

# Constructing multi-loop scattering amplitudes with manifest singularity structure

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# Outline

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# Gauge Theory Amplitude Singularity Structure

S. Catani, Phys. Lett. **B427** (1998) 161; S. Mert Aybat *et. al.*, Phys. Rev. **D74** (2006) 074004

T. Becher and M. Neubert, JHEP **0906** (2009) 081; E. Gardi and L. Magnea, JHEP **0903** (2009) 079

The IR divergences of the simplest non-Abelian gauge theory, planar  $SU(N_c)$   $\mathcal{N} = 4$  super Yang-Mills, are believed to be of the form:

$$\mathcal{A}_1^{\mathcal{N}=4}(p_1, \dots, p_n) = \exp \left\{ -\frac{1}{2} \sum_{L=1}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^L \mu_\epsilon^{2L\epsilon} \int_0^{\mu_\epsilon^2} d\mu^2 (\mu^2)^{-1-L\epsilon} \right. \\ \left. \sum_{\substack{i,j=1 \\ i < j}}^n \left( \Gamma_{1;L}^{\mathcal{N}=4} \ln \left( \frac{\mu^2}{-s_{ij}} \right) + \mathcal{G}_{1;L}^{\mathcal{N}=4} \right) \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{N_c} \right\} \sum_{L=0}^{\infty} \mathbf{H}_{1;L}^{\mathcal{N}=4}(\epsilon; p_1, \dots, p_n)$$

At four points, this structure has been realized explicitly at strong coupling (L. F. Alday and J. Maldacena, JHEP **0706** (2007) 064). In a nutshell, the dipole conjecture is the suggestion that, with minor modifications, the above structure could hold for more general gauge theories like QCD.

# What Can We Hope To See In A Two-Loop Integrand?

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In the spirit of Catani's work, let's start with QCD form factors!

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The QCD form factors in dimensional regularization satisfy a renormalization group equation which was understood long ago:

L. Magnea and G. Sterman, Phys. Rev. **D42** (1990) 4222

$$q^2 \frac{\partial}{\partial q^2} \ln (\mathcal{F}(-q^2/\mu^2, \alpha_s, \epsilon)) = 1/2\mathcal{K}(\alpha_s) + 1/2\mathcal{G}(-q^2/\mu^2, \alpha_s, \epsilon)$$
$$\left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) \mathcal{G}(-q^2/\mu^2, \alpha_s, \epsilon) = \sum_{L=1}^{\infty} \Gamma_L \alpha_s^L$$
$$\left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) \mathcal{K}(\alpha_s) = - \sum_{L=1}^{\infty} \Gamma_L \alpha_s^L$$

At  $L$  loops,  $\Gamma_L$  characterizes the leading  $1/\epsilon^2$  IR divergences which cannot be understood as exponentiated lower-loop contributions.

# Pole Predictions For One- And Two-Loop Form Factors

From the renormalization group equation one can show that:

S. Moch *et. al.*, JHEP **0508** (2005) 049

$$\begin{aligned} \mathcal{F}_1^q &= -\frac{1}{2} \frac{1}{\epsilon^2} \Gamma_1 - \frac{1}{2} \frac{1}{\epsilon} G_1(\epsilon) \\ \mathcal{F}_2^q &= \frac{1}{8} \frac{1}{\epsilon^4} \Gamma_1^2 + \frac{1}{8} \frac{1}{\epsilon^3} \Gamma_1 \left( 2G_1(0) - \beta_0 \right) \\ &\quad - \frac{1}{8} \frac{1}{\epsilon^2} \left( \Gamma_2 + G_1(0) \left( 2\beta_0 - G_1(0) \right) - 2\Gamma_1 G_1'(0) \right) + \dots \end{aligned}$$

where we have  $\mathcal{G}(1, \alpha_s, \epsilon) = G_1(\epsilon)\alpha_s + \dots$

$$\begin{aligned} \Gamma_1 &= 4C_F & \Gamma_2 &= -\frac{40}{9}C_F N_f + \left( \frac{268}{9} - 8\zeta_2 \right) C_A C_F \\ G_1(0) &= 6C_F & G_1'(0) &= \left( 16 - 2\zeta_2 \right) C_F & \beta_0 &= \frac{11}{3}C_A - \frac{2}{3}N_f \end{aligned}$$

# Explicit Results For One- And Two-Loop Form Factors

F. Tkachov, Phys. Lett. **B100** (1981) 65; K. Chetyrkin and F. Tkachov, Nucl. Phys. **B192** (1981) 159;

S. Laporta, Int. J. Mod. Phys. **A15** (2000) 5087; A. von Manteuffel and C. Studerus, arXiv:1201.4330

T. Matsuura *et. al.*, Z. Phys. **C38** (1988) 623; T. Gehrmann *et. al.*, Phys. Lett. **B622** (2005) 295

With respect to the conventional basis of integrals:

$$\begin{aligned}
 \mathcal{F}_1^q &= C_F \left\{ \frac{f_1}{\epsilon} \right\} \\
 \mathcal{F}_2^q &= C_F^2 \left\{ \frac{f_2}{\epsilon^3} + \frac{1}{\epsilon^2} \left[ f_3 + f_4 \right] + f_5 \right\} \\
 &+ C_F C_A \left\{ \frac{f_6}{\epsilon^3} + \frac{f_7}{\epsilon^2} + f_8 \right\} + C_F N_f \left\{ \frac{f_9}{\epsilon} \right\}
 \end{aligned}$$



# Explicit Results For One- And Two-Loop Form Factors

G. Heinrich *et. al.*, Phys. Lett. **B678** (2009) 359;

R. N. Lee *et. al.*, Nucl. Phys. Proc. Suppl. **205-206** (2010) 308; JHEP **1102** (2011) 102

With respect to a **uniform weight** basis of integrals:


$$\begin{aligned}
 \mathcal{F}_1^q &= C_F \left\{ \frac{a_1}{\epsilon} \text{Diagram}_1 \right\} \\
 \mathcal{F}_2^q &= C_F^2 \left\{ \frac{1}{\epsilon^2} \left[ a_2 \text{Diagram}_2 + a_3 \text{Diagram}_3 \right] + \frac{a_4}{\epsilon} \text{Diagram}_4 + a_5 \text{Diagram}_5 \right\} \\
 &+ C_F C_A \left\{ \frac{a_6}{\epsilon^2} \text{Diagram}_6 + \frac{a_7}{\epsilon} \text{Diagram}_7 + a_8 \text{Diagram}_8 \right\} + C_F N_f \left\{ a_9 \text{Diagram}_9 \right\}
 \end{aligned}$$


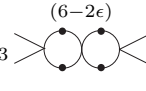
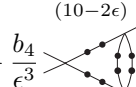
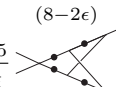
The diagrams represent various two-loop form factor topologies with external lines and a central vertex. Diagram 1 is a bubble with a dot. Diagram 2 is a bubble with two dots. Diagram 3 is two bubbles with two dots. Diagram 4 is a triangle with a dot. Diagram 5 is a triangle with two dots. Diagram 6 is a bubble with two dots. Diagram 7 is a triangle with a dot. Diagram 8 is a triangle with two dots. Diagram 9 is a triangle with a dot.


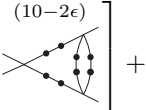
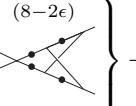
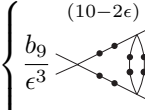
# Explicit Results For One- And Two-Loop Form Factors

A. von Manteuffel *et. al.*, Phys. Rev. **D93** (2016) no. 12, 125014

With respect to a **finite** basis of integrals:

$$\mathcal{F}_1^q = C_F \left\{ \frac{b_1}{\epsilon^2} \text{Diagram}_1 \right\}$$


$$\mathcal{F}_2^q = C_F^2 \left\{ \frac{1}{\epsilon^4} \left[ b_2 \text{Diagram}_2 + b_3 \text{Diagram}_3 \right] + \frac{b_4}{\epsilon^3} \text{Diagram}_4 + \frac{b_5}{\epsilon} \text{Diagram}_5 \right\}$$





$$+ C_F C_A \left\{ \frac{1}{\epsilon^4} \left[ b_6 \text{Diagram}_6 + b_7 \text{Diagram}_7 \right] + \frac{b_8}{\epsilon} \text{Diagram}_8 \right\} + C_F N_f \left\{ \frac{b_9}{\epsilon^3} \text{Diagram}_9 \right\}$$





# Explicit Results For One- And Two-Loop Form Factors

$$\begin{aligned}
 \mathcal{F}_1^q &= C_F \left\{ \frac{c_1}{\epsilon^2} (1-2\epsilon) \text{Diagram}_1^{(6-2\epsilon)} \right\} \\
 \mathcal{F}_2^q &= C_F^2 \left\{ \frac{c_2}{\epsilon^4} (1-2\epsilon)^2 \text{Diagram}_2^{(6-2\epsilon)} + \frac{1}{\epsilon} \left[ c_3 (1-2\epsilon) \text{Diagram}_3^{(4-2\epsilon)} + c_4 (1-2\epsilon)(1-3\epsilon) \text{Diagram}_4^{(6-2\epsilon)} \right. \right. \\
 &\quad \left. \left. + c_5 (1-4\epsilon) \left( 2(1-3\epsilon) \text{Diagram}_5^{(6-2\epsilon)} - \text{Diagram}_6^{(6-2\epsilon)} \right) \right] \right\} + C_F C_A \left\{ \frac{c_6}{\epsilon^3} (1-2\epsilon)^2 \text{Diagram}_7^{(6-2\epsilon)} \right. \\
 &\quad \left. + \frac{c_7}{\epsilon^2} (1-2\epsilon)(1-3\epsilon) \text{Diagram}_8^{(6-2\epsilon)} + \frac{c_8}{\epsilon} (1-4\epsilon) \left( 2(1-3\epsilon) \text{Diagram}_9^{(6-2\epsilon)} - \text{Diagram}_{10}^{(6-2\epsilon)} \right) + c_9 (1-2\epsilon) \text{Diagram}_{11}^{(4-2\epsilon)} \right\} \\
 &\quad + C_F N_f \left\{ \frac{c_{10}}{\epsilon^3} (1-2\epsilon)^2 \text{Diagram}_{12}^{(6-2\epsilon)} + \frac{c_{11}}{\epsilon} (1-2\epsilon)(1-3\epsilon) \text{Diagram}_{13}^{(6-2\epsilon)} + c_{12} (1-2\epsilon) \text{Diagram}_{14}^{(4-2\epsilon)} \right\}
 \end{aligned}$$

# If One Works Strictly With The Conventional Master Topologies, It Appears That Nothing Is Gained

$$\begin{aligned}
 -\epsilon^2 \text{ (Diagram with 2 external lines, 2 internal lines, and 2 dots) }^{(4-2\epsilon)} &= -8(1-3\epsilon)(2-3\epsilon) \text{ (Diagram with 2 external lines, 4 internal lines, and 4 dots) }^{(8-2\epsilon)} = \frac{e^{2\gamma_E \epsilon} \Gamma^3(1-\epsilon) \Gamma(1+2\epsilon)}{\Gamma(1-3\epsilon)} \\
 &= 1 - \zeta_2 \epsilon^2 - \frac{32}{3} \zeta_3 \epsilon^3 - \frac{57}{10} \zeta_2^2 \epsilon^4 + \left( \frac{32}{3} \zeta_2 \zeta_3 - \frac{272}{5} \zeta_5 \right) \epsilon^5 + \mathcal{O}(\epsilon^5)
 \end{aligned}$$

# If One Works Strictly With The Conventional Master Topologies, It Appears That Nothing Is Gained

$$\begin{aligned}
 -\epsilon^2 \text{ (Diagram: circle with two dots) }^{(4-2\epsilon)} &= -8(1-3\epsilon)(2-3\epsilon) \text{ (Diagram: circle with four dots) }^{(8-2\epsilon)} = \frac{e^{2\gamma_E \epsilon} \Gamma^3(1-\epsilon) \Gamma(1+2\epsilon)}{\Gamma(1-3\epsilon)} \\
 &= 1 - \zeta_2 \epsilon^2 - \frac{32}{3} \zeta_3 \epsilon^3 - \frac{57}{10} \zeta_2^2 \epsilon^4 + \left( \frac{32}{3} \zeta_2 \zeta_3 - \frac{272}{5} \zeta_5 \right) \epsilon^5 + \mathcal{O}(\epsilon^5)
 \end{aligned}$$

⇒ To construct better candidate master integrals with higher leading weights, one should consider **reducible topologies** as well!

$$\begin{aligned}
 -\frac{1}{6}(1-2\epsilon) \text{ (Diagram: circle with vertical line) }^{(4-2\epsilon)} &= \zeta_3 + \frac{3}{5} \zeta_2^2 \epsilon + (-\zeta_2 \zeta_3 + 7\zeta_5) \epsilon^2 + \left( \frac{99}{35} \zeta_2^3 - \frac{23}{3} \zeta_3^2 \right) \epsilon^3 \\
 + \left( -\frac{17}{2} \zeta_2^2 \zeta_3 - 7\zeta_2 \zeta_5 + 49\zeta_7 \right) \epsilon^4 &+ \left( \frac{3777}{350} \zeta_2^4 + \frac{23}{3} \zeta_2 \zeta_3^2 - \frac{1306}{15} \zeta_3 \zeta_5 \right) \epsilon^5 + \mathcal{O}(\epsilon^6)
 \end{aligned}$$

# What Uniformly-Finite Integrals Have To Offer

A uniformly-finite integral basis simultaneously realizes all the advantages of uniform weight and finite integral bases.

- Multi-scale uniformly-finite integrals satisfy especially simple differential equations, which usually makes their analytical evaluation easier.

A. V. Kotikov, "Subtleties in quantum field theory," 150, (2010);

J. M. Henn, Phys. Rev. Lett. **110** (2013) 251601; J. Broedel *et. al.*, JHEP **1901** (2019) 023

- Single-scale uniformly-finite integrals may be evaluated analytically using **HyperInt** (E. Panzer, Comput. Phys. Commun. **188** (2015) 148) in many cases.

E. Panzer, JHEP **1403** (2014) 071; A. von Manteuffel *et. al.*, JHEP **1502** (2015) 120

- Numerical evaluations of uniformly-finite integrals are possible in situations where integrals with subdivergences (*i.e.* conventional or uniform weight integrals) cause problems for **FIESTA 4** and **pySecDec**.

P. Marquard *et. al.*, Phys. Rev. **D94** (2016) no. 7, 074025;

S. Borowka *et. al.*, JHEP **1610** (2016) 107; A. von Manteuffel and RMS, JHEP **1704** (2017) 129;

S. P. Jones *et. al.*, Phys. Rev. Lett. **120** (2018) no.16, 162001

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S. P. Jones *et. al.*, Phys. Rev. Lett. **120** (2018) no.16, 162001
- Integrands for virtual corrections expressed in terms of a uniformly-finite basis may be fixed such that one can see immediately that the deeper poles in  $\epsilon$  have their origin in lower-loop contributions. (conjecturally for now)

The relation:

$$-\frac{1}{6}(1-2\epsilon) \text{ (circle diagram)}^{(4-2\epsilon)} = -\frac{1}{3}(1-2\epsilon)^2 \left( \text{triangle diagram}_1^{(6-2\epsilon)} + \text{triangle diagram}_2^{(6-2\epsilon)} \right)$$



The relation:

$$-\frac{1}{6}(1-2\epsilon) \text{ (Diagram: circle with vertical line)}^{(4-2\epsilon)} = -\frac{1}{3}(1-2\epsilon)^2 \left( \text{Diagram: triangle with dot}^{(6-2\epsilon)} + \text{Diagram: triangle with dot}^{(6-2\epsilon)} \right)$$

immediately suggests that, if the conjecture holds, **all** higher-order poles of the three-loop form factors ( $\epsilon^{-6} - \epsilon^{-3}$ ) are generated by:

$$\begin{aligned}
 & -(1-2\epsilon)^2(1-3\epsilon) \text{ (Diagram: circle with dot and triangle)}^{(6-2\epsilon)} - \frac{1}{3}(1-2\epsilon)^3 \left( \text{Diagram: circle with dot and triangle}^{(6-2\epsilon)} + \text{Diagram: circle with dot and triangle}^{(6-2\epsilon)} \right) \\
 & (1-2\epsilon)^3 \text{ (Diagram: three circles)}^{(6-2\epsilon)} - \frac{1}{6}(1-2\epsilon)(1-4\epsilon) \left( 2(1-3\epsilon) \text{ (Diagram: circle with dot and triangle)}^{(6-2\epsilon)} - \text{Diagram: circle with dot and triangle}^{(6-2\epsilon)} \right)
 \end{aligned}$$

The relation:

$$-\frac{1}{6}(1-2\epsilon) \text{ (Diagram: circle with vertical line)}^{(4-2\epsilon)} = -\frac{1}{3}(1-2\epsilon)^2 \left( \text{Diagram: triangle with dot on left}^{(6-2\epsilon)} + \text{Diagram: triangle with dot on right}^{(6-2\epsilon)} \right)$$

immediately suggests that, if the conjecture holds, **all** higher-order poles of the three-loop form factors ( $\epsilon^{-6} - \epsilon^{-3}$ ) are generated by:

$$\begin{aligned}
 & -(1-2\epsilon)^2(1-3\epsilon) \text{ (Diagram: circle with dot, triangle with dot)}^{(6-2\epsilon)} - \frac{1}{3}(1-2\epsilon)^3 \left( \text{Diagram: circle with dot, triangle with dot on left}^{(6-2\epsilon)} + \text{Diagram: circle with dot, triangle with dot on right}^{(6-2\epsilon)} \right) \\
 & (1-2\epsilon)^3 \text{ (Diagram: three circles in a row)}^{(6-2\epsilon)} - \frac{1}{6}(1-2\epsilon)(1-4\epsilon) \left( 2(1-3\epsilon) \text{ (Diagram: circle with dot, triangle with dot)}^{(6-2\epsilon)} - \text{Diagram: circle with dot, triangle with dot}^{(6-2\epsilon)} \right)
 \end{aligned}$$

# Selected Results For Three-Loop Form Factor Integrals

R. N. Lee *et. al.*, Nucl. Phys. Proc. Suppl. **205-206** (2010) 308; JHEP **1102** (2011) 102

$$\begin{aligned}
 \frac{1}{20}(1-2\epsilon) \text{ (Diagram: Circle with two internal lines forming an X)}^{(4-2\epsilon)} &= \zeta_5 + \left( \frac{4}{7}\zeta_2^3 + \frac{17}{5}\zeta_3^2 \right) \epsilon + \left( \frac{102}{25}\zeta_2^2\zeta_3 - \frac{3}{2}\zeta_2\zeta_5 + \frac{45}{2}\zeta_7 \right) \epsilon^2 \\
 &+ \left( \frac{175149}{4375}\zeta_2^4 - \frac{51}{10}\zeta_2\zeta_3^2 - \frac{647}{5}\zeta_3\zeta_5 - \frac{2268}{25}\zeta_{5,3} \right) \epsilon^3 + \mathcal{O}(\epsilon^4)
 \end{aligned}$$

$$\begin{aligned}
 -\frac{1}{12}(1-6\epsilon) \text{ (Diagram: Triangle with internal lines)}^{(4-2\epsilon)} &= \zeta_2\zeta_3 + \frac{5}{6}\zeta_5 + \left( \frac{11}{9}\zeta_2^3 + \frac{3}{2}\zeta_3^2 \right) \epsilon + \left( \frac{13}{10}\zeta_2^2\zeta_3 - \frac{7}{12}\zeta_2\zeta_5 \right. \\
 &\left. + \frac{307}{24}\zeta_7 \right) \epsilon^2 + \left( -\frac{30239}{15750}\zeta_2^4 - \frac{185}{4}\zeta_2\zeta_3^2 - \frac{199}{6}\zeta_3\zeta_5 - \frac{62}{15}\zeta_{5,3} \right) \epsilon^3 + \mathcal{O}(\epsilon^4)
 \end{aligned}$$

## Selected Results For Three-Loop Form Factor Integrals

$$\begin{aligned}
 -\frac{1}{20}(1-2\epsilon) \int_{\text{triangle}}^{(4-2\epsilon)} &= \zeta_5 + \left( \frac{4}{7}\zeta_2^3 - \frac{1}{5}\zeta_3^2 \right) + \left( -\frac{6}{25}\zeta_2^2\zeta_3 - \frac{3}{2}\zeta_2\zeta_5 + \frac{359}{20}\zeta_7 \right) \epsilon^2 \\
 &+ \left( \frac{2712}{625}\zeta_2^4 + \frac{3}{10}\zeta_2\zeta_3^2 - \frac{49}{5}\zeta_3\zeta_5 + \frac{162}{25}\zeta_{5,3} \right) \epsilon^3 + \mathcal{O}(\epsilon^4)
 \end{aligned}$$

$$\begin{aligned}
 -(1-2\epsilon)^2(1-4\epsilon) \int_{\text{triangle}}^{(6-2\epsilon)} &= \zeta_2^2 + 4\zeta_2\zeta_3\epsilon + \left( \frac{59}{10}\zeta_2^3 + 4\zeta_3^2 \right) \epsilon^2 + \left( -\frac{41}{5}\zeta_2^2\zeta_3 \right. \\
 &\left. + 28\zeta_2\zeta_5 \right) \epsilon^3 + \left( \frac{27621}{1400}\zeta_2^4 - 86\zeta_2\zeta_3^2 + 56\zeta_3\zeta_5 \right) \epsilon^4 + \mathcal{O}(\epsilon^5)
 \end{aligned}$$

Outline

Background And Proof Of Concept

Uniformly-Finite Integrals For Scattering Amplitudes

What About At Higher Loops?

What To Look At Next

# What's Next?

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- If everything goes according to plan for massless gauge theory amplitudes, it may then be worthwhile to study processes of recent phenomenological interest with many mass scales, probably even those which involve integrals which cannot yet be evaluated analytically in any reasonable way.



## What's Next?

- Complete the construction of such a basis for the three-loop form factors and check the conjecture for the higher-order poles.
- Find a fully-systematic way to construct a uniformly-finite basis.
- If everything goes according to plan for massless gauge theory amplitudes, it may then be worthwhile to study processes of recent phenomenological interest with many mass scales, probably even those which involve integrals which cannot yet be evaluated analytically in any reasonable way.
- Even if a natural generalization of the pole-term conjecture fails to hold in general, it is still quite possible that a uniformly-finite basis of integrals may be constructed for an arbitrary hard scattering process. If such a basis does exist in general, how canonical is the construction and how cost-effective is it?