

# Non-leptonic B decays, discussion

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## Items

- Charm loops, duality & factorization
- Observations on PQCD
- Observations on FAT

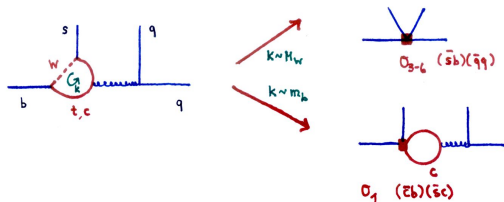


# I. Charm loops, duality & factorization

[BBNS, hep-ph/0411171, 0902.4446]

## Penguins with charm

$b \rightarrow s$  transitions are dominated by loop diagrams. Among them a leading non-local contribution due to a charm-quark loop. Some controversy in the past about how well this can be calculated and about factorization as such.



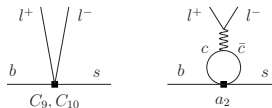
Two separate issues:

- Factorizes, but power corrections could be important [Ciuchini et al. '97; '01 – their fit to  $B \rightarrow \pi K$  (0811.0341 [hep-ph]) seems to be consistent with power-suppressed deviations from factorization]
- No factorization in the heavy-quark limit [Bauer et al. '04; '05]  
Loss of predictivity for direct CP asymmetries and penguin modes. (final-state dependent phenomenological parameter)  
Non-factorization arises from the charm threshold region  $k_s^2 = m_b^2 \bar{u} \approx 4m_c^2$ . Large resonant effects well-known from  $B \rightarrow X_s \ell \ell$ . Why should charm loops in non-leptonic decays be different?

## $B \rightarrow X_s \ell \ell$

Define “SD” as quark-level calculation including charm-loop, no resummations. Then

$$R_\psi \equiv \frac{\text{B}(B \rightarrow X_s \psi \rightarrow X_s \ell^+ \ell^-)}{\text{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{SD}}} \approx 90$$



### Parton-hadron duality violation!

Consider toy theory with Hamiltonian  $\mathcal{H}_{\text{eff}} = \frac{G}{\sqrt{2}} [(\bar{l}_2 l_1)_{V-A} (\bar{c} c)_{V-A} - (\bar{l}_2 l_1)_{V-A} (\bar{t} t)_{V-A}]$   
(drops inessential reconnections to  $b$  and  $s$  lines)

$$\frac{d\Gamma(l_1 \rightarrow l_2 e^+ e^-)}{ds} = \frac{G^2 \alpha^2 m_1^5}{27\pi} (1-s)^2 (1+2s) |\Pi(q^2)|^2$$

Resonance contribution:

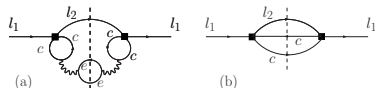
$$\Pi(q^2) = -\frac{f_\psi^2}{q^2 - M_\psi^2 + iM_\psi \Gamma_\psi}$$

$$\int_0^{m_1^2} dq^2 \text{Im} \Pi(q^2) \approx \pi f_\psi^2 \ll m_1^2 \quad \int_0^{m_1^2} dq^2 |\Pi(q^2)|^2 \approx \pi f_\psi^2 \times \frac{f_\psi^2}{M_\psi \Gamma_\psi}$$

Small contribution for averages of  $\text{Im} \Pi(q^2)$  and  $\Pi(q^2)$ , but (in principle arbitrarily large) enhancement by the small resonance width for averages of  $|\Pi(q^2)|^2$ .

## No duality for $B \rightarrow X_s \ell \ell$

$\text{Br}(l_1 \rightarrow l_2 e^+ e^-)$  is not a complete sum over cuts [compare the  $l_1$  inclusive width].  
Duality should not be expected!



- $J/\psi$  resonance in  $B \rightarrow X_s \ell \ell$

$$R_\psi \equiv \frac{\text{B}(B \rightarrow X_s \psi \rightarrow X_s l^+ l^-)}{\text{B}(B \rightarrow X_s l^+ l^-)_{\text{SD}}} = \underbrace{\frac{512\pi^5 \kappa^2 a_2^2 (1-r)^2 (1+2r)}{9(|C_9|^2 + |C_{10}|^2)}}_{\approx 0.16} \times \frac{f_\psi^2}{m_b^2} \times \underbrace{\frac{f_\psi^2}{M_\psi \Gamma_\psi}}_{560} \approx 90$$

In the heavy quark limit  $m_b \gg \Lambda$ ,  $m_c/m_b$  fixed, this is  $O(1)$ , but enhanced:

$$R_\psi = \frac{512\pi^5 \kappa^2 a_2^2 (1-r)^2 (1+2r)}{9(|C_9|^2 + |C_{10}|^2)} \times \frac{54}{5\pi(\pi^2 - 9)} \left( \frac{\alpha_s(m_c v)}{\alpha_s(M_\psi)} \right)^3 \times \frac{m_c^2}{m_b^2}$$

- $\rho$  resonance in  $B \rightarrow X_{s,d} \ell \ell$  – very small

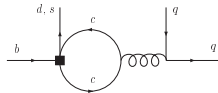
$$R_\rho \approx [10-50] \times \frac{f_\rho^2}{m_b^2} \times \underbrace{\frac{f_\rho^2}{M_\rho \Gamma_\rho}}_{0.38} \approx [0.007-0.036]$$

## Comparison with $B \rightarrow \pi\pi (K\pi)$

Charm-loop contribution to the EW penguin amplitude

$$\Delta a_{10}^p = \frac{C_1 + NC_2}{N} \frac{\alpha}{9\pi} \left[ -\frac{4}{3} \ln \frac{\mu}{m_b} + \frac{2}{3} - G_\pi(s_p) \right]$$

$$G_\pi(s) = \int_0^1 dx \phi_\pi(x) \left\{ -\frac{2}{3} \ln s + \frac{8\pi^2}{3} \left[ \Pi_q(\bar{x}m_b^2) - \Pi_q(0) \right] \right\}$$



The resonance contribution is

$$G_\pi^\psi = -\frac{8\pi^2 f_\psi^2}{m_b^2} \left[ 2r_\psi (1 - r_\psi) \left( \ln \frac{1 - r_\psi}{r_\psi} - i\pi \right) + 1 - 2r_\psi \right] \quad (r_\psi = M_\psi^2/m_b^2)$$

Suppressed in the heavy quark limit and no width enhancement. The contribution from the threshold region is of order  $G_\pi^\psi$ :

$$\frac{A_{c\bar{c}}}{A_{LO}} \sim \alpha_s(2m_c) f \left( \frac{2m_c}{m_b} \right) v \times \frac{4m_c^2 v^2}{m_b^2}$$

The last suppression factor is missing in [Bauer et al. '05] since in  $\int_{r(1-v^2)}^{r(1+v^2)} du C_I^{\text{prod}}(u) \dots$  they set  $C_I^{\text{prod}}(u) \sim \delta(\bar{u} - 4m_c^2/m_b^2)$  rather than the correct  $C_I^{\text{prod}}(u) \sim 1$ .

## II. Observations on PQCD

Main conceptual difference:  $B \rightarrow M_1$  form factor ( $\bar{\xi}\Gamma h_\nu$ , SCET<sub>I</sub> operator) assumed to factorize in LCDAs after self-consistent regularization of endpoint divergences by transverse-momentum-dependent Sudakov resummation

$$\begin{aligned}\langle M_1 M_2 | Q_i | \bar{B} \rangle &= F^{BM_1} T_i^I \star \Phi_{M_2} + \Phi_B \star [H_i^{II} \star J^{II}] \star \Phi_{M_1} \star \Phi_{M_2} \\ &\rightarrow \phi_B \star [T_i^{\text{PQCD}} \star J^{\text{PQCD}}] \star \phi_{M_1} \star \phi_{M_2} \\ &\rightarrow \phi_B \star [H_i] \star \phi_{M_1} \star \phi_{M_2}\end{aligned}$$

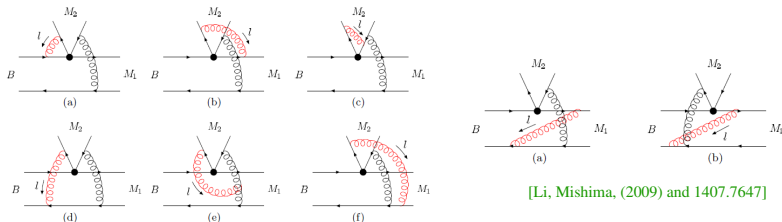
- Applied to *all* diagrams, spectator-scattering, weak annihilation, ...  
Scales not completely factorized,  $1/m_b$  power counting of contributions never clarified.
- Two questions: 1) theory of  $k_t$  resummation, 2) does it work for  $m_b = 5 \text{ GeV}$ ?  
[Descotes-Genon, Sachrajda, hep-ph/0109260]
- Phases from (resummed) tree diagrams in the endpoint region

$$\frac{1}{x\bar{y}m_b^2 - k_\perp^2 + i\epsilon}$$

If  $k_\perp^2 \sim m_b \Lambda$ , the effect is part of the perturbative loop function. If  $k_\perp^2 \sim \Lambda^2$ , it can't be computed. Need to argue that resummation forces self-consistently  $m_b \Lambda \gg k_\perp^2 \gg \Lambda^2$ .



# NLO computations in PQCD and Glauber singularities (1/2)



[Li, Mishima, (2009) and 1407.7647]

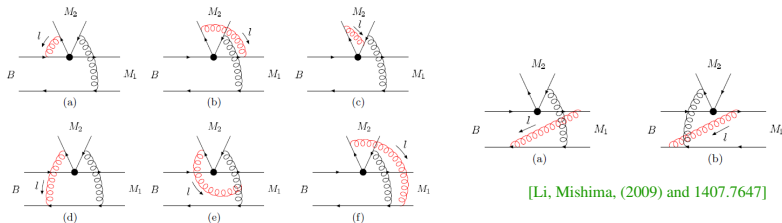
FIG. 2: NLO diagrams for Fig. 1(a) that are relevant to the factorization of the  $M_2$  meson wave function. Figures 2(d)-2(f) contribute to the Glauber divergences.

- Glauber scaling  $n+l, n-l \ll l_\perp$ .
- Glauber often subsumed in soft by contour deformation.  
Glauber effects in SCET [Rothstein, Stewart (2016)]. Do not expect Glauber singularities here?
- Here Glauber supposed to result in non-factorization whenever there is an integral

$$\int_{-a}^b \frac{d(n-l)}{n-l+i\epsilon}$$

which contains  $n-l = 0$ .

## NLO computations in PQCD and Glauber singularities (2/2)



[Li, Mishima, (2009) and 1407.7647]

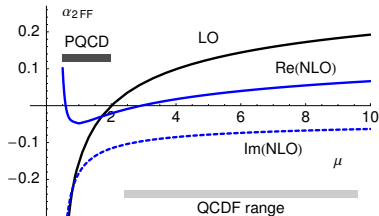
FIG. 2: NLO diagrams for Fig. 1(a) that are relevant to the factorization of the  $M_2$  meson wave function. Figures 2(d)-2(f) contribute to the Glauber divergences.

- Same graphs appear in the calculation of spectator scattering [MB, Yang (2005), MB, Jäger (2005)]. The IR singularities could be consistently factorized into B-meson and light-meson LCDAs as predicted by the QCD factorization formula.
  - Glauber divs noted for (d)-(f) when  $l$  is  $M_2$ -collinear. However, these diagrams are all power-suppressed, except for one term in (e), which is Glauber-free.
- ⇒ Effectively a parameterization of an arbitrary power-suppressed contribution to the colour-suppressed tree amplitude.
- Also for penguin diagrams, ...?

# Phenomenology

- Weak effective Lagrangian Wilson coefficients evolved to low scale. But their running stops at  $\mathcal{O}(m_b)$  and is replaced by factorized SCET<sub>I</sub> running.

$$a_{2FF}(\mu) = C_2 + \frac{C_1}{N_c} + \underbrace{\frac{\alpha_s C_F}{4\pi} \frac{C_1}{N_c} \left[ 12 \ln \frac{m_b}{\mu} - \frac{37}{2} - 3i\pi \right]}_{\text{NLO correction (asymptotic LCDA)}}$$



- Assumptions on and form of B meson LCDA
- Scale variations and theory parameter uncertainties not realistically estimated, sometimes determined from fit to data.
- Too low scales of  $\alpha_s$  etc. Not in the regime where PT is reliable.