Depolarisation at e+e- colliders due to strong fields at the interaction point

A. Hartin

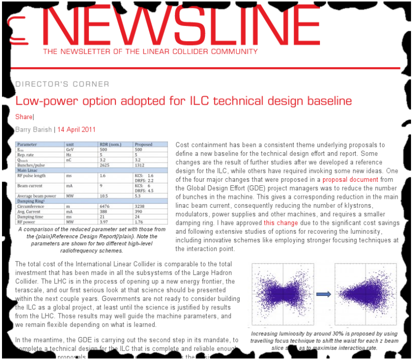
DESY

EUCARD Workshop
Spin Optimisation at Lepton Colliders
Mainz, Feb 13, 2014

Synopsis

1. The Low power option and luminosity recovery
2. 1st order IP depolarisation processes
3. Depolarisation and the waist shift
4. Higher order depolarisation processes
5. Theoretical development - Furry picture & new solutions
6. Beamstrahlung process rexamined
7. Issues in second order strong field processes
8. We are in an era of experimental tests of this phenomenology
9. A new strong field event generator

Luminosity recovery



NEWSLINE

THE NEWSLETTER OF THE LINEAR COLLIDER COMMUNITY

DIRECTOR'S CORNER

Low-power option adopted for ILC technical design baseline

Share|

Barry Barish | 14 April 2011

Parameter	unit	RDR (nom.)	Proposed
E_{cm}	GeV	500	500
Rep. rate	Hz	5	5
Q_{bunch}	nC	3.2	3.2
Bunches/pulse		2625	1312
Main Linac			
RF pulse length	ms	1.6	KCS: 1.6 DRFS: 2.2
Beam current	mA	9	KCS: 6 DRFS: 4.5
Average beam power	MW	10.5	5.3
Damping Ring[1]			
Circumference	m	6476	3238
Avg. Current	mA	388	390
Damping time	ms	21	24
RF power	MW	3.97	1.76

A comparison of the reduced parameter set with those from the {plain}Reference Design Report{/plain}. Note the parameters are shown for two different high-level radiofrequency schemes.

Cost containment has been a consistent theme underlying proposals to define a new baseline for the technical design effort and report. Some changes are the result of further studies after we developed a reference design for the ILC, while others have required invoking some new ideas. One of the four major changes that were proposed in a proposal document from the Global Design Effort (GDE) project managers was to reduce the number of bunches in the machine. This gives a corresponding reduction in the main linac beam current, consequently reducing the number of klystrons, modulators, power supplies and other machines, and requires a smaller damping ring. I have approved this change due to the significant cost savings and following extensive studies of options for recovering the luminosity, including innovative schemes like employing stronger focusing techniques at the interaction point.

The total cost of the International Linear Collider is comparable to the total investment that has been made in all the subsystems of the Large Hadron Collider. The LHC is in the process of opening up a new energy frontier, the terascale, and our first serious look at that science should be presented within the next couple years. Governments are not ready to consider building the ILC as a global project, at least until the science is justified by results from the LHC. Those results may well guide the machine parameters, and we remain flexible depending on what is learned.

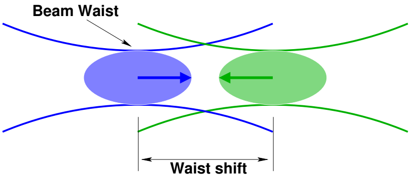
Increasing luminosity by around 30% is proposed by using travelling focus technique to shift the waist for each z beam slice so as to maximise interaction rate.

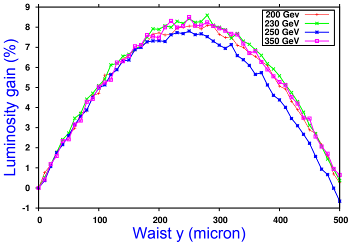
In the meantime, the GDE is carrying out the second step in its mandate, to complete a technical design for the ILC that is complete and reliable enough ... proposals ... the ...

- Low power option adopted as new baseline
  - Damping ring half the size
  - Half the RF power required
  - Luminosity decrease
- Luminosity to be recovered by travelling focus - difficult to implement
- How else to recover luminosity?

beam waist shifts I - EDMS study

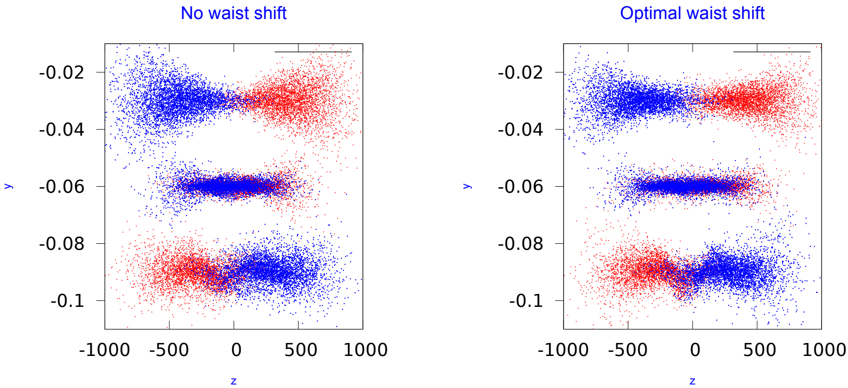
http://ilc-edmsdirect.desy.de/ilc-edmsdirect/item.jsp?edmsid=D00000000973835,B,1,2





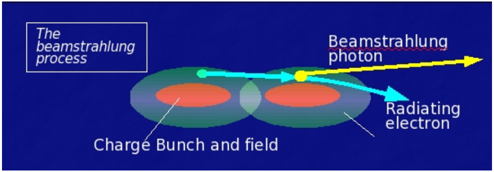
- final quads focus the e+e- beams to the nominal interaction point
- the beam waist shift is a shift in focus along the z-axis
- there are separate X and Y waist focal points
- Y Waist focus more important for flat beams - more particles in smaller space
- Luminosity will clearly vary with the beam waist shift
- EDMS study performed -Y waist optimisation
- 9% gain in luminosity for optimised Y Waist

beam waist shifts II



- Optimal waist shift, more focussed through collision, greater disruption
- Do the vertical fields (Upsilon parameter) increase?
- Does the depolarisation increase?

Strong fields at the collider Interaction Point



$$\Upsilon = \frac{e|\vec{a}|}{mE_{\text{cr}}}(k \cdot p)$$

$$\Upsilon_{\text{av}} \approx \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

$\Upsilon > 0.1$ **strong field regime**

- Υ depends on collider bunch parameters and the pinch effect
- Future linear colliders will have "strong" IP fields
- For polarised particles, beamstrahlung entails "spin-flip"

Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	1.74	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.335, 0.0027	0.045, 0.001
σ_z (mm)	20	1.1	0.225	0.044
Υ_{av}	0.00015	0.001	0.2	4.9

1st order strong field depolarisation processes

Spin precession

Beam field strengths

Spin vector

$$\frac{d\boldsymbol{s}}{dt} = -\left[(a+\gamma^{-1})(\boldsymbol{B} - \boldsymbol{v}\times\boldsymbol{E}) - \boldsymbol{v}\frac{a\gamma}{\gamma+1}\boldsymbol{v}\cdot\boldsymbol{B}\right]\times\boldsymbol{s}$$

Anomalous magnetic moment in external field

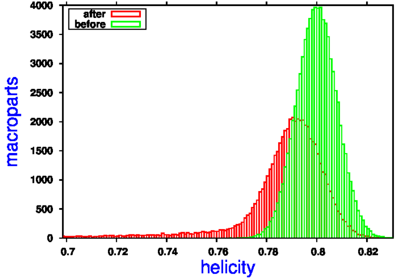
Spin-flip process

Constant crossed field – Airy functions

Spin vector

$$W(\Upsilon,\xi) = \frac{\alpha m_e^2}{\pi\epsilon}\int_0^\infty \frac{du}{(1+u)^3}\left[\frac{e}{m^3}F^{*\mu\nu}p_\mu s_\nu \frac{z\mathrm{Ai}(z)}{1+u} - \mathrm{Ai}_1(z) - \frac{2+2u+u^2}{z\,(1+u)}\mathrm{Ai}'(z)\right]$$

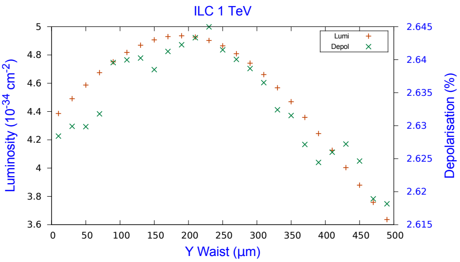
beam field tensor

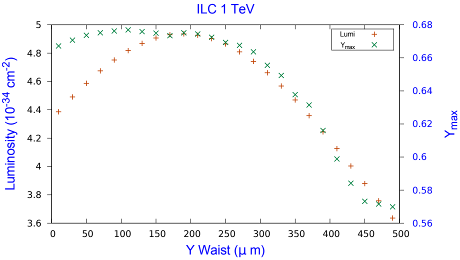


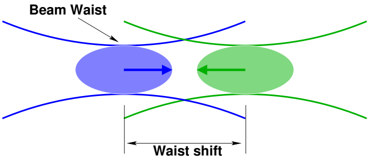
Machine	ILC	ILC	CLIC
E (GeV)	250	500	1500
Υ_{av}	0.1	0.27	4.9
Υ_{max}	0.26	0.66	11.8
ΔP	1.49%	2.64%	4.8%

- Simulation with Guinea-Pig++
- ΔP is the final depolarisation, E is the beam energy
- For gaussian beams $\Upsilon_{max} \approx \frac{12}{5}\Upsilon_{av}$

Depolarisation and waist shift

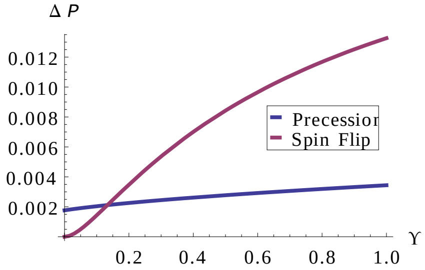






- 1 TeV parameter set has the strongest fields/effects
- Y waist=0 is the nominal
- Y waist=190 optimal
- Υ about a 3% increase
- ΔP about a 0.02% increase
- so, depol under control?

Depolarisation and field stength



$$\Delta P(\text{precession}) = 0.006\frac{n_\gamma}{U_0}$$

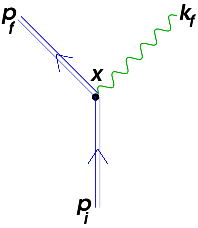
$$\Delta P(\text{spinflip}) = n_\gamma \frac{U_0}{U_f}$$

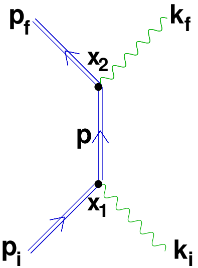
$$n_\gamma = \text{beamstr photons per particle}$$

$$U_0, U_f = \text{functions of} \Upsilon$$

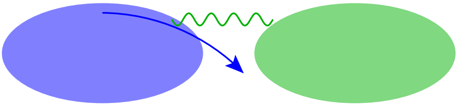
- Approximate formulae for 1st order effects
- $n_\gamma = 1.97$ for ILC 1TeV
- Spin flip contribution indicates the relative influence of quantum non-linear effects
- At $\Upsilon_{\text{av}} = 0.2$ already quantum strong field effects dominate

Higher order beam-beam depolarisation processes

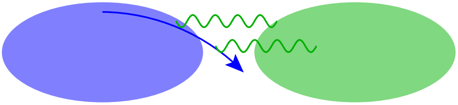




1st Order Effects

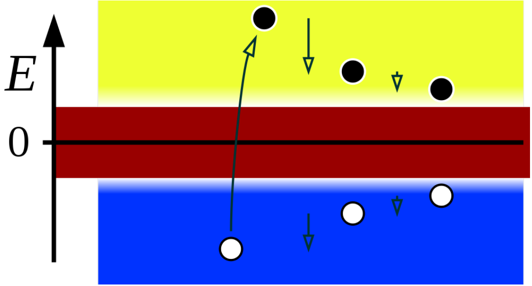


2nd Order Effects



- Average number of photons emitted per particle: 1.72 (500 GeV) 1.97 (1 TeV) - so second order effects possible
- coupling constant a function of α and Υ
- 2nd order effects contain strong field propagator
- How are virtual particles affected by the IP fields?

Vacuum changes due to intense fields

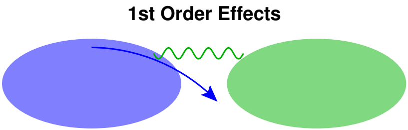


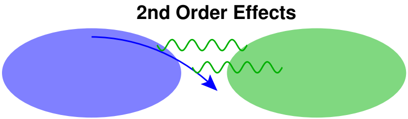
" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

- The Schwinger limit ($E_{\text{cr}} = 10^{18}$ V/m)
- Particles in future linear colliders will see $E \to E_{\text{cr}}$ (i.e. $\Upsilon \to 1$
- How do we incorporate these vacuum changes in a QFT $\to$ phenomenology?

What do we need to extend strong field analysis?





- We would like to account for the strong fields **exactly** $\rightarrow$ non-perturbative QFT
- Propagators are integrated over **all** momenta $\rightarrow$ solutions in two external fields
- Helicity amplitudes of second order process in two external fields
- **All** collider processes are strong field processes
- Event generation using new calculations in EM simulation of bunch collision
- Experimental validation using intense lasers

Furry Picture

Furry Picture

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar\psi(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar\psi(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar\psi^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar\psi^{\text{FP}} A\!\!\!/\,\psi^{\text{FP}}$$

Equations of Motion

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

Wavefunction

$$\psi^{\text{FP}} = E_p\, e^{-ipx}\, u_p, \qquad E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}} k\!\!\!/ + i2e(A^e p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Propagator

$$G^{\text{FP}} = \int \frac{d^4p}{(2\pi)^4} E_p(x)\frac{p\!\!\!/ + m}{p^2 - m^2}\bar E_p(x')\, e^{-ip\cdot x}\, u_p$$

(Volkov) Solution of the FP Dirac equation

Solution of the 2nd order Dirac equation with external 4-potential A_μ^{ext}

$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\text{FP}} = 0, \quad D_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$$

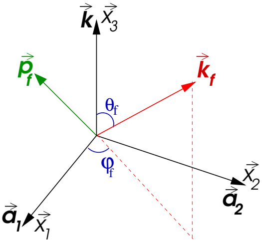
$$\psi^{\text{FP}} = e^{-i[p\cdot x + \$^p(k\cdot x)]}\, u_p$$

$$\$^p(k\cdot x) = \frac{1}{2(k\cdot p)}\int^{kx} \boxed{2eA^{\text{ext}}\cdot p - e^2 A^{\text{ext}\,2}} - \boxed{e\not{A}^{\text{ext}}\not{k}}$$

Volkov phase

Volkov spinor

Lorenz gauge with condition
$A^0 = 0 \implies \vec{a}_1 \perp \vec{a}_2 \perp \vec{k}$



(Volkov) Solution of the FP Dirac equation

Solution of the 2nd order Dirac equation with external 4-potential A_μ^{ext}

Lorenz gauge with condition $A^0 = 0 \implies \vec{a}_1 \perp \vec{a}_2 \perp \vec{k}$

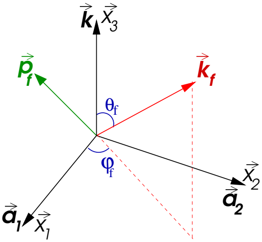
$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\text{FP}} = 0, \quad D_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$$

$$\psi^{\text{FP}} = e^{-i[p\cdot x + S^p(k\cdot x)]}\, u_p$$

$$S^p(k\cdot x) = \frac{1}{2(k\cdot p)}\int^{kx} \underbrace{2eA^{\text{ext}}\cdot p - e^2 A^{\text{ext}\,2}}_{\text{Volkov phase}} - \underbrace{e\not{A}^{\text{ext}}\not{k}}_{\text{Volkov spinor}}$$

Volkov phase

Volkov spinor



Orthornormality and Completeness of Volkov solutions **[Ritus, Ann Phys 69 552 (1971), Bergou and Varro, J Phys A 13 2823 (1980), Zakowicz JMathPhys 46 032304 (2005)]**

$$\int \frac{d^4x}{(2\pi)^4}\; e^{i[S^p(kx) - S^q(kx)]} = \delta^{(4)}(q-p)$$

$$\int \frac{d^4p}{(2\pi)^4}\; e^{i[S^p(kx) - S^p(ky)]} = \delta^{(4)}(x-y)$$

Volkov-type solutions

known solutions

- Single plane wave field **[Volkov, Z Phys 1935]**
- Circ/Linearly polarised field, constant field **[Nikishov and Ritus, JETP 1964]**
- Elliptically polarised field **[Lyulka, JETP 40 p815 1975]**
- 2 collinear orthogonal fields **[Lyulka 1975, Pardy 2004]**
- Coulomb fields + combinations **[Bagrov Gitman, Exact sols of Rel wave eqns 1990]**

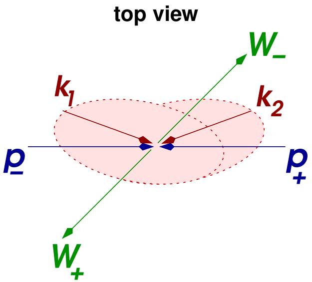
General procedure

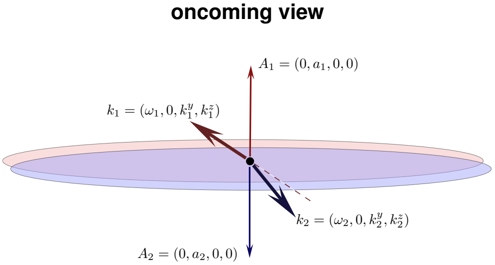
Klein-Gordon: $\left(D^2 + m^2\right)\phi_e = 0$ → Volkov phase

2nd order Dirac: $\left(D^2 + m^2 \pm \frac{ie}{2} F^{\mu\nu}\sigma_{\mu\nu}\right)\psi_e = 0$ → Volkov spinor

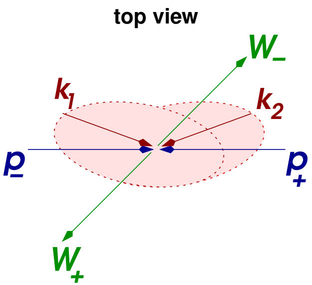
Dirac: $\left(i\not{D} - m\right)\psi_e = 0$ → particular solution

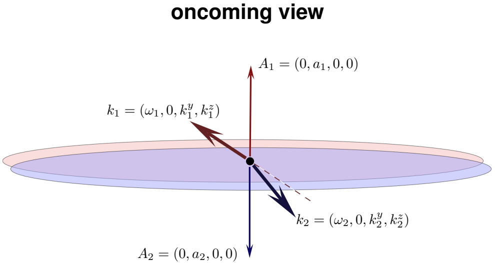
Solution of the FP Dirac equation in two fields





Solution of the FP Dirac equation in two fields



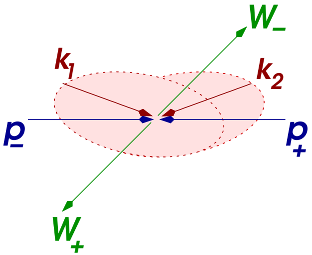


- Transition probabilities are covariant, so choose collinear $\vec{k}_1 \,||\, \vec{k}_2$ reference frame
- external field is a superposition; rewrite as orthogonal components

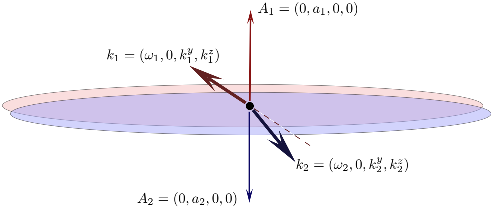
$$A_\mu = A_{1\mu}(k_1 \cdot x) + A_{2\mu}(k_2 \cdot x) \rightarrow A_{+\mu} + A_{-\mu} \quad \text{where} \quad A_+ \cdot A_- = 0$$

Solution of the FP Dirac equation in two fields

top view



oncoming view



- Transition probabilities are covariant, so choose collinear $\vec{k}_1 || \vec{k}_2$ reference frame
- external field is a superposition; rewrite as orthogonal components

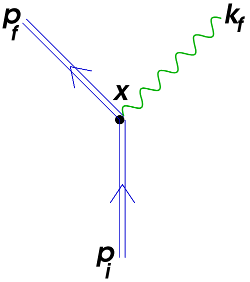
$$A_\mu = A_{1\mu}(k_1 \cdot x) + A_{2\mu}(k_2 \cdot x) \to A_{+\mu} + A_{-\mu} \quad \text{where} \quad A_+ \cdot A_- = 0$$

- solution is a product of Volkov solutions

$$\left[i\partial\!\!\!/ - eA\!\!\!/_+ - eA\!\!\!/_- - m\right] \psi^{\mathsf{FP}} = 0 \implies \psi^{\mathsf{FP}} = e^{-i\left[p\cdot x + \$^p_+ + \$^p_-\right]} u_r(p)$$

$$\text{where} \quad \$^p_+ = \int \frac{2eA_+(\phi)\cdot p - e^2 A_+(\phi)^2 - eA\!\!\!/_+(\phi)k\!\!\!/_1}{2k_1 \cdot p}\, d\phi$$

1st order Furry picture process and dressed vertex



FP Feynman diagrams only require a dressed vertex

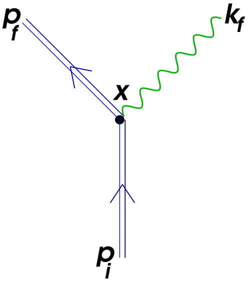
$$\gamma_\mu^{\text{FP}}(p_f,p_i) = e^{i\left[\$_+^{p_f}+\$_-^{p_f}\right]}\gamma_\mu\, e^{-i\left[\$_+^{p_i}+\$_-^{p_i}\right]}$$

$$\gamma_\mu^{\text{FP}}(p_f,p_i) \to \int dr_1 dr_2\; \mathcal{F}^{-1}\left[\gamma_\mu^{\text{FP}}(p_f,p_i)\right]\; e^{i(r_1k_1+r_2k_2)x}$$

contribution r_1k_1, r_2k_2 from external field enters into the conservation of momentum, allowing 1 vertex process

$$\delta^4(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

1st order Furry picture process and dressed vertex



FP Feynman diagrams only require a dressed vertex

$$\gamma_\mu^{\text{FP}}(p_f,p_i) = e^{i\left[\not{S}_+^{p_f} + \not{S}_-^{p_f}\right]}\gamma_\mu\, e^{-i\left[\not{S}_+^{p_i} + \not{S}_-^{p_i}\right]}$$

$$\gamma_\mu^{\text{FP}}(p_f,p_i) \to \int dr_1 dr_2\ \mathcal{F}^{-1}\left[\gamma_\mu^{\text{FP}}(p_f,p_i)\right]\ e^{i(r_1k_1+r_2k_2)x}$$

contribution r_1k_1, r_2k_2 from external field enters into the conservation of momentum, allowing 1 vertex process

$$\delta^4(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

two constant crossed fields leads to BesselK functions

$$A_\mu^{\text{ext}} = a_{1\mu}(k_1\cdot x)+a_{2\mu}(k_2\cdot x) : \quad \mathcal{F}^{-1}\left[\gamma_\mu^{\text{FP}}(p_f,p_i)\right] \propto K_{\frac{1}{3},\frac{2}{3}}(z)$$

Traces are more complicated, and integration over final states needs care **[Hartin and Moortgat-Pick EPJC (2011)]**

$$\frac{|M_{fi}|^2}{VT} = -e^2\int dr_1dr_2\ \text{Tr}[..r_1..r_2..]\ \frac{d\vec{p}_f\,d\vec{k}_f}{4\omega_f\epsilon_f}\ \delta^{(4)}(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

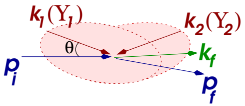
Beamstrahlung total transition probability

We get a modification to the standard beamstrahlung transition probability

$$W = -\frac{e^2 m}{2\epsilon_i}\int_0^\infty \frac{du}{(1+u)^2}\left[\int dz + \frac{1+(1+u)^2}{1+u} X \frac{d}{dz}\right] \mathrm{Ai}(z), \quad u = \frac{k_{1,2}.k_f}{k_{1,2}.(p_i - k_f)}$$

1 field: $z = \frac{u^{2/3}}{(k_2 \cdot p_i)^{2/3}}, \quad X = \frac{(k_2 \cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$

2 fields: $z = \frac{u^{2/3}}{[(k_1 \cdot p_i)^2 + (k_2 \cdot p_i)^2]^{1/3}}, \quad X = \frac{(k_1 \cdot p_i)^2 + (k_2 \cdot p_i)^2 + 2 a_1 \cdot a_2 (k_1 \cdot p_i)(k_2 \cdot p_i)}{u^{2/3}\,[(k_1 \cdot p_i)^2 + (k_2 \cdot p_i)^2]^{2/3}}$



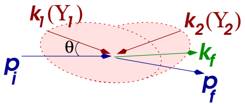
Beamstrahlung total transition probability

We get a modification to the standard beamstrahlung transition probability

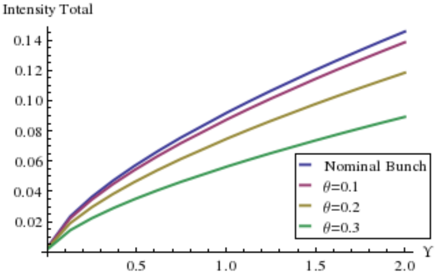
$$W = -\frac{e^2 m}{2\epsilon_i}\int_0^\infty \frac{du}{(1+u)^2}\left[\int dz + \frac{1+(1+u)^2}{1+u} X \frac{d}{dz}\right]\mathrm{Ai}(z), \quad u = \frac{k_{1,2}.k_f}{k_{1,2}.(p_i - k_f)}$$

1 field: $$z = \frac{u^{2/3}}{(k_2\cdot p_i)^{2/3}}, \quad X = \frac{(k_2\cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$$

2 fields: $$z = \frac{u^{2/3}}{[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{1/3}}, \quad X = \frac{(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2 + 2a_1\cdot a_2 (k_1\cdot p_i)(k_2\cdot p_i)}{u^{2/3}\,[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{2/3}}$$



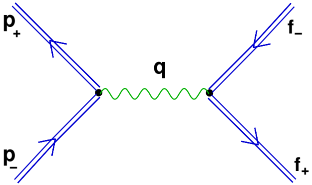
- Total intensity depends on field strength and angles
- θ depends on bunch disruption
- for ultra-relativistic bunches θ small
- Expt test with laser fields where θ can be large



(e.g.) Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 ds_1 ds_2 \; \bar{v}_{p_+} \gamma^{\mathrm{FP}\mu} u_{p_-} \bar{\epsilon}_{f_+} \gamma^{\mathrm{FP}}_{\mu} \epsilon_{f_-} \; \frac{\delta(F - I - (r_1+s_1)k_1 + (r_2+s_2)k_2)}{(I + r_1 k_1 + r_2 k_2)^2}$$

- final states momentum $F \equiv f_- + f_+$ initial state momentum $I \equiv p_- + p_+$
- spin and polarisation sums as usual
- two dressed vertices γ^{FP}
- r_1, r_2, s_1, s_2 momentum contribution from two external fields at two vertices
- Phase integral not (much) more complicated than for 1 vertex process

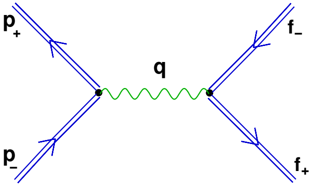


$$\frac{|M_{fi}|^2}{VT} = (g_1 g_2)^2 \int dr_1 dr_2 dl_1 dl_2 \;\; \mathrm{Tr}[..r_1..r_2..] \frac{d\vec{f_-}\, d\vec{f_+}}{4\omega_{f_-}\omega_{f_+}} \frac{\delta(F - I - l_1 k_1 + l_2 k_2)}{(I + r_1 k_1 + r_2 k_2)^4}$$

(e.g.) Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 ds_1 ds_2 \; \bar{v}_{p_+} \gamma^{\mathsf{FP}\mu} \, u_{p_-} \bar{\epsilon}_{f_+} \gamma^{\mathsf{FP}}_{\mu} \, \epsilon_{f_-} \; \frac{\delta(F - I - (r_1 + s_1)k_1 + (r_2 + s_2)k_2)}{(I + r_1 k_1 + r_2 k_2)^2}$$

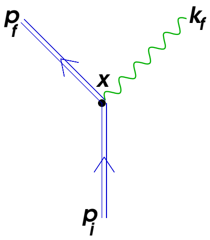
- final states momentum $F \equiv f_- + f_+$ initial state momentum $I \equiv p_- + p_+$
- spin and polarisation sums as usual
- two dressed vertices γ^{FP}
- r_1, r_2, s_1, s_2 momentum contribution from two external fields at two vertices
- Phase integral not (much) more complicated than for 1 vertex process

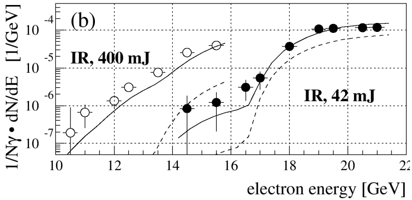


$$\frac{|M_{fi}|^2}{VT} = (g_1 g_2)^2 \int dr_1 dr_2 dl_1 dl_2 \;\; \mathrm{Tr}[..r_1..r_2..] \frac{d\vec{f_-} \, d\vec{f_+}}{4\omega_{f_-} \omega_{f_+}} \frac{\delta(F - I - l_1 k_1 + l_2 k_2)}{(I + r_1 k_1 + r_2 k_2)^4}$$

The pole structure depends on r_1, r_2 and is not standard need careful consideration of loops

Experimental tests - SLAC E144 - 1990s

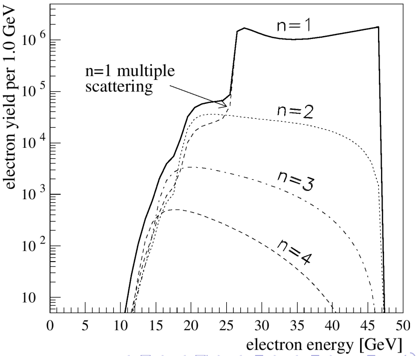




- Collided intense laser (10^{18} W/cm^2) with 46.6 GeV electrons
- effective momentum $q = p - \frac{e^2 a^2}{2k \cdot p} k$

$$\left(\sum_n\right) \quad q_i + nk \rightarrow q_f + k_f$$

- Compton-like scattering (HICS)
- Compton edge shifted by multiphoton effects



A strong field experiment at the ILC?

The actual proposal:

That we use some part of the extraction line to interact a terawatt LASER with the spent electron beam to do strong field physics

What we would like to measure/discover

- The mass shift, multiphoton effects to higher precision
- dependence of nonlinear effects on radiation angle, polarisation
- Discover higher order resonances
- Draw conclusions with expected primary IP effects

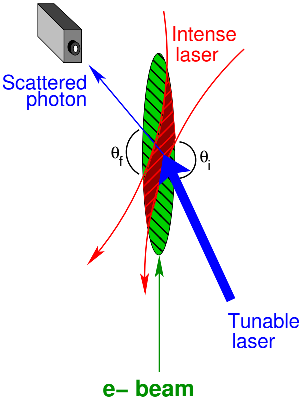
Issues/benefits

- Ideally be at a post IP beam focus
- We will have to think about possible backgrounds - situating of detectors
- Should be no interference with current extraction line diagnostics
- Dont need primary IP collisions
- Data collection time at the level of days for basic strong field phenomena

Reaching the critical field at the ILC

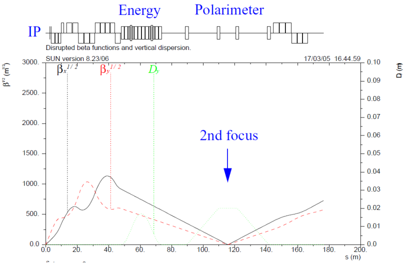
Experiment	$\lambda(\mu m)$	E_{laser}	focus	pulse	I(W/cm^2)	E_{e^-} (GeV)	Υ
E144 (SLAC)	1	2 J	60 μm^2	1.5 ps	$\approx 10^{18}$	46.6	0.27
ILC (E144 laser)	1	2 J	60 μm^2	1.5 ps	$\approx 10^{18}$	125-500	0.72-2.9
ILC (PL 9000)	1	3 J	40 μm^2	0.5 ps	6.75×10^{18}	125-500	1.87-7.54

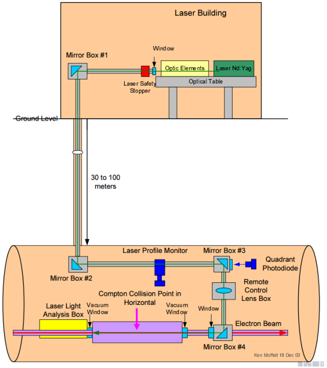
$$I = \frac{E_{\text{laser}}}{\text{spot} \times \text{pulse}}$$

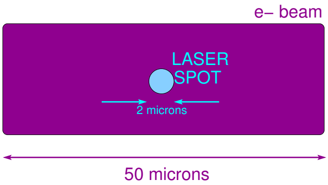


- Onset of detectable FP Compton scattering effects from $\nu^2 = 0.1$
- Can detect with today's technology
- The ratio of photon energies, incident, scattered angles is what's important
- Works too for two photon pair production, no e- beam, 2 tunable lasers

Possible intense LASER IP in the extraction line







- At secondary focus, illuminate centre of spent beam
- analysing magnets and detector
- Run parasitically with downstream polarimeter?
- Avoid damaging optical elements

Simulation of strong field effects

REQUIREMENTS:

To simulate a charge bunch collision and calculate the field strength at each point of production

To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc.

To perform a relatively complex cross-section calculation at each point of production

To have full spin tracking

To be flexible enough to include new higher order processes

SOLUTION:

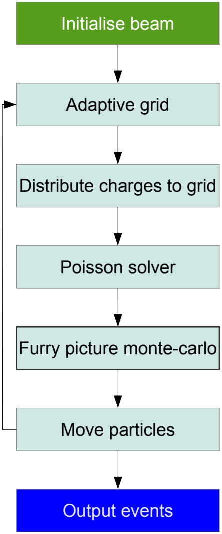
A PIC code using an efficient method for modeling the electrodynamics – crosscheck with CAIN/GP

MPI using openMPI or GPU programming

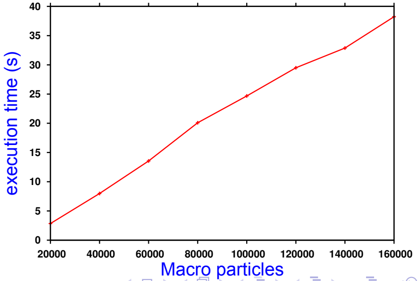
T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes

Allow new processes to be loaded externally

IPstrong - towards a strong field event generator



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs



Summary

- The charge bunch fields at future linear colliders will be strong
- **All** collider processes potentially affected
- Luminosity enhancement by Y waist optimisation $\rightarrow$ small increase in Υ, ΔP
- $n_\gamma = 1.72$ so higher order depolarisation processes are possible

Summary

- The charge bunch fields at future linear colliders will be strong
- **All** collider processes potentially affected
- Luminosity enhancement by Y waist optimisation $\rightarrow$ small increase in Υ, ΔP
- $n_\gamma = 1.72$ so higher order depolarisation processes are possible

- Use the Furry picture to incorporate bunch fields **exactly**
- Need to take into account both fields
- Use collinear reference frame and linear combination of 4-potentials - **New solutions obtained!**

Summary

- The charge bunch fields at future linear colliders will be strong
- **All** collider processes potentially affected
- Luminosity enhancement by Y waist optimisation $\rightarrow$ small increase in Υ, ΔP
- $n_\gamma = 1.72$ so higher order depolarisation processes are possible

- Use the Furry picture to incorporate bunch fields **exactly**
- Need to take into account both fields
- Use collinear reference frame and linear combination of 4-potentials - **New solutions obtained!**

- application to beamstrahlung alters transition probability
- strong field propagator requires attention to the pole structure
- radiative corrections and FP renormalisation required
- Extension to 2nd order & helicity amplitudes in progress

Summary

- The charge bunch fields at future linear colliders will be strong
- **All** collider processes potentially affected
- Luminosity enhancement by Y waist optimisation $\rightarrow$ small increase in Υ, ΔP
- $n_\gamma = 1.72$ so higher order depolarisation processes are possible

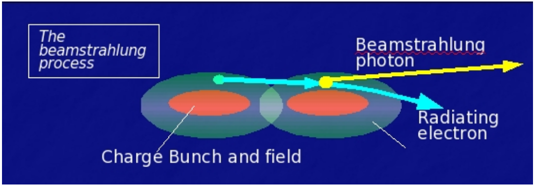
- Use the Furry picture to incorporate bunch fields **exactly**
- Need to take into account both fields
- Use collinear reference frame and linear combination of 4-potentials - **New solutions obtained!**

- application to beamstrahlung alters transition probability
- strong field propagator requires attention to the pole structure
- radiative corrections and FP renormalisation required
- Extension to 2nd order & helicity amplitudes in progress

- opportunities to experimentally test FP effects becoming available
- laser IP in extraction line at ILC would test strong field effects in situ
- Need new event generator for FP monte carlo during real bunch collision

Backup Slides

Collider strong field physics



" Strong field processes are physics processes calculated simultaneously in the normal perturbation theory as well as exactly with respect to a strong electromagnetic field. "

" Such calculations are necessary when the external field seen by a particle approaches or exceeds E_{cr}. "

(W.H.) Furry Picture

- Separate gauge field into external A_μ^{ext} and quantum A_μ parts

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}\,A\!\!\!/\,\psi^{\text{FP}}$$

- Euler-Lagrange equation $\rightarrow$ new equations of motion requires exact (w.r.t. A^{ext}) solutions ψ^{FP}

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

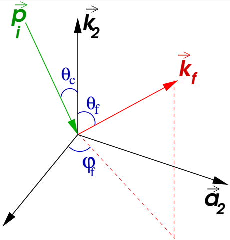
- For certain classes of external fields (plane waves, Coloumb fields and combinations) exact solutions exist **[Volkov Z Physik 94 250 (1935), Bagrov and Gitman Exact solutions of Rel wave equations (1990)]**
- A QFT which is non-perturbative wrt external gauge field A^{ext} and perturbative wrt ψ^{FP}, A

theoretical aspects of the Furry Picture

- External field makes space-time inhomogeneous so propagator depends on separate space-time points rather than on the difference between them **[Berestetski Lifshitz Pitaevski, QED $\S 109$]**
- Normalised IN and OUT states can be formed and LSZ extended to include such states **[Meyer, J Math Phys 11 312 (1970)]**
- Vanishing field strength at $t = \pm\infty \to$ stable vacuum
- Vacuum can be polarised so must include tadpole diagrams **[Schweber Relativistic QFT $\S$15g]**
- Operator and path integral representations for generating functional **[Fradkin, QED in an unstable vacuum]**
- Anomalous magnetic moment (one-loop) in a const crossed field varies from $\frac{\alpha}{2\pi}$ **[Ritus, JETP 30 1181 (1970)]**

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi\, dx}{(1+x)^3} \left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$

Beamstrahlung differential transition probability



- Return to an earlier stage of the calculation **[Nikishov, Trudy FIAN 111 p152 1979]**
- Divergence when $u = \dfrac{\omega_f(1-\cos\theta_f)}{2\epsilon_i - \omega_f(1-\cos\theta_f)} \to 0$
- i.e. **IR** condition ($\omega_f = 0$) and **backscattering** ($\theta_f = 0$). Why?

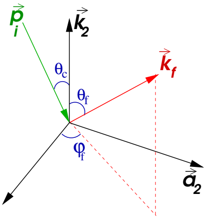
$$\frac{dW}{d\omega_f\, d\Omega} = -\frac{e^2 m}{2\epsilon_i}\frac{\omega_f \sin\theta_f}{1-\cos\theta_f}\left[X + \frac{1+(1+u)^2}{1+u}X^2\frac{d^2}{dz^2}\right]\mathrm{Ai}(z)^2, \quad u = \frac{k_2.k_f}{k_2.p_i - k_2.k_f}$$

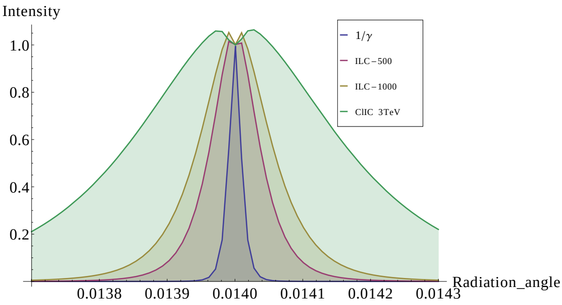
$$z = \frac{u^{2/3}}{(k_2\cdot p_i)^{2/3}}, \quad X = \frac{(k_2\cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$$

- modification to LSZ to accommodate Furry picture, means that the field has been acting since $t = -\infty$. **We have to limit the action**
- formation length can provide a limit to the action
- Can formally provide a limit using light cone slices **[Neville & Rohrlich, Phys Rev D, Trudy FIAN 3(8) p1692 1971]**

Beamstrahlung radiation angle (preliminary)

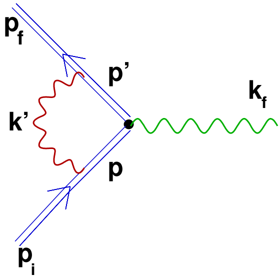
- examine plane of radiation orthogonal to field (x-z plane)
- 14 mrad crossing angle, normalised radiation intensity
- distribution peaked around direction of incoming electron
- vanishing external field coincides with classical result
- order of magnitude spread for ILC-1000
- will feed into greater angular spread for pair backgrounds





Vertex function in (one) external field

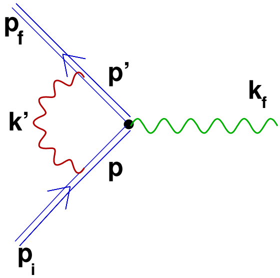
$$\Gamma^{\mathsf{FP}} = 2ie^2 \int drdsdl \int \frac{d^4k'}{k'^2}\, \gamma^{\mathsf{FP}\nu} \frac{\not{p}' + m}{(q_f - k' - rk)^2 - m_*^2}\, \gamma_\mu^{\mathsf{FP}} \frac{\not{p} + m}{(q_i - k' + sk)^2 - m_*^2}\, \gamma_\nu^{\mathsf{FP}}\, \delta(q_f + k_f - q_i - lk)$$



- Examine pole structure of the vertex function

Vertex function in (one) external field

$$\Gamma^{\mathsf{FP}} = 2ie^2 \int dr\,ds\,dl \int \frac{d^4k'}{k'^2}\, \gamma^{\mathsf{FP}\nu} \frac{\not{p}' + m}{(q_f - k' - rk)^2 - m_*^2} \gamma_\mu^{\mathsf{FP}} \frac{\not{p} + m}{(q_i - k' + sk)^2 - m_*^2} \gamma_\nu^{\mathsf{FP}}\, \delta(q_f + k_f - q_i - lk)$$



- Examine pole structure of the vertex function

- We combine denominators using Feynman parameters as normal,

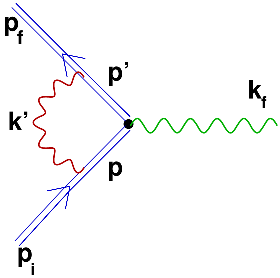
$$\int \frac{d^4k'}{k'^2[(q_f - k' - rk)^2 - m_*^2][(q_i - k' + sk)^2 - m_*^2]}$$
$$= \int_0^1 dx\,dy\,dz \frac{d^4k'}{(k'^2 - \Delta)^3}\, \delta(x + y + z - 1)$$

- Numerator more complicated than the usual case - need new tricks, but apart from the usual divergences we end up with additional poles in the residual

$$\frac{1}{\Delta(r, s, x, y, z)}$$

Vertex function in (one) external field

$$\Gamma^{\text{FP}} = 2ie^2 \int drdsdl \int \frac{d^4k'}{k'^2} \gamma^{\text{FP}\nu} \frac{\not{p}' + m}{(q_f - k' - rk)^2 - m_*^2} \gamma_\mu^{\text{FP}} \frac{\not{p} + m}{(q_i - k' + sk)^2 - m_*^2} \gamma_\nu^{\text{FP}} \, \delta(q_f + k_f - q_i - lk)$$



- Examine pole structure of the vertex function

- We combine denominators using Feynman parameters as normal,

$$\int \frac{d^4k'}{k'^2[(q_f - k' - rk)^2 - m_*^2][(q_i - k' + sk)^2 - m_*^2]}$$
$$= \int_0^1 dxdydz \frac{d^4k'}{(k'^2 - \Delta)^3} \delta(x+y+z-1)$$

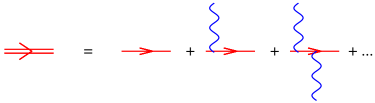
- Numerator more complicated than the usual case - need new tricks, but apart from the usual divergences we end up with additional poles in the residual

$$\frac{1}{\Delta(r, s, x, y, z)}$$

- Additional poles in the residue which match those in the tree level FP processwa
- Vertex function can be same order as tree-level diagram - must include!

Equiv Photon Approx and Perturbation expansion

- decompose external field into n equivalent photons
- sum the series to desired order of accuracy



$$G^e = G + G\hat{V}G + G\hat{V}G\hat{V}G + \ldots$$

$$G = (p^2 - m^2)^{-1}$$

$$\hat{V} = 2eA^e \cdot p - e^2 A^{e\,2}$$

- within certain constraints:
  - scalar particle
  - monochromatic photons

  the summation can be performed (Reiss Eberly 1966)

- Can the entire summation be performed in general ?

- The alternative is the Furry/Feynman method...

Infinite momentum frame

- QED can be formulated in a Lorentz frame moving at the limit of the speed of light (Kogut & Soper Phys Rev D 1(10) 2901 (1970))
- regular coordinates (t, x, y, z) can be expressed in light cone coordinates $x_\pm = \dfrac{1}{2}(t \pm z)\ ;\ x_\perp = (x, y)$
- light cone dirac matrices separate into sub-algebras whose members anti-commute $\gamma_\pm \gamma_\perp = -\gamma_\perp \gamma_\pm$
- light cone scalar products are $a.b = 2a_+b_- + 2a_-b_+ - a_\perp.b_\perp$

Strong fields at the collider IP

- moving charge has longitudinal length contraction

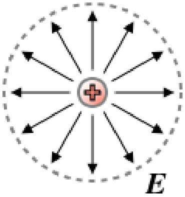
- relativistic charge bunch produces constant crossed plane wave field

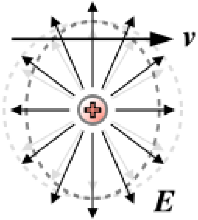
$$A_\mu = a_{1\mu}(k \cdot x)$$

$$a_{1\mu} = (0, \vec{a})$$

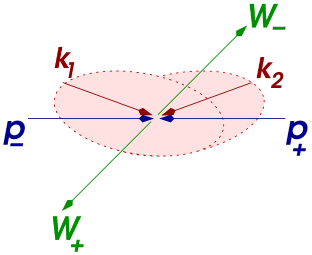
- particle p sees a field strength parameter Υ

$$\Upsilon = \frac{e|\vec{a}|}{mE_{\text{cr}}}(k \cdot p)$$





Volkov-type solutions in two external fields



- both incoming bunches contribute external fields
- external field wavevectors are generally anti-collinear
- Need new Volkov-type solution

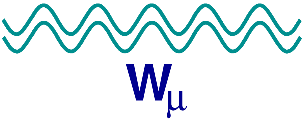
- strategy is to first solve Klein-Gordon equation $(D^2 + m_W^2)\phi_e^{\pm}$

$$\phi_e^{\pm} = \frac{1}{\sqrt{2\epsilon_p V}} \int dr\, \exp\left[-ib\, p\cdot x - ireA_e - \frac{(r-f)^2}{2|z|}\right]$$

- For constant crossed field Dirac equation solution proceeds from the Klein-Gordon solution

W boson Volkov Solution

- Equation of motion for the W boson



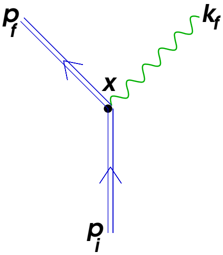
$$(D^2 + m_W^2) W_\nu + i2eF^\mu{}_\nu W_\mu = 0, \quad D^\mu W_\mu = 0$$

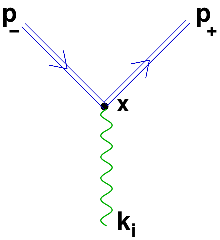
- with solution $W_\mu = E_p^W \, e^{-ip\cdot x} \, w_p$ where

$$E_p^W = \left(g_{\mu\nu} + \frac{e}{k\cdot p} \int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{e2} k_\mu k_\nu \right)$$
$$\cdot \exp\left[-\frac{i}{2(k\cdot p)} \left(2e(A^e \cdot p) - e^2 A^{e2} \right) \right]$$

- similar solutions can be found for other particles that couple to A^e

Beamstrahlung, incoherent/coherent pair production

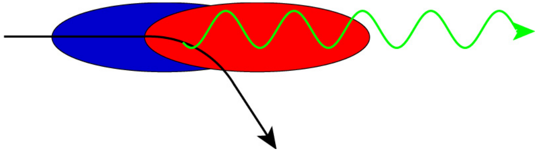




- IP beam-beam simulators - CAIN, Guinea-Pig
- beamstrahlung & coherent pair production calculated via quasi-classical approx
- incoherent pairs calculated with beamstrahlung photon and equivalent photon approx (EPA)
- **more exactly** these are 1st and 2nd order Furry picture processes

bkgd pairs	current	proposed
coherent	quasi-classical	1 vertex Furry picture
incoherent	EPA	2 vertex Furry picture

Formation length



" distance travelled by a charged particle while a radiated photon moves one wavelength in front of it "

A bad argument: *" If the bunch is sufficiently short we dont need to worry about strong field effects"*

- classical argument that only applies to the beamstrahlung
- strong field propagator integrated over all length scales