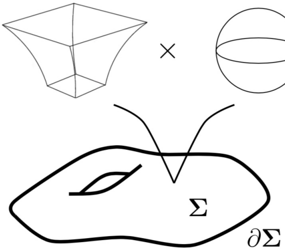


Dual operator and supermultiplets

Can we find the whole multiplet in supergravity ?

- Use supersymmetry transformation on $h_{\mu\nu}$. We can easily go up the multiplet.
- Full KK reduction is difficult, maybe not hopeless in M-theory for scalar primary since symmetry restricts the ansatz

Holographic duals of $d = 5$ SCFTs



Ansatz for type IIB supergravity solution realizes $SO(2,5) \times SU(2)_R$ as isometries of a warped product of $AdS_6 \times S^2$ over two dimensional Riemann surface Σ, with boundary ∂Σ.

$$ds^2 = f_6^2(z,\bar{z})ds^2_{AdS_6} + f_2^2(z,\bar{z})ds^2_{S^2} + \rho^2(z,\bar{z})dz \otimes d\bar{z}$$

The complex three form field strength (NS-NS and RR 2 form potential) takes the form

$$G = g_z e^z \wedge \omega_{S^2} + g_{\bar{z}} e^{\bar{z}} \wedge \omega_{S^2}$$

and the dilaton ϕ and axion χ only depend on coordinates $z, \bar{z}$ of Σ.

Local solutions

- The local solution is completely determined (up to one additional constant of integration) by two holomorphic functions $A_\pm(w)$.
- From $A_\pm(w)$. we can form two functions κ^2 and G

$$\kappa^2 = -|\partial_w A_+|^2 + |\partial_w A_-|^2, \quad G = |A_+|^2 - |A_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

  where

$$\partial_w \mathcal{B} = A_+ \partial_w A_- - A_- \partial_w A_+$$

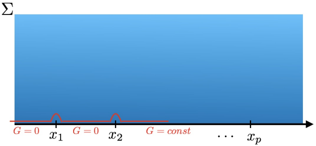
- regular solutions

$$\kappa^2 > 0, \quad G > 0, \quad \kappa^2|_{\partial\Sigma} = 0, \quad G|_{\partial\Sigma} = 0$$

Global solutions

- A large class of regular solutions can be constructed from the following ansatz: Σ is the upper half plane, $\partial_w A_\pm$ have L simple poles on the real line with complex residues.

$$A_\pm(w) = A_\pm^0 + \sum_{\ell=1}^{L} Z_\pm^\ell \ln(w - p_\ell), \quad \overline{Z_\pm^\ell} = -Z_\mp^\ell, \quad \sum_{\ell=1}^{L} Z_\pm^\ell = 0$$



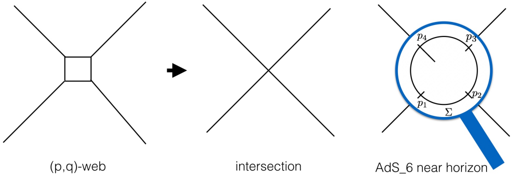
Vanishing of G on $\partial\Sigma$ implies L jump conditions (one for each pole) with $Z^{[\ell,\ell']} = Z_+^\ell Z_-^{\ell'} - Z_+^{\ell'} Z_-^\ell$

$$A^0 Z_-^k + \bar{A}^0 Z_+^k + \sum_{\ell \neq k} Z^{[\ell,k]} \ln|p_\ell - p_k| = 0, \quad k = 1, 2, \cdots L$$

- Number of moduli of our solutions: 2L-2 free real parameters, they can be chosen to be $L-1$ complex residues Z_p^+ correspond to (p, q) 5-brane charges.

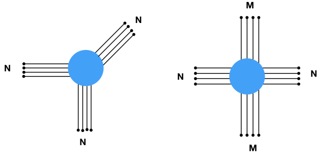
5-brane intersection

- The poles are remnants of semi-infinite fivebranes of a 5-brane intersection



- $L \geq 3$ - Minimum 3 (p,q) 5-brane intersection.
- $L - 1$ (p,q) 5-brane charges completely specify intersection and sugra solution.

Examples with low number of poles



- 3 Poles: 5-brane web of N D5-branes, N NS5-branes and N $(1,1)$ 5-branes. Field theories are 5dim versions of T_N theories. Long quivers Benini et al, 0906.0359

$$[2] \overset{y_1}{-} (2) \overset{x_1}{-} (3) - \cdots - (N-2) \overset{x_{N-3}}{-} (N-1) \overset{y_2}{-} [N]$$

- 4 Poles: 5-brane web of N D5-branes, M NS5-branes. Long quiver Aharony et al hep-th/9710116.

$$[N] \overset{y_1}{-} (N) \overset{x_1}{-} \cdots \overset{x_{M-2}}{-} (N) \overset{y_2}{-} [N]$$

Massive spin two fluctuations

- Same approach as for the $d = 4$ $N = 2$ holographic duals. Warped space time over Σ_2 fluctuation

$$ds^2 = f_6^2 \left(ds^2_{AdS_6} + h_{\mu\nu} dx^\mu dx^\nu \right) + \; f_2^2 ds^2_{S^2} + 4\rho^2 |dw|^2 \;,$$

with

$$h_{\mu\nu}(x, y) = h^{[tt]}_{\mu\nu}(x)\psi(y), \quad \Box^{(2)}_{AdS_6} h^{[tt]}_{\mu\nu} = (m^2 - 2) h^{[tt]}_{\mu\nu} \;.$$

Massive spin two fluctuations

- Equation for ψ on $S^2 \times \Sigma$ becomes

$$\frac{1}{f_6^4 f_2^2 \rho^2}\partial_a\left(f_6^6 f_2^2 \eta^{ab}\partial_b\psi\right) + \frac{f_6^2}{f_2^2}\nabla^2_{S^2}\psi + m^2\psi = 0\ ,$$

- expand ψ in spherical harmonics on S^2 $\psi(y) = \phi_\ell(w,\bar{w})Y_{\ell m}(S^2)$

$$6\partial_a\left(G^2\eta^{ab}\partial_b\phi_\ell\right) - \ell(\ell+1)\left(9\kappa^2 G + 6|\partial G|^2\right)\phi_\ell + m^2\kappa^2 G\phi_\ell = 0\ .$$

- looks horrible but there are two simple solutions simple solution

$$\phi_\ell = G^\ell, \qquad m^2 = 3\ell(3\ell+5)$$

$$\phi_\ell = G^\ell(A_+ - \bar{A}_-), \qquad m^2 = 3\ell(3\ell+6)$$

- This works because κ and G satisfy $\partial_w\partial_{\bar{w}}G = -\kappa^2$ and $A_\pm$ are holomorphic

Massive spin two fluctuations

- Using $m^2 = \Delta(\Delta - 5)$ we see that this solution is dual to a spin two operators of dimension

$$\Delta_{B_2} = 5 + 3\ell, \qquad \Delta_{A_4} = 6 + 3\ell$$

- The spin 2 operators are Q^4 descendants in a short multiplets denoted B_2 and A_4 in the notation of Cordova, Dumitrescu and Intriligator arXiv:1612.00809
- The dimension of the scalar primaries in the two multiplets are

$$\Delta_{B_2} = 3 + 3\ell \ , \qquad \Delta_{A_4} = 4 + 3\ell$$

Universally present for all IIB solutions

$F(4)$ Supermultiplets

B_2 and A_4 short mutliplets in $F(4)$ SCFT

$\boxed{\boldsymbol{B_2}}$ $\boxed{[0,0]_\Delta^{(R)}\ ,\ \Delta = \frac{3}{2}R + 3}$

$Q:$	$[1,0]^{(R\pm1)}$
$Q^2:$	$[2,0]^{(R)}\ ,\ [0,1]^{(r\pm2)\oplus(R)}\ ,\ [0,0]^{(R)\oplus(R-2)}$
$Q^3:$	$[1,1]^{(R\pm1)}\ ,\ [1,0]^{(R+1)\oplus2(R-1)\oplus(R-3)}$
$Q^4:$	$[2,0]^{(R)\oplus(R-2)}\ ,\ [0,2]^{(R)}$ $[0,1]^{(R)\oplus(R-2)}\ ,\ [0,0]^{(R)\oplus(R-2)\oplus(R-4)}$
$Q^5:$	$[1,1]^{(R-1)}\ ,\ [1,0]^{(R-1)\oplus(R-3)}$
$Q^6:$	$[0,1]^{(R-2)}$

$\boxed{\boldsymbol{A_4}}$ $\boxed{[0,0]_\Delta^{(R)}\ ,\ \Delta = \frac{3}{2}R + 4}$

$Q:$	$[1,0]^{(R\pm1)}$
$Q^2:$	$[2,0]^{(R)}\ ,\ [0,1]^{(R\pm2)\oplus(R)}\ ,\ [0,0]^{(R\pm2)\oplus(R)}$
$Q^3:$	$[1,1]^{(R\pm1)}\ ,\ [1,0]^{(R\pm3)\oplus2(R\pm1)}$
$Q^4:$	$[2,0]^{(R\pm2)\oplus(R)}\ ,\ [0,2]^{(R)}\ ,\ [0,1]^{(R\pm2)\oplus2(R)}\ ,\ [0,0]^{(R+2)\oplus2(r)\oplus(R-}$
$Q^5:$	$[1,1]^{(R\pm1)}\ ,\ [1,0]^{2(R\pm1)\oplus(R-3)}$
$Q^6:$	$[2,0]^{(R)}\ ,\ [0,1]^{(R)\oplus(R-2)}\ ,\ [0,0]^{(R)\oplus(R-2)}$
$Q^7:$	$[1,0]^{(R-1)}$