Mathematics Of Linear Relations Between Feynman Integrals

18-22 March 2019

Direct Reduction of Amplitude

ArXiv:1901.09390[hep-ph]

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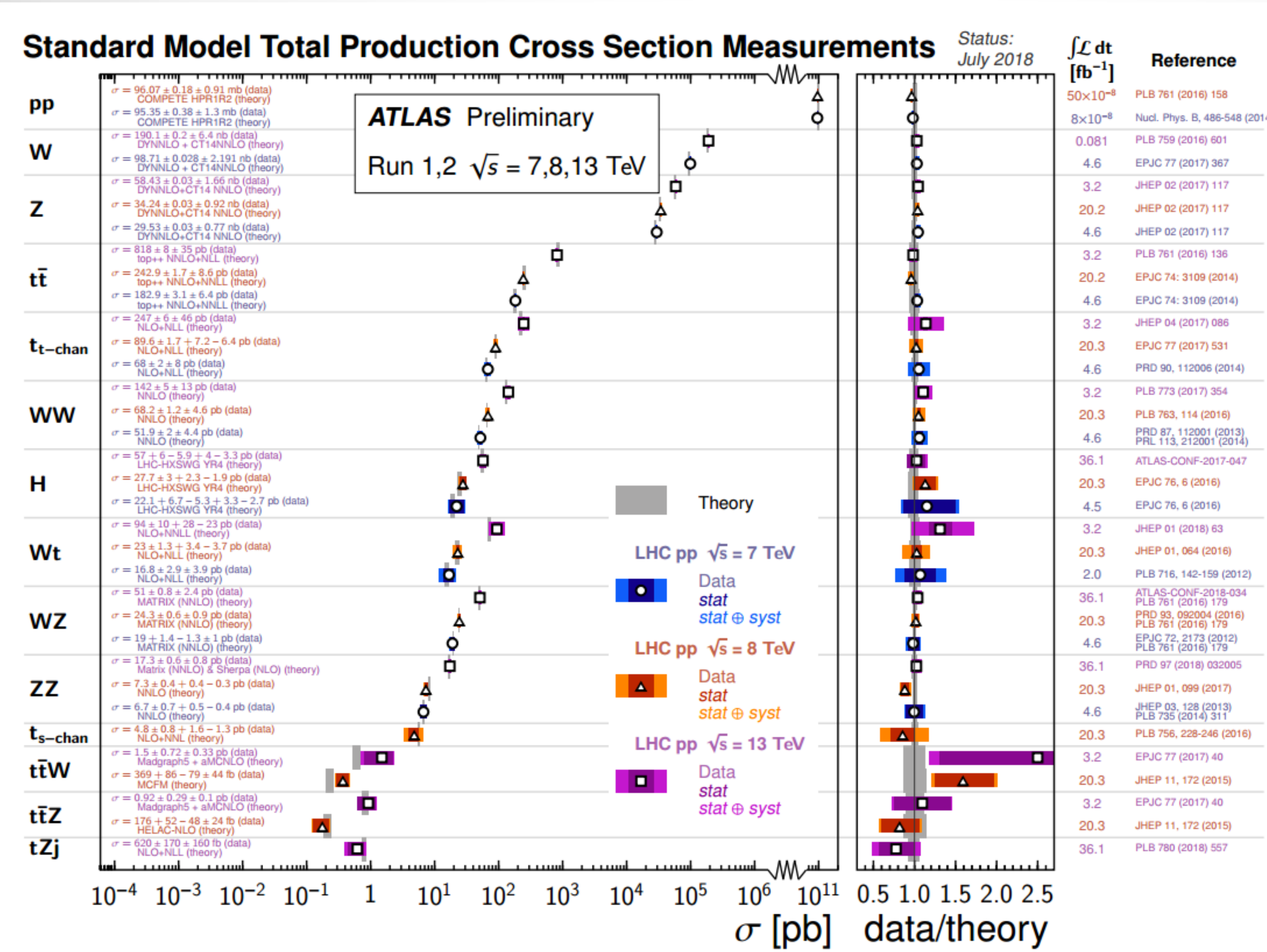
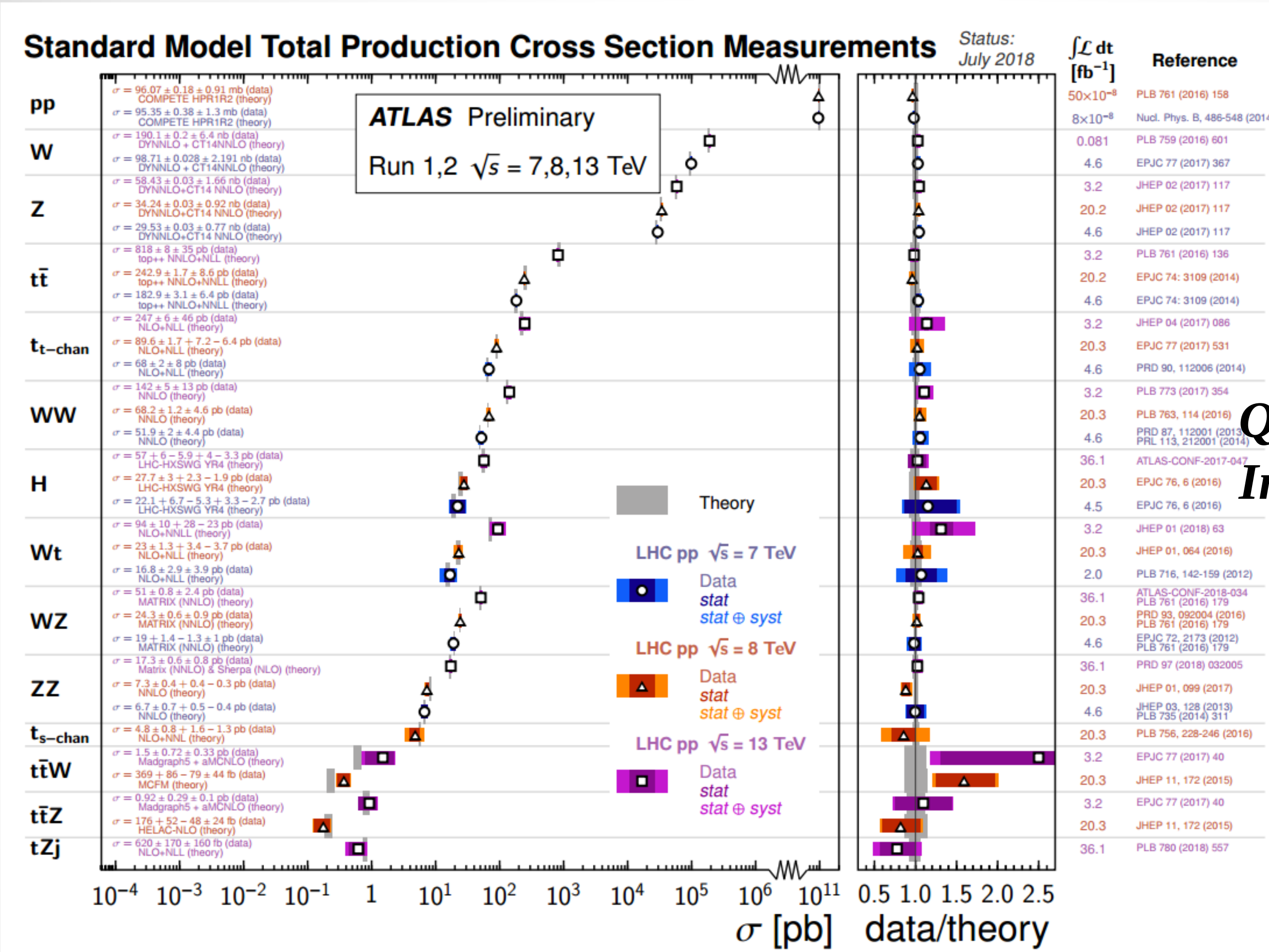
Introduction



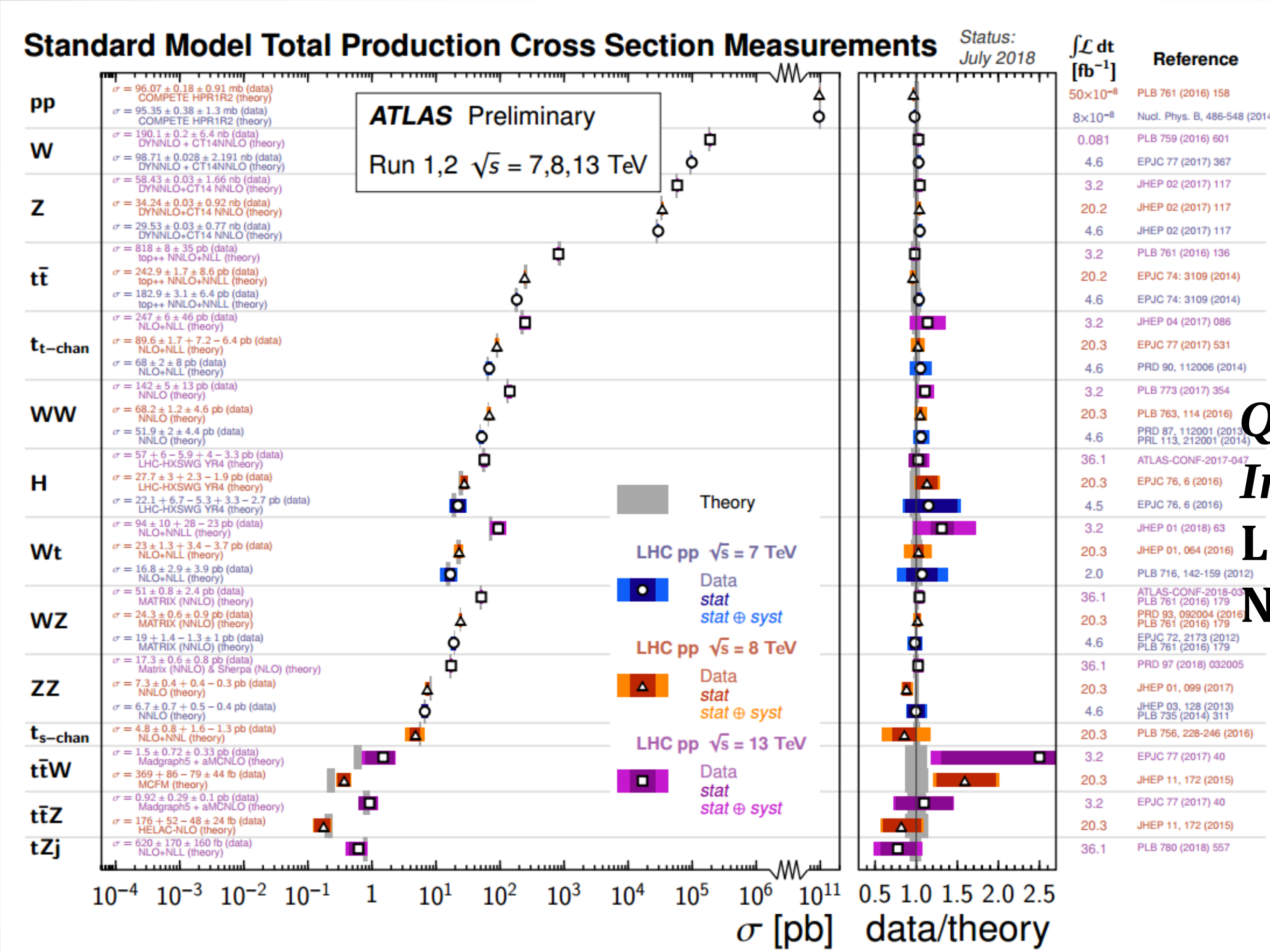
Introduction



$$gg \to H$$

QCD Cross Section Increase:

Introduction



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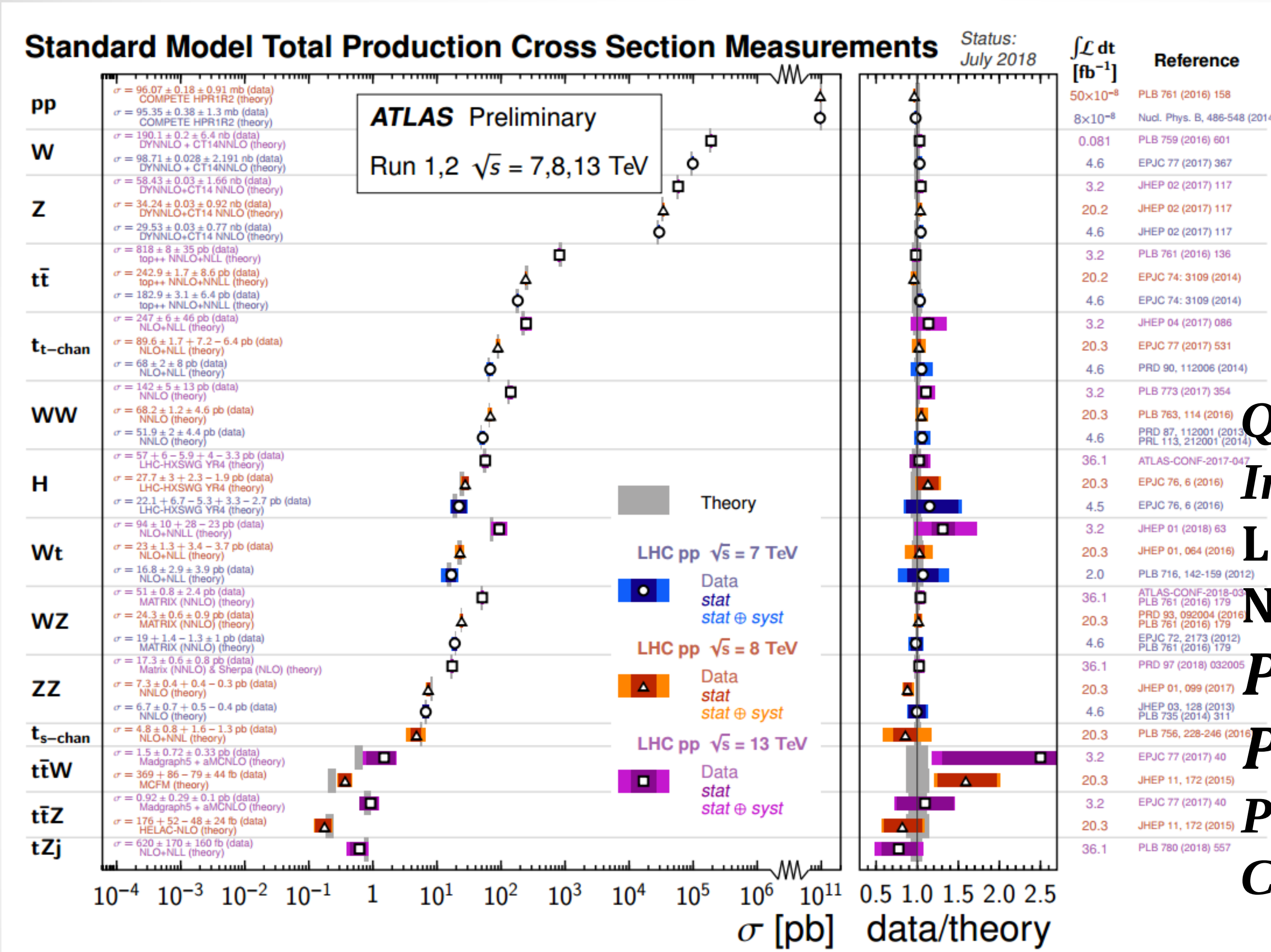
QCD Cross Section Increase:

LO → ***NLO*****: 80→100%**

NLO → ***NNLO*****: 25%**

Introduction



$$gg \to H$$

QCD Cross Section Increase:

LO → ***NLO*****: 80→100%**

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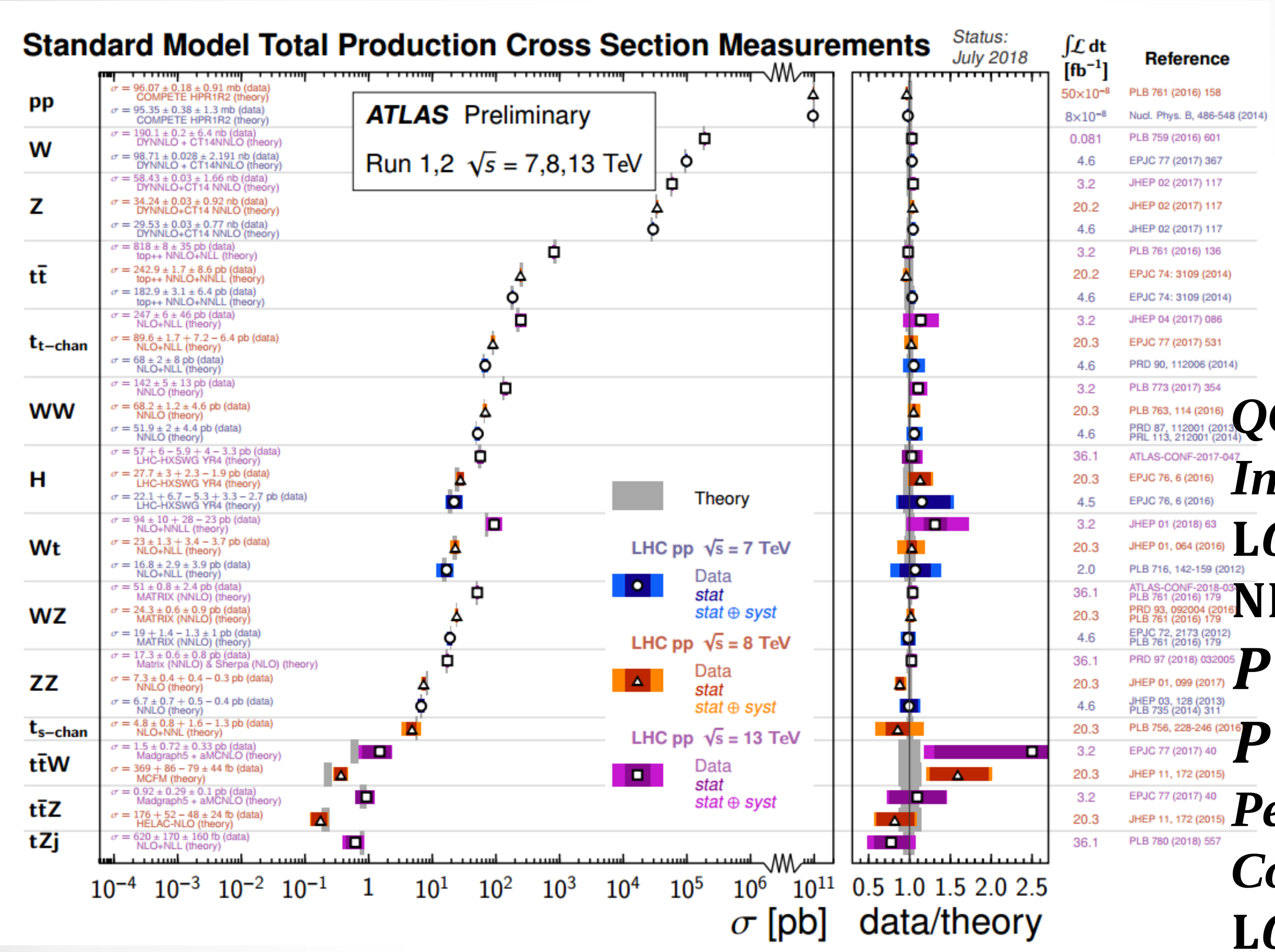
$$P\bar{P} \to H + X,$$

$$PP \to H + X$$

Perturbative Convergence:

Introduction



$gg \rightarrow H$

QCD Cross Section Increase:

$\mathbf{LO} \rightarrow \boldsymbol{NLO}$: **80→100%**

$\mathbf{NLO} \rightarrow \boldsymbol{NNLO}$: **25%**

$\boldsymbol{P\bar{P} \rightarrow H + X,}$

$\boldsymbol{PP \rightarrow H + X}$

Perturbative Convergence:

$\mathbf{LO} \rightarrow \boldsymbol{NLO}$: **70%**

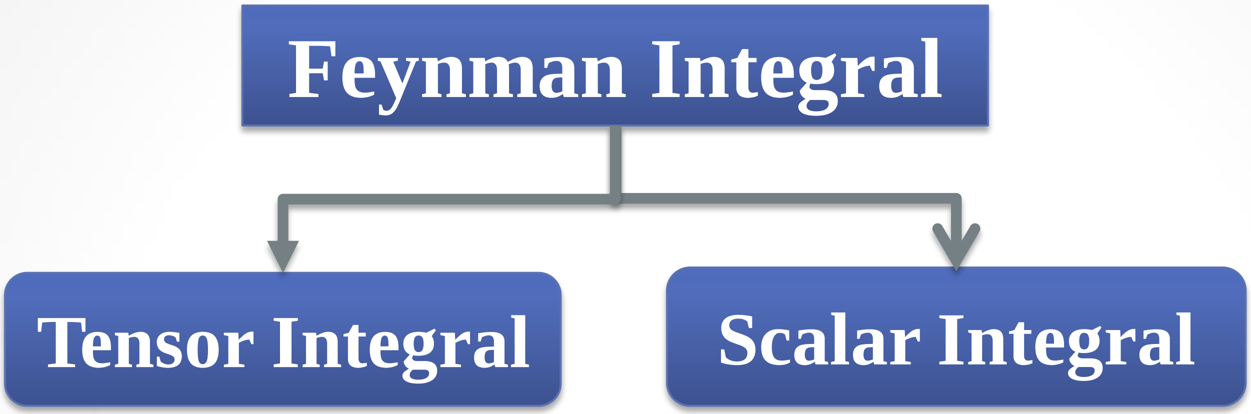
$\mathbf{NLO} \rightarrow \boldsymbol{NNLO}$: **30%**

Virtual Correction

Virtual Correction

Feynman Integral

Virtual Correction



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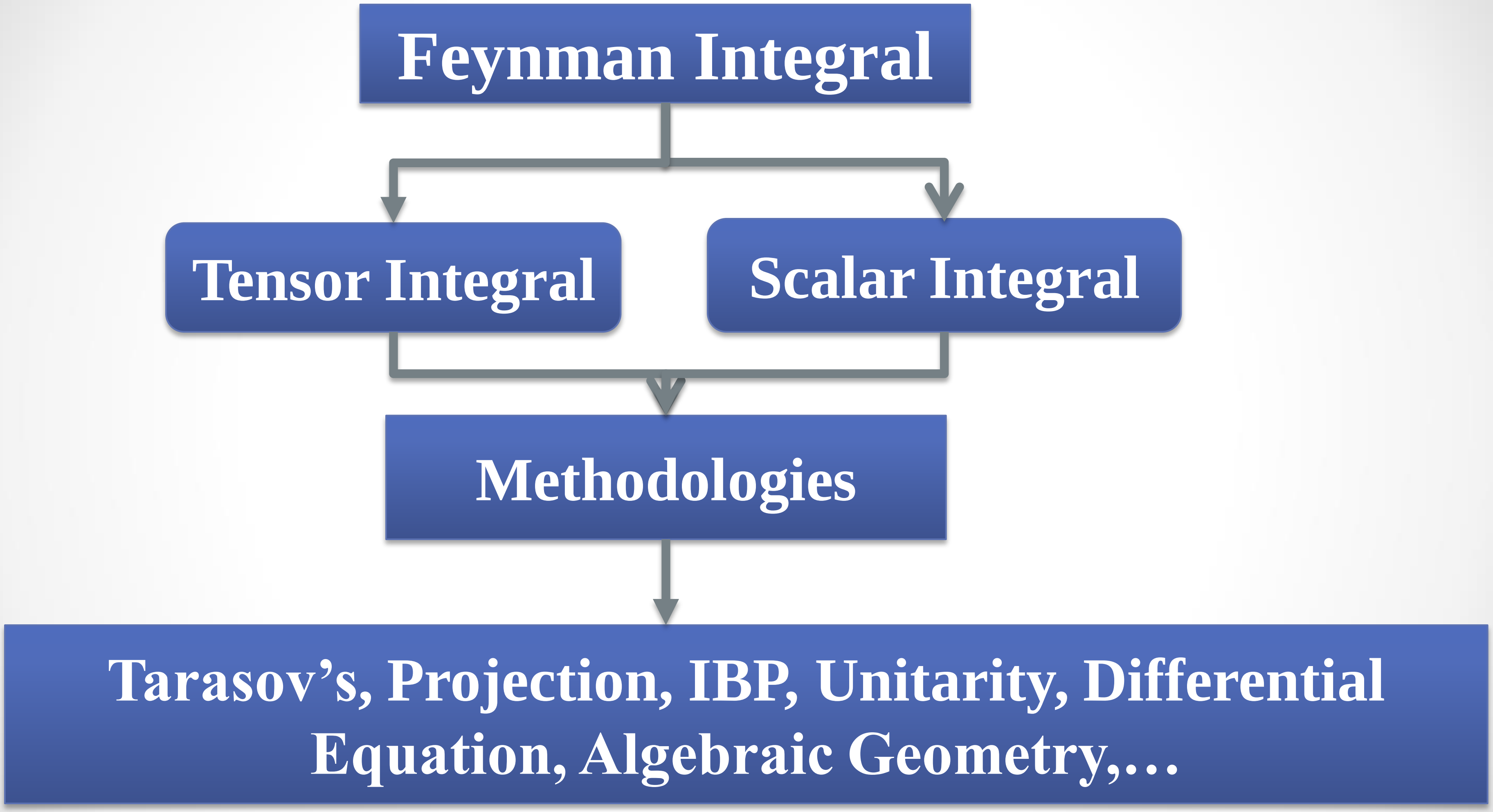

Feynman Integral Representations

Feynman Integral Representations

- Series…*Phys.Lett.B779(2018)353-357*
- Baikov…*Phys.Lett.B385(1996)404-410*
- Feynman...*Int.J.Mod.Phys.A23(2008)1457-1486*
- Alpha …..*Springer Tracts Mod. Phys.211(2004)1–244*
- Mellin-Barnes…*Vladimir A. Smirnov by Feynman Integral Calculus, 2006*
- Many Others ….

Projection Method *...Phys.Rev.D90(2014)no.11,114024*

General tensor structure for an amplitude:

$$A = \sum_{i=1}^{n} C_i \; T_i$$

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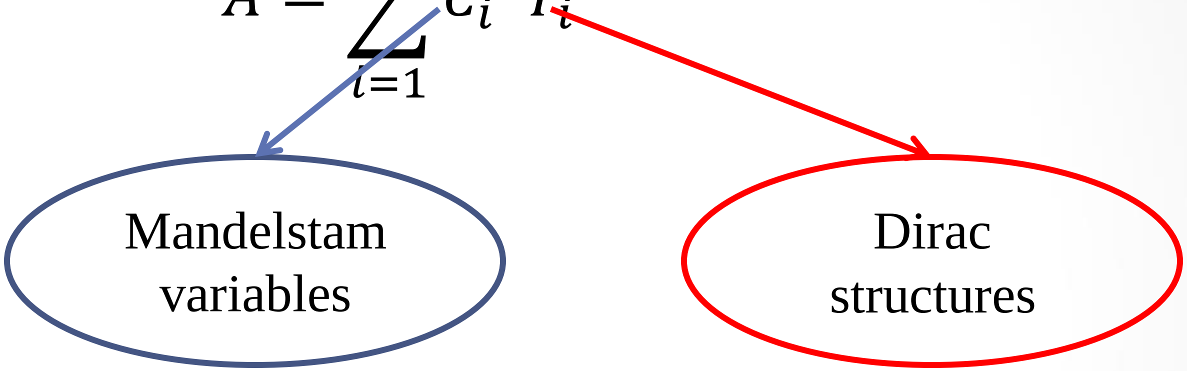

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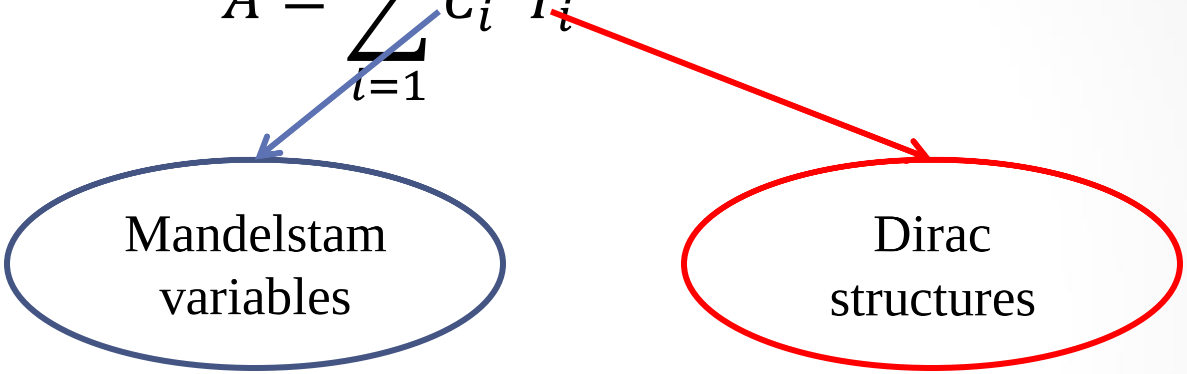

Implies

$$C_i = \sum_j (M_{ij})^{-1} (A T_j^{\dagger}) \,,$$

$$M_{ij} = T_i T_j^{\dagger}$$

If large number of Dirac structures,

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$$larger\ Dirac\ structure \rightarrow Complication\ in\ M_{ij}^{-1}$$

Finally very hard to evaluate the coefficients of Dirac structures.

Tarasov's Method *…Phys.Rev.D54(1995)6479-6490*

Tensor integrals and integrals with irreducible numerator can be reduce by generalized recurrence relations.

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Tensor integrals and integrals with irreducible numerator can be reduce by generalized recurrence relations.

$$I^{(d-2)} \propto D\left(\frac{\partial}{\partial m_i^2}\right) I^{(d)}$$ ... *Nucl.Phys.B502(1997)455-482*

$$I^{(d+2)} = C(d,k) I^{(d)}$$ ... *J.Phys.Conf.Ser.523(2014)012059*

Apply one of these equations on multiple of master integrals;

$$\begin{bmatrix} I_1^{(d+2)} \\ \vdots \\ . \\ I_i^{(d+2)} \end{bmatrix}_{Master} = M_{ii} \begin{bmatrix} I_1^{(d)} \\ \vdots \\ . \\ I_i^{(d)} \end{bmatrix}_{Master}$$

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M_{ii}^{-1} → Complicated

Design Methodology

General amplitude for multi-scale multi-loop Feynman diagram:

$$A = \int \mathbb{D}^L q \frac{N\left(\{q_j\}_{j=1}^{L}, \{k_e\}_{e=1}^{E}\right)}{\prod_{i=1}^{n} D_i^{v_i}},$$

where $\mathbb{D}^L q = \prod_{q=1}^{L} d^D q_l$, L number of loops, $\{q_j\}_{j=1}^{L}$ and $\{k_e\}_{e=1}^{E}$ run over number of loop momenta and external momenta respectively and $\prod_{i=1}^{n} D_i^{v_i}$ is the product of loop propagators for full loop amplitude. $\mathrm{N}\left(\{q_j\}_{j=1}^{L}, \{k_e\}_{e=1}^{E}\right)$ may consist of fermionic chain, polarization vector or both.

Series Representation:

Denominator:

$$D_i \to P_i^2 - m_i^2 + \iota\eta,$$

with $P_i = Q_i + K_i$ is momentum of i-th propagator. Q_i and K_i are linear combination of loop and external momenta respectively.

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Modified amplitude:

$$A \xrightarrow{Modification} \mathcal{A} = \sum_{\{u_1 \ldots u_R, l_1 \ldots l_R\}} N_{u_1 \ldots u_R, l_1 \ldots l_R}(\{k_e\}_{e=1}^{E})\, I_{l_1 \ldots l_R}^{u_1 \ldots u_R}$$

Summation runs over tensor structures.

where

$$I_{l_1 \ldots l_R}^{u_1 \ldots u_R} = \int \mathbb{D}^L q \frac{q_{l_1}^{u_1} \ldots q_{l_R}^{u_R}}{\prod_{i=1}^{n}\left[(Q_i + K_i)^2 - m_i^2 + \iota\eta\right]^{v_i}}$$

Feynman Parametrization:

Feynman parametrization form for tensor integral,

$$I_{l_1 \ldots l_R}^{u_1 \ldots u_R} \propto \int \prod_{j=1}^{N} dx_j x_j^{v_j - 1} \delta\left(1 - \sum_{l=1}^{N} x_l\right) \times \sum_{m=0}^{[R/2]} \frac{\Gamma(N_v^{(m)})}{(-2)^m} \left[(\widetilde{M}^{-1} \otimes g)^{(m)} \tilde{l}^{(R-2m)}\right]^{\Gamma_1, \ldots, \Gamma_R}$$

$$\times U^{-\frac{D}{2}+m-R} \left(\frac{F}{U} - \iota\eta\right)^{-N_v^{(m)}}$$

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$U(x_j)v\,(x_j,k)$

$U(x_j)M^{-1}\,(x_j)$

Rank of Loop in Numerator

$$I_{l_1\ldots l_R}^{u_1\ldots u_R} \propto \int \prod_{j=1}^{N} dx_j x_j^{v_j-1} \delta\left(1-\sum_{l=1}^{N} x_l\right) \times \sum_{m=0}^{[R/2]} \frac{\Gamma(N_v^{(m)})}{(-2)^m} \left[(\widetilde{M}^{-1} \otimes g)^{(m)} \tilde{l}^{(R-2m)}\right]^{\Gamma_1,\ldots,\Gamma_R}$$

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$$\times\, U^{-\frac{D}{2}+m-R} \left(\frac{F}{U} - \iota\eta\right)^{-N_v^{(m)}}$$

F *is* $2-$ *tree graph*, U is $1-$ tree graph

By Taylor series expansion

$$\left(\frac{F}{U} - \iota\eta\right)^{-N_v^{(m)}} = (-\iota\eta)^{-N_v^{(m)}} \sum_{n=0}^{\infty} \binom{-N_v^{(m)}}{n} \frac{F^n}{U^n(-\iota\eta)^n}$$

form factors can be constructed by

$$\sum_{m=0}^{[R/2]} \frac{\Gamma(N_v^{(m)})}{(-2)^m} \left[(\widetilde{M}^{-1} \otimes g)^{(m)} \tilde{l}^{(R-2m)} \right]^{\Gamma_1,\ldots,\Gamma_R} \times U^{-\frac{D}{2}+m-R} \left(\frac{F}{U} - \iota\eta \right)^{-N_v^{(m)}}$$

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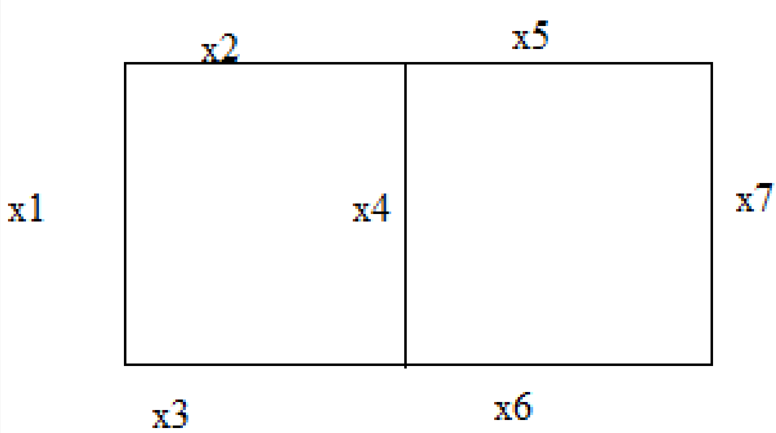
And the coefficients of obtained form factors are Feynman parameters $\{x_1, \ldots, x_N\}$ dependent integral

$$\int \prod_{j=1}^{N} dx_j x_j^{v_j - 1} \delta\left(1 - \sum_{l=1}^{N} x_l\right) U^{-\frac{\mathcal{D}}{2}},$$

Remind $\mathcal{D}$ is different than space-time dimension D.

If we get U by using 1-tree definition



cuts to obtain U
$\longrightarrow$

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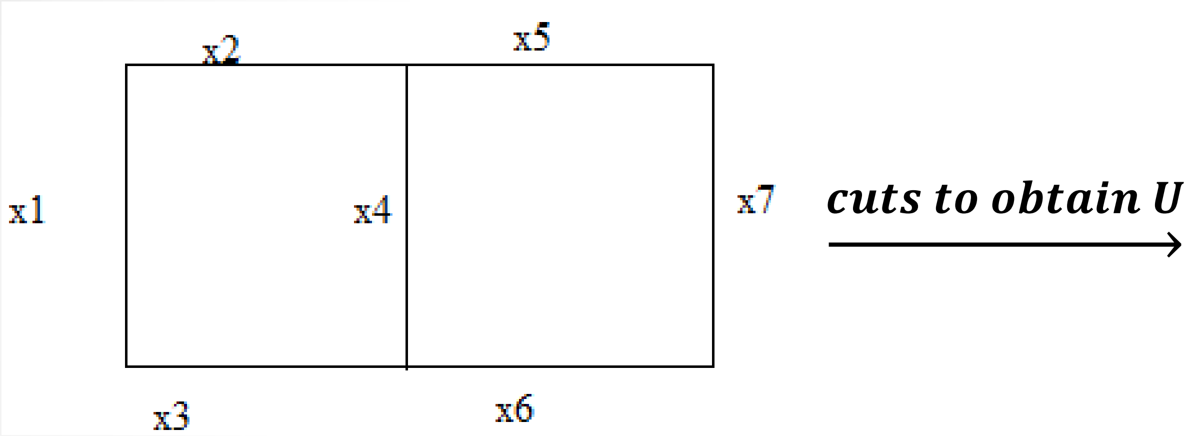
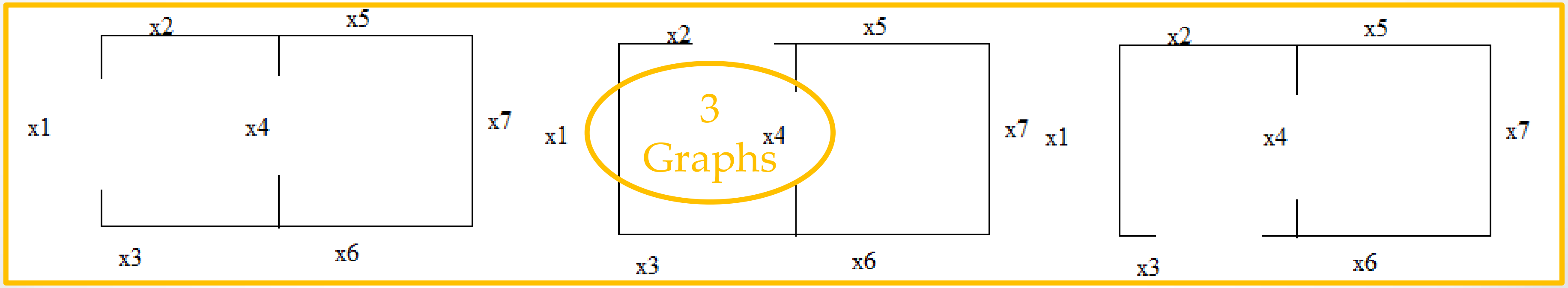
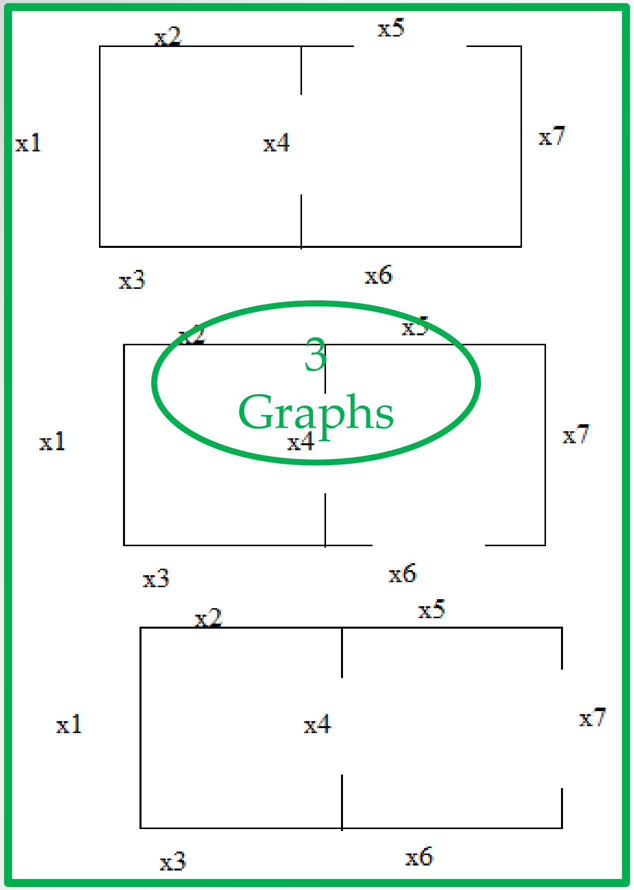
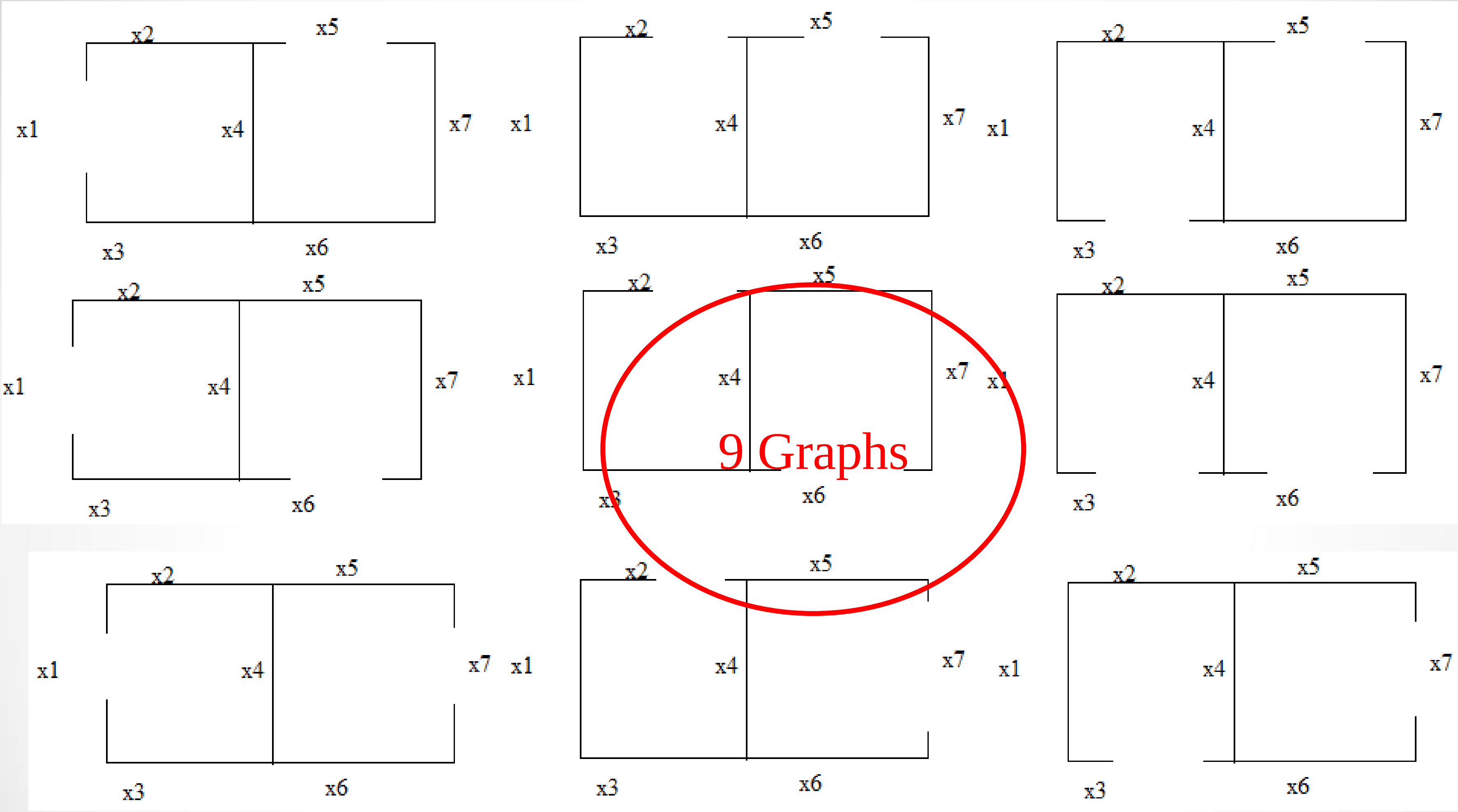




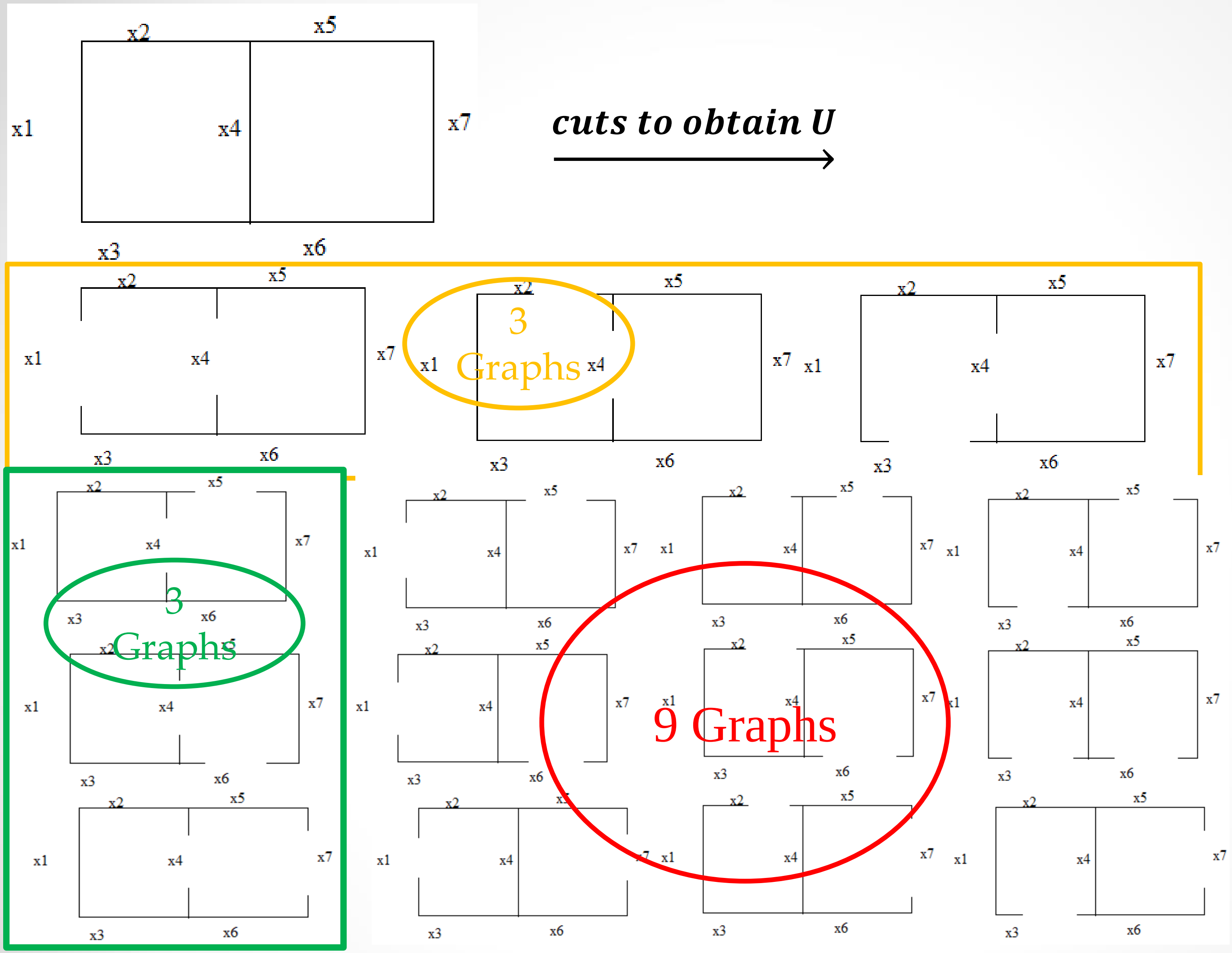
x4 is cut with left loop X_i



x4 is cut with right loop X_i



Left and right X_i are cut and no cut of x4.


cuts to obtain U
3 Graphs
3 Graphs
9 Graphs

Unique cut condition:

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1. Middle with left loop
2. Middle with right loop
3. Left and right loop without using middle.

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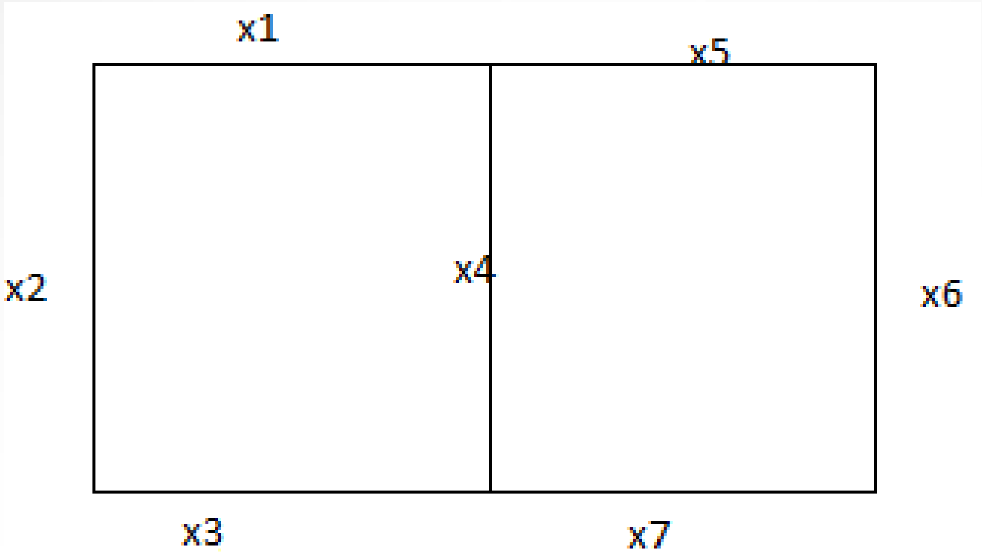
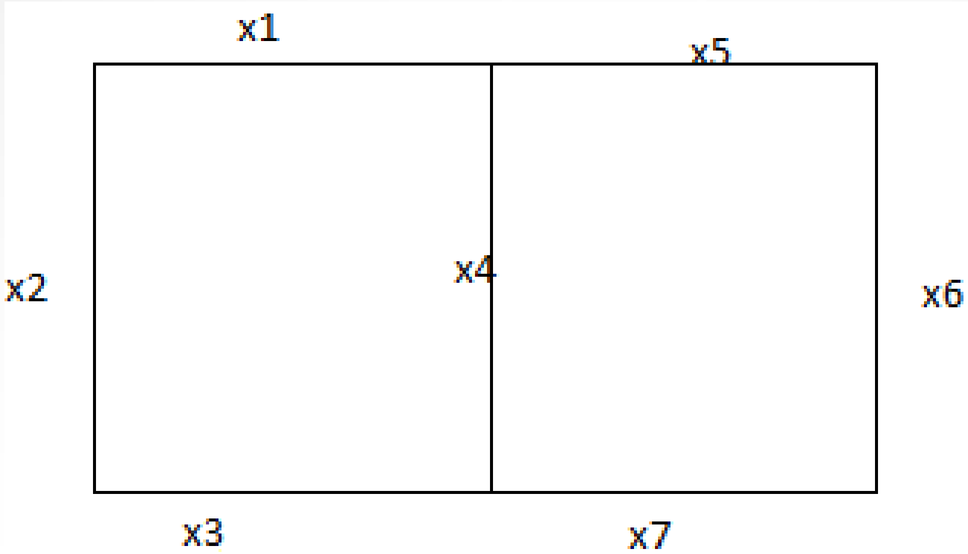
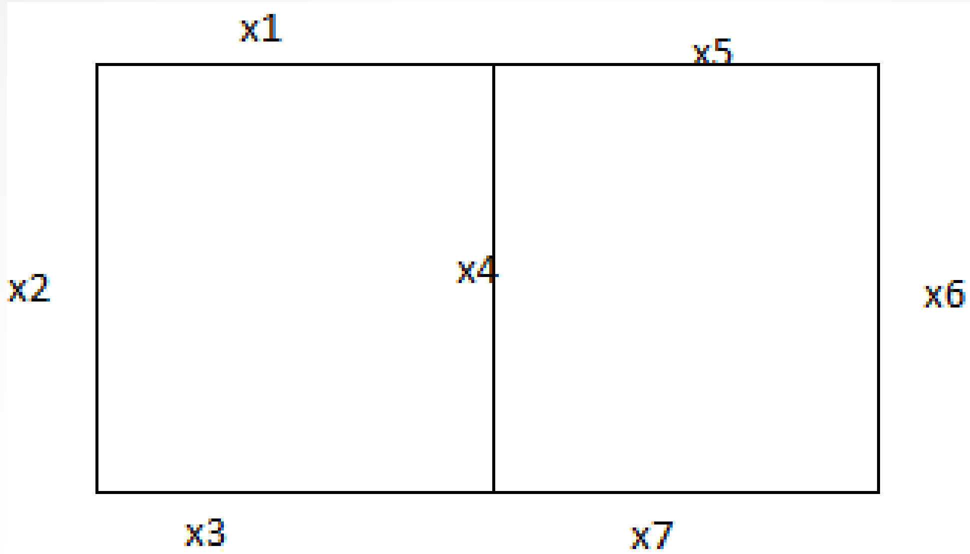
By mapping Feynman parameters in such a way that Symanzik parameter U found to be only dependent of new parameters $\{y_i\}$.e.g.,

$$f : \mathrm{x} \rightarrow y \quad such\ that\ \left\{f(x_i) = t_i y_j\ ; \quad t_i \in [0, \infty)\right\}$$


x1
x5
x2
x4
x6
x3
x7



$$x1 = t_1 y1, x2 = t_2 y1, x3 = t_3 y1$$
$$x5 = t_5 y2, x6 = t_6 y2, x7 = t_7 y2$$
$$x4 = t_4 y3$$



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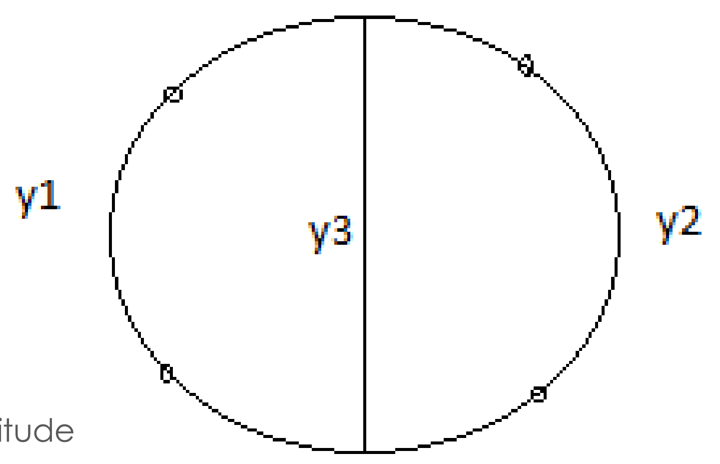

Therefore one can easily evaluate,

$$\int dx_{i_1} \ldots dx_{i_k} x_{i_1}^{n_{i_1}} \ldots x_{i_k}^{n_{i_k}} \int dy_i \delta\left(y_i - x_{i_1} - \cdots - x_{i_k}\right)$$

$$= \int dy_i y_i^{n_{i_1}+\cdots+n_{i_k}+k-1} \frac{\Gamma\left(n_{i_1}+1\right)\ldots\Gamma\left(n_{i_k}+1\right)}{\Gamma\left(n_{i_1}+\cdots+n_{i_k}+1\right)}.$$

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$$C_i = \eta^{\frac{LD}{2} - N_v + \left\lfloor \frac{r_i^{max}}{2} \right\rfloor} \sum_j \sum_{p=0}^{\infty} B_{ijp} I_{L,j}^{(vac),D} \eta^{-p},$$

r_i^{max} is the maximum rank of loop momenta in the form factor, $I_{L,j}^{(vac),D}$ is j-*th* L-loop vacuum bubble master integral. B_{ijp} is series coefficient.

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Mass dimension of C_i,

$$\dim(C_i) = 2\left(\frac{LD}{2} - N_v + \left\lfloor \frac{r_i^{max}}{2} \right\rfloor\right) + \dim(B_{i10}),$$

$\dim(B_{i10})$ is mass dimension of B_{i10}.

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$\dim(B_{i10})$ is mass dimension of B_{i10}.

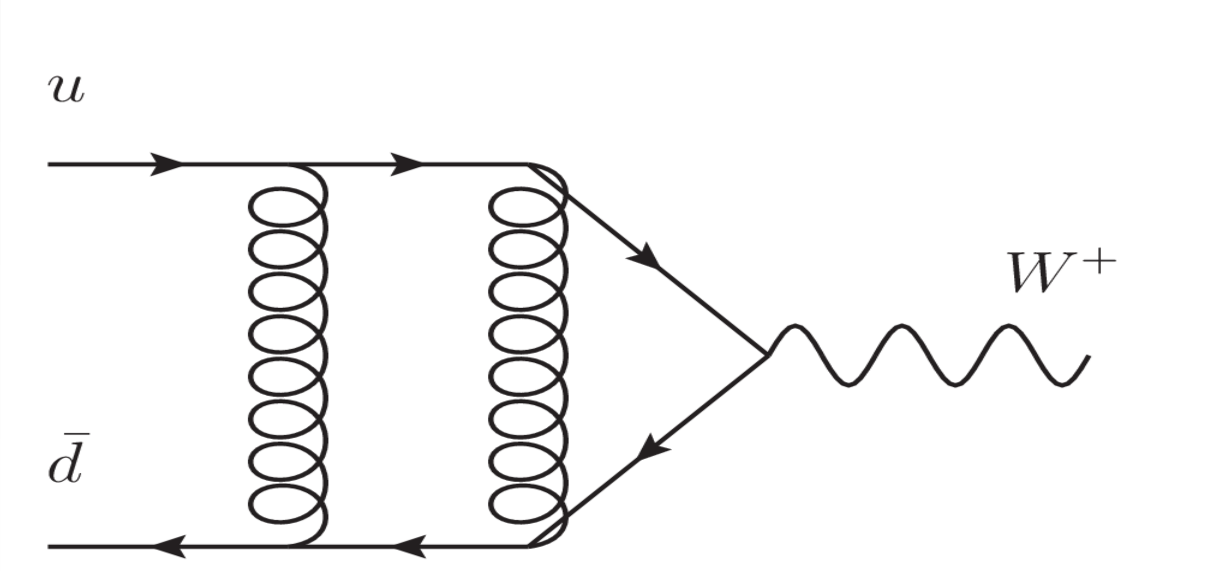
Thus we can choose that suitable master integrals which having mass dimension closer to the mass dimension of C_i.

Finally, final reduction will be finalized over the limit $\eta \to 0^+$.

$$Amplitude = \lim_{\eta \to 0^+} Modified\ Amplitude$$

$$= \sum_i \lim_{\eta \to 0^+} C_i(\eta)(Modified\ Master\ Integrals)_i$$

$u\bar{d} \rightarrow W^+$ Process



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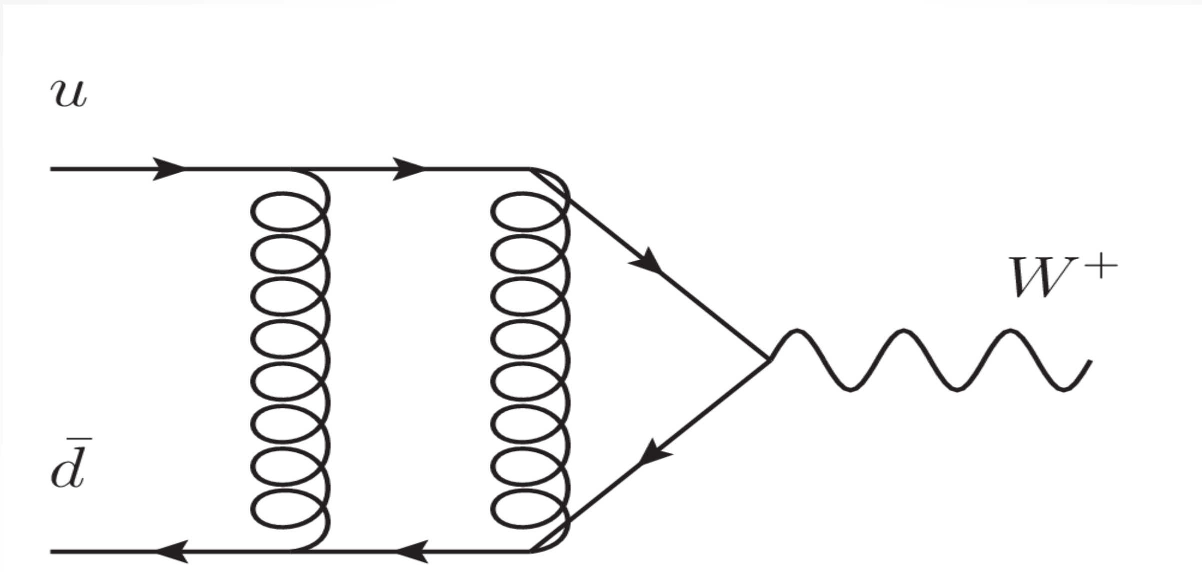

- Numerator

$$\frac{16}{9}\bar{v}(k_2)\gamma^{\alpha}(\not{k}_2+\not{q}_1)\gamma^{\beta}(\not{k}_3+\not{q}_2)\times\not{\epsilon}(k_3)P_L\not{q}_2\gamma^{\beta}(\not{k}_1-\not{q}_1)\gamma^{a}u(k_1)$$

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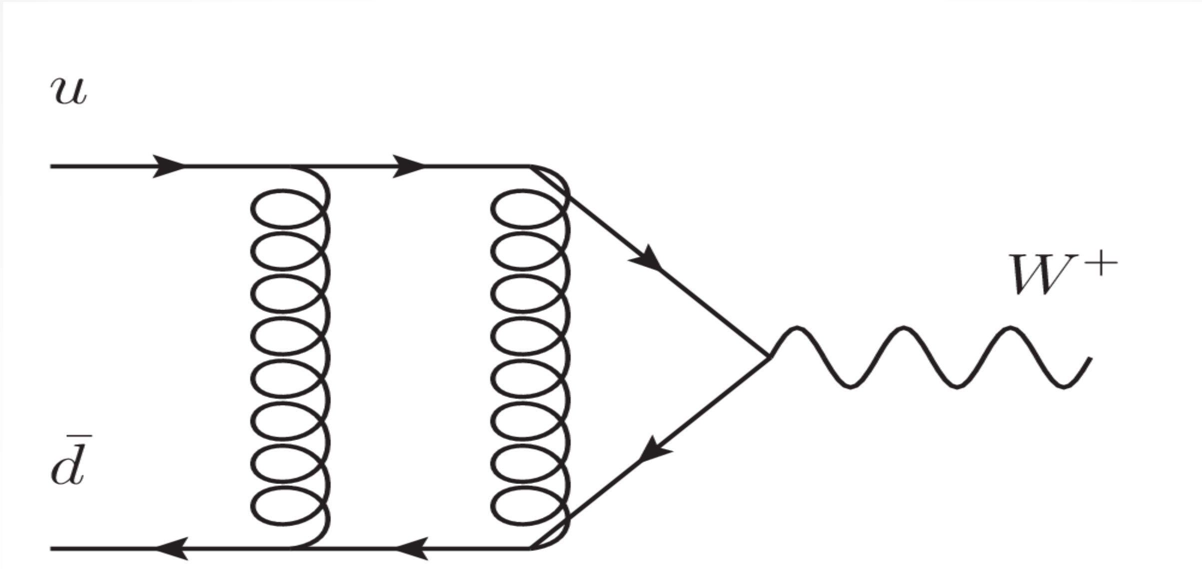

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- Form factor

$$\bar{v}(k_2)\not{\epsilon}(k_3)P_L u(k_1)$$

Modified amplitude

$$\widetilde{M}(\eta) = \int d^D q_1 d^D q_2 \frac{N(q_i,k_j)}{\widetilde{D}_1\widetilde{D}_2\widetilde{D}_3\widetilde{D}_5\widetilde{D}_6\widetilde{D}_7},$$

$$\widetilde{D}_1 = (q_1 - q_2 - k_1)^2 + \iota\eta, \widetilde{D}_2 = (q_1 + k_2)^2 + \iota\eta, \widetilde{D}_3 = (q_2 + k_1 + k_2)^2 + \iota\eta,$$

$$\widetilde{D}_5 = (q_1)^2 + \iota\eta, \widetilde{D}_6 = (q_2)^2 + \iota\eta, \widetilde{D}_7 = (q_1 - k_1)^2 + \iota\eta$$

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For complete amplitude one more denominator term

$$\widetilde{D}_4 = (q_2 + k_1)^2 + \iota\eta.$$

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Vacuum Bubble Master Integrals

$$I_{2,1}^{(Vac),D} = \int \frac{d^D q_1 d^D q_2}{[q_1^2 + \iota][q_2^2 + \iota]}, \qquad I_{2,2}^{(Vac),D} = \int \frac{d^D q_1 d^D q_2}{[q_1^2 + \iota][q_2^2 + \iota][(q_1 + q_2)^2 + \iota]}$$

Finally modified loop amplitude $\widetilde{M}(\eta)$ in series representation form,

$$= \frac{\iota e g_s^4}{\sqrt{2} s_W} \mathcal{F}_1 \eta^{D-4} \left\{ -\frac{8(D-3)(D-2)^2(D^3-3D^2+11D-6)}{243D} \iota I_{2,1}^{(vac),D} \right.$$

$$+ \frac{(D-2)^4(D^2-16D+12)}{81D} I_{2,2}^{(vac),D}$$

$$- \frac{4(D-3)(5D^7-53D^6+319D^5-638D^4-1844D^3+4552D^2+2528D+3456)}{6561D(D+2)} \frac{m_W^2}{\eta} I_{2,1}^{(vac),D}$$

$$- \frac{(D-2)^2(83D^6-724D^5-976D^4+15968D^3-7600D^2-51904D-27648)}{17496D(D+2)} \frac{m_W^2}{\eta} \iota I_{2,2}^{(vac),D}$$

$$\left. + \mathcal{O}\left(\frac{1}{\eta^2}\right) \right\}$$

Matching Criteria for MIs

By using the mass dimension criteria

$$\dim(C_i) = 2\left(\frac{LD}{2} - N_v + \left\lfloor \frac{r_i^{max}}{2} \right\rfloor\right) + \dim(B_{i10})$$

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Under this mass dimension condition we found 25 number of master integrals.

$$\tilde{I}_1(\eta) \equiv \tilde{I}_{0,1,1,0,0,1,1}(\eta), \tilde{I}_2(\eta) \equiv \tilde{I}_{0,1,1,0,1,1,1}(\eta), \tilde{I}_3(\eta) \equiv \tilde{I}_{0,0,1,0,1,0,1}(\eta),$$
$$\tilde{I}_4(\eta) \equiv \tilde{I}_{1,0,1,0,0,0,1}(\eta), \tilde{I}_5(\eta) \equiv \tilde{I}_{1,0,1,0,1,0,1}(\eta), \tilde{I}_6(\eta) \equiv \tilde{I}_{1,0,1,0,1,1,0}(\eta),$$
$$\tilde{I}_7(\eta) \equiv \tilde{I}_{1,0,1,0,1,1,1}(\eta), \tilde{I}_8(\eta) \equiv \tilde{I}_{1,0,1,0,1,2,0}(\eta), \tilde{I}_9(\eta) \equiv \tilde{I}_{1,0,2,0,1,0,1}(\eta),$$
$$\tilde{I}_{10}(\eta) \equiv \tilde{I}_{0,0,1,0,1,1,1}(\eta), \tilde{I}_{11}(\eta) \equiv \tilde{I}_{1,1,0,0,1,0,1}(\eta), \tilde{I}_{12}(\eta) \equiv \tilde{I}_{1,0,0,0,1,1,1}(\eta),$$
$$\tilde{I}_{13}(\eta) \equiv \tilde{I}_{1,1,1,0,0,0,1}(\eta), \tilde{I}_{14}(\eta) \equiv \tilde{I}_{1,1,1,0,0,1,1}(\eta), \tilde{I}_{15}(\eta) \equiv \tilde{I}_{1,1,1,0,1,0,1}(\eta),$$
$$\tilde{I}_{16}(\eta) \equiv \tilde{I}_{1,1,1,0,1,1,1}(\eta), \tilde{I}_{17}(\eta) \equiv \tilde{I}_{1,1,1,0,1,2,0}(\eta), \tilde{I}_{18}(\eta) \equiv \tilde{I}_{1,1,1,0,2,1,0}(\eta),$$
$$\tilde{I}_{19}(\eta) \equiv \tilde{I}_{2,0,1,0,0,0,1}(\eta), \tilde{I}_{20}(\eta) \equiv \tilde{I}_{2,0,1,0,1,1,0}(\eta), \tilde{I}_{21}(\eta) \equiv \tilde{I}_{1,-1,1,-1,1,1,1}(\eta),$$
$$\tilde{I}_{22}(\eta) \equiv \tilde{I}_{1,0,1,-1,1,1,1}(\eta), \tilde{I}_{23}(\eta) \equiv \tilde{I}_{1,0,1,-2,1,1,1}(\eta), \tilde{I}_{24}(\eta) \equiv \tilde{I}_{1,1,1,-1,1,1,1}(\eta),$$
$$\tilde{I}_{25}(\eta) \equiv \tilde{I}_{1,1,1,-2,1,1,1}(\eta)$$

Also under $\eta \to 0^+$ some master integrals $\tilde{I}_3(\eta), \tilde{I}_{10}(\eta), \tilde{I}_{11}(\eta)$ and $\tilde{I}_{12}(\eta)$ found to be zero

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Therefore, remaining 19 master integrals could be reduced into final set of master integrals.

Conclusion & Prospective

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- We propose an alternative reduction approach to directly reduce loop amplitude into linear combination of master integrals and extract the form factors meanwhile.

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- Our series result contain rational function as a coefficients which can be further simplified by mathematical techniques.

- Reliability of our approach on single top production and Higgs production channels is our future work.

Thank for Attention