CP violation and strong pion-pion interactions in the weak $B^{\pm} \to \pi^{\pm}\pi^{\mp}\pi^{\pm}$ decays

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Final state interactions and CP violation in $B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$ decays, Acta Phys. Pol. B **42**, 2013 (2011); arXiv:1011.0960v2 [hep-ph].

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Long range form factors

- Factorized form for the decay amplitudes:

$$\langle \pi^\pm(p_1)[\pi^+(p_2)\pi^-(p_3)]_{S,P,D}\, |j_1 \otimes j_2| B^\pm(p_B)\rangle \equiv \langle \pi^\pm |j_1| B^\pm\rangle \langle [\pi^+\pi^-]_{S,P}\, |j_2|0\rangle$$
$$\text{or}\ \langle [\pi^+\pi^-]_{S,P,D}|j_1|B^\pm\rangle \langle \pi^\pm\, |j_2|0\rangle$$

- The first term leads to the long distance functions:

$$Y_{S,P} \equiv \langle \pi^\pm(p_1)|j_1|B^\pm(p_B)\rangle \langle [\pi^+(p_2)\pi^-(p_3)]_{S,P}|j_2|0\rangle$$
$$\propto F_{0,1}^{B\pi}(q^2) F_{0,1}^{\pi\pi}(q^2),\ \text{with}\ q = p_B - p_1 = p_2 + p_3.$$

- $F_{0,1}^{B\pi}(q^2)$: scalar or vector B to π transition form factor $\rightarrow$ light cone sum rule [P. Ball, R. Zwicky, PRD **71**, 014015 (2005)],
- $F_{0,1}^{\pi\pi}(q^2)$: scalar or vector pion form factor $\rightarrow$ see later.

Model for the vertex functions

- The second term leads to the long distance functions:

$$\begin{aligned} X_{S,P,D} &\equiv \langle[\pi^+(p_2)\pi^-(p_3)]_{S,P,D}|j_1|B^\pm\rangle\,\langle\pi^\pm(p_1)|j_2|0\rangle \\ &\propto G^n_{R_{S,P,D}\pi^+\pi^-}(q^2)\,\langle R_{S,P,D}|j_1|B^\pm\rangle\,\langle\pi^\pm|j_2|0\rangle\,. \end{aligned}$$

- Vertex functions $G^n_{R_{S,P,D}\pi^+\pi^-}(q^2)$: resonance $R_{S,P,D}$ decays into $\pi^+\pi^-$ pairs.
- $\langle R_{S,P,D}|j_1|B^\pm\rangle$: $B^\pm$ to $R_{S,P,D}$ transition form factors, $R_S \equiv f_0(980)$, $R_P \equiv \rho(770)^0$, $R_D \equiv f_2(1270)$.
- $\langle\pi^\pm|j_2|0\rangle$: $f_\pi = 0.1304$ MeV: pion decay constant.
- Model:

$$\begin{aligned} G^n_{R_{S,P}\pi^+\pi^-}(q^2) &\propto \text{scalar or vector pion form factor } F_{0,1}(q^2) \\ G^n_{R_D\pi^+\pi^-}(q^2) &\propto \text{relativistic Breit} - \text{Wigner for } f_2(1270) \end{aligned}$$

Total symmetrized amplitude

$\to$ For $B^- \to \pi^+\pi^-\pi^-$ decays, with $s_{i,j} = (p_i + p_j)^2, i, j = 1, 2, 3, i < j$:

$$\mathcal{M}^-_{sym}(s_{12}, s_{23}) = \frac{1}{\sqrt{2}} \Big[\mathcal{M}^-_S(s_{12}) + \mathcal{M}^-_S(s_{23}) + \mathcal{M}^-_P(s_{12})(s_{13} - s_{23})$$
$$+ \mathcal{M}^-_P(s_{23})(s_{13} - s_{12}) + \mathcal{M}^-_D(s_{12})D(s_{23}, s_{12}) + \mathcal{M}^-_D(s_{23})D(s_{12}, s_{23})\Big],$$

$$\mathcal{M}^-_S(s_{ij}) = \frac{G_F}{\sqrt{3}} \Big[-\chi_S f_\pi \left(M_B^2 - s_{ij}\right) F_0^{BR_S}(m_\pi^2)u(R_S\pi^-)$$
$$+ B_0 \frac{M_B^2 - m_\pi^2}{m_b - m_d} F_0^{B\pi}(s_{ij})v(\pi^- R_S)\Big] \Gamma_1^{n*}(s_{ij}),$$

$$\mathcal{M}^-_P(s_{ij}) = \frac{G_F}{\sqrt{2}} \left[N_P \frac{f_\pi}{f_{R_P}} A_0^{BR_P}(m_\pi^2)u(R_P\pi^-) + F_1^{B\pi}(s_{ij})w(\pi^- R_P)\right] F_1^{\pi\pi}(s_{ij}),$$

$$\mathcal{M}^-_D(s_{ij}) = -\frac{G_F}{\sqrt{6}} u(R_D\pi^-) f_\pi F^{BR_D}(m_\pi^2) \frac{G_{f_2}}{m_{R_D}^2 - s_{ij} - i m_{R_D}\Gamma(s_{ij})}.$$

• Short distance terms $u(R_S\pi^-)$, $v(\pi^- R_S)$, $w(\pi^- R_P)$, $u(R_D\pi^-)$: linear combination of $\lambda_u = V_{ub}V_{ud}^*$, $\lambda_c = V_{cb}V_{cd}^*$ and effective LO +NLO Wilson coefficients $a_i(M_1 M_2)$.

$\to$ Similar expressions for $B^+ \to \pi^-\pi^+\pi^+$ amplitudes. In red: fitted quantities.

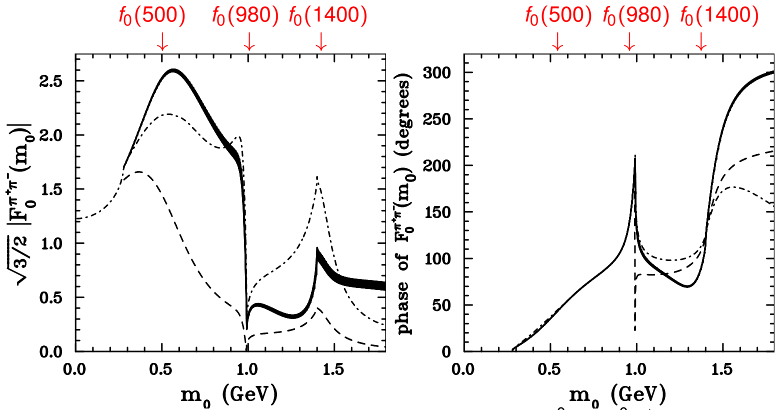
$\Gamma_1^{n*}(q^2) \propto F_0(q^2) = \langle [\pi^+\pi^-]_S \, | u\bar{u} + d\bar{d} | 0 \rangle$

- *Model*: relativistic 3-coupled channel equation

$$\Gamma_i^{n*}(s) = R_i^n(E) + \sum_{j=1}^{3} R_j^n(E) H_{ij}(E), \quad i = 1, 2, 3,$$

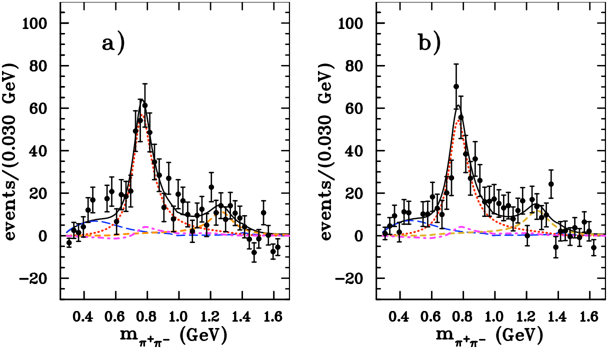
$$H_{ij}(E) = \int \frac{d^3p}{(2\pi)^3} T_{ij}(E, k_i, p) \frac{1}{E - 2\sqrt{p^2 + m_j^2} + i\epsilon} \frac{k_j^2 + \kappa^2}{p^2 + \kappa^2}.$$

- $E = \sqrt{s}$; p off-shell momentum; $i, j = 1, 2, 3$: $\pi\pi$, $K\bar{K}$, effective $(2\pi)(2\pi)$ channels; CMS $k_j = \sqrt{s/4 - m_j^2}$, $m_1 = m_\pi$, $m_2 = m_K$, $m_3 = m_{(2\pi)} = 700$ MeV.
$\rightarrow$ For $\pi\pi$ T matrix: solution *A* of R. Kamiński, L. Leśniak, B. Loiseau, Eur. Phys. J**C9**, 141 (1999).
- $\rightarrow$ $R_i^n(E) = (\alpha_i^n + \tau_i^n E + \omega_i^n E^2)/(1 + cE^4)$, $i = 1, 2, 3$, production functions.
$\rightarrow$ The fitted parameter c controls high energy behavior.
$\rightarrow$ The parameters α_i^n, τ_i^n, ω_i^n are calculated by requiring $\Gamma_i^n(s)$ to satisfy low energy behavior given by one loop calculation in NLO chiral-perturbation theory.
$\rightarrow$ The function $(k_j^2 + \kappa^2)/(p^2 + \kappa^2)$, reducing to 1 on shell, ensures convergence, here $\kappa = 2$ GeV.



⇒ $F_{0n}^{\pi\pi}(m)$: unitarity + analyticity + $\pi\pi$ data. Dark band: $D^0 \to K_S^0\pi^+\pi^-$ variation with error parameters [J.-P. Dedonder *et al.*, Phys. Rev. D **89**, 094018 (2014)]. Dashed line: $B \to 3\pi$ [J.-P. Dedonder *et al.* Acta Phys. Pol. B **42**, 2013 (2011)]. Dotted-dashed line: B. Moussallam [Eur. Phys. J. C. **14**, 111 (2000)] using Muskhelishvili-Omnès equations.

Results of fit on B^- and B^+ $\pi^+\pi^-$ mass distributions

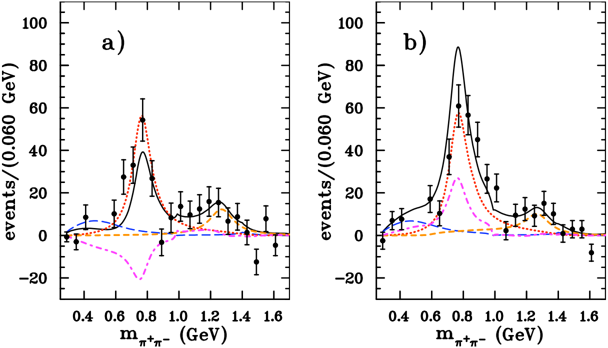


$B^- \to \pi^-\pi^+\pi^-$ $\qquad$ $B^+ \to \pi^+\pi^+\pi^-$

Fit to BABAR's data: Phys. Rev. D **79**, 072006 (2009): — — — *S*-wave; · · · *P*-wave; - - - *D*-wave; · - · interference term; ——— sum.

$A_{CP}[B^\pm \to \rho(770)^0\pi^\pm,\ \rho(770)^0 \to \pi^+\pi^-] = 3.6\% \pm 0.2\%$ [BABAR isobar model: $\left(18 \pm 7 \pm 5^{+2}_{-14}\right)\%$]

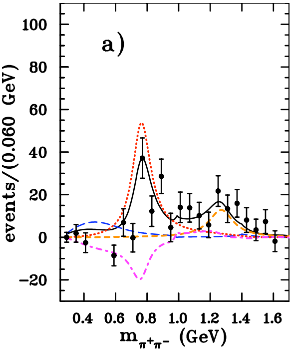
B^- decay distributions with $\cos\theta < 0$ and > 0



$B^- \to \pi^-\pi^+\pi^-$, $\cos\theta < 0$ $B^- \to \pi^-\pi^+\pi^-$, $\cos\theta > 0$

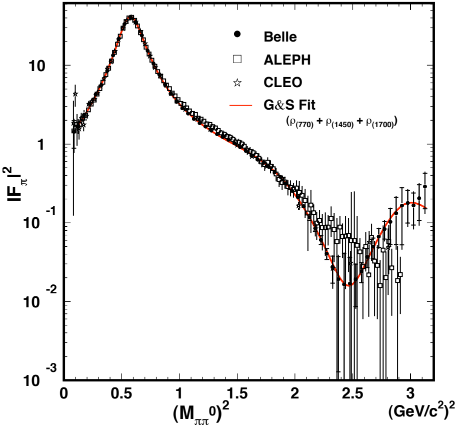
Fit to BABAR's data: Phys. Rev. D **79**, 072006 (2009): — — — *S*-wave; · · · *P*-wave; - - - *D*-wave; · - · interference term; ——— sum.

B^+ decay distributions with $\cos\theta < 0$ and > 0



$B^+ \to \pi^+\pi^+\pi^-$, $\cos\theta < 0$ $B^+ \to \pi^+\pi^+\pi^-$, $\cos\theta > 0$

Fit to BABAR's data: Phys. Rev. D **79**, 072006 (2009): — — — *S*-wave; · · · *P*-wave; - - - *D*-wave; · - · interference term; ——— sum.



Fit by Belle Collaboration of $\tau^- \to \pi^-\pi^0\nu_\tau$
with Gounaris-Sakurai model $\rho(770) + \rho(1450 + \rho(1700)$
[Phys. Rev. D **78** 072006 (2008)]

[Backup] B^- branching fractions, data and fit parameters

- The double and single differential $B^- \to \pi^-\pi^+\pi^-$ branching fractions:

$$\frac{d^2\mathcal{B}^-}{dm_{12}\, d\cos\theta} = \frac{1}{\Gamma_B}\frac{m_{12}|\overrightarrow{p_2}||\overrightarrow{p_3}|}{8(2\pi)^3 M_B^3}\left|\mathcal{M}^-_{sym}(s_{12}, s_{23})\right|^2, \ \Gamma_B \text{ total width of } B^-$$

$$\frac{d\mathcal{B}^-}{dm_{12}} = \int_{-1}^{\cos\theta_g} \frac{d^2\mathcal{B}^-}{dm_{12}\, d\cos\theta} d\cos\theta, \text{ with } \cos\theta_g = \cos\theta\ (m_{12} = m_{23}),$$

  θ: helicity angle between $\overrightarrow{p_2}$ and $\overrightarrow{p_3}$.

- χ^2 fit to 170 data points from BABAR's invariant mass distributions + experimental branching ratio $\mathcal{B}_P$ for $B^\pm \to \rho(770)^0\pi^\pm,\ \rho(770)^0 \to \pi^+\pi^-$.
- $\chi^2/d.o.f. = 231.6/(171-4) = 1.39$ with,
  $\to \kappa = 2$ GeV (fixed): regulator parameter of $\Gamma_1^{n*}(q^2)$,
  $\to c = 19.5 \pm 4.2$ (fit): high-energy cutoff of the production function $R_i^n(E)$,
  $\to \chi_S = -19.4 \pm 2.5\ \text{GeV}^{-1}$ (fit): strength of the S-wave amplitude,
  $\to N_P = 1.122 \pm 0.034$ (fit): deviation from 1 of strength f_π/f_{R_P} of P-wave amplitude,
  $\to F^{BR_D}(m_\pi^2) = 0.0977 \pm 0.0070$ (fit): transition form factor between B and $f_2(1270)$,
  $\mathcal{B}_P = (8.1 \pm 0.5) \times 10^{-6}$ versus $\mathcal{B}_P^{exp} = (8.1 \pm 0.5 \pm 1.2^{+0.4}_{-1.1}) \times 10^{-6}$.

[Backup] Summary: *CP* violation and $\pi\pi$ interaction in $B^\pm \to \pi^\pm\pi^\mp\pi^\pm$

- Analysis: quasi two-body $B^\pm \to \pi^\pm[\pi^+\pi^-]_{S,P,D}$ QCD factorization approach.
- Short distance amplitude NLO in α_S with vertex and penguin corrections.
- $\pi^+\pi^-$ final state interactions: pion non-strange scalar and vector form factors for *S* and *P* waves and relativistic Breit-Wigner formula for the *D* wave.
- Pion scalar form factor: unitary relativistic coupled-channel model including $\pi\pi$, $K\bar{K}$ and effective $(2\pi)(2\pi)$ forces.
- Pion vector form factor: Belle Collaboration analysis of $\tau^- \to \pi^-\pi^0\nu_\tau$ data.
- Fit recent $B^\pm \to \pi^\pm\pi^\mp\pi^\pm$ BABAR Collaboration data: 3 parameters for *S* wave, 1 for *P* wave and 1 for *D* wave.

[Backup] Summary: Good agreement with BABAR's data

- Sizable S wave near $\pi\pi$ threshold: $f_0(600)$ - significant interference between S and P waves under $\rho(770)$ peak.
- $f_0(980)$ not visible as a peak since pion scalar form factor has a dip near 1 GeV.
- For B to $f_2(1270)$ transition form factor we predict: $F^{Bf_2}(m_\pi^2) = 0.098 \pm 0.007$.
- Unified unitary description of $f_0(600)$, $f_0(980)$ and $f_0(1400)$ in terms of pion non-strange scalar form factor.
- Strong interaction phases constrained by unitarity and meson-meson data: help in extraction of weak phase γ or $\phi_3 = \arg(-\lambda_u^*/\lambda_c^*)$.
- New experimental analysis with better statistics welcome: Belle Collaboration data exist and in future LHCb and near term super B factories.