

Exceptional geometry of supersymmetric AdS vacua and their consistent truncations

Emanuel Malek

Max-Planck-Institut für Gravitationsphysik
(Albert-Einstein-Institut).

Holography, Generalized Geometry and Duality,
6th May 2019

Based on EM: [arXiv:1612.01692](https://arxiv.org/abs/1612.01692), [arXiv:1612.01990](https://arxiv.org/abs/1612.01990), [arXiv:1707.00714](https://arxiv.org/abs/1707.00714),
EM, Samtleben, Vall Camell: [arXiv:1808.05597](https://arxiv.org/abs/1808.05597), [arXiv:1901.11039](https://arxiv.org/abs/1901.11039)

Motivation

- AdS/CFT: window into strongly-coupled gauge theories
- AdS_D × M_{int} of 10-/11-dim SUGRA ↔ CFT_{D-1}

CFT _{D-1}	String theory on AdS _D × M _{int}
(Exactly) marginal deformations	(Finite) moduli
RG flow	Domain-wall solutions
Thermal field theory	Asymptotically AdS black hole
Operators in short representations	Kaluza-Klein spectrum

- Geometry of M_{int} controls many properties of CFT.
- Practical applications: useful to study AdS_D × M_{int} using D-dimensional (gauged) SUGRA via “truncation”.

Consistent truncations

- Which modes to keep on M_{int} ?
- No scale separation \Rightarrow No effective action! [Kim, Romans, Nieuwenhuizen '85]

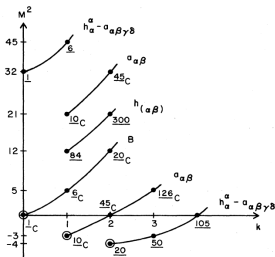


FIG. 2. Mass spectrum of scalars.

- Instead use “consistent truncation” (c.f. Henning Samtleben’s talk)
- Conjecture [Gauntlett, Varela '07]: For any SUSY AdS_D \times_w M_{int} sol of 10-/11-dim SUGRA \exists consistent truncation to only D -dim grav. supermultiplet (“minimal”).
- Consistent truncations with matter multiplets?

Outline

- SUSY AdS vacua & consistent truncations in ExFT
- Exceptional geometry of supersymmetric AdS_{6,7}
- Minimal consistent truncations around SUSY AdS_{6,7}
- Classification of consistent truncations with matter multiplets

SUSY AdS_{6,7} vacua of type II SUGRA

- AdS_{6,7} vacua preserve 1/2-maximal SUSY; R-symmetry: $SU(2)_R$
- IIA: AdS₇ × S² × I defined by $\ddot{t}(z) = m/2$, m Romans' mass.
[Apruzzi, Fazzi, Rosa, Tomasiello '13], [Gaiotto, Tomasiello '14], [Apruzzi, Fazi, Passias, Rota, Tomasiello '15], [Cremonesi, Tomasiello '15]
 - Minimal consistent truncation [Passias, Rota, Tomasiello '15]
 - All AdS₇ vacua reduce to same 7-d vacuum → vector multiplets?

SUSY AdS_{6,7} vacua of type II SUGRA

- AdS_{6,7} vacua preserve 1/2-maximal SUSY; R-symmetry: $SU(2)_R$
- IIA: AdS₇ × S² × I defined by $\ddot{t}(z) = m/2$, m Romans' mass.
[Apruzzi, Fazzi, Rosa, Tomasiello '13], [Gaiotto, Tomasiello '14], [Apruzzi, Fazi, Passias, Rota, Tomasiello '15], [Cremonesi, Tomasiello '15]
 - Minimal consistent truncation [Passias, Rota, Tomasiello '15]
 - All AdS₇ vacua reduce to same 7-d vacuum → vector multiplets?
- IIB: AdS₆ × S² × Σ₂ vacua defined by 2 holomorphic functions on Σ
[Lozano, O Colgain, Rodriguez-Gomez, Sftesos '12], [Apruzzi, Fazzi, Passias, Rosa, Tomasiello '14], [Kim, Kim, Suh '15], [D'Hoker, Gutperle, Karch, Uhlemann '16] .
 - Minimal consistent truncation?
 - Vector multiplets?

SUSY AdS_{6,7} vacua of type II SUGRA

- AdS_{6,7} vacua preserve 1/2-maximal SUSY; R-symmetry: $SU(2)_R$
- IIA: AdS₇ × S² × I defined by $\ddot{t}(z) = m/2$, m Romans' mass.
[Apruzzi, Fazzi, Rosa, Tomasiello '13], [Gaiotto, Tomasiello '14], [Apruzzi, Fazi, Passias, Rota, Tomasiello '15], [Cremonesi, Tomasiello '15]
 - Minimal consistent truncation [Passias, Rota, Tomasiello '15]
 - All AdS₇ vacua reduce to same 7-d vacuum → vector multiplets?
- IIB: AdS₆ × S² × Σ₂ vacua defined by 2 holomorphic functions on Σ
[Lozano, O Colgain, Rodriguez-Gomez, Sftesos '12], [Apruzzi, Fazzi, Passias, Rosa, Tomasiello '14], [Kim, Kim, Suh '15], [D'Hoker, Gutperle, Karch, Uhlemann '16] .
 - Minimal consistent truncation?
 - Vector multiplets?
- Systematics? Simplified construction? Consistent truncations?

Why ExFT?

- 10-/11-d SUGRA = GR + fluxes + spinors
- SUSY vacua $Mink_D / AdS_D \times M_{int} \Rightarrow$ On M_{int}

$$\delta_\epsilon \psi \sim \nabla \epsilon + \not{F} \epsilon = 0.$$

- Generic properties for $Mink_D$ & $F = 0$ are well-understood:
 - Special holonomy (Calabi-Yau, G_2 , etc.)
 - Moduli \rightarrow cohomology of M_{int}
 - Effective action from integrating out massive modes
- SUSY AdS vacua have $F \neq 0$
 - Special holonomy??
 - Moduli – very difficult problem
 - Consistent truncations?

Why ExFT?

- 10-/11-d SUGRA = GR + fluxes + spinors
- SUSY vacua $Mink_D / AdS_D \times M_{int} \Rightarrow$ On M_{int}

$$\delta_\epsilon \psi \sim \nabla \epsilon + \not{F} \epsilon = \nabla_{ExFT} \epsilon = 0.$$

- Generic properties for $Mink_D$ & $F = 0$ are well-understood:
 - Special holonomy (Calabi-Yau, G_2 , etc.)
 - Moduli \rightarrow cohomology of M_{int}
 - Effective action from integrating out massive modes
- SUSY AdS vacua have $F \neq 0$
 - Special holonomy??
 - Moduli – very difficult problem
 - Consistent truncations?
- Unify fluxes + geometry into “larger” geometry \rightarrow ExFT

Half-maximal SUSY in ExFT

- Half-maximal spinors define natural structures in ExFT [EM '16], [EM' 17]

D	7	6
$E_{d(d)}$	SL(5)	Spin(5, 5)
\tilde{K}_d	USp(4)	USp(4) \times USp(4)
Spinors	4	(4, 1) \oplus (1, 4)
G_{stab}	SU(2)	SU(2) \times SU(2)
G_R	SU(2)	SU(2) \times SU(2)

- Spinor bilinears define “differential forms”:

$$J_u{}^M \in \Gamma(\mathcal{R}_1), \quad \hat{K}_{MN} \in \Gamma(\mathcal{R}_{D-2}),$$

$$u = 1, \dots, d-1 \text{ of } G_R = \text{Spin}(d-1)_R,$$

Half-maximal SUSY in ExFT

- Half-maximal spinors define natural structures in ExFT [EM '16], [EM' 17]

D	7	6
$E_{d(d)}$	SL(5)	Spin(5, 5)
\tilde{K}_d	USp(4)	USp(4) × USp(4)
Spinors	4	(4, 1) ⊕ (1, 4)
$G_{stab} = \text{Spin}(d - 1)$	SU(2)	SU(2) × SU(2)
$G_R = \text{Spin}(d - 1)_R$	SU(2)	SU(2) × SU(2)

- Spinor bilinears define “differential forms”:

$$J_u{}^M \in \Gamma(\mathcal{R}_1), \quad \hat{K}_{MN} \in \Gamma(\mathcal{R}_{D-2}),$$

$$u = 1, \dots, d - 1 \text{ of } G_R = \text{Spin}(d - 1)_R,$$

Half-maximal SUSY in ExFT

- Half-maximal spinors define natural structures in ExFT [EM '16], [EM' 17]

D	7	6
$E_{d(d)}$	SL(5)	Spin(5, 5)
\tilde{K}_d	USp(4)	USp(4) \times USp(4)
Spinors	4	(4, 1) \oplus (1, 4)
$G_{stab} = \text{Spin}(d - 1)$	SU(2)	SU(2) \times SU(2)
$G_R = \text{Spin}(d - 1)_R$	SU(2)	SU(2) \times SU(2)

- Spinor bilinears define “differential forms”:

$$J_u^M = v_u + \text{differential forms}, \quad \hat{K}_{MN} = \text{differential forms},$$

$$u = 1, \dots, d - 1 \text{ of } G_R = \text{Spin}(d - 1)_R,$$

Half-maximal SUSY in ExFT

- Half-maximal spinors define natural structures in ExFT [EM '16], [EM' 17]

D	7	6
$E_{d(d)}$	SL(5)	Spin(5, 5)
\tilde{K}_d	USp(4)	USp(4) \times USp(4)
Spinors	4	(4, 1) \oplus (1, 4)
$G_{stab} = \text{Spin}(d - 1)$	SU(2)	SU(2) \times SU(2)
$G_R = \text{Spin}(d - 1)_R$	SU(2)	SU(2) \times SU(2)

- Spinor bilinears define “differential forms”:

$$J_u^M = v_u + \text{differential forms}, \quad \hat{K}_{MN} = \text{differential forms},$$

$u = 1, \dots, d - 1$ of $G_R = \text{Spin}(d - 1)_R$, satisfying

$$\left(\delta_u^w \delta_v^x - \frac{1}{d-1} \delta_{uv} \delta^{wx} \right) J_w^M J_x^N Y_{MN}^{PQ} = 0,$$

$$\delta^{uv} J_u^M J_v^N \hat{K}_{MN} > 0, \quad \hat{K} \times_{R_c} \hat{K} = 0. \quad \left(R_c = \begin{cases} \emptyset, & D=7 \\ \mathbf{1}, & D=6 \end{cases} \right)$$

Half-maximal background in ExFT

- SUGRA fields captured by generalised metric (c.f. Henning Samtleben's talk)

[EM, Samtleben, Vall Camell '18]

$$\{\text{SUGRA fields}\} = \mathcal{M}_{MN} \sim J_u^P J^{u,Q} \hat{K}_{MP} \hat{K}_{NQ} + \hat{K}_{MN} \\ + \epsilon^{u_1 \dots u_{d-1}} (J_{u_1} \dots J_{u_{d-1}})_{MN} .$$

- Spin($d - 1$)_R invariant combination of J_u and \hat{K} .

BPS equations

- BPS conditions \Leftrightarrow differential conditions on J_u , \hat{K} :

[EM' 17]

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w, \quad \mathcal{L}_{J_u} \hat{K} = 0,$$

$R_{uvw} R^{uvw} = -\Lambda$, and

- $D = 7$: $d\hat{K}^{MN} = \epsilon^{uvw} R_{uvw} J^{xP} J_x^Q Y_{PQ}^{MN}$,
- $D = 6$: $d\hat{K} = \epsilon^{uvw} R_{uvw} J_x$,

BPS equations

- BPS conditions \Leftrightarrow differential conditions on J_u , \hat{K} :

[EM' 17]

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w, \quad \mathcal{L}_{J_u} \hat{K} = 0,$$

$R_{uvw} R^{uvw} = -\Lambda$, and

- $D = 7$: $d\hat{K}^{MN} = \epsilon^{uvw} R_{uvw} J^{xP} J_x^Q Y_{PQ}^{MN}$,
- $D = 6$: $d\hat{K} = \epsilon^{uvw} R_{uvw} J_x$,
- R_{uvw} breaks $\text{Spin}(d-1)_R \rightarrow \text{SU}(2)$ R-symmetry of AdS vacuum

BPS equations

- BPS conditions \Leftrightarrow differential conditions on J_u , \hat{K} :
[EM' 17]

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w, \quad \mathcal{L}_{J_u} \hat{K} = 0,$$

$$R_{uvw} R^{uvw} = -\Lambda, \text{ and}$$

- $D = 7$: $d\hat{K}^{MN} = \epsilon^{uvw} R_{uvw} J^x P J_x^Q Y_{PQ}^{MN}$,
- $D = 6$: $d\hat{K} = \epsilon^{uvw} R_{uvw} J_x$,
- R_{uvw} breaks $\text{Spin}(d-1)_R \rightarrow \text{SU}(2)$ R-symmetry of AdS vacuum
- **R-symmetry must be realised by isometries of M_{int} .**
c.f. [Ashmore, Petrini, Waldram '16], [Coimbra, Strickland-Constable '17]
- $J_u \supset$ Killing vector fields, generate R-symmetry
- \hat{K} invariant under R-symmetry

AdS consistent truncation: “minimal”, no matter

- [EM '16], [EM '17] : **Proof of 1/2-max version of** [Gauntlett, Varela '07]
- Y coordinates on M_{int} , x coordinates on M_D .
- Given $J_u(Y)$, $\hat{K}(Y)$ of 1/2-max AdS, the linear Ansatz

$$\begin{aligned} \mathcal{J}_u(x, Y) &= X^{-1}(x) J_u(Y), & \hat{\mathcal{K}}(x, Y) &= X^2(x) \hat{K}(Y), \\ \mathcal{A}_\mu(x, Y) &= A_\mu{}^u(x) J_u(Y), & \dots, & \end{aligned}$$

gives a *consistent* truncation.

- Consistency follows from BPS conditions of J_u , \hat{K} .
- $X(x)$ scalar, $A_\mu{}^u(x)$ $d - 1$ vector fields of grav. supermultiplet.
- “Minimal” truncation \Rightarrow no matter multiplets.
- Spacetime fields from $\mathcal{M}_{MN} \longrightarrow$ non-linear truncation Ansatz.

Consistent truncations with vector multiplets

[EM '16], [EM '17]

- Half-maximal SUSY \Rightarrow N vector multiplets $\mathcal{M}_{scalar} = \frac{SO(d-1, N)}{SO(d-1) \times SO(N)} \times \mathbb{R}^+$
- Need extra tensors to expand in:

$$\mathcal{J}_u(x, Y) = \phi_I(x) T^I(Y), \quad \dots$$

- $G_{stab}(T^I)$ dictates scalar manifold:

$$\mathcal{M}_{scalar} = \frac{Comm(G_{stab}, E_{d(d)})}{Comm(G_{stab}, K(E_{d(d)}))} \Rightarrow G_{stab} = Spin(d - 1 - N).$$

Consistent truncations with vector multiplets

[EM '16], [EM '17]

- Half-maximal SUSY \Rightarrow N vector multiplets $\mathcal{M}_{scalar} = \frac{SO(d-1, N)}{SO(d-1) \times SO(N)} \times \mathbb{R}^+$
- Need extra tensors to expand in:

$$\mathcal{J}_u(x, Y) = \phi_I(x) T^I(Y), \quad \dots$$

- $G_{stab}(T^I)$ dictates scalar manifold:

$$\mathcal{M}_{scalar} = \frac{Comm(G_{stab}, E_{d(d)})}{Comm(G_{stab}, K(E_{d(d)}))} \Rightarrow G_{stab} = Spin(d-1-N).$$

- $d-1+N$ generalised vector fields:

$$J_A^M = (J_u^M, J_{\bar{u}}^M) \in \Gamma(\mathcal{R}_1), \quad \hat{K}_{MN} \in \Gamma(\mathcal{R}_{D-3}),$$

$A = (u, \bar{u}) = 1, \dots, d-1+N$ of $SO(d-1, N)$, s.t.

$$\left(\delta_A^C \delta_B^D - \frac{1}{d-1+N} \eta_{AB} \eta^{CD} \right) J_C^M J_D^N Y_{MN}^{PQ} = 0, \quad \hat{K} \times_{R_c} \hat{K} = 0, \quad \eta^{AB} J_A^M J_B^N \hat{K}_{MN} > 0.$$

- **Can only have $N \leq d-1$ vector multiplets.**

Truncation Ansatz with vector multiplets

[EM '16], [EM '17]

- Consistent truncation

$$\mathcal{J}_u(x, Y) = X^{-1}(x) b_u^A(x) J_A(Y), \quad \hat{\mathcal{K}}(x, Y) = X^2(x) \hat{K}(Y),$$

$$\mathcal{A}_\mu(x, Y) = A_\mu^A(x) J_A(Y), \quad \dots,$$

- $b_u^A(x)$ constrained by

$$b_u^A b_v^B \eta_{AB} = \delta_{uv} \Rightarrow b_u^A \in \frac{\text{SO}(d-1, N)}{\text{SO}(d-1) \times \text{SO}(N)}.$$

- Consistency requires differential conditions

$$\mathcal{L}_{J_A} J_B = f_{AB}^C J_C, \quad \text{with } f_{AB}^C \text{ constant,}$$

$$\mathcal{L}_{J_A} \hat{K} = 0.$$

- AdS vacua: $f_{uvw} = R_{uvw}$, $f_{uv\bar{w}} = 0$, $J_{\bar{u}}$ organise into reps of $\text{SU}(2)_R$
R-symmetry \rightarrow can fully classify!

Applications:
AdS₇ of mIIA
AdS₆ of IIB.

[EM, Samtleben, Vall Camell '18] : Solutions & derive “minimal” consistent truncation

[EM, Samtleben, Vall Camell '19] : Classify and construct consistent truncations with vector multiplets

Constructing AdS solutions from ExFT

- Geometric Ansatz $\rightarrow J_u, \hat{K}$
 - Algebraic conditions (existence of spinors)
 - Differential conditions (SUSY AdS vacuum)
- Construct generalised metric
$$\mathcal{M}_{MN} = \hat{J}_{u,M} \hat{J}^u{}_N - \hat{K}_{MN} + (J^{d-1})_{MN}.$$
- ExFT dictionary: $\mathcal{M}_{MN} = \{\text{SUGRA fields}\}$.
- Minimal consistent truncation for free.
- Classify vector multiplets.

Example 1: AdS₇ from ExFT

[EM, Samtleben, Vall Camell '18]

- AdS₇ of mIIA in SL(5) ExFT.
- SU(2)_R must be realised by isometries of M_{int}
 - $M_{int} = S^2 \times I \quad (S^2 \rightarrow SU(2)_R)$
 - $M_{int} = S^3/\mathbb{Z}_k \quad (S^3/\mathbb{Z}_k \rightarrow SU(2)_R \times U(1))$
- 3 J_u , 1 \hat{K} satisfying algebraic conditions and

$$\mathcal{L}_{J_u} J_v = \sqrt{-\Lambda} \epsilon_{uvw} J^w, \quad \mathcal{L}_{J_u} \hat{K} = 0, \quad d\hat{K} = \sqrt{-\Lambda} J_u \wedge J^u.$$

- Write most general $J_u \rightarrow SU(2)_R$ triplets, $\hat{K} \rightarrow SU(2)_R$ singlet.

$$J_u \in \Gamma(TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M),$$

$$\hat{K} \in \Gamma(\Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M).$$

$$J_u = v_u + \dots$$

Example 1: AdS₇ from ExFT

[EM, Samtleben, Vall Camell '18]

- AdS₇ of mIIA in SL(5) ExFT.
- SU(2)_R must be realised by isometries of M_{int}
 - $M_{int} = S^2 \times I$ ($S^2 \rightarrow SU(2)_R$)
 - ~~$M_{int} = S^3/\mathbb{Z}_k$~~ (~~$S^3/\mathbb{Z}_k \rightarrow SU(2)_R \times U(1)$~~)
- 3 J_u , 1 \hat{K} satisfying algebraic conditions and

$$\mathcal{L}_{J_u} J_v = \sqrt{-\Lambda} \epsilon_{uvw} J^w, \quad \mathcal{L}_{J_u} \hat{K} = 0, \quad d\hat{K} = \sqrt{-\Lambda} J_u \wedge J^u.$$

- Write most general $J_u \rightarrow SU(2)_R$ triplets, $\hat{K} \rightarrow SU(2)_R$ singlet.

$$J_u \in \Gamma(TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M),$$

$$\hat{K} \in \Gamma(\Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M).$$

$$J_u = v_u + (g(z)dy_u + k(z)y_u dz + l(z)\epsilon_{uvw}y^v dy^w) + \dots, \quad y_u y^u = 1.$$

G_{half} structure on $S^2 \times I$

[EM, Samtleben, Vall Camell '18]

- Algebraic conditions $\rightarrow J_u, \hat{K}$ depend on 5 functions on I : $g(z), h(z), q(z), s(z), t(z)$; $t(z)h(z) \geq 0$.
- Differential conditions:

$$\dot{g}(z) = -\frac{h(z)}{q(z)}, \quad 2q(z)\dot{q}(z) = mh(z),$$

$$\dot{s}(z) = h(z), \quad \dot{t}(z) = -\frac{h(z)s(z)}{q(z)}.$$

- Gauge choice $h(z) = q(z)$, wlog

$$g(z) = -z, \quad s(z) = -\dot{t}(z), \quad q(z) = -\ddot{t}(z),$$

$$\text{and } \ddot{\dot{t}}(z) = -m/2.$$

AdS₇ solutions

[EM, Samtleben, Vall Camell '18]

Generalised metric $\mathcal{M}(J_u, \hat{K})$ and EFT \leftrightarrow IIA SUGRA dictionary:

$$ds_{10}^2 = 4\sqrt{-\frac{t}{\ddot{t}}} ds_{AdS_7}^2 + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}} \left(\frac{t^2}{\dot{t}^2 - 2\ddot{t}t} ds_{S^2}^2 + dz^2 \right),$$

$$e^\psi = \left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{\dot{t}^2 - 2\ddot{t}t}},$$

$$B_2 = \frac{1}{2\sqrt{2}} \left(z - \frac{\dot{t}t}{\dot{t}^2 - 2\ddot{t}t} \right) vol_2,$$

$$F_2 = \frac{1}{2\sqrt{2}} \left(2\ddot{t} + \frac{m\dot{t}t}{\dot{t}^2 - 2\ddot{t}t} \right) vol_2,$$

with $\ddot{t} = -m/2$, $t \geq 0$ with equality at ∂I .These are the general SUSY AdS₇ solutions of mIIA [Apruzzi, Fazzi, Rosa, Tomasiello] in coords of [Cremonesi, Tomasiello].

AdS₇ minimal consistent truncation

- Consistent truncation for AdS₇ vacua [EM, Samtleben, Vall Camell '18].

$$\mathcal{J}_u(x, Y) = X^{-1}(x)J_u(Y), \quad \hat{K}(x, Y) = X^2(x)\hat{K}(Y):$$

$$ds_{10}^2 = 4\sqrt{-\frac{t}{\ddot{t}}}X^{1/2}ds_7^2 + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}}\left[X^{-5/2}dz^2 + X^{5/2}\frac{t^2}{\dot{t}^2X^5 - 2\ddot{t}t}ds_{S^2}^2\right],$$

$$e^\psi = X^{5/4}\left(-\frac{t}{\ddot{t}}\right)^{3/4}\frac{1}{\sqrt{X^5\dot{t}^2 - 2\ddot{t}t}},$$

$$B_2 = \frac{1}{2\sqrt{2}}\left(z - \frac{\dot{t}tX^5}{\dot{t}^2X^5 - 2\ddot{t}t}\right)vol_{S^2},$$

$$F_2 = \frac{1}{2\sqrt{2}}\left(2\ddot{t} + X^5\frac{m\dot{t}t}{\dot{t}^2X^5 - 2\ddot{t}t}\right)vol_{S^2},$$

$$\ddot{\dot{t}} = -m/2.$$

- Reproduces [Passias, Rota, Tomasiello].
- Explains universality of consistent truncation!

Example 2: AdS₆ of IIB from ExFT

[EM, Samtleben, Vall Camell '18]

- AdS₆ of IIB in SO(5, 5) ExFT. $SU(2)_R$ must be realised by isometries of M_{int}
 - $M_{int} = S^2 \times \Sigma_2 \quad (S^2 \rightarrow SU(2)_R)$
 - $M_{int} = S^3/\mathbb{Z}_k \times I \rightarrow (S^3/\mathbb{Z}_k \rightarrow SU(2)_R \times U(1))$
- Write most general Ansatz for J_u, \hat{K}
- Algebraic + differential conditions + gauge choice: AdS₆ solutions defined by pair of harmonic functions $f^\alpha = -p^\alpha + ik^\alpha$ on Σ_2 with

$$df^\alpha \wedge d\bar{f}_\alpha \geq 0, \quad (\text{equality on } \partial\Sigma_2).$$

AdS₆ solutions of IIB

[EM, Samtleben, Vall Camell '18]

- Compute SUGRA fields from $\mathcal{M}_{MN}(J_u, \hat{K})$:

$$ds^2 = \frac{\sqrt{2} r^{5/4} \Delta^{1/4}}{3^{3/4} |dk|^{1/4}} \left(\frac{12}{r} ds_{AdS_6}^2 + \frac{|dk|}{\Delta} ds_{S^2}^2 + \frac{4}{r^2} dk^\alpha \otimes dp_\alpha \right), \quad C_{(4)} = 0,$$

$$C_{(2)}^\alpha = -\frac{4}{3} vol_{S^2} \left(k^\alpha + \frac{r p_\gamma \partial_\beta k^\gamma \partial^\beta p^\alpha}{2\Delta} |dk|^{1/2} \right), \quad H_{\alpha\beta} = \frac{|dk|^{1/2} p_\alpha p_\beta + 6 r \partial_\gamma k_\alpha \partial^\gamma p_\beta}{2\sqrt{3} \Delta r},$$

$$\Delta = \frac{3}{4} r |dk| + \frac{1}{2} |dk|^{1/2} p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta, \quad H_{\alpha\beta} = \frac{1}{\text{Im } \tau} \begin{pmatrix} |\tau|^2 & \text{Re } \tau \\ \text{Re } \tau & 1 \end{pmatrix},$$

$$r = -p_\alpha dk^\alpha.$$

- Matches [D'Hoker, Gutperle, Karch, Ulhemann 2016] upon field redefinition
- Consistent truncation??

AdS₆ minimal consistent truncations

- Find new minimal consistent truncation for free

[EM, Samtleben, Vall Camell '18]:

$$\mathcal{J}_u(x, Y) = X^{-1}(x) J_u(Y), \quad \hat{\mathcal{K}}(x, Y) = X^2(x) \hat{K}(Y).$$

- From generalised metric and ExFT/IIB dictionary:

$$ds^2 = \frac{\sqrt{2} r^{5/4} \Delta^{1/4}}{3^{3/4} |dk|^{1/4}} \left(\frac{12}{r} ds_6^2 + \frac{X^2 |dk|^{1/2}}{\Delta} ds_{S^2}^2 + \frac{4}{X^2 r^2} dk^\alpha \otimes dp_\alpha \right), \quad C_{(4)} = 0,$$

$$C_{(2)}^\alpha = -\frac{4}{3} \text{vol}_{S^2} \left(k^\alpha + \frac{X^4 r p_\gamma \partial_\beta k^\gamma \partial^\beta p^\alpha}{2\Delta} |dk|^{1/2} \right), \quad H_{\alpha\beta} = \frac{X^4 |dk|^{1/2} p_\alpha p_\beta + 6 r \partial_\gamma k_\alpha \partial^\gamma p_\beta}{2\sqrt{3} \Delta r},$$

$$\Delta = \frac{3}{4} r |dk| + \frac{1}{2} X^4 |dk|^{1/2} p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta.$$

- Universal consistent truncation.
- Allows us to study AdS₆ solution using minimal 6d gSUGRA.
- c.f. [Hong, Liu, Mayerson '18] .

Minimal consistent truncation vs vector multiplets

- For AdS₇, AdS₆, “minimal” consistent truncations universal for whole family.
- All AdS₇, AdS₆ vacua are same vacuum in minimal 7-D/6-D SUGRA!
- Cannot construct flows between different AdS_{6,7} vacua.
- Consistent truncation with vector multiplets to differentiate? [De Luca, Gnechhi, Lo Monaco, Tomasiello '18]

Consistent truncations with vector multiplets for AdS₇

[EM, Samtleben, Vall Camell '19]

- Need N additional generalised vector fields s.t.

$$J_A^M J_B^N Y_{MN}^{PQ} = \frac{1}{d-1+N} \eta_{AB} \eta^{CD} J_C^M J_D^N Y_{MN}^{PQ},$$

$$\mathcal{L}_{J_A} J_B = f_{AB}{}^C J_C,$$

$$\mathcal{L}_{J_A} \hat{K} = 0.$$

- $N \leq d - 1$ vector multiplets; organise themselves into reps of $SU(2)_R$ symmetry.
- Fully classify possible consistent truncations with vector multiplets around AdS₇:
 - **NO** consistent truncations with vector multiplets when $m \neq 0$
 - 1 vector multiplet when $m = 0$.

Consistent truncations with vector multiplets for AdS₆

[EM, Samtleben, Vall Camell '19]

- Fully classify possible consistent truncations with vector multiplets around AdS₆:

N	SU(2) _R rep	Consistent truncation
1	1	Only if $\exists \chi: \partial(e^{i\chi}\partial f^\alpha) \in \text{Real functions on } \Sigma_2$
2	1 \oplus 1	NO (no globally regular solutions)
3	1 \oplus 1 \oplus 1	NO
3	3	Only if $d\pi^\alpha = 0$
4	3 \oplus 1	Only if $\exists \mathbf{3}$ and $\exists \mathbf{1}$ with $e^{i\chi} = \left(\frac{p_\alpha \bar{\partial} \bar{f}^\alpha}{p_\beta \partial f^\beta}\right)$

$$\pi^\alpha = \frac{1}{2} r^2 (g \partial f^\alpha d\bar{z} + \bar{g} \bar{\partial} \bar{f}^\alpha dz), \quad g = i \left(\frac{p_\alpha \bar{\partial} \bar{f}^\alpha}{p_\beta \partial f^\beta} \right), \quad dr = -p_\alpha dk^\alpha$$

- Easily construct full non-linear truncation Ansätze.
- Can use results to uplift 6-D $F(4)$ gSUGRA results to IIB, e.g. [Gutperle, Kaidi, Raj '18], [Gutperle, Kaidi, Raj '17]

AdS₆ uplift formulae for 1 vector multiplet

Scalar fields $\in \frac{SO(4,1)}{SO(4)} \times \mathbb{R}^+$:

X and $m_A = (m_I, m_4, m_5)$, s.t. $\eta^{AB} m_A m_B = -1$, $\eta_{AB} = \text{diag}(1, 1, 1, 1, -1)$.

$$ds^2 = \frac{r^{5/4} \lambda^{3/2}}{2^{5/4} \bar{\Delta}^{3/4}} \left[\frac{2\sqrt{2}}{r \lambda^2} ds_6^2 + X^2 \left(ds_{S^2}^2 + w \otimes w - \frac{1}{r^2} p_\alpha p_\beta n^\alpha \otimes n^\beta \right) + \frac{2}{X^2 r} n_\alpha \otimes X^\alpha + \frac{4\tilde{\Delta}}{X^2 r^2 \lambda^2} dk^\alpha \otimes dp_\alpha \right],$$

$$C_{(2)}^\alpha = -\text{vol}_{S^2} \left(k^\alpha + \frac{X^4 r \lambda}{2\bar{\Delta}} p_\beta \left[m_5 \partial_\gamma k^\beta \partial^\gamma p^\alpha + n^{\beta\gamma} \omega^\alpha_\gamma \right] \right) + \frac{\lambda^2}{4\bar{\Delta}} \left(2r [m_5 n^\alpha - \omega^\alpha] - X^4 p^\alpha p_\beta \star n^\beta \right) \wedge dy^I y^J m^K \epsilon_{IJK},$$

$$H^{\alpha\beta} = \frac{X^4 p^\alpha p^\beta \lambda + 4r (m_5 \partial_\gamma k^\alpha \partial^\gamma p^\beta + n^{\alpha\gamma} \omega^\beta_\gamma)}{2\sqrt{2} r \bar{\Delta}},$$

$$\omega^\alpha = m_I y^I dk^\alpha + m_4 dp^\alpha, \quad X^\alpha = m_I y^I dp^\alpha - m_4 dk^\alpha, \quad w = m_I dy^I + \frac{p_\alpha}{r} (m_5 n^\alpha - \omega^\alpha),$$

$$\bar{\Delta} = \frac{1}{2} r \lambda^2 \left(m_5^2 - m_4^2 - (m_I y^I)^2 \right) + \frac{1}{2} X^4 \lambda p_\alpha p_\beta \left(m_5 \partial_\gamma k^\alpha \partial^\gamma p_\alpha + n^{\alpha\gamma} \omega^\beta_\gamma \right),$$

$$\tilde{\Delta} = \frac{1}{2} r m_5 \lambda^2 + \frac{1}{2} X^4 \lambda p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta, \quad n^\alpha = \frac{1}{2} \left(e^{i\chi} \partial f^\alpha d\bar{z} + \text{c.c.} \right).$$

Conclusions

- Exceptional geometry underlying SUSY AdS vacua.
- ExFT useful new tool to construct AdS vacua.
- Reproduce all mIIA AdS₇ × S³, IIB AdS₆ × S² × Σ₂ solutions.
- Reproduce / discover consistent truncation (grav. multiplet).
- Classification and construction of consistent truncations with vector multiplets.
- Useful for studying & finding new solutions (e.g. domain-wall solutions for holographic RG flow, ...)
- **Caveat:** consistent truncations beyond ExFT? Higher-derivative corrections? [De Luca, Guechi, Lo Monaco, Tomasiello '18]

Outlook

- Lower dimensions.
- Study/uplift of moduli, e.g. $\mathcal{N} = 4$ AdS₅ vacua. Zamolodchikov metric, ...
- Construct new AdS vacua.
- Classification of AdS vacua?
- Other amounts of SUSY, e.g. $\mathcal{N} = 2$ [Ashmore, Gabella, Graña, Petrini, Waldram '16].

In the absence of extra isometries beyond R-symmetry, infinitesimal deformations (marginal) → finite deformations (exactly marginal).

- Other holographic properties: “a” / “c”-minimisation, etc.