Black holes and (0,4)CFT's in string and F-theory

C. Couzens, H. het Lam, K. Mayer, S.V. 1904.05361
T. Grimm, H. het Lam, K. Mayer, S.V. 1808.05228
B. Haghighat, S. Murthy, C. Vafa, S.V. 1509.00455

Recap of Black holes and

Black Strings

Strominger-Vafa '96

IIB on T^4 (or K3) + D1-D5 (2,2) D=6 SUGRA $AdS_3 \times S^3$ (4,4) CFT

Maldacena-Strominger-Witten '97

M-theory on $CY_3 + M5$ N=2 D=5 SUGRA $AdS_3 \times S^2$ (0,4) CFT

Black holes in F-theory [Hagighat, Murthy, Vafa, S.V., '15]

F-theory on $CY_3 + D3$ (1,0) D=6 SUGRA $AdS_3 \times S^3$ (0,4) CFT with SU $(2)_{1}$

D3 wrapping Riemann Surface D3 on curve $C = \Sigma_q$ $(0,4) \& SU(2)_1 \text{ with } k_1 = g$

5D Spinning Black Holes

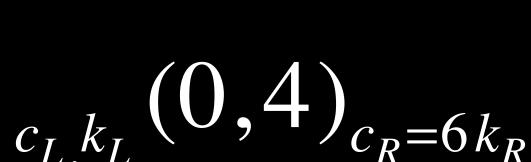
Central charges [Vafa '97]

$c_L = 6g + 12c_1(B_4).C$ $c_R = 6g + 6c_1(B_4).C$

 $2g(C)-2 = C.C - c_1(B_4).C = q^a q^b \eta_{ab} - q^a c_a$

New AdS/CFT duality (HMVV)

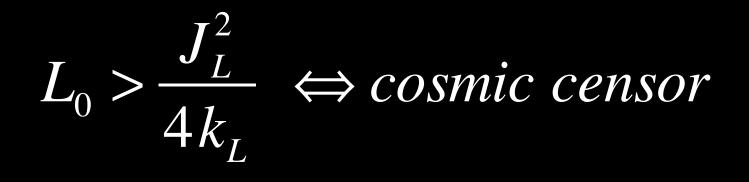
IIB on $AdS_3 \times S^3 \times B_4$



See also Couzens, Lawrie, Martelli, Schafer-Nameki, Wong, '17

Cardy formula for BH entropy

 $S = 2\pi \sqrt{\frac{c_L}{6}(n - \frac{J_L^2}{4k_I})}$



4D black holes from **F-theory** Grimm, het Lam, Mayer, S.V. '18

4D Black Holes from F-theory

• Put D3 into background

$$\mathbb{R} \times S^1 \times TN_m \times CY_3$$

- Compactify on the (asymptotic) TN circle. Break isometries to U(1)xSU(2).
- This is the 4D/5D correspondence [Gaiotto, Strominger, Yin, '05], but now applied to F-theory

D3 and Taub-NUT

• Near horizon limit of 6D black string:

$$AdS_3 \times S^3 / \mathbb{Z}_m$$

- (0,4) CFT with only U(1)_L x SU(2)_R.
- Central charges of the dual CFT should now also depend on *m*.

Microscopics

 Bena, Diaconescu, Florean ['06] using MSW techniques. E.g.:

$$c_{L} = 3mC^{2} - 3m^{2}c_{1}(B) \cdot C + m^{3}c_{1}(B)^{2}$$
$$+ 12c_{1}(B) \cdot C + 12m - 2mc_{1}(B)^{2}$$

$$k_{L} = \frac{1}{2}mC^{2} - \frac{1}{2}m^{2}c_{1}(B) \cdot C$$

Macroscopics

Reduce 6D sugra on $AdS_3 \times S^3 / \mathbb{Z}_m$ to three dimensions.

Coeffcients in front of the 3D CS terms (gauge and gravitational) determine the levels and central charges

6D (1,0) SUGRA

 $L = \frac{1}{2}R - \frac{1}{A}g_{\alpha\beta}G^{\alpha}_{\mu\nu\rho}G^{\beta,\mu\nu\rho} - \frac{1}{2}g_{\alpha\beta}\partial_{\mu}j^{\alpha}\partial^{\mu}j^{\beta}$ $-\overline{h_{\mu\nu}}\partial_{\mu}q^{\mu}\partial^{\mu}\overline{q^{\nu}}$ $-\frac{1}{8}\eta_{\alpha\beta}c^{\alpha}B^{\beta}\wedge tr\,R\wedge R$

 $g_{\alpha\beta} = 2j_{\alpha}j_{\beta} - \eta_{\alpha\beta}$ $\eta = diag(1, -1, ..., -1)$

3 steps...

- 1) Reduce 2-derivative action to 3D
- 2) Reduce GS-term to 3D
- 3) (chiral) KK states at one-loop contribute to 3D CS terms

Step 1

 Reduce (1,0) sugra on S³/Z_m to 3D. Produces U(1)xSU(2) gauge fields in 3D:

$$S_{CS} = \frac{k_L}{8\pi} \int A_L \wedge F_L + \frac{k_R}{4\pi} \int tr \left(A_R \wedge F_R + \frac{2}{3} A_R^2 \right)$$

$$k_L = k_R = \frac{1}{2} m \eta_{\alpha\beta} Q^{\alpha} Q^{\beta} \quad c_L = c_R = 6k_R$$

Step 2

 Reduce GS term to 3D. The spin connection produces a gravitational 3D CS term:

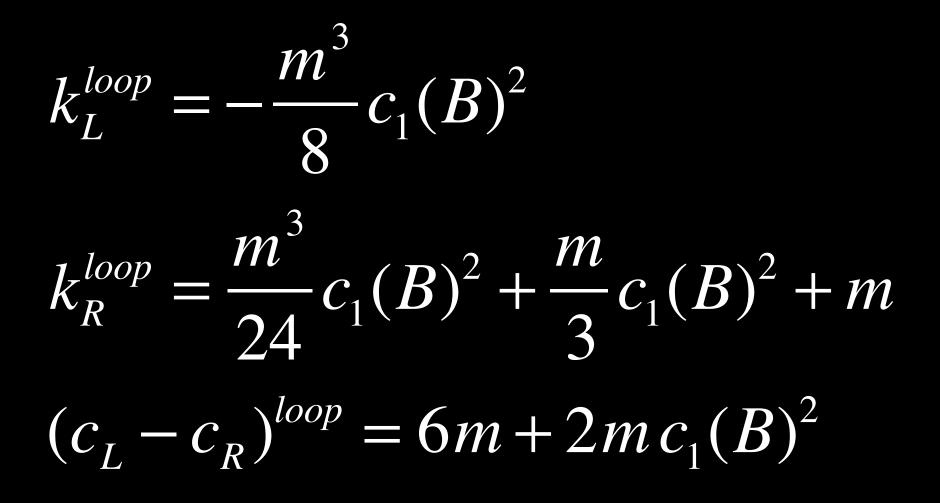
$$L = \frac{1}{16\pi} \eta_{\alpha\beta} c^{\alpha} Q^{\beta} \left[\omega_{grav}^{CS} + 4\omega^{CS} (A_R) \right]$$
$$(c_L - c_R)^{4-der} = 6\eta_{\alpha\beta} c^{\alpha} Q^{\beta} \quad k_R^{4-der} = \eta_{\alpha\beta} c^{\alpha} Q^{\beta}$$

Step 3

- Integrating out massive KK modes of chiral fields contribute to 3D CS terms:
- (A)SD Tensors: 3D massive KK chiral vectors [Townsend, Pilch, van Nieuwenhuizen, '84]
- Hyperini and tensorini: 3D massive spin 1/2
- Gravitino: 3D massive spin 3/2

Calculation related to parity anomaly and APS index theorems [Alvarez-Gaume, Della Pietra, Moore, '85, ...]. Zeta function regularization.

Result Step 3



F-inal Result

 Adding up everything, the macroscopics matches the microscopics, leading and subleading terms, upon making the identification

$$Q^{\alpha} = q^{\alpha} - \frac{m}{2}c^{\alpha}$$

This can be derived separately from GS term.

Other F- theory backgrounds...

F-theory on

$$\mathbb{R} \times S^1 \times HK_4 \times CY_3$$

Equivalently IIB on

$$\mathbb{R} \times S^1 \times HK_4 \times B_4$$

with HK_4 = ALE or ALF spaces

New IIB set up

 $\Gamma \subset \overline{SU(2)} \ e.g. \mathbb{Z}_m$

• IIB on

 $\mathbb{R} \times S^1 \times HK_{A} \times K3$

• ALE: $\mathbb{R}^+ \times S^3 / \Gamma \longleftrightarrow \mathbb{R}^4 / \Gamma$

• ALF: $\mathbb{R}^+ \times S^3 / \Gamma \leftarrow \mathcal{R}^3 \times S^1) / \Gamma$

[Couzens, het Lam, Mayer, S.V.,'19]

ADE

$\Gamma \subset \mathrm{SU}(2)$	Г	singularity type
cyclic group \mathbb{Z}_m	m	A_{m-1}
binary dihedral \mathbb{D}_m^*	4m	D_{m+2}
binary tetrahedral \mathbb{T}^*	24	E_6
binary octahedral \mathbb{O}^*	48	E_7
binary icosahedral \mathbb{I}^*	120	E_8

Table 2.1: The freely acting discrete subgroups of SU(2)

Macroscopics

Near horizon geometry of black string is

$$AdS_3 \times S^3 / \Gamma$$

• Consider ALE for concreteness. For A-series ($\Gamma \subset SU(2) \ e.g. \mathbb{Z}_m$), there is still a U(1)_L x SU(2)_R isometry, and for D-E, only SU(2)_R. Compactify to 3D as before to read of levels and central charges. [No HD-terms and no-loops!]

Macroscopics (ALE)

$k_{L} = \frac{1}{2} |\Gamma| C \cdot C \quad (for \ \Gamma = \mathbb{Z}_{m})$ $c_{L} = c_{R} = 6k_{R} = \frac{1}{2} |\Gamma| C \cdot C$

Microscopics (ALE)

D3 brane wraps curve C inside K3. The transverse space contains the ALE space, and so the D3 brane probes the ADE singularity. The 4D gauge theories are known to be N=2 quiver gauge theories with $n_v = n_H = |\Gamma|$.

Г	Gauge multiplets	
\mathbb{Z}_m	$U(1)^m$	
\mathbb{D}_m^*	$U(1)^4 \times U(2)^{m-1}$	
\mathbb{T}^*	$U(1)^3 \times U(2)^3 \times U(3)$	
\mathbb{O}^*	$U(1)^2 imes U(2)^3 imes U(3)^2 imes U(4)$	2
	$U(1) \times U(2)^2 \times U(3)^2 \times U(4)^2 \times U(5) \times U(6)$	

Table 4.1: $\mathcal{N} = 2$ multiplets for the different quive

Microscopics (ALE)

- Compactify 4D theory on curve C and topologically twist to get spectrum of IR (0,4) 2D CFT. [Benini & Bobev '13,....]
- Compute the levels and central charges from 't Hooft anomaly matching conditions.

$$c_{L} - c_{R} = Tr(\gamma_{3}) = 0$$

$$k_{R} = Tr[\gamma_{3}Q_{R}^{2}] = |\Gamma|(g-1) = \frac{1}{2}|\Gamma|C \cdot C$$

Microscopics (ALE)

For the A-series, there is an additional U(1)_L, which produces a left-moving current with level

$$k_{L} = -Tr[\gamma_{3}Q_{L}^{2}] = |\Gamma|(g-1) = \frac{1}{2}|\Gamma|C \cdot C$$

 All of this matches with the macroscopics, so there is perfect agreement!

Conclusions

• Wealth of new BH solutions with ALE or ALF asymptotics.

 Microscopics described by (0,4) CFTs that descend from quiver gauge theories on a Riemann surface.

Macroscopics - Sugra

F-theory on $X = CY_3 \longrightarrow B_4$ gives D=6 (1,0) sugra with [Vafa, Vafa & Morrison, '96] *Tensor multiplets:* $n_T = h^{1,1}(B_4) - 1$

Vector multiplets: $n_V = h^{1,1}(X) - h^{1,1}(B_4) - 1$ Hyper multiplets: $n_H = h^{2,1}(X) + 1$

For simplicity, we take $n_V = 0$.

BPS Black String

$$ds_{6}^{2} = 2H^{-1}du(dv - \frac{1}{2}H_{5}du) + Hds_{TN_{m}}^{2}$$
$$Q^{\alpha} = \frac{1}{(2\pi)^{2}}\int G^{\alpha} \qquad j^{\alpha} = \frac{H_{1}^{\alpha}}{H}$$
$$H = \left(\eta_{\alpha\beta}H_{1}^{\alpha}H_{1}^{\beta}\right)^{1/2}$$

Attractor flow on the 6D tensor branch

Near horizon geometry is $AdS_3 \times S^3/Z_m$ But the entropy does not reproduce the microscopics?

How come?

Near horizon geometry is $AdS_3 \times S^3/Z_m$ But the entropy does not reproduce the microscopics?

How come?

1) Did not include GS terms...

2) Loop effects....