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# $\mathcal{N}=2$ AdS<sub>4</sub> IIA solutions

arXiv:1805.03661

with Daniël Prins and Alessandro Tomasiello

arXiv:1807.06031

with Ibrahima Bah and Peter Weck

# motivation

## AdS/CFT correspondence

Conformal Field Theories       $\longleftrightarrow$       Theories of Gravity  
on anti-de Sitter spacetime

## AdS/CFT correspondence

space and properties  
of CFTs



space and properties  
of AdS solutions

## AdS<sub>4</sub>/CFT<sub>3</sub> correspondence

$\mathcal{N} = 2$  supersymmetry :  
computational control + variety of theories

AdS<sub>4</sub>/CFT<sub>3</sub> correspondence

M-theory

[Gabella,Martelli,AP,Sparks '12]

AdS<sub>4</sub>/CFT<sub>3</sub> correspondence

M-theory

[Gabella,Martelli,AP,Sparks '12]

Type IIB

[AP,Solard,Tomasiello '17]

## AdS<sub>4</sub>/CFT<sub>3</sub> correspondence

### M-theory

[Gabella,Martelli,AP,Sparks '12]

Type IIB

[AP,Solard,Tomasiello '17]

Type IIA

[AP,Prins,Tomasiello '18]

## AdS<sub>4</sub>/CFT<sub>3</sub> in M-theory

M2-branes on  $\mathbb{C}^4$

AdS<sub>4</sub> × S<sup>7</sup> / ABJM

AdS<sub>4</sub>/CFT<sub>3</sub> in M-theory

M2-branes on CY<sub>4</sub>

AdS<sub>4</sub> × SE<sub>7</sub> / Chern–Simons–matter

## Old Status

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| reductions of  $\text{AdS}_4 \times \text{SE}_7$  solutions

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- | solutions with non-zero Romans mass

[Petrini, Zaffaroni '09], [Lüst, Tsimpis '09], [Tomasiello, Zaffaroni '10]

## Old Status

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[Petrini, Zaffaroni '09], [Lüst, Tsimpis '09], [Tomasiello, Zaffaroni '10]

[Guarino, Jafferis, Varela '15]

# methodology

## Background

$$ds_{10}^2 = e^{2A} \left( ds_{AdS_4}^2 + ds_{M_6}^2 \right)$$

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$$ds_{10}^2 = e^{2A} (ds_{AdS_4}^2 + ds_{M_6}^2)$$

+

$$\phi, H, F_{p_{\text{even}}}$$

preserving the symmetries of  $AdS_4$

## Supersymmetry

$$\exists \epsilon_{1,2} : \delta_{\epsilon_{1,2}} \psi = 0 = \delta_{\epsilon_{1,2}} \lambda$$

# Supersymmetry

$$\begin{aligned}\epsilon_1 &= \sum_{I=1}^2 \chi_+^I \otimes \eta_{1+}^I + \sum_{J=1}^2 \chi_-^J \otimes \eta_{1-}^J \\ \epsilon_2 &= \sum_{I=1}^2 \chi_+^I \otimes \eta_{2-}^I + \sum_{J=1}^2 \chi_-^J \otimes \eta_{2+}^J\end{aligned}$$

# Supersymmetry

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$$\nabla_\mu \chi_\pm^I = \frac{1}{2} \gamma_\mu \chi_\mp^I$$

# Supersymmetry

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# Supersymmetry

$$\eta_{1+}^I \quad \eta_{2+}^J$$

$$so(2)_R \simeq u(1)_R$$

R-symmetry

## G-structure

$SO(6)$



$G$

stabilizer

## G-structure

$M_6$  acquires a G-structure characterized by a set of tensors  
constructed as bilinears of  $\{\eta_{1+}^I, \eta_{2+}^J\}$

## G-structure

$$G = \begin{cases} \text{SU}(2) \\ \text{identity} \end{cases}$$

## SU(2)-structure

$$\{w,\, j,\, \omega\}$$

$$\iota_w j = 0 = \iota_w \omega \qquad j \wedge \omega = 0 \qquad j \wedge j = \tfrac{1}{2} \omega \wedge \overline{\omega}$$

## Supersymmetry Equations

$$\Phi_{\pm}^{IJ} \equiv \eta_{1+}^I \bar{\eta}_{2\pm}^J$$

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↓

polyforms

$$\Phi_{\pm}^{IJ}(w, j, \omega)$$

## Supersymmetry Equations

$$\delta_{\epsilon_{1,2}} \psi = 0 = \delta_{\epsilon_{1,2}} \lambda$$



constraints on  $\{\eta_{1+}^I, \eta_{2+}^J\}$



constraints on  $\Phi_{\pm}^{IJ}(w, j, \omega)$



constraints on SU(2)-structure  $\{w, j, \omega\}$

geometry

## Generic Geometry

$$ds_{M_6}^2 = \overline{w}w(y, \psi) + ds_{M_4}^2(x^i)$$

Base

## Generic Geometry

$$ds_{M_6}^2 = \frac{1}{e^{4A} - y^2} dy^2 + \frac{1}{4}(1 - e^{-4A}y^2)(d\psi + \rho)^2 + ds_{M_4}^2$$



## Generic Geometry

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|  $\partial_\psi$  generates  $U(1)_R$

|

## Generic Geometry

$$ds_{M_6}^2 = \frac{1}{e^{4A} - y^2} dy^2 + \frac{1}{4}(1 - e^{-4A}y^2)(d\psi + \rho)^2 + g_{ij}(y, x^i) dx^i dx^j$$

- |  $\partial_\psi$  generates  $U(1)_R$
- |  $g_{ij}$  conformally Kähler

family of solutions #1

# Complex M<sub>6</sub>

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$$g(y, x^i) = \begin{cases} \end{cases}$$

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$$g(y, x^i) = \begin{cases} f(y) \textcolor{red}{g_{KE_4}}(x^i) \\ f_1(y) \textcolor{red}{g_{\Sigma_1}}(x^i) + f_2(y) \textcolor{red}{g_{\Sigma_2}}(x^i) \end{cases}$$

## Complex M<sub>6</sub>

$$g(y, x^i) = \begin{cases} f(y) g_{KE_4}(x^i) \\ f_1(y) g_{\Sigma_1}(x^i) + f_2(y) g_{\Sigma_2}(x^i) \end{cases}$$

solution determined by A(y) subject to a Riccati ODE

$$\mathsf{M}_4 = \mathsf{KE}_4$$

$$ds^2_{\mathsf{M}_6}=-\frac{1}{4}\frac{\mathsf{q}'}{\mathsf{xq}}dx^2-\frac{\mathsf{q}}{\mathsf{xq}'-4\mathsf{q}}(d\psi+\rho)^2+\frac{\kappa\mathsf{q}'}{3\mathsf{q}'-\mathsf{xq}''}ds^2_{\mathsf{KE}_4}\,,$$

$$e^{2A}=\sqrt{\frac{x^2q'-4xq}{q'}}\qquad e^{2\phi}=\frac{xq'}{(3q'-xq'')^2}\left(\frac{x^2q'-4xq}{q'}\right)^{3/2}$$

$$q^{\beta,\gamma}(x)=x^6+3(2\gamma^2-\beta)x^4+8\gamma x^3+3x^2-\beta$$

$$M_4 = KE_4$$

| positivity restricts  $x$  to lie in an interval

|

$$M_4 = KE_4$$

- | positivity restricts  $x$  to lie in an interval
- | compactness: the  $\psi$ -circle shrinks at the endpoints

$$M_4 = KE_4$$

- | positivity restricts  $x$  to lie in an interval
- |  $q$  has zeros at the endpoints

$x_0$	$q(x_0)$	$q'(x_0)$	$q''(x_0)$	interpretation
0				regular

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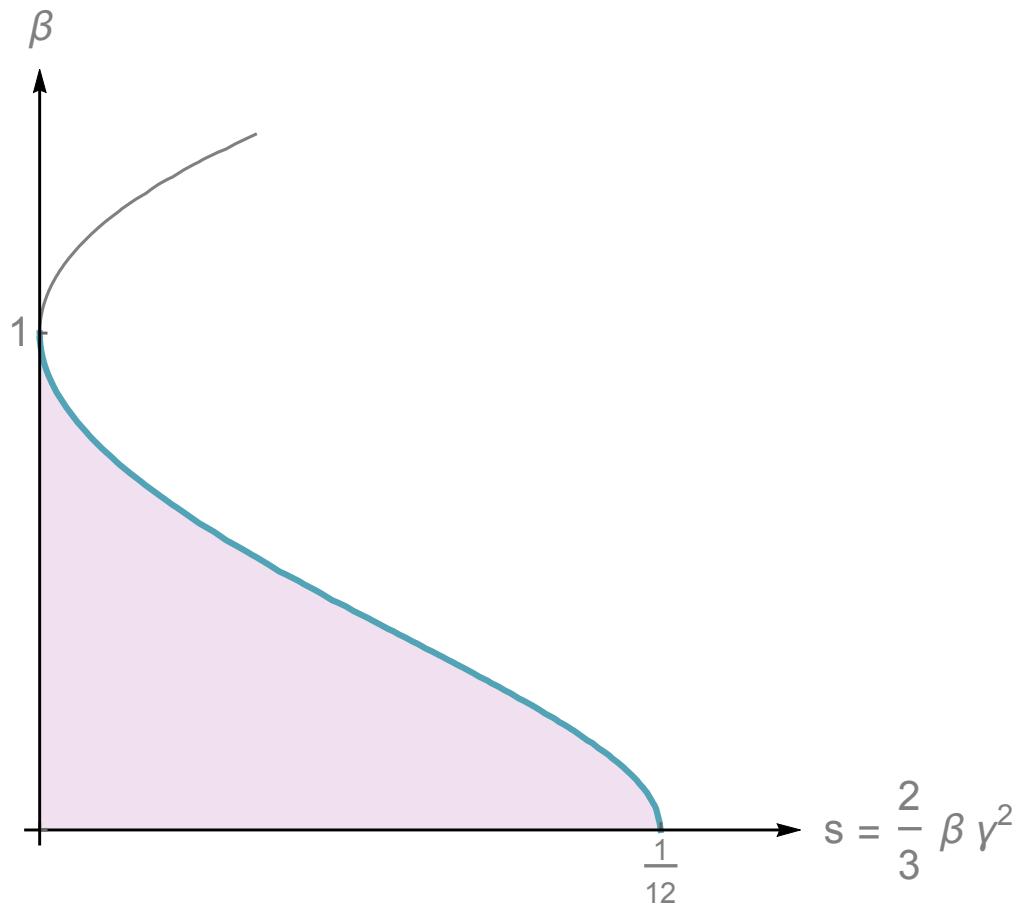
$$ds_{M_6}^2 \sim \underbrace{dr^2 + r^2(d\psi + \rho)^2}_{\mathbb{R}^2 \text{ for } \psi \in [0, 2\pi]} + ds_{KE_4}^2$$

$x_0$	$q(x_0)$	$q'(x_0)$	$q''(x_0)$	interpretation
0				regular
0	0	0		conical CY

$$ds_{M_6}^2 \sim dr^2 + r^2 \underbrace{\left( \frac{1}{9}(d\psi + \rho)^2 + \frac{1}{6}ds_{KE_4}^2 \right)}_{SE_5}$$

$x_0$	$q(x_0)$	$q'(x_0)$	$q''(x_0)$	interpretation
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$$ds_{M_6}^2 \sim dr^2 + r^2 \underbrace{\left( \frac{1}{9}(d\psi + \rho)^2 + \frac{1}{6}ds_{\mathbb{CP}^2}^2 \right)}_{S^5 \text{ for } \psi \in [0, 6\pi]}$$



$$\mathsf{M}_4 = \Sigma_1 \times \Sigma_2$$

$$ds^2_{\mathsf{M}_6}=-\frac{1}{4}\frac{\mathsf{q}'}{\mathsf{x}\mathsf{q}}dx^2-\frac{\mathsf{q}}{\mathsf{x}\mathsf{q}'-4\mathsf{q}}(d\psi+\rho)^2+\frac{\kappa_1\mathsf{q}'}{\mathsf{x} u_1}ds^2_{\Sigma_1}+\frac{\kappa_2\mathsf{q}'}{\mathsf{x} u_2}ds^2_{\Sigma_2}$$

$$e^{2A}=\sqrt{\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}}\qquad e^{2\Phi}=\frac{\mathsf{q}'}{\mathsf{x} u_1 u_2}\left(\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}\right)^{3/2}.$$

$$e^{-2A}=e^{-2\Phi}=\sqrt{\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}}\qquad e^{2\Phi}=\frac{\mathsf{q}'}{\mathsf{x} u_1 u_2}\left(\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}\right)^{3/2}.$$

$$\mathsf{q}=\mathsf{q}^{\beta,\gamma_1,\gamma_2}(x),\qquad u_1=u_1^{\gamma_1}(x),\qquad u_2=u_2^{\gamma_2}(x)$$

$$e^{-2A}=e^{-2\Phi}=\sqrt{\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}}\qquad e^{2\Phi}=\frac{\mathsf{q}'}{\mathsf{x} u_1 u_2}\left(\frac{x^2\mathsf{q}'-4\mathsf{x}\mathsf{q}}{\mathsf{q}'}\right)^{3/2}.$$

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$$\mathsf{M}_4 = \Sigma_1 \times \Sigma_2$$

$$ds^2_{M_6}=-\frac{1}{4}\frac{q'}{xq}dx^2-\frac{q}{xq'-4q}(d\psi+\rho)^2+\frac{\kappa_1 q'}{xu_1}ds^2_{\Sigma_1}+\frac{\kappa_2 q'}{xu_2}ds^2_{\Sigma_2}$$

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$$q^{\beta,\gamma_1,\gamma_2}(x)=x^6+3(2\gamma_1\gamma_2-\beta)x^4+4(\gamma_1+\gamma_2)x^3+3x^2$$

$$\mathsf{M}_4 = \Sigma_1 \times \Sigma_2$$

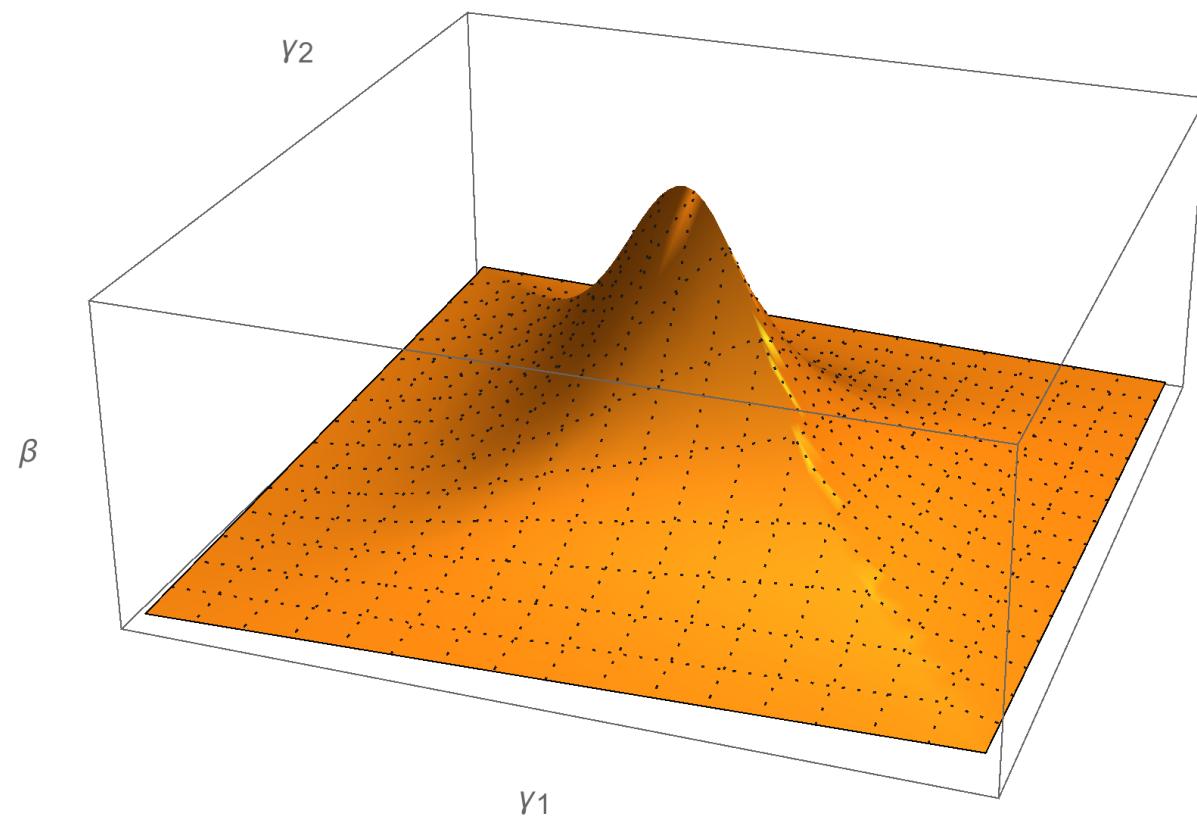
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$x_0$	$q(x_0)$	$q'(x_0)$	$q''(x_0)$	$u_1(x_0)$	$u_2(x_0)$	interpretation
0	0		0			$\mathbb{R}^4/\mathbb{Z}_2$

$$ds_{M_6}^2 \sim dr^2 + \frac{1}{4}r^2 \left( (d\psi + \rho)^2 + ds_{S^2}^2 \right) + ds_{\Sigma_2}^2$$



family of solutions #2

## Compactifying Higher Dimensional Field Theories

- | large classes of theories in lower dimensions
- | their properties admit a description in terms of the geometry and topology of the compact manifold

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

Class S SCFTs in four dimensions

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

$$\text{AdS}_5 \times_w \mathcal{M}_6$$

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

$$\begin{array}{ccc} S^4 & \rightarrow & M_6 \\ & & \downarrow \\ & & \Sigma_g \end{array}$$

## (2,0) theory on a Riemann surface

[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12]

part of a domain wall geometry which at large distances  
asymptotes to  $\text{AdS}_7 \times S^4$

# D4-D8/O8 brane configuration

[Brandhuber, Oz '99]

$\mathcal{N} = 1$  SUSY  $USp(2N)$  gauge theory  
coupled to

- | 1 hyper in the antisymmetric representation ( $H_a$ )
- |  $N_f$  hypers in the fundamental ( $H_f$ )

| vector multiplet scalar parametrizes  $x_9$  fluctuations



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- | 4  $H_a$  scalars parametrize  $x_5-x_8$  fluctuations

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- |  $H_f$  from D4-D8 open strings

## Global Symmetries

$$SU(2)_R \times SU(2) \times SO(2N_f) \times U(1)_I$$

## Global Symmetries

$SU(2)_R \times SU(2)$   
 $\chi^5 - \chi^8$  rotations

## Global Symmetries

$SU(2)_R$   
 $R$ -symmetry

# Global Symmetries

$$\begin{matrix} \text{SU}(2) \\ H_a \end{matrix}$$

# Global Symmetries

$$\begin{array}{c} \mathrm{SO}(2N_f) \\ H_f \end{array}$$

# Global Symmetries

$$\begin{matrix} U(1)_I \\ j = *Tr(F \wedge F) \end{matrix}$$

| fixed point at strong coupling [Seiberg '96]

|

- | fixed point at strong coupling [Seiberg '96]
- | degrees of freedom scale as  $N^{5/2}$  at large  $N$  [Jafferis, Pufu '12]

$$\text{AdS}_6 \times_w S^4$$

[Brandhuber, Oz '99]

$$ds_{10}^2 = \ell_s^2 \Omega \left[ ds_{\text{AdS}_6}^2 + \frac{4}{9} d\alpha^2 + \cos^2(\alpha) ds_{S^3}^2 \right], \quad \Omega = \frac{18\pi^2 N}{n_0 \sin^{2/3}(\alpha)}$$

$$e^{-4\phi} = \frac{9N n_0^3 \sin^{10/3}(\alpha)}{8\pi^2}, \quad F_4 = \frac{80}{9} \ell_s^3 \pi N \cos^3(\alpha) \sin^{1/3}(\alpha) \text{vol}_{S^3}$$

$$\text{AdS}_6 \times_w S^4$$

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$$[-\frac{\pi}{2}, \frac{\pi}{2}]$$

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$$SO(5) \rightarrow SO(4) \equiv SU(2)_R \times SU(2)$$

are there 3D SCFTs produced by  
the 5D SCFT on a Riemann surface?

look for  $\text{AdS}_4$  solutions

# SUSY on $\mathbb{R}^{1,4}$

$$d\epsilon = 0$$

# SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\Bigl(d+\tfrac{1}{4}\omega^{ab}\gamma_{ab}\Bigr)\epsilon=0$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\left( d + \underbrace{\frac{1}{4}\omega^{ab}\gamma_{ab} + A_R}_{=0} \right) \epsilon = 0$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$\mathrm{U}(1)_{\text{holonomy}} \equiv \mathrm{U}(1)_R \subset \mathrm{SU}(2)_R$$

$$J_h=J_R$$

## SUSY on $\mathbb{R}^{1,2} \times \Sigma$

$$U(1)_{\text{holonomy}} \equiv U(1)_R, \, U(1) \subset SU(2)_R, \, SU(2)$$

$$\mathbf{J}_h = \mathbf{J}_R + z \mathbf{J}$$

$\mathrm{AdS}_4 \times \mathcal{M}_6$

$$\mathrm{AdS}_4\times \mathcal{M}_6$$

$$S^4_{\mathsf{U}(1)_R \times \mathsf{U}(1)} \rightarrow \mathcal{M}_6 \qquad \qquad \downarrow \\ \Sigma_g$$

## Ansatz for $M_4$

$$ds_{M_4}^2 = \underbrace{e^{2W}(dx_1^2 + dx_2^2)}_{\Sigma_g} + e^{2Z}[(d\tau + V_1)^2 + e^{2C}(d\varphi + V_2)^2]$$

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|  $\partial_\varphi$  Killing vector generating second  $U(1)$

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|  $\partial_\varphi$  Killing vector generating second  $U(1)$

| keep  $F_0, F_4, \phi$

## General System

$$ds_{10}^2 = e^{2A}[ds_{AdS_4}^2 + \frac{1}{4}H[(e^{2W}(dx_1^2 + dx_2^2) + 4h^{ij}\eta_i\eta_j + g_{ij}t^it^j)]$$

$$i,j=\pm, \quad \quad \eta_{\pm}=d\phi_{\pm}+\tfrac{1}{2}*_2d_2\partial_{\pm}D_0, \quad \quad (\partial_{x_1}^2+\partial_{x_2}^2)D_0=e^{2W}$$

## Constant Curvature

$$ds_{10}^2 = \frac{H^{-1/2}}{\frac{3^{1/6}}{2}\mu_0^{1/3}F_0^{2/3}}(ds^2(\text{AdS}_4) + e^{2\nu}ds^2(\Sigma_g) + ds^2(\mathcal{M}_4))$$

$$H=\frac{2}{3\mu_0^2+4(1-\mu_0^2)q(\theta)}\qquad q(\theta)=a_+\cos^2\theta+a_-\sin^2\theta$$

## Constant Curvature

$$ds^2(M_4) = \frac{1}{3} \frac{1}{q(\theta)} \frac{d\mu_0^2}{1 - \mu_0^2} + \frac{(1 - \mu_0^2)}{2a_+ a_-} H ds^2(M_3)$$

$$ds^2(M_3) = q(\theta) \left( d\theta - (a_+ + a_-) \frac{\sin(2\theta)}{2q(\theta)} \frac{\mu_0 d\mu_0}{1 - \mu_0^2} \right)^2 + a_- \cos^2(\theta) \eta_+^2 + a_+ \sin^2(\theta) \eta_-^2$$

$$\eta_{\pm} = d\phi_{\pm} - 2(\kappa \pm z)V, \quad \theta \in [0, \pi/2], \quad \mu_0 \in [0, 1]$$

## Free Energy

$$\mathcal{F} = \frac{\pi L_{AdS_4}^2}{2G_4} = \frac{16\pi^3}{(2\pi\ell_s)^8}\int e^{8A - 2\phi} vol(M_6)$$

## Free Energy

$$\mathcal{F}_{g \neq 1} = \frac{8\pi 2(1-g)}{5} \frac{\textcolor{red}{N^{5/2}}}{\kappa n_0^{1/2}} F_z(z, \kappa)$$

$$F_z(z, \kappa) = \frac{(|z^2 - \kappa^2|)^{3/2} (\sqrt{\kappa^2 + 8z^2} - \kappa)}{(14z^2 - \kappa^2 + \kappa\sqrt{\kappa^2 + 8z^2})^{3/2}}$$

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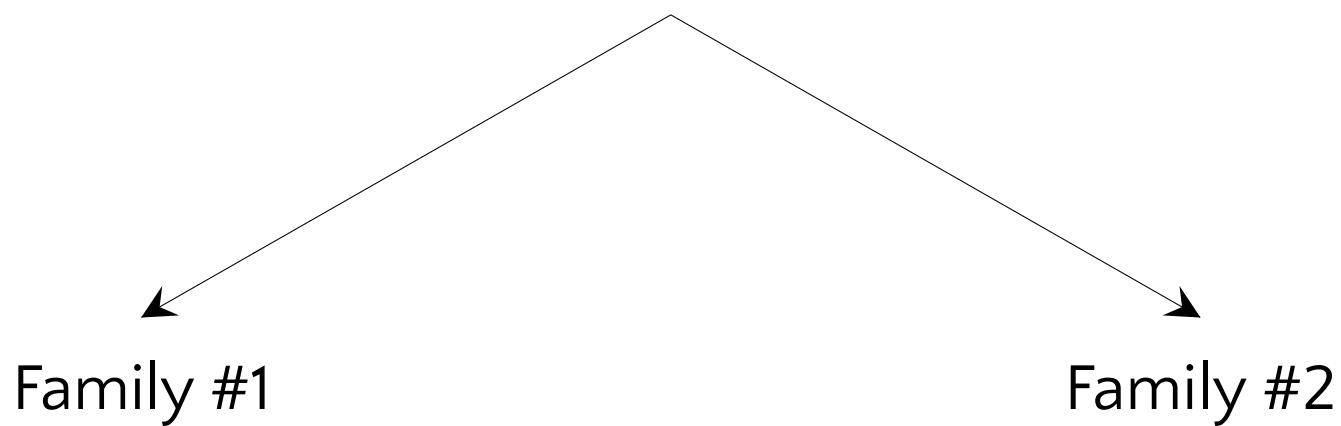
Reproduced by

[Crichigno, Jain, Willett '18] and [Hosseini, Yaakov, Zaffaroni '18]

# summary

## Summary

$\mathcal{N} = 2$   $\text{AdS}_4$  SU(2)-structure  
classification



## Summary

Family #1

$$\begin{array}{ccc} S^2 & \rightarrow & M_6 \\ & & \downarrow \\ & & M_4 \end{array}$$

Chern-Simons-matter SCFTs

## Summary

Family #2

$$\begin{array}{ccc} S^4 & \rightarrow & M_6 \\ & & \downarrow \\ & & \Sigma_g \end{array}$$

5D SCFTs on Riemann surface

The End