

Determination of γ using flavor SU(3)

Making a case for three-body decays

Bhubanjyoti Bhattacharya
(*bbhattach@ltu.edu*)

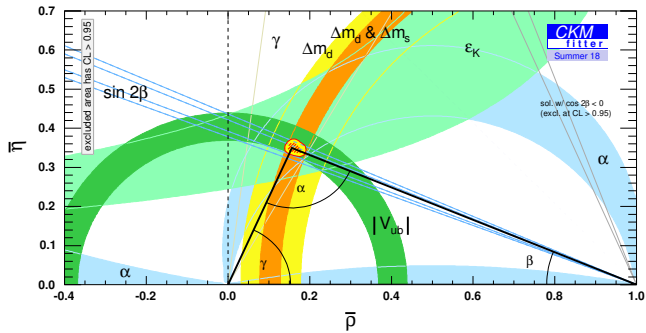


Prepared for:
Future Challenges in Non-Leptonic B Decays, 2019
Mainz Institute of Theoretical Physics
<https://wolfr.am/ADHlwXff>



Jan 14-18
2019

Motivation



CKM fitter, ICHEP 2018

- α, β quite precise; Still room in γ
- Precise γ from LHCb: Present error $\sim {}^{+5.0^\circ}_{-5.8^\circ}$ (LHCb-CONF-2018-002)
- B-factories + LHCb highly successful! More work necessary ...

Standard γ extraction: two-body charmed

Two-body (tree-level)

- $B^\pm \rightarrow DK^\pm, B^\pm \rightarrow D\pi^\pm,$
 $D \rightarrow 2P, 3P, \dots$

- Interfere: D^0, \bar{D}^0, D_{CP}

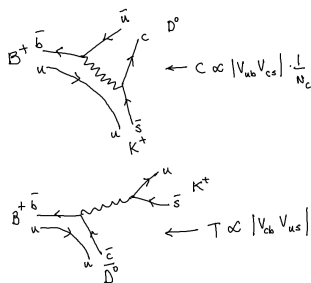
- Standard Methods:

GLW (Gronau-London-Wyler)

ADS (Atwood-Dunietz-Soni)

GGSZ (Giri-Grossman-Sofer-Zupan)

- n parameters: hadronic amplitudes + strong phases + $\gamma \sim n_B + n_D$
- m observables: BRs, direct CP asymmetries $\sim n_B n_D$
 $\Rightarrow m \geq n \Rightarrow$ solve to find γ : no hadronic input necessary.
- Theoretically clean $\delta\gamma \sim \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-7})$: Brod + Zupan, 1308.5663
- Experiment: statistics limited, small ratio of amplitudes



γ from three-body charmless hadronic decays

Three-body (tree + loop)

- Charmless B decays $B \rightarrow PPP$ ($P = \pi, K$)
- Observables: \mathcal{B}, A_{CP} (direct, indirect)
- Find “scenarios” with more observables than parameters
- Need CP eigenstate \rightarrow measure indirect CPV
3 body: final state \rightarrow momentum dependent \rightarrow not CP eigenstate
- Flavor symmetry approach
 - \rightarrow Relate different final states using symmetry
 - \rightarrow Dynamical assumptions? Flavor symmetry breaking effects?
 - \rightarrow How theoretically clean is it?
 - \rightarrow What information can this measurement provide?

Observables and Parameters

Example 3-body process: $B^0 \rightarrow K^0 \pi^+ \pi^-$ ($\bar{b} \rightarrow \bar{s}$ transition)

$$\begin{aligned} \text{Amplitude} \rightarrow \mathcal{A}(B^0 \rightarrow K^0 \pi^+ \pi^-) &= V_{tb} V_{ts}^* a_t + V_{ub} V_{us}^* a_u \\ &= \tilde{a}_t + e^{-i\gamma} \tilde{a}_u \end{aligned}$$

$|\tilde{a}_t|$, $|\tilde{a}_u|$, δ (relative strong phase) \rightarrow **momentum dependent**

Momentum dependent observables: need Dalitz amplitude analysis

$$X_{\text{DP}} = |\mathcal{A}_{\text{sym}}|^2 + |\overline{\mathcal{A}}_{\text{sym}}|^2 \quad \text{CP averaged branching fraction}$$

$$Y_{\text{DP}} = |\mathcal{A}_{\text{sym}}|^2 - |\overline{\mathcal{A}}_{\text{sym}}|^2 \quad \text{Direct CP asymmetry}$$

$$Z_{\text{DP}} = \text{Im}(\mathcal{A}_{\text{sym}}^* \overline{\mathcal{A}}_{\text{sym}}) \quad \text{Indirect CP asymmetry}$$

Momentum dependent hadronic parameters

Standard Model Weak Phase : momentum independent!

Hadronic Parameters

- Hadronic parameters are momentum dependent
- $\tilde{a}_{u,t} \equiv \tilde{a}_{u,t}(p_1, p_2, p_3) = \tilde{a}_{u,t}(s_{12}, s_{13})$
- Recent work on momentum dependence in context of CP asymmetries in $B \rightarrow 3\pi$
K. Keri Vos, et al. 1708.02047
- Disentangle weak and strong phases through resonances
Charles, et al. 1704.01596
- Need some external hadronic input from QCDF
- This work: treat hadronic parameters as observables
 - No external hadronic input (such as QCDF) used
 - Hadronic parameters may be extracted and studied independently

γ from U-spin

- In two-body decays:
 - U-spin pair: $B_s^0 \rightarrow K^+K^-$, $B_d^0 \rightarrow \pi^+\pi^-$ ($a_d = a_s$)
 - 4 observables: $\mathcal{B}_d, \mathcal{B}_s, A_{\text{CP}}^d, A_{\text{CP}}^s$; 4 unknowns: $|a_1|, |a_2|, \delta, \gamma$
 - U-spin relation: $A_{\text{CP}}^d \mathcal{B}_d \tau_s = -A_{\text{CP}}^s \mathcal{B}_s \tau_d$ (1 less observable)
 - Measure indirect A_{CP} and use $B - \bar{B}$ mixing (β, β_s)
Fleischer method, hep-ph/9903456
 - Enough info (5 observables, 4 unknowns): γ can be extracted
- Test U-spin breaking: check U-spin relation
- 1503.00737 (with D.London) → γ from 3-body U-spin pairs
 - $B_s^0 \rightarrow K_S K^+ K^-$, $B_d^0 \rightarrow K_S \pi^+ \pi^-$
 - Need time-dependent Dalitz analysis for indirect CPV
 - Test momentum dependence of U-spin breaking

Binning and averaging

- Local (momentum dependent) observables:

$$\rightarrow \Gamma(t) = \frac{\Gamma(B^0 \rightarrow f) + \Gamma(\bar{B}^0 \rightarrow f)}{2}, \quad A_{\text{CP}}(t) = \frac{\Gamma(B^0 \rightarrow f) - \Gamma(\bar{B}^0 \rightarrow f)}{\Gamma(B^0 \rightarrow f) + \Gamma(\bar{B}^0 \rightarrow f)}$$

$$\rightarrow |f\rangle \neq |\bar{f}\rangle \quad A_{\text{CP}} \text{ is not true CP Asymmetry}$$

- Time-dependent Dalitz analysis : extract coefficients
- Observables are coefficients of sin, cos, sinh, cosh of $\Delta m, \Delta\Gamma$

→ For fit use redefinitions:

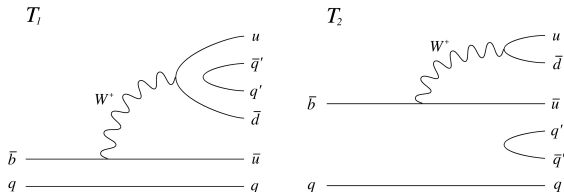
$$\int_{\text{bin}} |\mathcal{A}|^2 \equiv |A'|^2, \quad \int_{\text{bin}} |\tilde{\mathcal{A}}|^2 \equiv |\tilde{A}'|^2, \quad \int_{\text{bin}} \text{Im} \left[(q/p) \mathcal{A}^* \tilde{\mathcal{A}} \right] \equiv ?$$

$$\rightarrow \text{Approximate : } \int_{\text{bin}} \text{Im} \left[(q/p) \mathcal{A}^* \tilde{\mathcal{A}} \right] \approx \text{Im} \left[(q/p) \mathcal{A}'^* \tilde{\mathcal{A}}' \right]$$

- U-spin relation in Dalitz analysis: $a_{\text{CP}}^d b_d \tau_s = -a_{\text{CP}}^s b_s \tau_d$
 - Momentum-dependent relationship
 - Unequal U-spin breaking in pockets, but satisfied on average?

Flavor SU(3) diagrammatics

- 2-body $\bar{b} \rightarrow \bar{s}$ transitions: $T', C', P'_{tc}, P'_{uc}, P'_{EW}, P'_{EW}^C$
- No weak-exchange diagrams (suppressed, dynamical assumption)
- 2 choices for 3-body diagrams (subscripts 1 or 2)



- Three-body topologies: $T'_1, T'_2, C'_1, C'_2, \dots$
- In SU(3) limit: 3 identical particles in final state
- Equivalent to SU(3) matrix element for Fully Symmetric case

Flavor SU(3) diagrammatics

- Flavor SU(3): triplet of B mesons \rightarrow octet of π, K, η
- Final state with 3 identical particles: S_3 symmetry
 - \rightarrow Fully symmetric (FS), fully antisymmetric (FA)
 - \rightarrow Mixed symmetric (MS1, MS2, MS3, MS4)
- FS amplitude stays same when any 2 particles are interchanged
- 1402.2909 (with Gronau, Imbeault, London, Rosner) considered FS
 - \rightarrow 9 independent SU(3) matrix elements
 - \rightarrow *Already includes rescattering effects to all orders in α_s*
 - \rightarrow 16 $b \rightarrow s$ and 16 $b \rightarrow d$ 3-body channels
 - \rightarrow Several SU(3) relationships that can be tested

$$\text{Example: } \mathcal{A}(B^+ \rightarrow K^0 \pi^+ \pi^0)_{\text{FS}} = - \mathcal{A}(B^0 \rightarrow K^+ \pi^0 \pi^-)_{\text{FS}}$$

γ from 3-body B decays

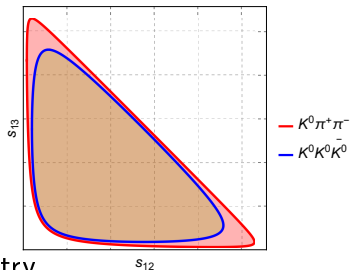
- SU(3) limit gives relations between tree and electroweak penguins
- 4 effective combinations (a, b, c, d) in the SU(3) limit
- SU(3) FS amplitudes expressed in terms of effective diagrams:

$$\begin{aligned}A(B^0 \rightarrow K^0 K^0 \bar{K}^0)_{\text{FS}} &= a \\ \sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{\text{FS}} &= -ce^{i\gamma} - a + \kappa b \\ 2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{\text{FS}} &= be^{i\gamma} - \kappa c \\ \sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{\text{FS}} &= -de^{i\gamma} - a + \kappa d\end{aligned}$$

- Construct momentum dependent observables (X, Y, Z)
- Amplitude analysis (Example: isobar) to get amplitude at a point
- One Dalitz plot point sufficient for γ : average over multiple points
(more details in the next talk by E. Bertholet; 1812.06194)

SU(3) breaking

- Quark masses are different: $m_s > m_{u,d}$
- Meson masses are different: $m_K > m_\pi$
 - ⇒ Kinematic boundaries are different
 - ⇒ Large regions with no overlap
- Resonances, quasi-2-body objects



Vector resonances absent due to symmetry

- Effects of adding an m_s spurion in SU(3)? (Future)
 - ⇒ Additional parameters and/or relationships? (Future)
- Symmetrize over “square” Dalitz plot? (Future)
 - ⇒ May provide some estimate of deviation from SU(3) (Future)

SU(3) breaking: How to test it?

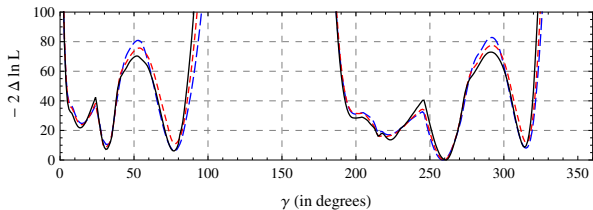
- Single parameter SU(3) breaking $\rightarrow |f_{SU(3)}|$

$$\begin{aligned}A(B^0 \rightarrow K^0 K^0 \bar{K}^0)_{\text{FS}} &= f_{SU(3)} a \\ \sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{\text{FS}} &= f_{SU(3)}(-ce^{i\gamma} - a + \kappa b) \\ 2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{\text{FS}} &= be^{i\gamma} - \kappa c \\ \sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{\text{FS}} &= -de^{i\gamma} - a + \kappa d \\ \sqrt{2}A(B^+ \rightarrow K^+ \pi^+ \pi^-)_{\text{FS}} &= -ce^{i\gamma} - a + \kappa b\end{aligned}$$

- Extract $|f_{SU(3)}|$ from a fit; Expect $|f_{SU(3)}| = 1$ for exact SU(3)
- $|f_{SU(3)}|$ should vary widely over allowed Dalitz area
- Average $|f_{SU(3)}| - 1$ may indicate the level of SU(3) breaking (Similar to U-spin breaking test by averaging over Dalitz area)

Results

- γ extracted by applying SU(3) to fully symmetric state
→ Full Analysis in 1303.0846 (with Imbeault, London)



- SM-like : 77°
- Other solutions : 32° , 259° , 315°
- Discrete Ambiguity

- Key : γ extraction by applying SU(3) to other symmetry states
→ Break discrete ambiguity : more information from other states
- Also key : estimate systematic uncertainties in γ extraction
- Interesting situation : γ widely different from SM value
→ Significant SU(3) breaking?
→ NP in three-body B decays? $K\pi\pi - KKK$ puzzle?

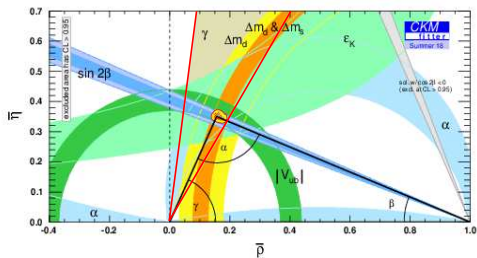
Conclusions

- γ from tree is theoretically very clean!
- Precise γ from LHCb will strongly test CKM unitarity
- γ from tree + loop can expose interesting physics
- Flavor symmetries provide effective path to study γ
- γ from U-spin + time-dependent Dalitz analysis
- γ from SU(3) + fully-symmetric amplitude analysis
- Discrete ambiguities may be broken using other symmetries
- γ tree + loop may present hints of new physics

Thank You!

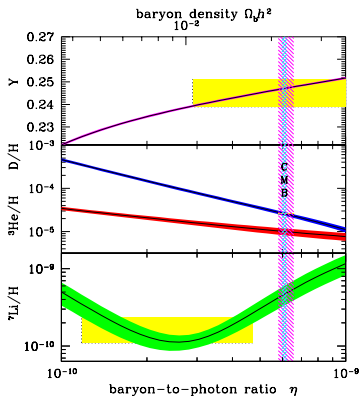
Back-up Slides

Motivation



CKM fitter, Summer 2018

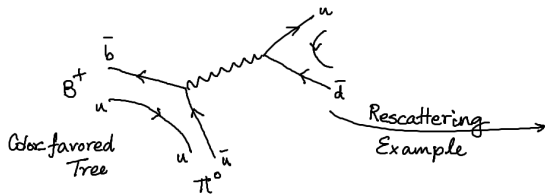
- α, β quite precise
- Still room in γ
- Precise γ from LHCb: soon?
- B-factory program
→ highly successful!



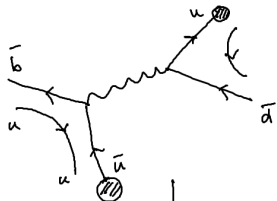
η from CMB & BBN, PDG

- CKM paradigm not enough
- Need new sources of CPV
- Deviations from CKM?

Rescattering

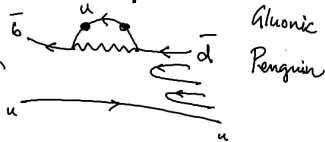


Short-distance contribution goes to respective diagram
 Long-distance contribution goes to the final diagram



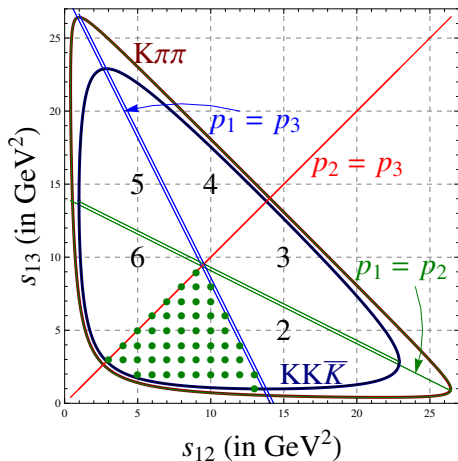
Long-distance effect

So this contributes to the penguin.



Three-body decays : Dalitz plots

- Three-body final state : $|P_1(p_1)P_2(p_2)P_3(p_3)\rangle$ $s_{ij} = (p_i + p_j)^2$
 \rightarrow Momentum dependent. One relation $s_{12} + s_{23} + s_{13} = \text{constant}$



Features of a Dalitz plot:

- Independent measurements at different points may be possible
- Same SM weak phase (γ);
Hadronic parameters are local
- Consistency checks :
Flavor symmetries (SU(3), U-spin)
provide amplitude relationships

Three-body SU(3) relations

$b \rightarrow s$ relationships from isospin and SU(3) symmetry :

$$\begin{aligned}\mathcal{A}(B^+ \rightarrow K^0 \pi^+ \pi^0)_{\text{FS}} &= -\mathcal{A}(B_d^0 \rightarrow K^+ \pi^0 \pi^-)_{\text{FS}} \\ \sqrt{2}\mathcal{A}(B_s^0 \rightarrow 3\pi^0)_{\text{FS}} &= -\sqrt{3}\mathcal{A}(B_s^0 \rightarrow \pi^0 \pi^+ \pi^-)_{\text{FS}} \\ \sqrt{2}\mathcal{A}(B^+ \rightarrow K^0 \pi^+ \pi^0)_{\text{FS}} &= \mathcal{A}(B_d^0 \rightarrow K^0 \pi^+ \pi^-)_{\text{FS}} + \sqrt{2}\mathcal{A}(B_d^0 \rightarrow K^0 \pi^0 \pi^0)_{\text{FS}} \\ \sqrt{2}\mathcal{A}(B_d^0 \rightarrow K^+ \pi^0 \pi^-)_{\text{FS}} &= \mathcal{A}(B^+ \rightarrow K^+ \pi^+ \pi^-)_{\text{FS}} + \sqrt{2}\mathcal{A}(B_d^0 \rightarrow K^0 \pi^0 \pi^0)_{\text{FS}} \\ \mathcal{A}(B^+ \rightarrow K^+ K^+ K^-)_{\text{FS}} &+ \sqrt{2}\mathcal{A}(B^+ \rightarrow K^+ K^0 \bar{K}^0)_{\text{FS}} \\ &= \sqrt{2}\mathcal{A}(B_d^0 \rightarrow K^0 K^+ K^-)_{\text{FS}} + \mathcal{A}(B_d^0 \rightarrow K^0 K^0 \bar{K}^0)_{\text{FS}} \\ \mathcal{A}(B_s^0 \rightarrow \pi^0 K^+ K^-)_{\text{FS}} &+ \sqrt{2}\mathcal{A}(B_s^0 \rightarrow \pi^0 K^0 \bar{K}^0)_{\text{FS}} \\ &= -\sqrt{2}\mathcal{A}(B_s^0 \rightarrow \pi^- K^+ \bar{K}^0)_{\text{FS}} - \mathcal{A}(B_s^0 \rightarrow \pi^+ K^- K^0)_{\text{FS}} \\ \sqrt{2}\mathcal{A}(B^+ \rightarrow K^+ \pi^+ \pi^-)_{\text{FS}} &= \mathcal{A}(B^+ \rightarrow K^+ K^+ K^-)_{\text{FS}}\end{aligned}$$

7 relations involving 16 amplitudes that depend on 9 matrix elements

Similar situation also in $b \rightarrow d$ FS amplitudes