Comments on Hadronic B Decays in QCD

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Future Challenges in Non-Leptonic B Decays
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QCD Factorization for Hadronic B-Meson Decays

- Factorization formulae (schematic form).
  - Collinear factorization:

$$\langle M_1\, M_2 | Q_i | B \rangle = F^{B M_1}(0)\, T_i^{\rm I} * f_{M_2}\, \phi_{M_2} + \underbrace{T_i^{\rm II}}_{H_i^{\rm II} * J} * f_B\, \phi_B * f_{M_1}\, \phi_{M_1} * f_{M_2}\, \phi_{M_2}\,.$$

  - TMD factorization:

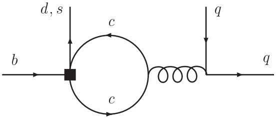
$$\langle M_1\, M_2 | Q_i | B \rangle = T * f_B\, \phi_B * f_{M_1}\, \phi_{M_1} * f_{M_2}\, \phi_{M_2}\,.$$

- Technical differences:
  - No soft contribution to $F^{B M_1}(0)$ in TMD factorization (Sudakov mechanism?).
  - Strong phases from the loop diagrams at the hard scale (LP) and from the weak annihilation as well as the twist-3 correction to the HSS (NLP) in collinear factorization.
  - Strong phases originate from the weak annihilation diagrams in TMD factorization.
- Conceptual aspects of TMD factorization for hard exclusive processes:
  - Power counting scheme for the intrinsic transverse momenta of hadrons.
    - ★ For $k_T \sim \mathcal{O}(\Lambda)$, the hard kernel independent of $k_T \Rightarrow$ collinear factorization.
    - ★ For $k_T \gg \Lambda$, the TMD wave function can be further factorized (power correction).
    - ★ Better to study TMD factorization for the pion-photon form factor in the first place.
  - Definition of TMD wavefunctions free of the rapidity divergence and the pinch singularity.

Charming Penguins in Hadronic B-Meson Decays

- Question: Whether charming-penguin diagrams factorize at leading power in Λ/m_b?
  Answer: Yes, irrespective of whether the charm quark is treated as a light or a heavy quark.

- Power counting for the threshold effect (BBNS, arxiv: 0902.4446).



Penguin contraction of the tree operators:

- Partonic result:

$$G_\pi(s) = \int_0^1 dx\, G(s, \bar{x})\, \phi_\pi(x) \ , s = \frac{m_c^2}{m_b^2} .$$

- Resonance effect:

$$G_\pi^\psi = -\frac{8\pi^2 f_\psi^2}{m_b^2} [...] .$$

- Coulombic limit:
  $G_\pi^\psi \sim \alpha_s^3 \, (m_c/m_b)^2$.
  Small m_c limit: $G_\pi^\psi \sim (\Lambda/m_b)^2$.
  Resonance effect suppressed parametrically and numerically.

Scaling behaviour for the nonperturbative $c\bar{c}$ loops:

$$\frac{A_{c\bar{c}}}{A_{\rm LO}} \sim \alpha_s(2m_c) f\left(\frac{2m_c}{m_b}\right) v \underbrace{\frac{4m_c^2 v^2}{m_b^2}}_{\text{missed in BPRS : hep-ph/0502094}} .$$

- For $m_c v^2 \sim \mathcal{O}(\Lambda) \to$ power suppression.

$$\frac{A_{c\bar{c}}}{A_{\rm LO}} \sim \frac{\alpha_s}{v} f\left(\frac{2m_c}{m_b}\right) \frac{\Lambda^2}{m_b^2} .$$

- For $m_c v^2 \gg \Lambda \Rightarrow$ can be treated in perturbation theory.
  Uncalculable nonperturbative effects are power suppressed (Voloshin, 1982).

Charming Penguins in $B \to X_s \ell\ell$ [BBNS, arxiv: 0902.4446]

- Parton-hadron duality does not work for the integrated $B \to X_s \ell\ell$ branching fraction.

$$R_\psi = \frac{\mathrm{BR}(B \to X_s\,\psi \to X_s\,\ell\ell)}{\mathrm{BR}(B \to X_s \ell\ell)_{\mathrm{SD}}} = \frac{7.8 \cdot 10^{-3} \times 0.06}{5.3 \cdot 10^{-6}} \simeq 90\,.$$

- A toy model: $l_1 \to l_2\, e^+\, e^-$ ($m_t > m_1 \gg m_c$ and $m_2 = 0$).
  - Effective weak Hamiltonian:

$$\begin{aligned} \mathscr{H}_{\mathit{eff}} &= \frac{G}{\sqrt{2}} \left[(\bar{l}_2 l_1)_{V-A}\, (\bar{c}c)_{V-A} - (\bar{l}_2 l_1)_{V-A}\, (\bar{t}t)_{V-A} \right] . \\ &\Downarrow \\ A(l_1 \to l_2\, e^+ e^-) &= -\frac{G}{\sqrt{2}}\, e_c e^2\, \Pi(q^2)\, \bar{l}_2 \gamma^\mu (1-\gamma_5) l_1\, \bar{e} \gamma_\mu e\,. \end{aligned}$$

  - The differential decay rate for $l_1 \to l_2\, e^+\, e^-$:

$$\frac{d\Gamma(l_1 \to l_2\, e^+ e^-)}{ds} = \frac{G^2 \alpha^2 m_1^5}{27\pi}\, (1-s)^2\, (1+2s)\, \left|\Pi(q^2)\right|^2.$$

Parton-hadron duality is not expected to apply to averages of the quantity $\left|\Pi(q^2)\right|^2$.

Charming Penguins in $B \to X_s \ell\ell$ [BBNS, arxiv: 0902.4446]

- Parton-hadron duality works for averages of the quantity $\mathrm{Im}\,\Pi(q^2)$ (or $\Pi(q^2)$).
  - Resonance effect:

$$\Pi(q^2) = -\frac{f_\psi^2}{q^2 - M_\psi^2 + iM_\psi\Gamma_\psi}\,,$$

$$\Rightarrow \mathrm{Im}\,\Pi(q^2) = \frac{f_\psi^2 M_\psi \Gamma_\psi}{(q^2 - M_\psi^2)^2 + M_\psi^2\Gamma_\psi^2}\,, \qquad \left|\Pi(q^2)\right|^2 = \overbrace{\frac{f_\psi^2}{M_\psi\Gamma_\psi}}^{\simeq 560!}\,\mathrm{Im}\,\Pi(q^2)\,.$$

  - Parton-hadron duality works for $\mathrm{Im}\,\Pi(q^2)$.

$$\int_0^{m_1^2} dq^2\,\mathrm{Im}\,\Pi(q^2) \approx \pi f_\psi^2 \ll m_1^2\,.$$

    Used in the determination of m_c from QCD sum rules.
  - Parton-hadron duality does not work for $\left|\Pi(q^2)\right|^2$.

$$\int_0^{m_1^2} dq^2\,|\Pi(q^2)|^2 \approx \pi f_\psi^2 \times \underbrace{\frac{f_\psi^2}{M_\psi\Gamma_\psi}}_{\text{Can be arbitrarily large.}}\,, \text{ compared to } m_1^2\,.$$

  - The absence of global duality can be also understood from the fact that there is no OPE for the total decay rate of $l_1 \to l_2\, e^+ e^-$ (exclusive cut of the $e^+ e^-$ loop needed!).