Black holes and (0,4)CFT’s in string and F-theory

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Recap of Black holes

and

Black Strings

Strominger-Vafa ’96

IIB on T^4 (or K3) + D1-D5

(2,2) D=6 SUGRA

AdS_3 x S^3

(4,4) CFT

Maldacena-Strominger-Witten '97

M-theory on CY_3 + M5

N=2 D=5 SUGRA

$AdS_3 \times S^2$

(0,4) CFT

Black holes in F-theory
[Hagighat, Murthy, Vafa, S.V., '15]

F-theory on CY_3 + D3

(1,0) D=6 SUGRA

$AdS_3 \times S^3$

(0,4) CFT with $SU(2)_L$

D3 wrapping Riemann Surface

D3 on curve $C = \Sigma_g$

$\Downarrow$

(0,4) & $SU(2)_L$ with $k_L = g$

5D Spinning Black Holes

Central charges [Vafa '97]

$$c_L = 6g + 12c_1(B_4).C$$

$$c_R = 6g + 6c_1(B_4).C$$

$$2g(C)-2 = C.C - c_1(B_4).C = q^a q^b \eta_{ab} - q^a c_a$$

New AdS/CFT duality (HMVV)

IIB on $AdS_3 \times S^3 \times B_4$

$$=$$

$${}_{c_L, k_L}(0,4)_{c_R = 6 k_R}$$

See also Couzens,Lawrie, Martelli, Schafer-Nameki, Wong, '17

Cardy formula for BH entropy

$$S = 2\pi\sqrt{\frac{c_L}{6}\left(n - \frac{J_L^2}{4k_L}\right)}$$

$$L_0 > \frac{J_L^2}{4k_L} \Leftrightarrow \textit{cosmic censor}$$

4D black holes from F-theory

[Grimm, het Lam, Mayer, S.V. ’18]

4D Black Holes from F-theory

- Put D3 into background

$$\mathbb{R} \times S^1 \times TN_m \times CY_3$$

- Compactify on the (asymptotic) TN circle. Break isometries to U(1)xSU(2).
- This is the 4D/5D correspondence [Gaiotto, Strominger, Yin, ’05], but now applied to F-theory

D3 and Taub-NUT

- Near horizon limit of 6D black string:

$$AdS_3 \times S^3 / \mathbb{Z}_m$$

- (0,4) CFT with only $U(1)_L$ x $SU(2)_R$.
- Central charges of the dual CFT should now also depend on *m.*

Microscopics

- Bena, Diaconescu, Florean ['06] using MSW techniques. E.g.:

$$c_L = 3mC^2 - 3m^2 c_1(B)\cdot C + m^3 c_1(B)^2$$
$$+12c_1(B)\cdot C + 12m - 2mc_1(B)^2$$

$$k_L = \frac{1}{2}mC^2 - \frac{1}{2}m^2 c_1(B)\cdot C$$

Macroscopics

Reduce 6D sugra on $AdS_3 \times S^3 / \mathbb{Z}_m$
to three dimensions.

Coeffcients in front of the 3D CS terms (gauge and gravitational) determine the levels and central charges

6D (1,0) SUGRA

$$L = \frac{1}{2}R - \frac{1}{4}g_{\alpha\beta}G^{\alpha}_{\mu\nu\rho}G^{\beta,\mu\nu\rho} - \frac{1}{2}g_{\alpha\beta}\,\partial_{\mu}\,j^{\alpha}\,\partial^{\mu}\,j^{\beta}$$

$$-\,h_{uv}\,\partial_{\mu}q^{u}\,\partial^{\mu}\,q^{v}$$

$$-\frac{1}{8}\eta_{\alpha\beta}c^{\alpha}B^{\beta}\wedge tr\,R\wedge R$$

$$g_{\alpha\beta} = 2\,j_{\alpha}\,j_{\beta} - \eta_{\alpha\beta} \quad \eta = diag(1,-1,\ldots,-1)$$

3 steps…

- 1) Reduce 2-derivative action to 3D
- 2) Reduce GS-term to 3D
- 3) (chiral) KK states at one-loop contribute to 3D CS terms

Step 1

- Reduce (1,0) sugra on S^3/Z_m to 3D. Produces U(1)xSU(2) gauge fields in 3D:

$$S_{CS} = \frac{k_L}{8\pi}\int A_L \wedge F_L + \frac{k_R}{4\pi}\int tr\left(A_R \wedge F_R + \frac{2}{3} A_R^2 \right)$$

$$k_L = k_R = \frac{1}{2} m \eta_{\alpha\beta} Q^\alpha Q^\beta \quad c_L = c_R = 6k_R$$

Step 2

- Reduce GS term to 3D. The spin connection produces a gravitational 3D CS term:

$$L = \frac{1}{16\pi}\eta_{\alpha\beta}c^{\alpha}Q^{\beta}\left[\omega^{CS}_{grav} + 4\omega^{CS}(A_R)\right]$$

$$(c_L - c_R)^{4-der} = 6\eta_{\alpha\beta}c^{\alpha}Q^{\beta} \quad k_R^{4-der} = \eta_{\alpha\beta}c^{\alpha}Q^{\beta}$$

Step 3

- Integrating out massive KK modes of chiral fields contribute to 3D CS terms:

- (A)SD Tensors: 3D massive KK *chiral* vectors [Townsend, Pilch, van Nieuwenhuizen, ’84]

- Hyperini and tensorini: 3D massive spin ½
- Gravitino: 3D massive spin 3/2

Calculation related to parity anomaly and APS index theorems [Alvarez-Gaume, Della Pietra, Moore, ’85, …]. Zeta function regularization.

Result Step 3

$$k_L^{loop} = -\frac{m^3}{8} c_1(B)^2$$

$$k_R^{loop} = \frac{m^3}{24} c_1(B)^2 + \frac{m}{3} c_1(B)^2 + m$$

$$(c_L - c_R)^{loop} = 6m + 2m\, c_1(B)^2$$

F-inal Result

- Adding up everything, the macroscopics matches the microscopics, leading and subleading terms, upon making the identification

$$Q^{\alpha} = q^{\alpha} - \frac{m}{2} c^{\alpha}$$

This can be derived separately from GS term.

Other F- theory backgrounds…

F-theory on

$$\mathbb{R} \times S^1 \times HK_4 \times CY_3$$

Equivalently IIB on

$$\mathbb{R} \times S^1 \times HK_4 \times B_4$$

with HK_4 =ALE or ALF spaces

New IIB set up

- IIB on

$$\mathbb{R} \times S^1 \times HK_4 \times K3$$

- ALE: $\mathbb{R}^+ \times S^3 / \Gamma \longleftrightarrow \mathbb{R}^4 / \Gamma$

- ALF: $\mathbb{R}^+ \times S^3 / \Gamma \longleftrightarrow (\mathbb{R}^3 \times S^1)/\Gamma$

$$\Gamma \subset SU(2) \;\; e.g.\, \mathbb{Z}_m$$

[Couzens, het Lam, Mayer, S.V.,'19]

ADE

$\Gamma \subset \mathrm{SU}(2)$	$\lvert\Gamma\rvert$	singularity type
cyclic group $\mathbb{Z}_m$	m	A_{m-1}
binary dihedral $\mathbb{D}_m^*$	$4m$	D_{m+2}
binary tetrahedral $\mathbb{T}^*$	24	E_6
binary octahedral $\mathbb{O}^*$	48	E_7
binary icosahedral $\mathbb{I}^*$	120	E_8

Table 2.1: The freely acting discrete subgroups of SU(2)

Macroscopics

- Near horizon geometry of black string is

$$AdS_3 \times S^3 / \Gamma$$

- Consider ALE for concreteness. For A-series

($\Gamma \subset SU(2)\ \ e.g.\, \mathbb{Z}_m$), there is still a $U(1)_L$ x $SU(2)_R$ isometry, and for D-E, only $SU(2)_R$. Compactify to 3D as before to read of levels and central charges. [No HD-terms and no-loops!]

Macroscopics (ALE)

$$k_L = \frac{1}{2} |\Gamma| C \cdot C \quad (for\ \Gamma = \mathbb{Z}_m)$$

$$c_L = c_R = 6k_R = \frac{1}{2} |\Gamma| C \cdot C$$

Microscopics (ALE)

D3 brane wraps curve C inside K3. The transverse space contains the ALE space, and so the D3 brane probes the ADE singularity. The 4D gauge theories are known to be N=2 quiver gauge theories with $n_V = n_H = |\Gamma|$.

Γ	Gauge multiplets	
$\mathbb{Z}_m$	$U(1)^m$	
$\mathbb{D}^*_m$	$U(1)^4 \times U(2)^{m-1}$	
$\mathbb{T}^*$	$U(1)^3 \times U(2)^3 \times U(3)$	
$\mathbb{O}^*$	$U(1)^2 \times U(2)^3 \times U(3)^2 \times U(4)$	2
$\mathbb{I}^*$	$U(1) \times U(2)^2 \times U(3)^2 \times U(4)^2 \times U(5) \times U(6)$	

Table 4.1: $\mathcal{N} = 2$ multiplets for the different quiv

Microscopics (ALE)

- Compactify 4D theory on curve C and topologically twist to get spectrum of IR (0,4) 2D CFT. [Benini & Bobev ’13,….]
- Compute the levels and central charges from ’t Hooft anomaly matching conditions.

$$c_L - c_R = Tr(\gamma_3) = 0$$

$$k_R = Tr[\gamma_3 Q_R^2] = |\Gamma|(g-1) = \frac{1}{2}|\Gamma| C \cdot C$$

Microscopics (ALE)

- For the A-series, there is an additional $U(1)_L$, which produces a left-moving current with level

$$k_L = -Tr[\gamma_3 Q_L^2] = |\Gamma|(g-1) = \frac{1}{2}|\Gamma| C \cdot C$$

- All of this matches with the macroscopics, so there is perfect agreement!

Conclusions

- Wealth of new BH solutions with ALE or ALF asymptotics.
- Microscopics described by (0,4) CFTs that descend from quiver gauge theories on a Riemann surface.

Macroscopics - Sugra

F-theory on $X = CY_3 \longrightarrow B_4$ gives D=6 (1,0) sugra with [Vafa, Vafa & Morrison, '96]

$$\textit{Tensor multiplets: } n_T = h^{1,1}(B_4) - 1$$

$$\textit{Vector multiplets: } n_V = h^{1,1}(X) - h^{1,1}(B_4) - 1$$

$$\textit{Hyper multiplets: } n_H = h^{2,1}(X) + 1$$

For simplicity, we take $n_V = 0$.

BPS Black String

$$ds_6^2 = 2H^{-1}du(dv - \frac{1}{2}H_5 du) + Hds^2_{TN_m}$$

$$Q^\alpha = \frac{1}{(2\pi)^2}\int G^\alpha \qquad j^\alpha = \frac{H_1^\alpha}{H}$$

$$H = \left(\eta_{\alpha\beta} H_1^\alpha H_1^\beta\right)^{1/2}$$

Attractor flow on the 6D tensor branch

Near horizon geometry is AdS_3 x S^3/Z_m But the entropy does not reproduce the microscopics?

How come?

Near horizon geometry is $AdS_3 \times S^3/Z_m$ But the entropy does not reproduce the microscopics?

How come?

1) Did not include GS terms...
2) Loop effects....