JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

Future Challenges in Non-Leptonic B Decays: Theory and Experiment

QCD factorisation inspired parameterization of $B \to K\pi^+\pi^-$

$$
\begin{aligned}
&\mathcal{M}(B^\pm \to K^\pm \pi^\pm \pi^\mp) = \\
&\quad = a_{K^*} e^{i\delta_{K^*}} (1 \pm b_{K^*} e^{i\varphi_{K^*}}) \mathcal{A}(K^*(892)^0) + a_{K_0^*} e^{i\delta_{K_0^*}} (1 \pm b_{K_0^*} e^{i\varphi_{K_0^*}}) \mathcal{A}(K_0^*(1430)^0) \\
&\quad + \ a_\rho e^{i\delta_\rho} (1 \pm b_\rho e^{i\varphi_\rho}) \mathcal{A}(\rho(770)^0) + a_\omega e^{i\delta_\omega} (1 \pm b_\omega e^{i\varphi_\omega}) \mathcal{A}(\omega(782)) \\
&\quad + \ a_{f_0} e^{i\delta_{f_0}} (1 \pm b_{f_0} e^{i\varphi_{f_0}}) \mathcal{A}_{\rm Flatte}(f_0(980)) + a_{f_2} e^{i\delta_{f_2}} (1 \pm b_{f_2} e^{i\varphi_{f_2}}) \mathcal{A}(f_2(1270)) \\
&\quad + \ a_{f_X} e^{i\delta_{f_X}} (1 \pm b_{f_X} e^{i\varphi_{f_X}}) \mathcal{A}(f_X) + a_{\chi_{c0}} e^{i\delta_{\chi_{c0}}} (1 \pm b_{\chi_{c0}} e^{i\varphi_{\chi_{c0}}}) \mathcal{A}(\chi_{c0}) \\
&\quad + \ \mathcal{A}_{\rm nr}(K^\pm \pi^\pm \pi^\mp)
\end{aligned}
$$

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QCD factorisation inspired parameterization of $B \to K\pi^+\pi^-$

$\mathcal{M}(B^\pm \to K^\pm\pi^\pm\pi^\mp) =$

$= a_{K^*} e^{i\delta_{K^*}}(1 \pm b_{K^*} e^{i\varphi_{K^*}})\mathcal{A}(K^*(8$ [illegible] $_{K_0^*} e^{i\delta_{K_0^*}}(1 \pm b_{K_0^*} e^{i\varphi_{K_0^*}})\mathcal{A}(K_0^*(1430)^0)$

$+\ a_\rho e^{i\delta_\rho}(1 \pm b_\rho e^{i\varphi_\rho})\mathcal{A}(\rho(770)^0) +$ [illegible] $(1 \pm b_\omega e^{i\varphi_\omega})\mathcal{A}(\omega(782))$

$+\ a_{f_0} e^{i\delta_{f_0}}(1 \pm b_{f_0} e^{i\varphi_{f_0}})\mathcal{A}_{\mathrm{Flatte}}(f$ [illegible] $a_{f_2} e^{i\delta_{f_2}}(1 \pm b_{f_2} e^{i\varphi_{f_2}})\mathcal{A}(f_2(1270))$

$+\ a_{f_X} e^{i\delta_{f_X}}(1 \pm b_{f_X} e^{i\varphi_{f_X}})\mathcal{A}(f$ [illegible] $a_{\chi_{c0}}$ [illegible] $(1 \pm b_{\chi_{c0}} e^{i\varphi_{\chi_{c0}}})\mathcal{A}(\chi_{c0})$

$+\ \mathcal{A}_{\mathrm{nr}}(K^\pm\pi^\pm\pi^\mp)$

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Motivations: why study three-body hadronic B and D decays?

- Three-body hadronic B and D decays rich field : Standard Model, CP violation, QCD, Hadron Physics ….
- Hadron Physics component : 2-body resonances + interferences ⇔ weak observables.
- Final state meson-meson interactions: constrained by unitarity, analyticity, chiral symmetry + data other than B and D decays.
- Basic Dalitz-plot analyzes: isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances + non resonant background. *S-wave resonance contributions difficult to fit* — beyond isobar?
- Proposal: replace by parametrizations of amplitudes in terms of unitary two-meson form factors including the weak-interaction dynamics of flavor-changing diagrams.
- Parametrizations based on published results and motivated by analyzes of high-statistics present and forthcoming data: BES III, LHC*b*, Belle II …
- No three-body decays factorization theorem but major contributions from intermediate resonances $\rho(770)$, $K^*(892)$, $\phi(1020)$ ⇒ *quasi-two-body decays*.

QCD factorization (two body)

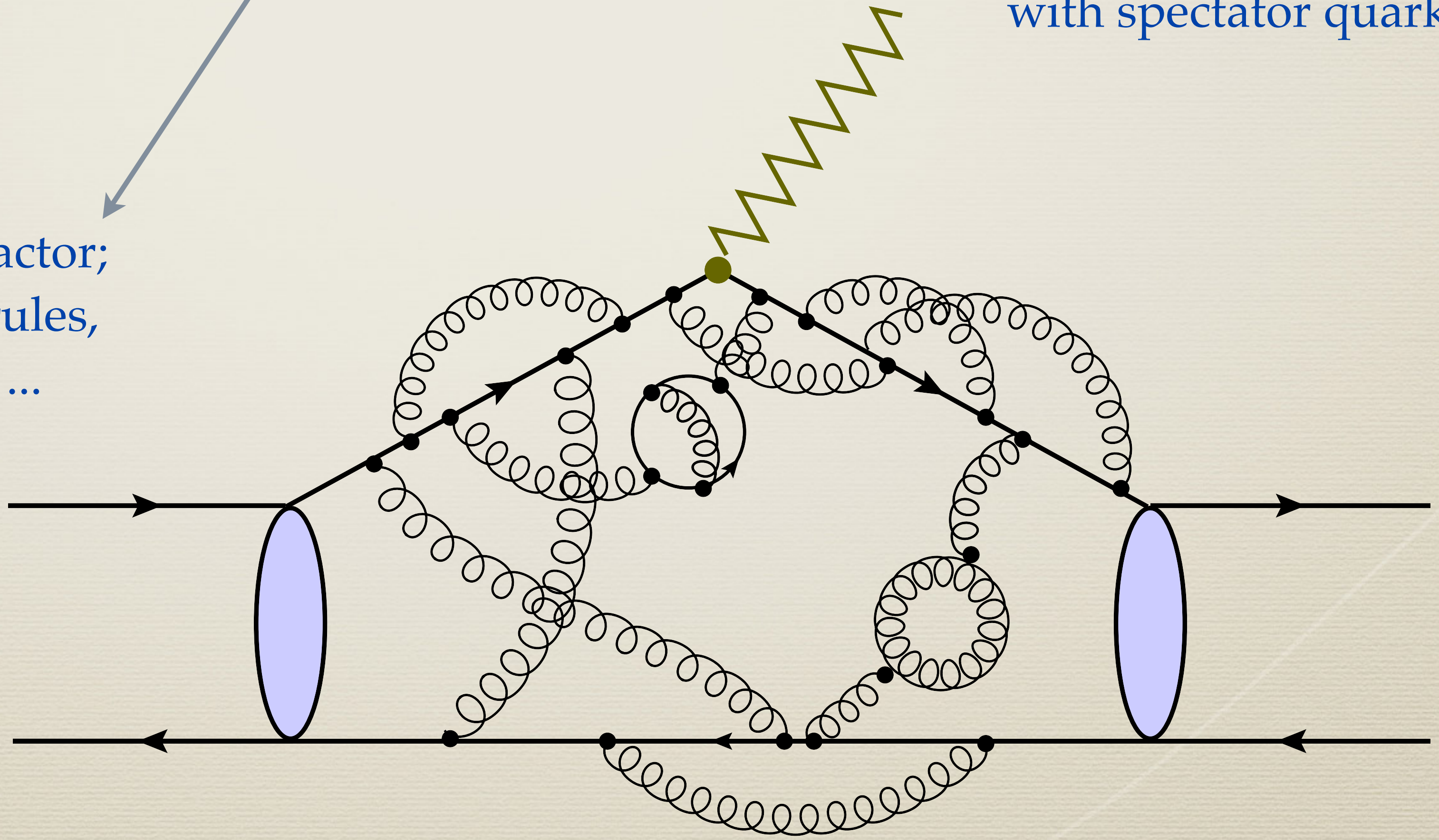
Beneke, Buchalla, Neubert, Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); *Beneke & Neubert*, Nucl. Phys. B 675, 333 (2003).

$$\langle M_1 M_2 | Q_k(\mu) | B \rangle \sim \langle M_2 | J_1 | 0 \rangle \otimes \langle M_1 | J_2 | B \rangle \times \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\mathbf{QCD}}/m_b) \right]$$

Decay constant
(mostly known experimentally)

Radiative vertex corrections
and hard gluon exchange
with spectator quark

Hadronic transition form factor;
estimated with QCD sum rules,
lattice QCD, quark models ...



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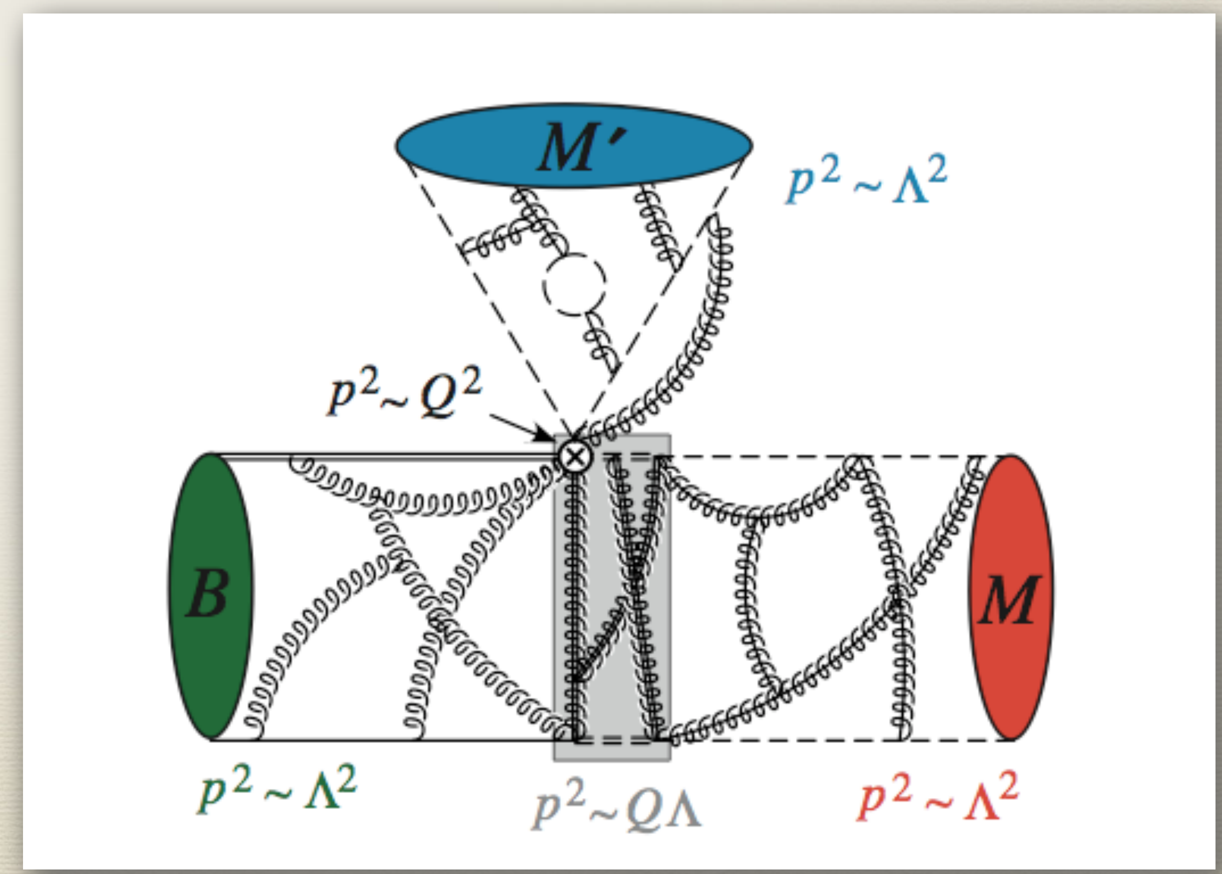


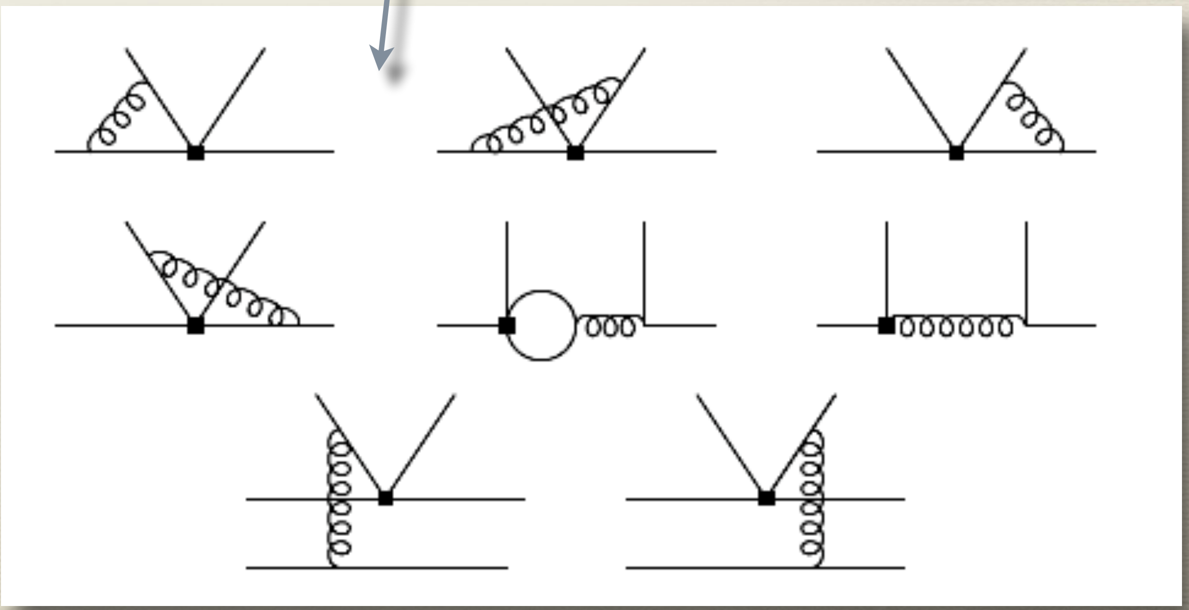
QCD factorization (quasi-two body)

B. E. , A. Furman, R. Kamiński, L. Leśniak, B. Loiseau and B. Moussallam, Phys. Rev. D79, 094005 (2009)

$$\langle (M_1 M_2)_{S,P} M_3 | Q_k(\mu) | B \rangle \sim \langle (M_1 M_2)_{S,P} | J_1 | 0 \rangle \otimes \langle M_3 | J_2 | B \rangle \times \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\mathbf{QCD}}/m_b) \right]$$

Radiative vertex corrections and hard gluon exchange with spectator quark

Scalar or vector form factor; their definition allows for inclusion of pion-pion and kaon-pion form factors and the calculation of a "resonance decay constant"



Meson-Meson Form Factors

$\langle M^* | J^i_\mu | 0 \rangle \propto \langle M_1 M_2 | J^i_\mu | 0 \rangle$: form factor, creation from a $\bar{q}q$ pair.

Usage of dispersion relations and field theory $\Rightarrow$ *form factors known if M_1M_2 interactions known at all energies* [G. Barton, Introduction to dispersion techniques in field theory, W. A. Benjamin, INC, New York (1965)].

Method: Two-body data + unitarity + asymptotic QCD + chiral symmetry at low energies.

Example: scalar and vector πK form factors

The hadronic matrix element that describes the creation of a kaon-pion pair can be written in terms of the usual Lorentz invariants:

$$\langle K^-(p_{K^-})\pi^+(p_{\pi^+})|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle = f_+^{K^-\pi^+}(q^2)\ (p_{K^-}+p_{\pi^+})_\mu + f_-^{K^-\pi^+}(q^2)\ (p_{K^-}-p_{\pi^+})_\mu$$

with $t=(p_k-p_\pi)^2$ and which can be re-expressed in terms of a scalar $F_1\,(t=q^2)$ and a vector $G_1\,(t=q^2)$ form factor, such as:

in *S*- and *P*-wave

$$\langle K^-(p_{K^-})\pi^+(p_{\pi^+})|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle =$$

$$= \left[(p_{K^-}-p_{\pi^+})_\mu - \frac{m_K^2-m_\pi^2}{q^2}q_\mu\right] G_1^{K^-\pi^+}(q^2) + \frac{m_K^2-m_\pi^2}{q^2}q_\mu\, F_1^{K^-\pi^+}(q^2)$$

Two sets of form factors are related by: $$F_1(t) = \sqrt{2}\left[f_+^{K^-\pi^+}(t) + \frac{t}{m_K^2-m_\pi^2}f_-^{K^-\pi^+}(t)\right]$$

$$G_1(t) = \sqrt{2}f_+^{K^-\pi^+}(t)$$

These form factors also appear in semileptonic decays $\tau\to K\pi\nu_\tau$ and $K\to\pi\ell\nu_\ell$

B. Moussallam, Eur.Phys.J.C53:401-412 (2008); M. Jamin, J. Oller & A. Pich, Nucl.Phys.B622:279-308 (2002).

Mushkelishvili-Omnès Equations

Analyticity and asymptotic conditions: *dispersion relation*

$$\text{Re } F_1(t) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{\text{Im } F_1(t')}{t'-t}\, dt'$$ (scalar form factor)

$$\text{Re } G_1(t) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{\text{Im } G_1(t')}{t'-t}\, dt'$$ (vector form factor)

Unitarity equations and *T*-invariance:

$$\text{Im } F_m(t) = \frac{1}{2}\sum_n T^*_{mn}(t) F_n(t) \quad \text{Im } G_m(t) = \frac{1}{2}\sum_n T^*_{mn} G_n(t)$$

with the approximation of the truncation $|n\rangle = |K\pi\rangle, |K\eta'\rangle$ for the scalar $F_1(t)$ and $|n\rangle = |K\pi\rangle, |K^*\pi\rangle, |K\rho\rangle$ for the vector $G_1(t)$.

Combining the dispersion relations with the unitarity equations yields a set of integral equations (Mushkelishvili-Omnès) which can be solved numerically with *initial conditions*.

➟ Find an effective parametrization of $T_{mn}(t)$ constrained by theory (chiral symmetry) at low energies and experimental data on phase shifts and inelasticities at higher energies.

An (incomplete) collection of meson-meson form factors

- Scalar $\pi\pi$ form factor:
  J. T. Daub, C. Hanhart, B. Kubis, *A model-independent analysis of final-state interactions in $\bar{B}^0_{d/s} \to J/\psi\pi\pi$*, JHEP 1602, 009 (2016).
- Vector $\pi\pi$ form factor:
  C. Hanhart, *A new parametrization for the vector pion form factor*, Phys. Lett. B 715, 170 (2012).
  D. Gómez Dumm and P. Roig, *Dispersive representation of the pion vector form factor in $\tau \to \pi\pi\nu_\tau$ decays*, Eur. Phys. J. C 73, 2528 (2013).
  A. Celis, V. Cirigliano, E. Passemar, *Lepton flavor violation in the Higgs sector and the role of hadronic τ-lepton decays*, Phys. Rev. D 89, 013008 (2014).
- Scalar $K\pi$ form factors: M. Jamin, J. A. Oller and A. Pich, *Scalar $K\pi$ form factor and light quark masses*, Phys. Rev. D 74, 074009 (2006).
- Vector $K\pi$ form factor: D. R. Boito, R. Escribano and M. Jamin, *$K\pi$ vector form factor constrained by $\tau \to K\pi\nu_\tau$ and K_{l3} decays*, JHEP 1009, 031 (2010).
- Scalar KK form factors: B. Moussallam, *N_f dependence of the quark condensate from a chiral sum rule*, Eur. Phys. J. C 14, 111 (2000).

Alternatives to Isobar model

"Parametrizations of three-body hadronic B- and D-decay amplitudes in terms of analytic and unitary meson-meson form factors"

D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R. Kamiski, L. Lesniak, B. Loiseau, Phys. Rev. D 96, 113003 (2017)

- Isobar parametrizations do not respect unitarity and extraction of strong CP phases should be taken with caution. S-wave resonance contribution hard to fit.
- Our parametrizations, while not fully three-body unitary, are based on a sound theoretical application of QCD factorization to a hadronic quasi-two-body decay.
- Assume final three-meson state preceded by intermediate resonant states, justified by phenomenological and experimental evidence.
- Analyticity, unitarity, chiral symmetry + correct asymptotic behavior of the two-meson scattering amplitude in S and P waves implemented.

Parametrized amplitudes in terms of analytic and unitary meson-meson form factors

D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R. Kamiński, L. Leśniak, B. Loiseau, Phys. Rev. D **96**, 113003 (2017), gives parametrizations, based on quasi-two-body factorization, for the following three-body hadronic amplitudes.

$B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ J.-P. Dedonder *et al.*, Acta Phys. Pol. B **42**, 2013 (2011).

$B \to K\pi^+\pi^-$: A. Furman *et al.*, Phys. Lett. B **622**, 207 (2005); B. El-Bennich *et al.*, Phys. Rev. D **74**, 114009 (2006); B. El-Bennich *et al.*, Phys. Rev. D **79**, 094005 (2009); Erratum-ibid, Phys. Rev. D **83**, 039903 (2011).

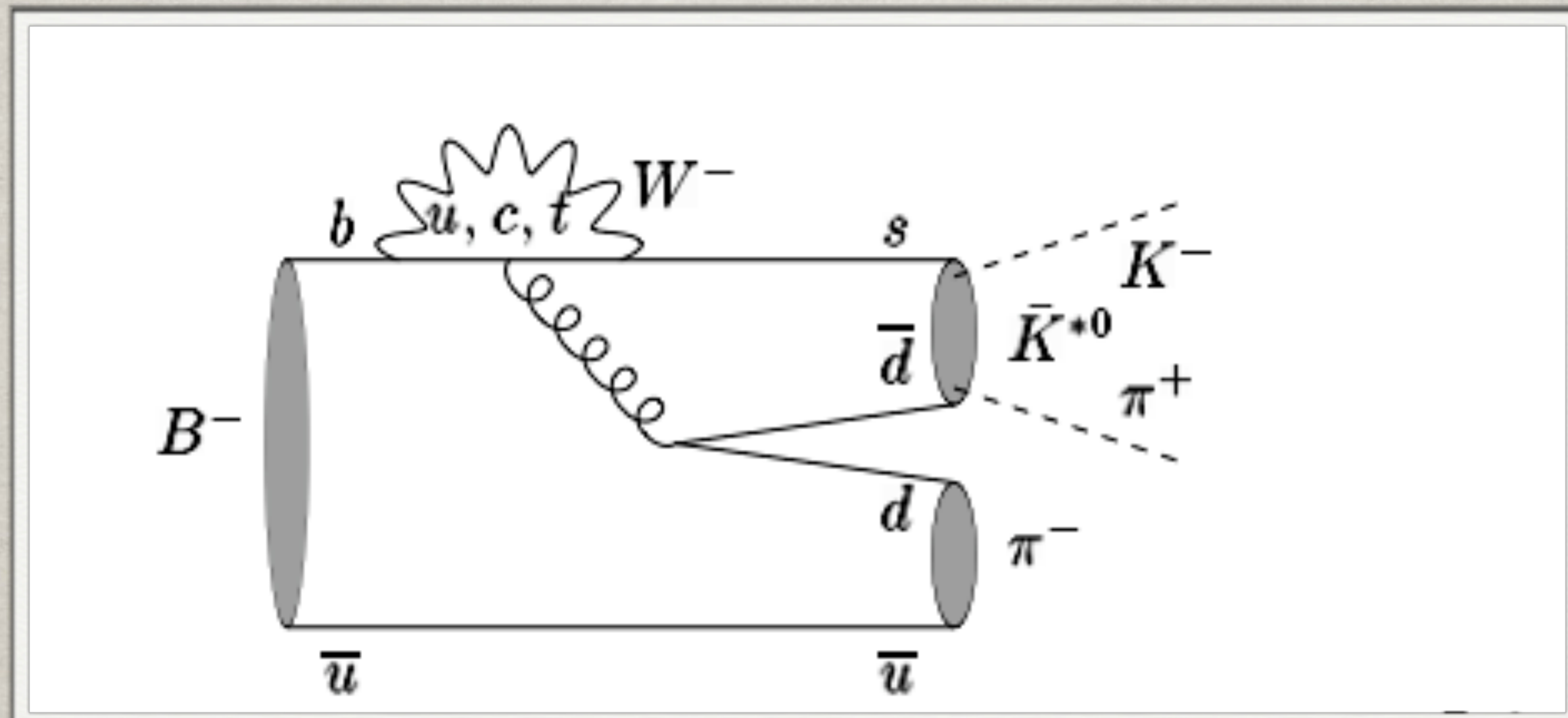
$B^{\pm} \to K^+K^-K^{\pm}$: A. Furman *et al.*, Phys. Lett. B **699**, 102 (2011); L. Leśniak and P. Żenczykowski, Phys. Lett. B **737**, 201 (2014).

$D^+ \to \pi^+\pi^-\pi^+$: D. Boito *et al.*, Phys. Rev. D **79**, 034020 (2009).

$D^+ \to K^-\pi^+\pi^+$: D. R. Boito and R. Escribano, Phys. Rev. D **80**, 054007 (2009); D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59**, 821 (2009).

$D^0 \to K^0_S\pi^+\pi^-$: J.-P. Dedonder *et al.*, Phys. Rev. D **89**, 094018 (2014).

$D^0 \to K^0_S K^+ K^-$: J.-P. Dedonder *et al.*, work in progress .



b
u, c, t
W⁻
s
K⁻
$\bar{K}^{*0}$
$\bar{d}$
π⁺
B⁻
d
π⁻
$\bar{u}$
$\bar{u}$

Invariant $m_{\pi K}$ mass distributions

$$\frac{d^2\Gamma^-}{d\cos\theta\, dm_{K^-\pi^+}} = \frac{m_{K^-\pi^+}|\mathbf{p}_{\pi^+}|\ |\mathbf{p}_{\pi^-}|}{8(2\pi)^3 M_B^3}|\mathcal{M}^-|^2$$

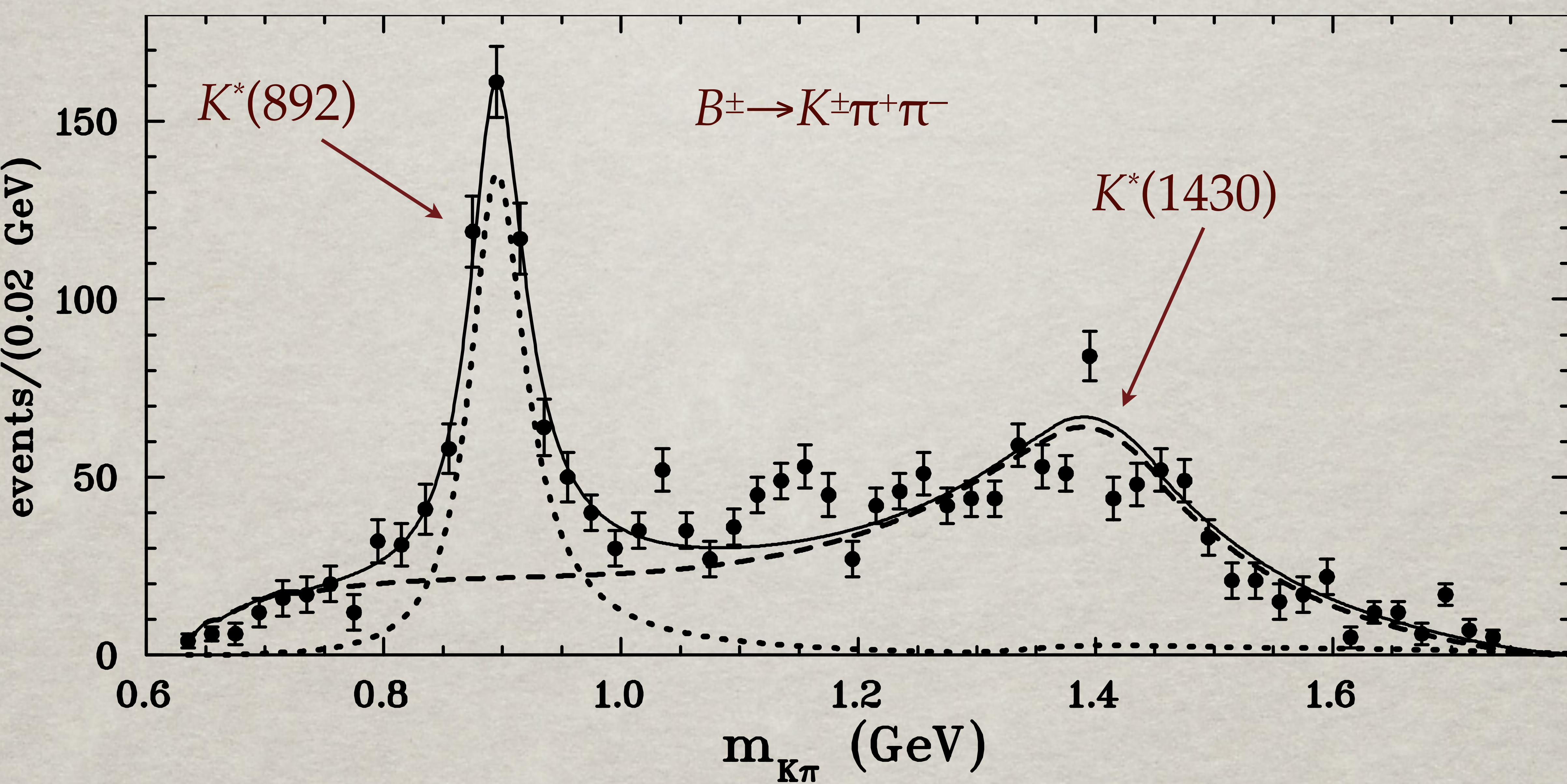
$$\frac{d\mathcal{B}^-}{dm_{K^-\pi^+}} = \frac{1}{\Gamma_B^-}\frac{m_{K^-\pi^+}|\mathbf{p}_{\pi^+}|\ |\mathbf{p}_{\pi^-}|}{4(2\pi)^3 M_B^3}\left(|\mathcal{M}_S^-|^2 + \frac{1}{3}|\mathbf{p}_{\pi^+}|^2\ |\mathbf{p}_{\pi^-}|^2|\mathcal{M}_P^-|^2\right)$$

$$\frac{d\mathcal{B}^-}{d\cos\theta} = A + B\cos\theta + C\cos^2\theta$$

with the helicity angle related to $m_{\pi\pi}$:

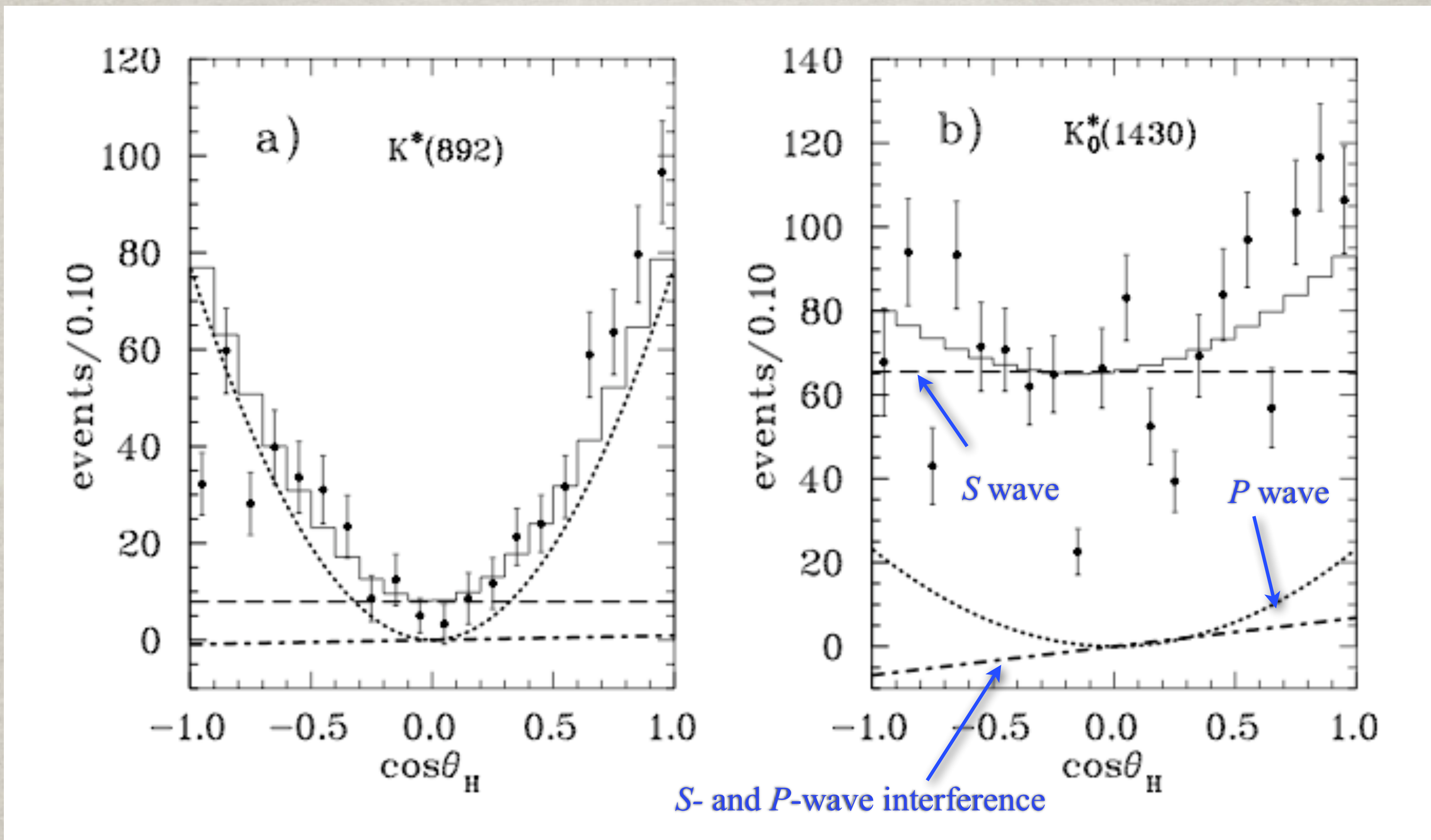
$$\cos\theta = \frac{p_{\pi^+}\cdot p_{\pi^-}}{|p_{\pi^+}||p_{\pi^-}|}$$

Pion-kaon invariant mass distributions



Data: A. Garmash *et al*. (Belle), PRL 96, 251803 (2006)

$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ helicity distributions



$B \to [K\pi^{\pm}]_S \pi^{\mp}$ amplitude (S wave)

- In terms of the two complex parameters c_1^S, c_2^S

$$\mathcal{A}_S(s_{12}) \equiv \langle \pi^- \ [K^-\pi^+]_S | \mathcal{H}_{\rm eff} | B^- \rangle = \left(c_1^S + c_2^S s_{12}\right) \frac{F_0^{B\pi}(s_{12}) F_0^{K\pi}(s_{12})}{s_{12}},$$

$F_0^{K\pi}(s)$ [contains $K_0^*(800)$ or κ, $K_0^*(1430)$], $F_0^{B\pi}(s)$, $K\pi$, $B\pi$ scalar form factors.

- Parametrization used with success by R. Aaij *et al.* [LHCb Collaboration], Amplitude analysis of the decay $\bar{B} \to K_S^0 \pi^+\pi^-$ and first observation of the CP asymmetry in $\bar{B} \to K^*(892)^-\pi^+$, arXiv: 1712.09320 [hep-ex].

- From $B^- \to [K^-\pi^+]_S \pi^-$ [B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

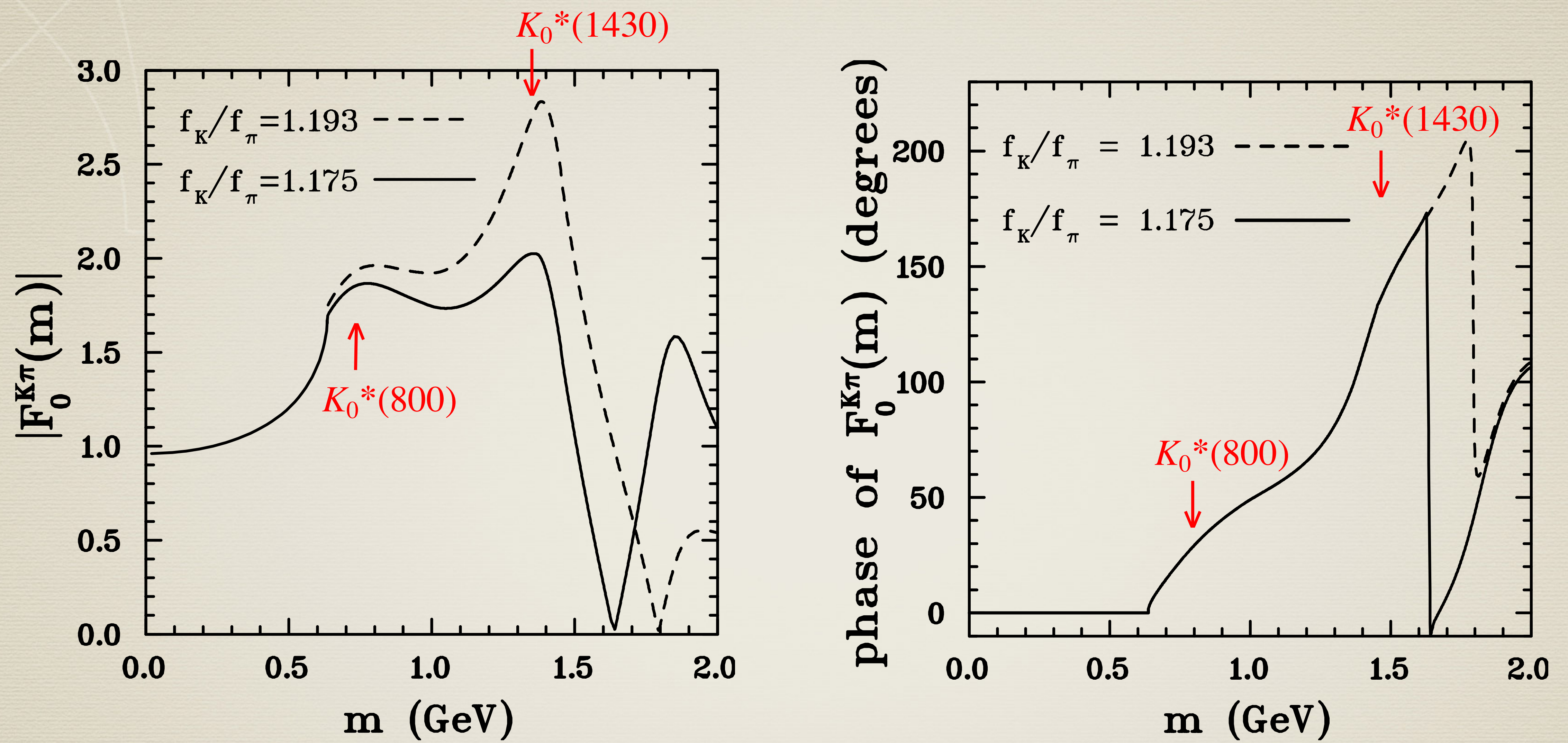
$$\begin{aligned} c_1^{-S} &= \frac{G_F}{\sqrt{2}} (M_B^2 - m_\pi^2)(m_K^2 - m_\pi^2) \\ &\times \left[\lambda_u \left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c \right) \right], \end{aligned}$$

$\Rightarrow$ $\lambda_c = V_{cb} V_{cs}^*$; $a_i^{u(c)}(S)$, $i = 4, 10$: leading order effective Wilson coefficients + vertex + penguin corrections; $c_4^{u(c)}$ free fitted parameters: non-perturbative + higher order contributions to the penguin diagrams.

$B \to [K\pi^{\pm}]_S \pi^{\mp}$ amplitude (S wave)

$$\mathcal{A}_S(s_{12}) \equiv \langle \pi^- \ [K^-\pi^+]_S | \mathcal{H}_{\text{eff}} | B^- \rangle = \left(c_1^S + c_2^S s_{12}\right) \frac{F_0^{B\pi}(s_{12}) F_0^{K\pi}(s_{12})}{s_{12}}$$

Scalar $K\pi$ form factors $F_0^{K\pi}(\sqrt{s})$: $f_K/f_\pi = 1.193$ in fit to $B \to [K\pi]\pi$; $f_K/f_\pi = 1.175$ in fit to $D^0 \to K_S^0\pi^+\pi^-$.



$\left|F_0^{K^0\pi^-}(m)\right|$ scalar $K\pi$ form factor

Phase of $F_0^{K^0\pi^-}(m)$

⇒ Unitary scalar $K\,\pi$ form factor: Muskhelishvili-Omnès 2 coupled-channel ($K\,\pi$, $K\,\eta'$) equations with experimental $K\pi$ T-matrix + chiral symmetry + asymptotic QCD constraints, variation with f_K/f_π ;

⇒ See work by B. Moussallam in B. El-Bennich et al. Phys. Rev. D 79, 094005 (2009).

$B \to [K\pi^{\pm}]_P \pi^{\mp}$ amplitude (P wave)

- In terms of one complex parameters c_1^P

$$\mathcal{A}_P(s_{12}, s_{23}) \equiv \langle \pi^- \ [K^- \pi^+]_P | \mathcal{H}_{\text{eff}} | B^- \rangle$$
$$= c_1^P \left(s_{13} - s_{23} - (M_B^2 - m_\pi^2) \, \frac{m_K^2 - m_\pi^2}{s_{12}} \right) F_1^{B\pi}(s_{12}) F_1^{K\pi}(s_{12}).$$

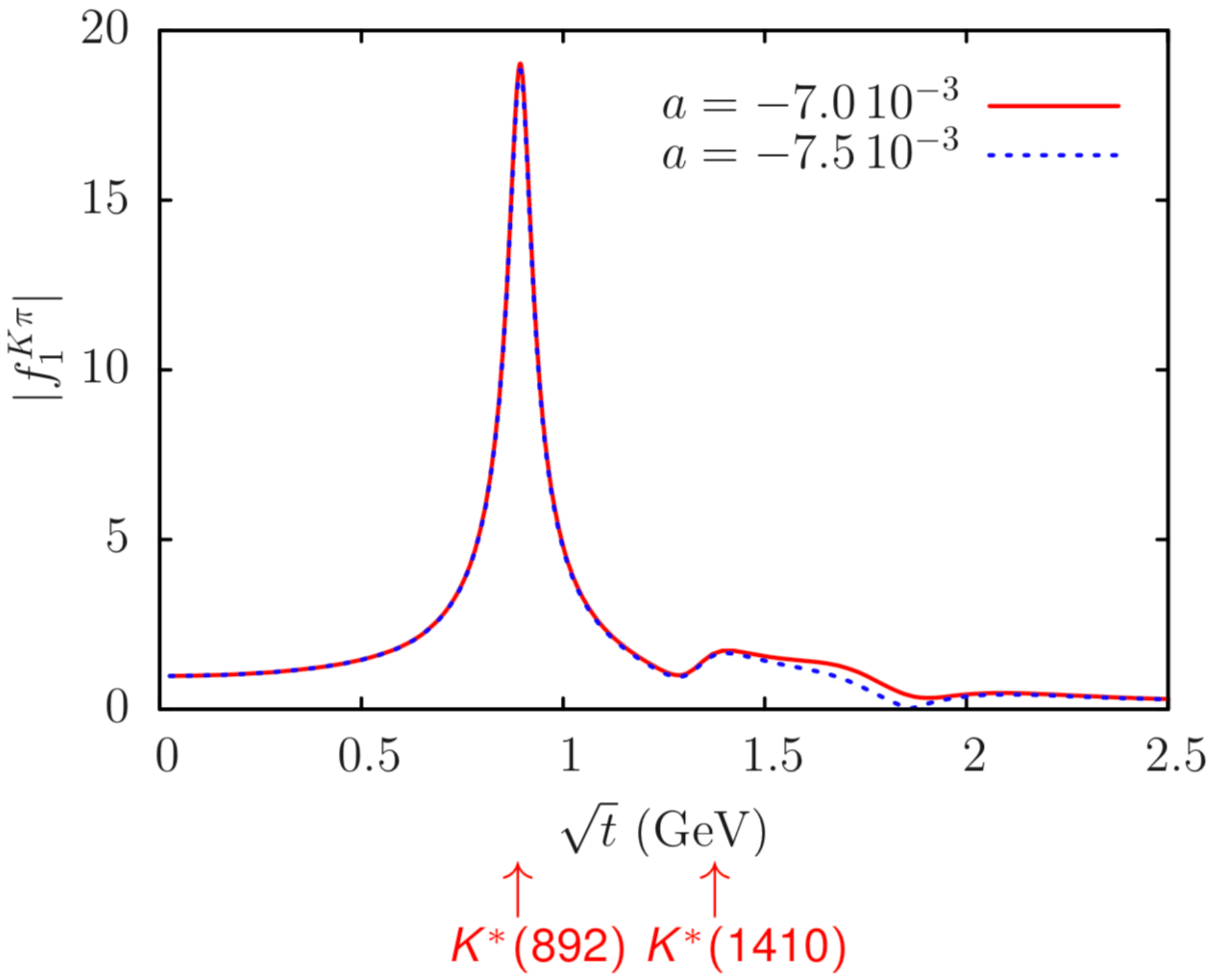
⇒ $F_1^{K\pi}(s)$ [contains $K^*(892), K^*(1410)$], $F_1^{B\pi}(s)$, $K\pi$, $B\pi$ vector form factors.

- From $B^- \to [K^-\pi^+]_S \pi^-$ [B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

$$c_1^{-P} = \frac{G_F}{\sqrt{2}} \Bigg\{ \lambda_u \left(a_4^u(P) - \frac{a_{10}^u(P)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(P) - \frac{a_{10}^c(P)}{2} + c_4^c \right)$$
$$+ \ 2 \frac{m_{K^*}}{m_b} \frac{f_V^{\perp}(\mu)}{f_V} \left[\lambda_u \left(a_6^u(P) - \frac{a_8^u(P)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(P) - \frac{a_8^c(P)}{2} + c_6^c \right) \right]$$

⇒ $a_i^{u(c)}(S)$, $i = 4, 6, 10$: leading order effective Wilson coefficients + vertex + penguin corrections; $c_{4,6}^{u(c)}$ free fitted parameters: non-perturbative + higher order contributions to the penguin diagrams; $f_V^{\perp}(\mu)/f_V$ related to $K^*(892)$ decay constant.

Unitary vector $K\pi$ form factor



- Unitary model. P-wave coupled channels $K\pi, K^*\pi, K\rho$ + asymptotic QCD + chiral symmetry constraints + $K\pi$ elastic data+ $K^*(1410)$+ $K^*(1680) \to$ form factor: Muskhelishvili-Omnès equation (dispersion relation).

→ B. Moussallam, Analyticity constraints on the strangeness changing vector current and applications to $\tau \to K\pi\nu_\tau$ and $\tau \to K\pi\pi\nu_\tau$, Eur. Phys. J. C **53**, 401 (2008).

→ Variation with the flavor symmetry breaking parameter *a*.

Conclusions

- Isobar parametrizations do not respect unitarity and extraction of strong CP phases should be taken with caution. S-wave resonance contribution hard to fit.

- Our parametrizations, not fully three-body unitary, are based on a sound theoretical application of QCD factorization to a hadronic quasi-two-body decay.

- Assume final three-meson state preceded by intermediate resonant states, justified by phenomenological and experimental evidence.

⇒ Analyticity, unitarity, chiral symmetry + correct asymptotic behavior of the two-meson scattering amplitude in S and P waves implemented via analytical and unitary S- and P-wave $\pi\pi$, πK and $K\bar{K}$ form factors entering in hadronic final states of our amplitude parametrizations.

- Parametrized amplitudes can be readily used adjusting parameters in a least-square fit to the Dalitz plot — for a given decay channel — and employing tabulated form factors as functions of momentum squared or energy.

⇒ Explicit amplitude expressions for: $B^\pm \to \pi^+\pi^-\pi^\pm$, $B \to K\,\pi^+\pi^-$, $B^\pm \to K^+K^-K^\pm$, $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\,\pi^+$, $D^0 \to K^0_S\,\pi^+\,\pi^-$ and for $D^0 \to K^0_S\,K^+K^-$ [study in progress].

- In progress: $B^\pm \to K^+K^-\pi^\pm$ [Belle, LHCb] and $B^0 \to K^0_S\,K^+K^-$ [LHCb].

Backup

Scalar and vector resonances in the weak decays $B \to (\pi\pi)K$

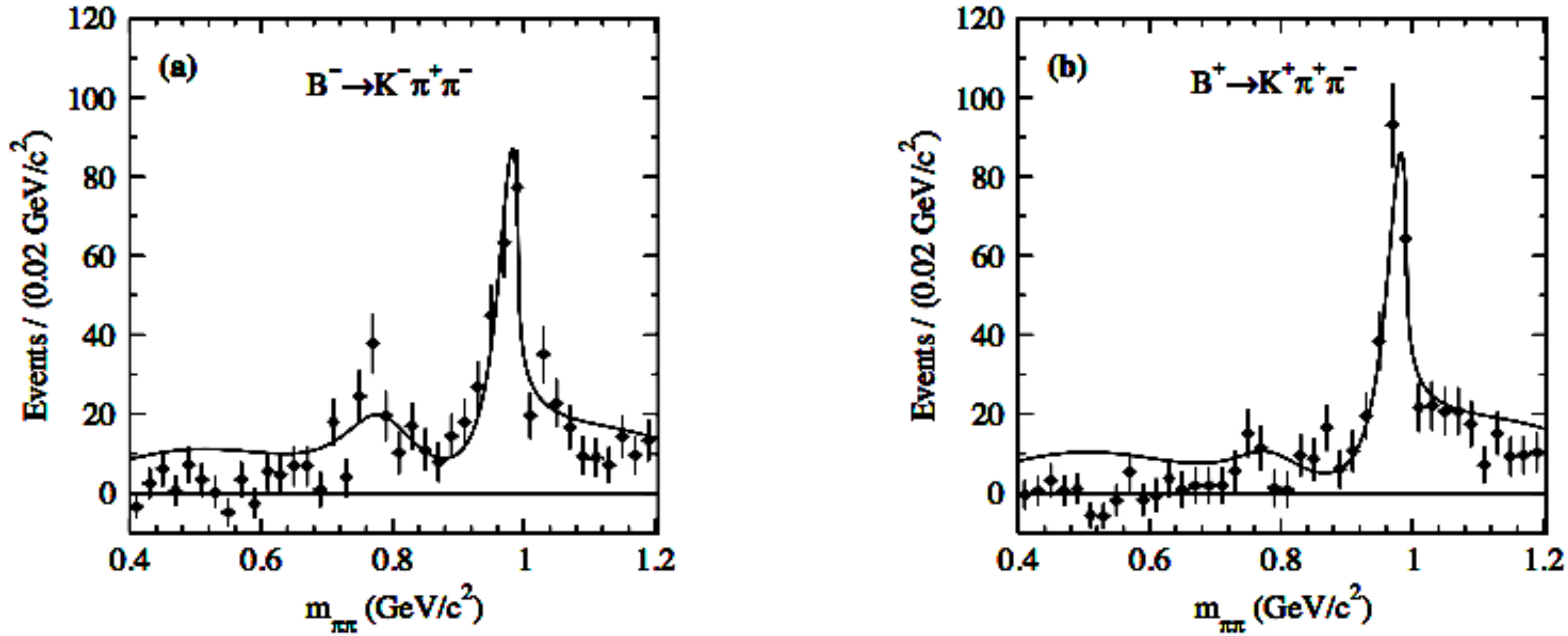


Fig. 2 $m_{\pi\pi}$ distributions **(a)** in $B^- \to \pi^+\pi^- K^-$ and **(b)** in $B^+ \to \pi^+\pi^- K^+$ decays. Data from Belle [1]. Solid lines: our model.

Example Parametrization : $B \to K\,[\pi^-\pi^+]_S$ amplitude [S wave]

$B(p_B) \to K(p_1)\pi^+(p_2)\pi^-(p_3)$, $s_{12} = (p_1+p_2)^2$, $s_{13} = (p_1+p_3)^2$, $s_{23} = (p_2+p_3)^2$
$s_{12} + s_{13} + s_{23} = m_B^2 + m_K^2 + 2m_\pi^2$.

- Parametrized in terms of three complex parameters, b_i^S, $i = 1, 2, 3$, for the different charges $B = B^\pm$, $K = K^\pm$ and $B = B^0(\bar{B}^0)$, $K = K^0(\bar{K}^0)$ or K_S^0,

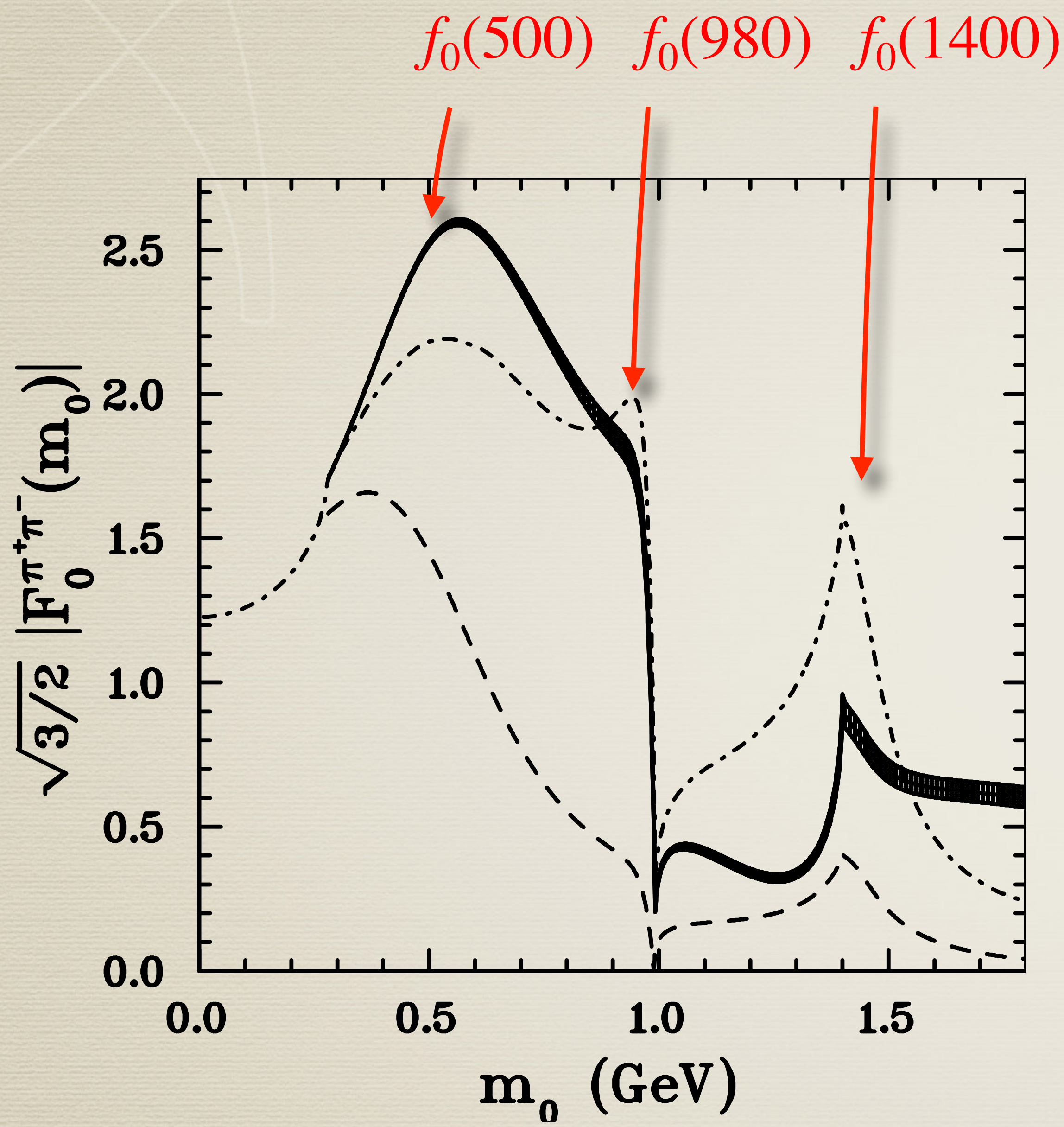
$$\mathcal{A}_S(s_{23}) \equiv \langle K\,[\pi^+\pi^-]_S|\mathcal{H}_{\text{eff}}|B\rangle$$
$$= b_1^S\left(M_B^2 - s_{23}\right) F_{0n}^{\pi\pi}(s_{23}) + \left(b_2^S F_0^{BK}(s_{23}) + b_3^S\right) F_{0s}^{\pi\pi}(s_{23}).$$

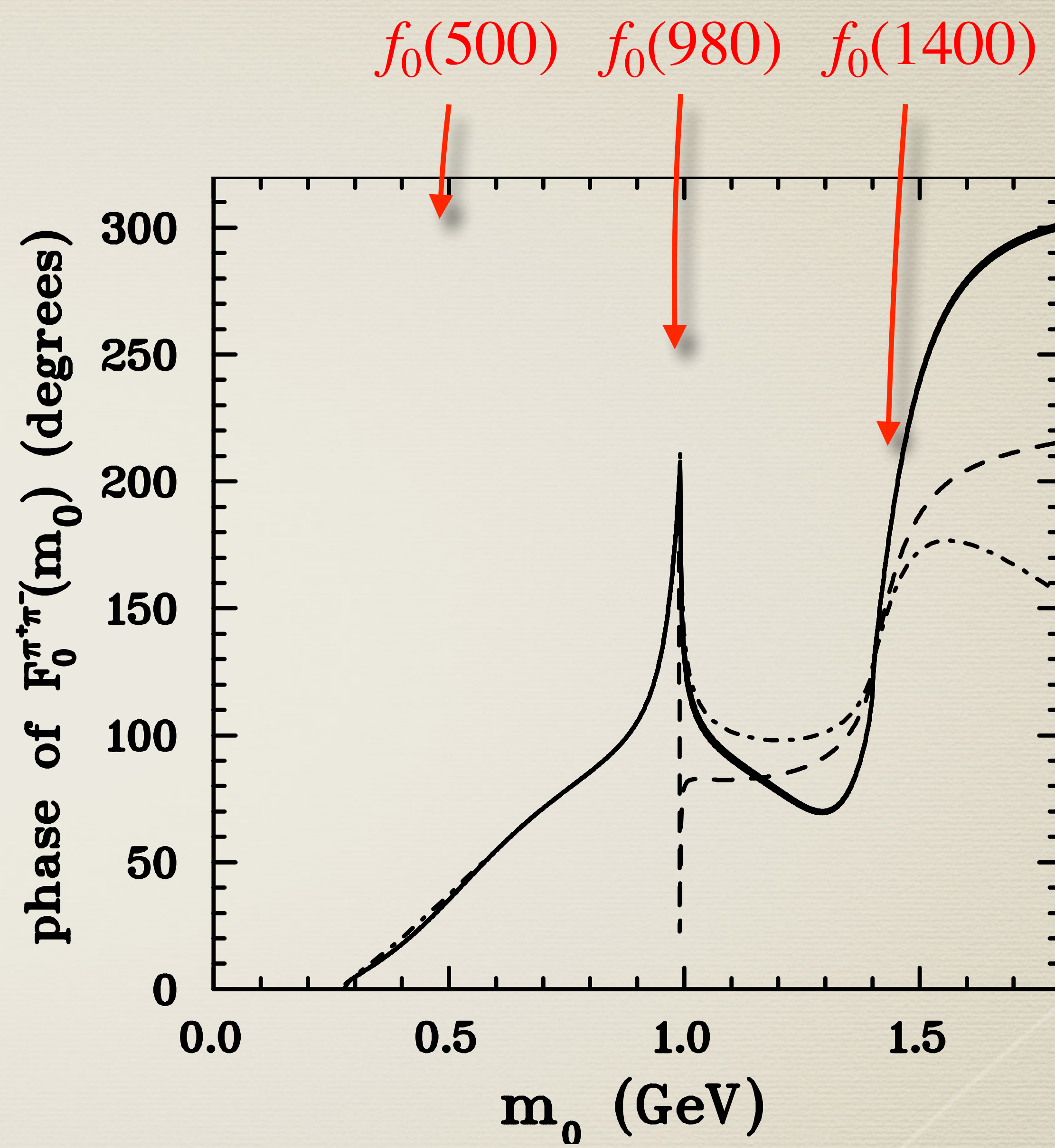
- Non-strange scalar form factor $F_{0n}^{\pi\pi}(s)$: $f_0(500)$, $f_0(980)$, $f_0(1400)$.
  Strange scalar form factor $F_{0s}^{\pi\pi}(s)$: $f_0(980)$, $f_0(1400)$.
- From $B^- \to K^-[\pi^+\pi^-]_S$ [A. Furman *et al.* Phys. Lett. B **622**, 207 (2005)]

$$b_1^{-S} = \frac{G_F}{\sqrt{2}}\left[\chi f_K F_0^{B\to(\pi\pi)_S}(m_K^2)\,U - \tilde{C}\right]$$

$\tilde{C} = f_\pi F_\pi\left(\lambda_u P_1^{GIM} + \lambda_t P_1\right)$, $\lambda_u = V_{ub}V_{us}^*$, $\lambda_t = V_{tb}V_{ts}^*$, F_π $B\pi$ form factor at $m_\pi^2 = 0$, P_1^{GIM}, P_1 complex charming penguin parameters, U short-distance contribution : CKM $\times$ effective Wilson coefficients. χ fitted free parameter.

Comparison of unitary non-strange scalar form factors $F_{0n}^{\pi\pi}$





Dark band: $D^0 \to K_S^0 \pi^+ \pi^-$ variation with error parameters [J.-P. Dedonder *et al.*, Phys. Rev. D **89**, 094018 (2014)]. Dashed line: $B \to 3\pi$ [J.-P. Dedonder *et al.* Acta Phys. Pol. B **42**, 2013 (2011)]. Dotted-dashed line: B. Moussallam [Eur. Phys. J. C. **14**, 111 (2000)] using Muskhelishvili-Omnès equations.