Exceptional Field Theories & Applications

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Holography, Generalized Geometry and Duality
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Lab PHYS
ENS de LYON

plan

exceptional field theory

- dimensional reduction and duality symmetries
- exceptional geometry & tensor hierarchy
- generalized Scherk-Schwarz reductions

applications

- consistent truncations
  - $AdS_5 \times S^5$
  - S^p, $H^{p,q}$ spheres and hyperboloids
  - $S^p \times H^{p,q}$ products of spheres: dyonic gaugings
- supergravity magic
- timelike dualities
- generalized IIB supergravity

based on work with Olaf Hohm, Arnaud Baguet, Hadi Godazgar, Mahdi Godazgar, Hermann Nicolai, Edvard Musaev, Gianluca Inverso, Marc Magro, Emanuel Malek, Mario Trigiante, Valentí Vall Camell

exceptional field theory (ExFT)

manifestly duality covariant formulation of maximal supergravity

- upon toroidal reduction on T^d, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$

  after proper dualisation/reorganisation of the fields

D=11 supergravity $\xrightarrow{T^7}$ D=4 supergravity

maximal supersymmetry, global $E_{7(7)}$

IIB supergravity $\xrightarrow{T^5}$ D=5 supergravity

maximal supersymmetry, global $E_{6(6)}$

[Cremmer, Julia]

- ExFT : reformulate D=11 supergravity such that $E_{d(d)}$ (or its remnants) becomes manifest **before** dimensional reduction

- example: $E_{6(6)}$: exceptional field theory

$E_{6(6)}$: exceptional field theory

D=5 maximal supergravity (torus reduction of D=11 or IIB)

after proper dualization of the dof's (different for IIA / IIB)
the D=5 Lagrangian takes the $E_{6(6)}$ invariant form

$$\mathcal{L} = R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\, F_{\mu\nu}{}^{M} F^{\mu\nu\, N} + \mathcal{L}_{\rm top}$$

$g_{\mu\nu}$: 5 x 5 external metric

$\mathcal{A}_\mu{}^M$: 27 vector fields $\longleftrightarrow$ 27 two-form fields $\mathcal{B}_{\mu\nu\, M}$

$\mathcal{M}_{MN}$: 27 x 27 internal metric, parametrizing $E_{6(6)}/USp(8)$

with $\mathcal{L}_{\rm top} = d_{KMN}\, F^M \wedge F^N \wedge A^K$

$E_{6(6)}$: exceptional field theory

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${\cal M}_{MN}$: 27 x 27 internal metric, parametrizing $E_{6(6)}/USp(8)$

exceptional field theory:

— same Kaluza-Klein reorganisation of the higher-dimensional fields

— keeping the dependence on all internal coordinates (non-abelian gauge structure)

$E_{6(6)}$: exceptional field theory

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^M F^{\mu\nu\,N} + \mathcal{L}_{\rm top}$$

exceptional field theory: [Hohm, H.S.]

– same Kaluza-Klein reorganisation of the higher-dimensional fields

– keeping the dependence on all internal coordinates (non-abelian gauge structure)

$$\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^M\,\mathcal{F}^{\mu\nu\,N} + \mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

– all fields live on external space-time : $\{x^\mu\}$ $\mu \;=\; 0,\ldots,4$

internal (exceptional) space : $\{Y^M\}$ $M = 1,\ldots,27$

– internal coordinates enhanced to $E_{6(6)}$ representation: 6 —> 27

subject to the section constraint (covariant restriction down to 6 coordinates)

$$d^{KMN}\,\partial_M \otimes \partial_N \;=\; 0 \qquad \left\{ \begin{array}{rcl} d^{KMN}\,\partial_M \partial_N f &=& 0 \\ d^{KMN}\,\partial_M f\,\partial_N g &=& 0 \end{array} \right.$$

[Berman, Godazgar, Perry, West, Coimbra, Strickland-Constable, Waldram, Cederwall, Kleinschmidt, Thompson]

$E_{6(6)}$: exceptional field theory

$$\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} + \mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

- dependence on internal coordinates induces non-abelian gauge structure

$$\mathcal{D}_\mu \;=\; \partial_\mu - \mathcal{L}_{A_\mu} \qquad\qquad \mathcal{L}_\Lambda V^M \;=\; \Lambda^N \partial_N V^M - \kappa \Big[\partial_N \Lambda^M\Big]_{\rm adj} V^N$$

generalized diffeomorphisms

[Coimbra, Strickland-Constable, Waldram]

- non-associativity of generalized diffeomorphisms induces modified YM field strengths

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu} A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_{\rm E}^M + 10\, d^{MNK} \partial_K B_{\mu\nu N}$$

two-form field equations compensated by the topological term

$$S_{\rm top} \;=\; \int d^{27}Y \int_{\mathcal{M}_6} \left(d_{MNK}\, \mathcal{F}^M \wedge \mathcal{F}^N \wedge \mathcal{F}^K - 40\, d^{MNK} \mathcal{H}_M \wedge \partial_N \mathcal{H}_K\right)$$

boundary term of a six-dimensional bulk

$E_{6(6)}$: exceptional field theory

$$\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^{M}\,\mathcal{F}^{\mu\nu N}$$
$$+\;\mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

- "potential"

$$V_{\rm pot} \;=\; \frac{1}{24}\,\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\,(12\,\partial_L \mathcal{M}_{NK} - \partial_N \mathcal{M}_{KL})$$
$$-\frac{1}{2}\,g^{-1}\partial_M g\,\partial_N \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}^{MN} g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu}$$

— invariant under generalized diffeomorphisms

— generalised (internal) curvature scalar

$E_{6(6)}$: exceptional field theory

$$
\begin{aligned}
\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^{M}\,\mathcal{F}^{\mu\nu N} \\
+\,\mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})
\end{aligned}
$$

- unique action with generalized diffeomorphism invariance in all $5{+}27$ coordinates (modulo section condition)
- section condition is solved upon breaking $E_{6(6)}$ –> $\mathrm{SL}(5)$ $\quad d^{KMN}\,\partial_M \otimes \partial_N \;=\; 0$

$$27 \longrightarrow 1 + \boxed{5'} + 10 + \boxed{5} + 5' + \boxed{1}$$

$\boxed{5'}$: reproduces full IIA supergravity (additional singlet $\boxed{1}$: D$=11$ supergravity)

$\boxed{5}$: reproduces full IIB supergravity (preserves an $\mathrm{SL}(2)$)

together with proper dictionary of ExFT fields into IIA/IIB

$$
\boxed{\mathcal{M}_{MN}} = \begin{pmatrix}
\mathcal{M}_{0,0} & \mathcal{M}_{0,m} & \mathcal{M}_0{}^{mn} & \mathcal{M}_0{}^{m} & \mathcal{M}_{0\bar{0}} \\
\mathcal{M}_{k,0} & \mathcal{M}_{k,m} & \mathcal{M}_k{}^{mn} & \mathcal{M}_k{}^{m} & \mathcal{M}_{k\bar{0}} \\
\mathcal{M}^{kl}{}_0 & \mathcal{M}^{kl}{}_m & \mathcal{M}^{kl,mn} & \mathcal{M}^{kl,m} & \mathcal{M}^{kl}{}_{\bar{0}} \\
\mathcal{M}^{k}{}_0 & \mathcal{M}^{k}{}_m & \mathcal{M}^{k,mn} & \mathcal{M}^{k,m} & \mathcal{M}^{k}{}_{\bar{0}} \\
\mathcal{M}_{k,0} & \mathcal{M}_{k,m} & \mathcal{M}_k{}^{mn} & \mathcal{M}_k{}^{m} & \mathcal{M}_{k\bar{0}} \\
\mathcal{M}_{\bar{0},0} & \mathcal{M}_{\bar{0},m} & \mathcal{M}_{\bar{0}}{}^{mn} & \mathcal{M}_{\bar{0}}{}^{m} & \mathcal{M}_{\bar{0}\bar{0}}
\end{pmatrix}
\qquad
\boxed{\mathcal{M}_{MN}} = \begin{pmatrix}
\mathcal{M}_{0,0} & \mathcal{M}_{0,m} & \mathcal{M}_0{}^{mn} & \mathcal{M}_0{}^{m} & \mathcal{M}_{0\bar{0}} \\
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\mathcal{M}^{kl}{}_0 & \mathcal{M}^{kl}{}_m & \mathcal{M}^{kl,mn} & \mathcal{M}^{kl,m} & \mathcal{M}^{kl}{}_{\bar{0}} \\
\mathcal{M}^{k}{}_0 & \mathcal{M}^{k}{}_m & \mathcal{M}^{k,mn} & \mathcal{M}^{k,m} & \mathcal{M}^{k}{}_{\bar{0}} \\
\mathcal{M}_{k,0} & \mathcal{M}_{k,m} & \mathcal{M}_k{}^{mn} & \mathcal{M}_k{}^{m} & \mathcal{M}_{k\bar{0}} \\
\mathcal{M}_{\bar{0},0} & \mathcal{M}_{\bar{0},m} & \mathcal{M}_{\bar{0}}{}^{mn} & \mathcal{M}_{\bar{0}}{}^{m} & \mathcal{M}_{\bar{0}\bar{0}}
\end{pmatrix}
$$

$E_{6(6)}$: exceptional field theory

manifestly duality covariant formulation of maximal supergravity

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

D=5+27 with section condition

dictionary

dictionary

D=11 sugra

IIB sugra

- upon solving the section constraint: reformulation of the original theories
- IIA and IIB supergravity accommodated in the same framework

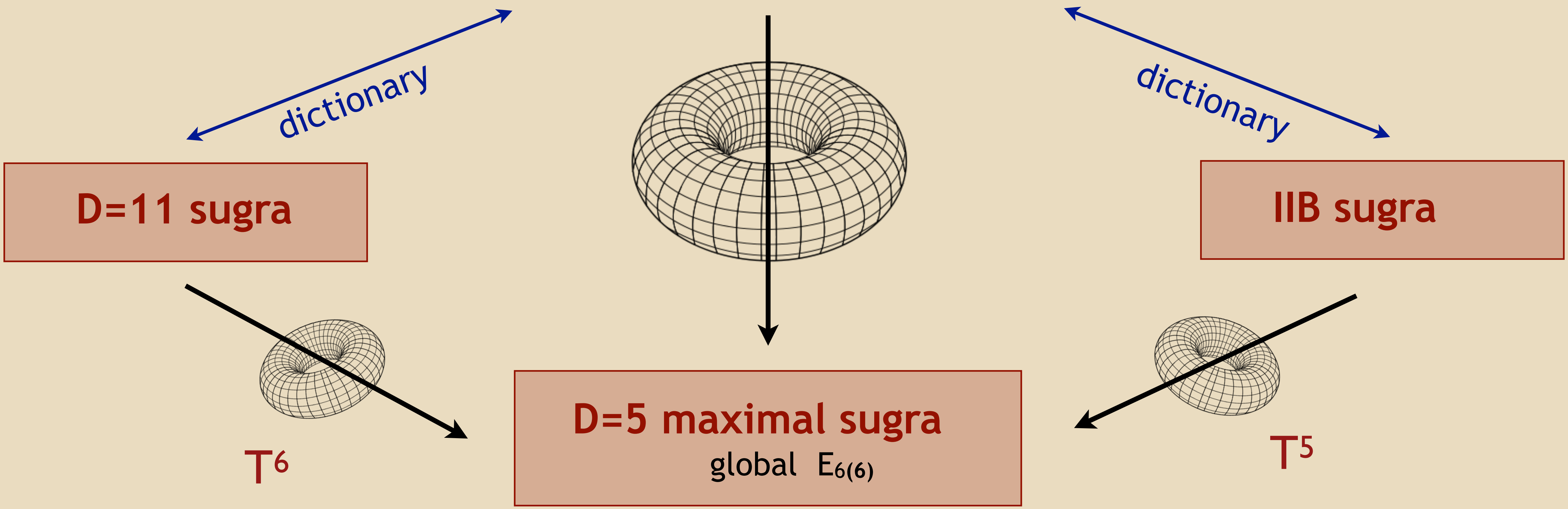
$E_{6(6)}$: exceptional field theory

manifestly duality covariant formulation of maximal supergravity

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

D=5+27 with section condition



makes the symmetry enhancement after torus reduction manifest

$E_{6(6)}$: exceptional field theory

manifestly duality covariant formulation of maximal supergravity

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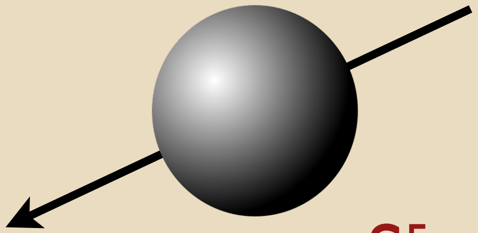
$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

D=5+27 with section condition

dictionary

IIB sugra



S^5 x AdS_5

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

also allows a compact description of complicated reductions

$E_{6(6)}$: exceptional field theory

manifestly duality covariant formulation of maximal supergravity

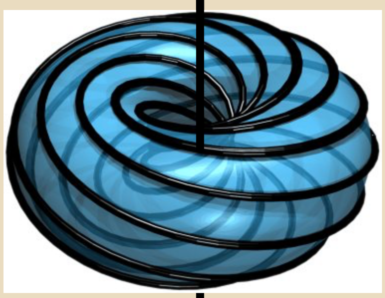
ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

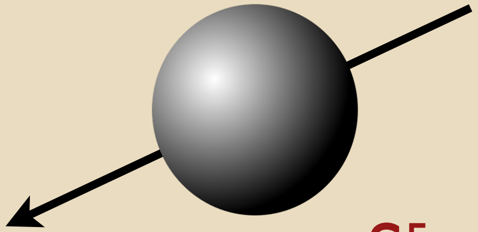
D=5+27 with section condition

dictionary

captured by a twisted torus (Scherk-Schwarz) reduction of ExFT



IIB sugra



S^5 x AdS_5

[Kaloper, Myers, Dabholkar, Hull, Reid-Edwards, Dall'Agata, Prezas, HS, Trigiante, Hohm, Kwak, Aldazabal, Baron, Nunez, Marques, Geissbuhler, Grana, Berman, Musaev, Thompson, Rosabal, Lee, Strickland-Constable, Waldram, Dibitetto, Roest, Malek, Blumenhagen, Hassler, Lust, Cho, Fernández-Melgarejo, Jeon, Park, Guarino, Varela, Inverso, Ciceri, ..., ...]

D=5 maximal sugra

global $E_{6(6)}$

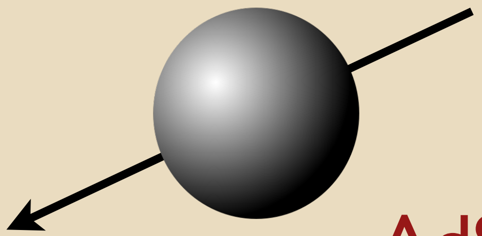
gauge group SO(6)

also allows a compact description of complicated reductions

- consistent truncation on $AdS_5 \times S^5$

consistent truncation on $AdS_5 \times S^5$

IIB sugra



$AdS_5 \times S^5$

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

[Gunaydin,Romans,Warner]
[Pernici,Pilch,vNieuwenhuizen]

- $AdS_5 \times S^5$: maximal supersymmetric solution of IIB
- fluctuations around the background: D=5 gauged supergravity
- explicit reduction formulas: highly non-trivial

$$
\begin{aligned}
ds^2 \;&=\; \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A^{ab}_\mu(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A^{cd}_\nu(x) dx^\nu\right) \\
& G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)
\end{aligned}
$$

D=11 on $AdS_4 \times S^7$: [de Wit, Nicolai] 1987

D=11 on $AdS_7 \times S^4$: [Nastase, van Nieuwenhuizen, Vaman] 1999

IIB on $AdS_5 \times S^5$:   shown for various sub-sectors...

consistent truncation on AdS$_5$ x S^5

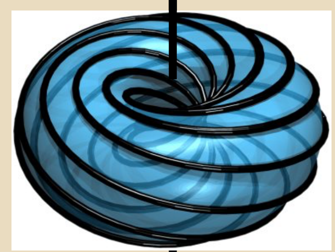
ExFT

dictionary

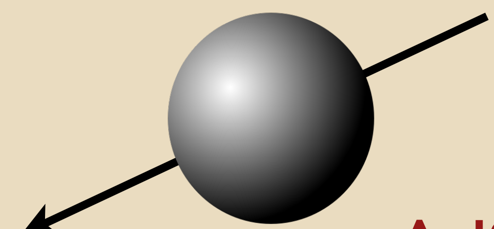
$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

$$\mathcal{B}_{\mu\nu\, M}(x,Y) \;=\; \rho^{-2}(Y)\, U_M{}^K(Y)\, B_{\mu\nu\, K}(x)$$



IIB sugra



AdS$_5$ x S^5

D=5 maximal sugra
global $E_{6(6)}$
gauge group SO(6)

[Gunaydin,Romans,Warner]
[Pernici,Pilch,vNieuwenhuizen]

- consistent truncation via generalized Scherk-Schwarz ansatz in ExFT
  in terms of an $E_{6(6)}$—valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

consistent truncation on AdS_5 x S^5

ExFT

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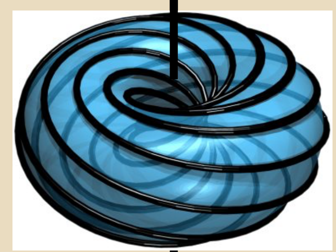
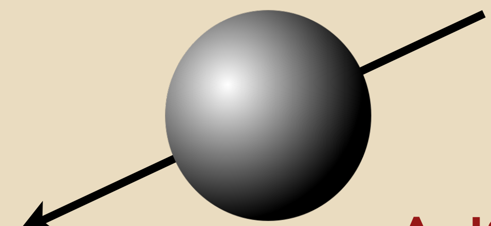


IIB sugra



AdS_5 x S^5

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

[Gunaydin,Romans,Warner]
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- consistent truncation via generalized Scherk-Schwarz ansatz in ExFT

  in terms of an $E_{6(6)}$—valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

  - system of consistency equations $\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{351}} \stackrel{!}{=} \rho\, X_{MN}{}^K$
  - generalized parallelizability
  - no general classification of its solutions (Lie algebras vs Leibniz algebras)

consistent truncation on AdS$_5$ x S^5

ExFT

dictionary

IIB sugra

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) = \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

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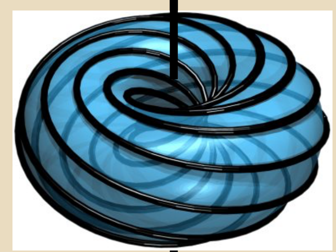
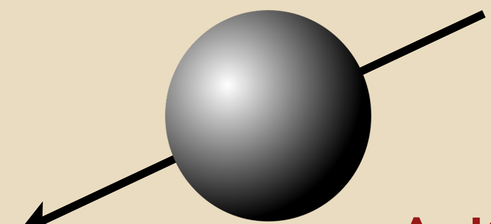




AdS$_5$ x S^5

D=5 maximal sugra
global $E_{6(6)}$
gauge group SO(6)

[Gunaydin,Romans,Warner]
[Pernici,Pilch,vNieuwenhuizen]

- twist matrix $U \in \mathrm{SL}(6)$ associated to SO(6)
  - background AdS$_5$ x S^5
  - full reduction formulas of IIB on AdS$_5$ x S^5
    in terms of sphere harmonics and the fields of D=5 maximal supergravity

$$U = \begin{pmatrix} g^{-1/2}\partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \in \mathrm{SL}(6)$$

consistent truncation on AdS₅ x S⁵

- e.g. metric (standard Kaluza-Klein form)

$$ds^2 \;=\; \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y)\mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

- e.g. 4-form (after reconstructing all components, in Kaluza-Klein basis)

$$C_{klmn} = \tilde{C}_{klmn} + \frac{1}{16}\tilde{\omega}_{klmnp}\Delta^{4/3} m_{\alpha\beta}\tilde{G}^{pq}\partial_q(\Delta^{-4/3}m^{\alpha\beta}),$$

$$C_{\mu kmn} = \frac{\sqrt{2}}{4}\mathcal{Z}_{[ab]kmn}A_\mu{}^{ab},$$

$$C_{\mu\nu mn} = \frac{\sqrt{2}}{4}\mathcal{K}_{[ab]}{}^k\mathcal{Z}_{[cd]kmn}A_{[\mu}{}^{ab}A_{\nu]}{}^{cd},$$

$$C_{m\mu\nu\rho} = -\frac{1}{32}\mathcal{K}_{[ab]m}\left(2\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}M_{ab,N}F^{\sigma\tau N} + \sqrt{2}\varepsilon_{abcdef}\Omega^{cdef}_{\mu\nu\rho}\right) - \frac{1}{4}\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{Z}_{[ef]mkl}(A_{[\mu}{}^{ab}A_\nu{}^{cd}A_{\rho]}{}^{ef}),$$

$$C_{\mu\nu\rho\sigma} = -\frac{1}{16}\mathcal{Y}_a\mathcal{Y}^b\left(\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D^\tau M_{bc,N}M^{Nca} + 2\sqrt{2}\varepsilon_{cdefgb}F_{[\mu\nu}{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{ga}\right)$$
$$+\frac{1}{4}\left(\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{K}_{[ef]}{}^n\mathcal{Z}_{[gh]kln} - \mathcal{Y}_h\mathcal{Y}^j\varepsilon_{abcegj}\eta_{df}\right)A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{gh} + \Lambda_{\mu\nu\rho\sigma}(x).$$

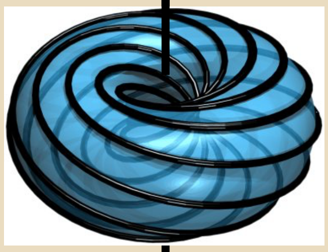
$$\begin{aligned}D_{[\mu}\Lambda_{\nu\rho\sigma\tau]} = &-\frac{1}{80}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D_\lambda(M^{Nac}D^\lambda M_{bc,N})\\ &+\frac{1}{40}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}F^{\kappa\lambda N}\left(M_{bc,N}F_{\kappa\lambda}{}^{ac} - \frac{1}{2}\sqrt{10}\varepsilon_{\alpha\beta}\eta_{db}M^{d\alpha}{}_N B_{\kappa\lambda}{}^{a\beta}\right)\\ &+\frac{1}{100}\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}\mathcal{Y}_a\mathcal{Y}^b(10M^{ac,fd} + \mathcal{X}^{(af)ec,d}{}_e)\eta_{cd}\eta_{bf}\\ &+\frac{1}{32}\sqrt{2}\varepsilon_{abcdef}F_{[\mu\nu}{}^{ab}F_{\rho\sigma}{}^{cd}A_{\tau]}{}^{ef} + \frac{1}{16}F_{[\mu\nu}{}^{ab}A_\rho{}^{cd}A_\sigma{}^{ef}A_{\tau]}{}^{gh}\varepsilon_{abcdeh}\eta_{fh}\\ &+\frac{1}{40}\sqrt{2}A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_\sigma{}^{gh}A_{\tau]}{}^{ij}\varepsilon_{abcegi}\eta_{df}\eta_{hj},\end{aligned}$$

- **proves the consistent truncation of IIB on AdS₅ x S⁵**

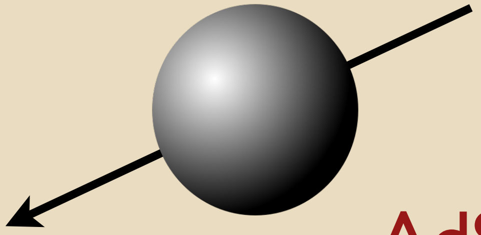
consistent truncation on $AdS_5 \times S^5$

ExFT

dictionary



IIB sugra



D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

$AdS_5 \times S^5$

[Gunaydin,Romans,Warner]
[Pernici,Pilch,vNieuwenhuizen]

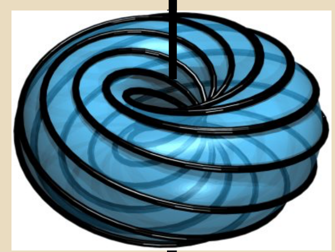
- twist matrix $U \in \mathrm{SL}(6)$ associated to SO(6)
  - background $AdS_5 \times S^5$
  - full reduction formulas of IIB on $AdS_5 \times S^5$

$$U = \begin{pmatrix} g^{-1/2}\partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \in \mathrm{SL}(6)$$

- **proves the consistent truncation of IIB on $AdS_5 \times S^5$**

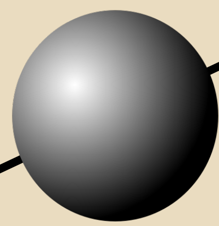
consistent truncation on $AdS_5 \times S^5$

ExFT



dictionary

IIB sugra



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- similar: twist matrix $U \in \mathrm{SL}(6)$ associated to SO(p,q) and CSO(p,q,r)

  built from sphere harmonics on SO(p,q)/SO(p–1,q)

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hyperboloid compactifications

ExFT

dictionary

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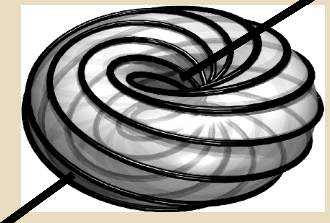

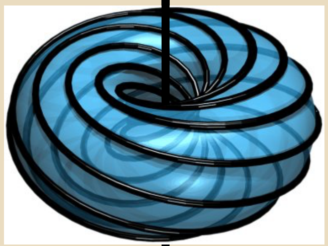

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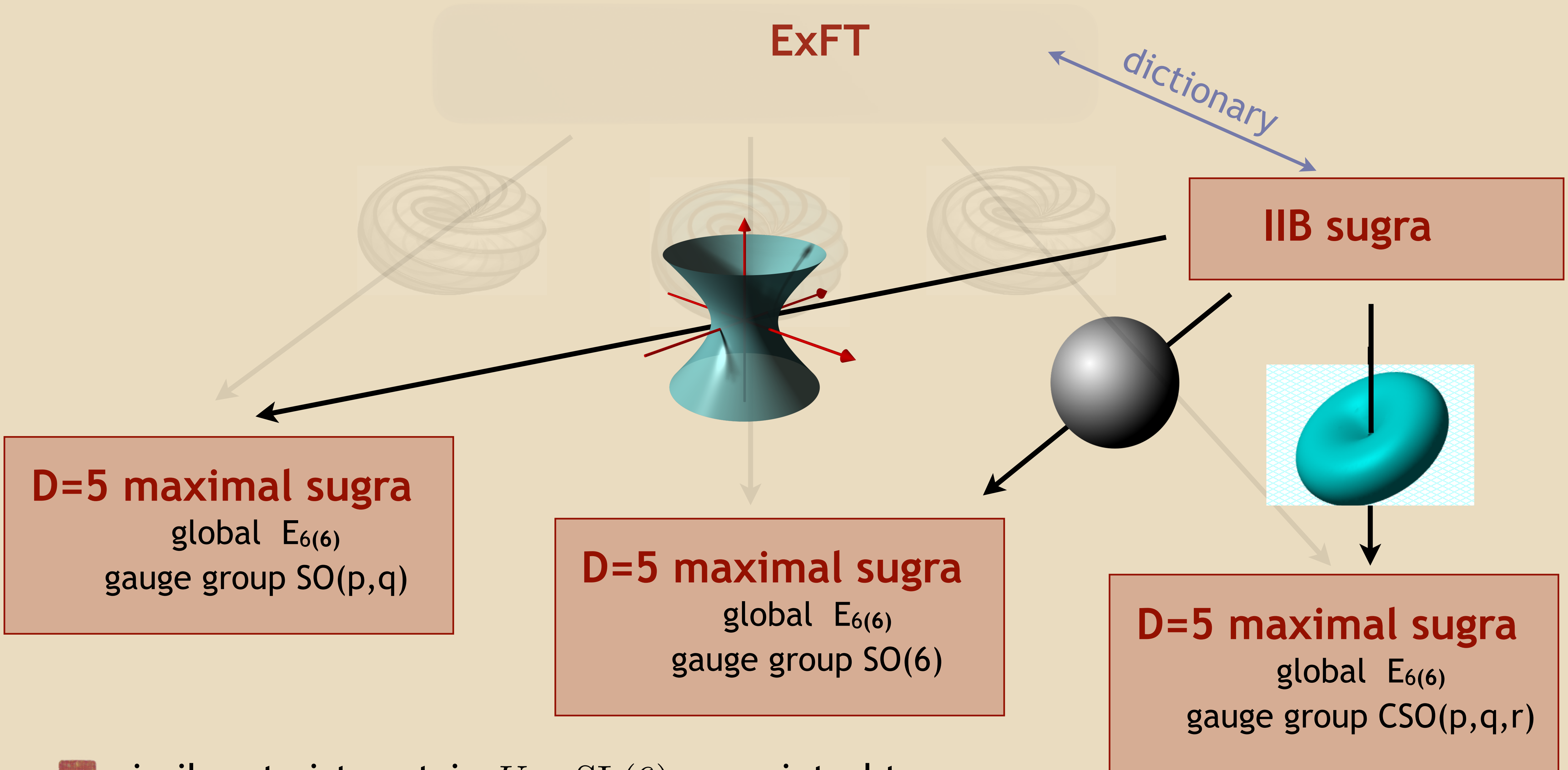
hyperboloid compactifications



- similar: twist matrix $U \in \mathrm{SL}(6)$ associated to SO(p,q) and CSO(p,q,r)
  - background: (warped) hyperboloids [Hull, Warner] [Baron, Dall'Agata]
  - in general no IIB solutions, still consistent truncations!

other examples of consistent truncations

- consistent truncations with smaller isometry groups [Inverso, HS, Trigiante, Malek]

  products of spheres and hyperboloids $S^p \times S^q$, $S^p \times H^q$

  specific D=4 construction, based on electric/magnetic split of internal coordinates

  inducing dyonic gaugings $(\mathrm{SO}(p,q) \times \mathrm{SO}(p',q')) \ltimes N$ [Dall'Agata, Inverso]

$$\text{IIB sugra} \xrightarrow{AdS_4 \times S^5 \times \mathbb{R}} \text{D=4 maximal sugra}$$

gauge group

$[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$

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IIB sugra

$\Big\downarrow$ $AdS_4 \times S^5 \times \mathbb{R}$

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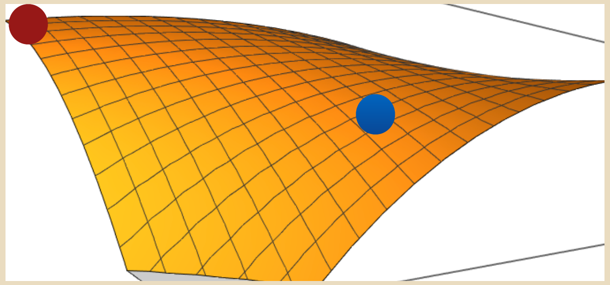
scalar potential



- SO(6) no ground state $AdS_4 \times S^5 \times \mathbb{R}$
- SO(4) N=4, AdS₄ vacuum $AdS_4 \times S^2 \times S^2 \times \Sigma$

$$ds^2 = \Delta^3 \sin^2 x\,(1+2\cos^2 x)\, d\mathcal{Y}_1^p d\mathcal{Y}_1^p + \Delta^3\,(1+2\sin^2 x)\cos^2 x\, d\mathcal{Y}_2^p d\mathcal{Y}_2^p$$
$$+\,\Delta^{-1}\,(dx dx + d\eta\, d\eta) + \frac{1}{2}\,\Delta^{-1}\, ds^2_{\mathrm{AdS}_4}\,,$$

Janus solution [D'Hoker, Estes, Gutperle]
with a maximally supersymmetric
consistent truncation around

other examples of consistent truncations

- consistent truncations with less supersymmetry via DFT
  [Baguet, Malek, Pope, HS, Sarioglu]
  - S^3 reduction of the bosonic string
  - more general: Pauli reduction of the bosonic string on group manifold G
  - example G = SO*(4), type II uplift of D=4 Minkowski vacua
  - $AdS_3 \times S^3$ reductions from 6D supergravity, N=(1,1) and N=(2,0) w tensor-multiplets

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- consistent truncations with less supersymmetry in ExFT (in type II sugra)
  [Malek] [Malek, HS, Vall Camell]

  embedding of half-maximal supergravity into ExFT

  construction and classification of supersymmetric AdS vacua

  half-maximal supersymmetric AdS vacua induce consistent truncations around

  $\longrightarrow$ [Emanuel's talk]

other applications / developments

- ExFT for all finite-dimensional exceptional groups $E_{d(d)}$, d<9

  [Hohm, HS] [Abzalov, Bakhmatov,Musaev, Hohm, Wang, Berman,Blair, Malek,Rudolph]

  based on the different splits external/internal coordinates $\{x^\mu, y^m\} \longrightarrow \{x^\mu, Y^M\}$

- ExFT embedding of massive IIA theory

  [Ciceri, Guarino, Inverso] [Cassani, de Felice, Petrini, Strickland-Constable, Waldram]

  — by deformations of ExFT

  — by Scherk-Schwarz reduction violating the section conditions

  —> more general theme: consistent theories from reductions violating section constraints

- ExFT embedding of ‘generalized IIB’ theory

  [Baguet, Magro, HS]

  — background from η-deformed AdS_5 x S^5 sigma model

  — T-dual of IIA with non-isometric dilaton

other applications / developments

- magical framework

higher-dimensional origin of 3D coset spaces

[Cremmer, Julia, Lu, Pope]

[Keurentjes]

11	$\times$								
10	$\mathbb{R}$ \ A_1	$\times$							
9	$A_1 \times \mathbb{R}$	$\mathbb{R}$							
8	$A_2 \times A_1$	$A_1 \times \mathbb{R}$ \ A_2	A_1						
7	A_4	$A_2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$	$\mathbb{R}$	$\times$				
6	D_5	$A_3 \times A_1$	$A_1^2 \times \mathbb{R}$	$\mathbb{R}^2$ \ A_1^2	$\mathbb{R}$				
5	E_6	A_5	A_2^2	$A_1^2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$	A_1			
4	E_7	D_6	A_5	$A_3 \times A_1$	$A_2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$ \ A_2	$\mathbb{R}$	$\times$	
3	E_8	E_7	E_6	D_5	A_4	$A_2 \times A_1$	$A_1 \times \mathbb{R}$	A_1 \ $\mathbb{R}$	$\times$
D \ r	8	7	6	5	4	3	2	1	0

all embedded within $E_{8(8)}$ ExFT: $$E_{8(8)} \longrightarrow SL(D-2) \times SL(9-r) \times U_{D,r}$$

magic

solution of section constraint: $$\{\, Y^M \,\} \longrightarrow \Big\{ \underbrace{Y^i{}_j}_{SL(D\text{-}2)} \,,\, \underbrace{Y^a{}_b}_{SL(9\text{-}r)} \Big\} \longrightarrow \{ y^i \equiv Y^i{}_0 \}$$

together with truncation to SL(9-r) singlets

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D \ r	8	7	6	5	4	3	2	1	0
11	$\times$								
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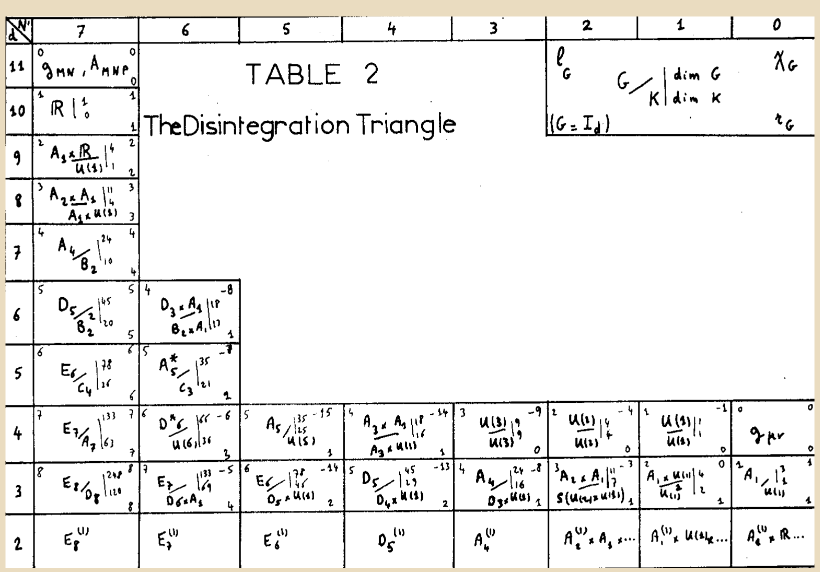
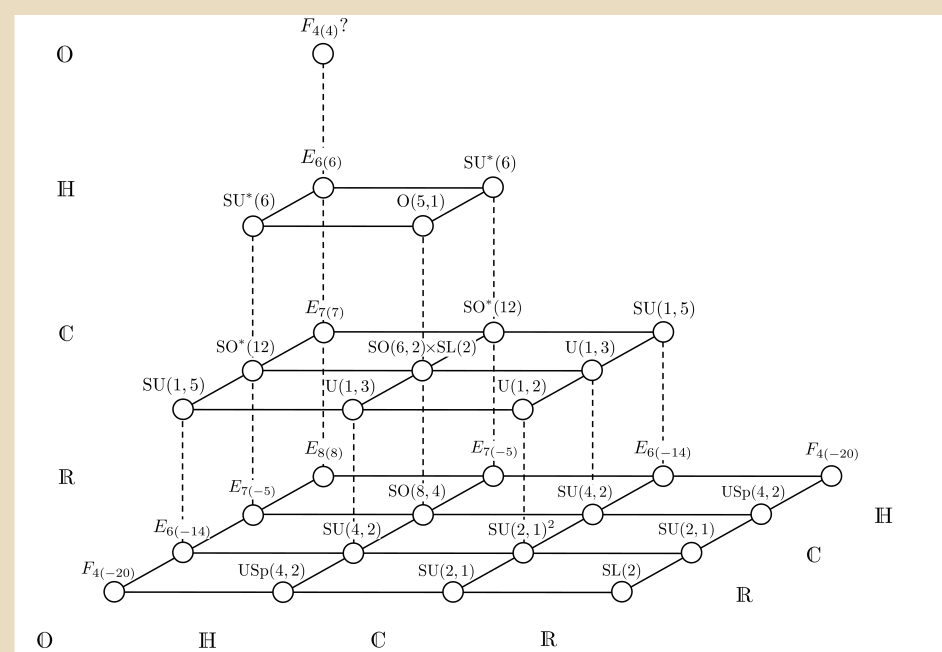
8	$A_2 \times A_1$	$A_1 \times \mathbb{R}$, A_2	A_1						
7	A_4	$A_2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$	$\mathbb{R}$	$\times$				
6	D_5	$A_3 \times A_1$	$A_1^2 \times \mathbb{R}$	$\mathbb{R}^2$, A_1^2	$\mathbb{R}$				
5	E_6	A_5	A_2^2	$A_1^2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$	A_1			
4	E_7	D_6	A_5	$A_3 \times A_1$	$A_2 \times \mathbb{R}$	$A_1 \times \mathbb{R}$, A_2	$\mathbb{R}$	$\times$	
3	E_8	E_7	E_6	D_5	A_4	$A_2 \times A_1$	$A_1 \times \mathbb{R}$	A_1, $\mathbb{R}$	$\times$

all embedded within $E_{8(8)}$ ExFT: $$E_{8(8)} \longrightarrow \underbrace{\mathrm{SL}(D-2) \times \mathrm{SL}(9-r)}_{\text{magic}} \times U_{D,r}$$

other magic triangles and pyramids ...



[Julia]



[Anastasiou, Borsten, Duff, Marrani, Nagy, Zoccali]

other applications / developments

the ExFT construction mainly relies on the (complex) algebra $\mathfrak{e}_6$

- different real forms: e.g. $\mathrm{E}_{6(6)}/\mathrm{USp}(8) \longrightarrow \mathrm{E}_{6(-26)}/\mathrm{F}_4$:

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

ExFT formally identical

different structure of (real) solutions to the section constraint

uplift to D=6 $\longrightarrow$ magic N=2 series

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different structure of (real) solutions to the section constraint

uplift to D=6 $\longrightarrow$ magic N=2 series

- different coset space: $\mathrm{E}_{6(6)}/\mathrm{USp}(8) \longrightarrow \mathrm{E}_{6(6)}/(\mathrm{USp}(4) \times \mathrm{USp}(4))$

reduction on torus including time-like circle

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\mathrm{top}} - V(\mathcal{M}, e).$$

ExFT formally identical, flip of external/internal signatures

(4,1) & (27,0) $\longleftrightarrow$ (5,0) & (16,11)

equivalent description, also captures Hull's *-theories ! SL(5): [Blair,Malek,Park]

other applications / developments

- unifying framework for brane solutions

  [Berman, Rudolph, Bakhmatov, Kleinschmidt, Musaev, Otsuki, Fernandez-Melgarejo, Kimura, Sakatani]

  1/2 BPS branes from a single ExFT solution, organisation of exotic branes

- orbifolds and orientifolds in ExFT

  [Blair, Malek, Thompson]

  unified approach in terms of generalized orbifolds (O-folds)

- exceptional string sigma model

  [Arvanitakis, Blair]

  string sigma model with ExFT background fields

- ExFT loop calculations

  [Bossard, Kleinschmidt]

  duality covariant graviton amplitudes

- underlying mathematical structures

  [Cederwall, Palmkvist][Hohm, Kupriyanov, Lüst, Traube]
  [Cagnacci, Codina, Marques][Arvanitakis]

  L_∞-algebras, Borchers superalgebras, tensor hierarchy algebras

conclusions

- **exceptional field theory**

- based on generalized diffeomorphisms in exceptional geometry
- unique theory with generalized diffeomorphism invariance
  in all coordinates   (modulo section condition)
- upon an explicit solution of the section condition
  the theory coincides with full D=11 supergravity or full D=10 IIB supergravity
- includes generalized type II supergravities and Hull's II*-theories
- powerful tool for construction & analysis of vacua & consistent truncations

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  - powerful tool for construction & analysis of vacua & consistent truncations

- **tool for analyzing existing theories**
  — or hints towards a more fundamental structure ..?

  - weaken / relax section constraints
  - decrease number of external dimensions $\longrightarrow$ unifying picture