Parametrized $\bar{B}^0 \to K_S^0 K^+ K^-$ amplitudes in terms of analytic and unitary kaon form factors

R. Kamiński[1], L. Leśniak[1], B. Loiseau[2], P. Żenczykowskii[1], preliminary study

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Future Challenges in Non-Leptonic B Decays: Theory and Experiment, MITP, Johannes Gutenberg University Mainz, January 14-25, 2019

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PARIS

This three-body decay has been analyzed within the isobar model approach by:

- B. Aubert *et al.*, (BABAR Collaboration), Measurements of *CP*-Violating Asymmetries in the Decay $\bar{B}^0 \to K_S^0 K^+ K^-$, Phys. Rev. L ett. **99**, 161802 (2007).
- Y. Nakahama *et al.*, (Belle Collaboration), Measurement of *CP* violating asymmetries in $\bar{B}^0 \to K^+ K^- K_S^0$ decays with a time-dependent Dalitz approach, Phys. Rev. D **82**, 073011 (2010).
- J. P. Lees *et al.* (BABAR Collaboration), Study of CP violation in Dalitz-plot analyses of $\bar{B}^0 \to K^+ K^- K_S^0$, $\bar{B}^0+ \to K^+ K^- K^+$, and $\bar{B}^+ \to K_S^0 K_S^0 K^+$, Phys. Rev. D **85**, 112010 (2012).

With the LHCb data, an isobar model Dalitz plot analysis has been started by the LPNHE LHCb subgroup, E. Bertholet, E. Ben-Haim, M.Charles.

- As just pointed out by Bruno:
  $\to$ Isobar parametrizations do not respect unitarity
  $\Rightarrow$ Extraction of strong CP phases to be taken with caution
  $\to$ *S*-wave resonance contribution hard to fit
  $\to$ Here difficult to disantangle $f_0(980) \leftrightarrow a_0(980)$ contributions.

$\Rightarrow$ Why not try to go beyond Isobar model Dalitz plot analysis ?

$\bar{B}^0 \to K^0_S K^+ K^-$ parametrization based on amplitudes derived in the quasi two-body QCD factorization approach ↔ Bruno El-Bennich's previous talk

- Parametrized amplitudes to be implemented in experimental analyses: alternative to the isobar model - Useful for interpretation CP asymmetries.

⇒ $\bar{B}^0(p_B) \to K^0_S(p_0)K^-(p_-)K^+(p_+)$ with $s_0 = (p_- + p_+)^2$, $s_- = (p_0 + p_-)^2$, $s_+ = (p_0 + p_+)^2$, and $s_0 + s_- + s_+ = m^2_{B^0} + m^2_{K^0} + 2m^2_K$.

$$\mathcal{A}^{\bar{0}}(s_0, s_-, s_+) = \sum_{L=S,P,D} \sum_{I=0,1} \mathcal{A}^{\bar{0}}_{L,I}(s_0, s_-, s_+).$$

- For isoscalar $[K^+K^-]_S$ pairs associated with $f_0(980)$, $f_0(1370)$ resonances

$$\mathcal{A}^{\bar{0}}_{S,0}(s_0, s_-, s_+) = \left(i_1^S + i_2^S s_0\right) F^{K\bar{K}}_{0n}(s_0) + \left(i_3^S + i_4^S s_0 + i_5^S F_0^{\bar{B}^0 K^0}(s_0)\right) F^{K\bar{K}}_{0s}(s_0).$$

- Isovector $[K^+K^-]_S$, $[K^0_S K^\pm]_S$ pairs ↔ $a_0(980)^{0(\mp)}$, $a_0(1450)^{0(\mp)}$ resonances

$$\begin{aligned}\mathcal{A}^{\bar{0}}_{S,1}(s_0, s_-, s_+) &= \left(i_6^S + i_7^S s_0\right) G_0^{K\bar{K}}(s_0) + \left(i_8^S + i_9^S s_-\right) G_0^{K\bar{K}}(s_-) \\ &+ \left(i_{10}^S + i_{11}^S s_+\right) G_0^{K\bar{K}}(s_+).\end{aligned}$$

⇒ 11 complex parameters $i^S_{j=1,11}$ to be fitted. Could be reduced to 5, fixing linear s value to square of dominant-resonance mass. If quasi- two-body QCD factorization amplitudes ⇒ 3 complex or even 3 real free parameters.

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- For example for i_{10}^S and i_{11}^S:

$$[i_{10}^S]_1 = -\frac{G_F}{\sqrt{2}}\,\chi^1\,f_K\,y(a_0^+K^-)\,F_0^{\bar{B}^0 a_0^+}(m_K^2),$$
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G_F Fermi decay constant; χ^1 free parameter: contribution strength of scalar-isovector $G_0^{K\bar{K}}(s)$ form factor; f_K, f_{B^0} kaon, meson B^0 decay constants; $F_0^{\bar{B}^0 a_0^+}(m_K^2)$, $F_0^{\bar{K}^0K^+}(m_{B^0}^2)$: $B^0 a_0^+$, $\bar{K}^0K^+$ transition form factors at m_K^2, $m_{B^0}^2$.

- Functions $y(a_0^+K^-)$, $z(a_0^+K^-)$, $\nu(a_0^+K^-)$ short distance contributions $\propto$ CKM matrix elements and effective Wilson coefficients

$\Rightarrow$ For instance, with $\lambda_u = V_{ub}V_{us}^*$, $\lambda_c = V_{cb}V_{cs}^*$; Wilson coefficient $a_i^{(p)}(M_1M_2)$; chiral factor: $r_\chi^K = 2m_K^2/[(m_b+m_d)(m_u+m_s)]$ the function $y(a_0^+K^-)$:

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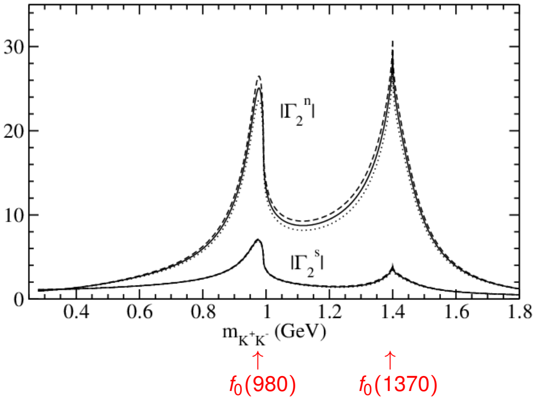
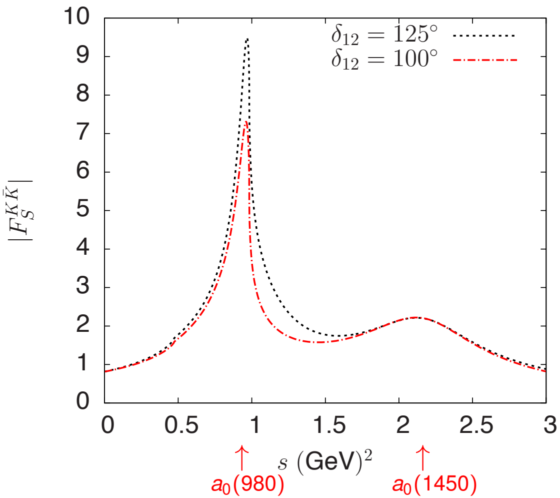
$\rightarrow$ Used in preliminary fit $D^0 \rightarrow K^0_S K^+ K^-$ [J.-P. Dedonder, R. R Kamiński, L. Leśniak and B. Loiseau, Dalitz plot studies of $D^0 \rightarrow K^0_S K^+ K^-$ decays in a factorization approach, work in progress.]



Solid lines:
$|\Gamma_2^n(m_{K^+K^-})| = |F_{0n}^{K\bar{K}}(\sqrt{s_{23}})/\sqrt{2}|$,
$|\Gamma_2^s(m_{K^+K^-})| = |F_{0s}^{K\bar{K}}(\sqrt{s_{23}})|$.
Derived by A. Furman, R. Kamiński, L. Leśniak, P. Żenczykowski, [Final state interaction in $B^\pm \rightarrow K^+K^-K^\pm$, Phys. Lett. B **699**, 102 (2011)] in
$\rightarrow$ solving three coupled channels $\pi\pi$, $K\bar{K}$, 4π (effective 2π-2π or $\sigma\sigma$ or $\eta\eta$ ...),
$\rightarrow$ imposing chiral symmetry constraints.

- Dashed and dotted lines: variations with parameter errors.



- Unitary model. S-wave coupled channels $\eta\pi$, $K\bar{K}$, asymptotic QCD + chiral symmetry constraints + $a_0(980)$, $a_0(1450)$ $\to$ form factor: Muskhelishvili-Omnès equation (dispersion relation). $\delta_{12} \equiv \delta_{11}(\sqrt{s}) + \delta_{22}(\sqrt{s})\big|_{\sqrt{s}=m_{a_0(1450)}}$. Channels $\to$ $1 \equiv \eta\pi, 2 \equiv K\bar{K}$.

$\to$ M. Albaladejo, B. Moussallam, Form factors of the isovector scalar current and the $\eta\pi$ scattering phase shifts, Eur. Phys. J. **C75** (2015), arXiv:1507.04526.

Some concluding remarks

- For Dalitz plot analyses for $\bar{B}^0 \to K_S^0 K^+ K^-$ decays we have presented parametrized amplitudes in terms of analytic and unitary kaon form factors.
- These amplitudes are based on amplitudes derived in quasi-two body QCD factorization approach.

$\Rightarrow$ These parametrizations can be readily implemented in experimental analyses as an alternative to the sum of Breit-Wigner type amplitudes.

- The LPNHE LHCb subgroup (E. Bertholet, E. Ben-Haim, M.Charles, ...) intend to probe them within their Dalitz plot analysis in progress.
- Preliminary parametrized amplitudes for $B^\pm \to K^+ K^- \pi^\pm$ [Belle, LHCb] are also available.

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BACKUP MATERIAL

- For isoscalar $[K^+K^-]_P$ pairs, contributions from $\omega(782)$, $\omega(1420)$, $\omega(1670)$, $\phi(1020)$, $\phi(1680)$ resonance:

$$\mathcal{A}^{\bar{0}}_{P,0}(s_0, s_-, s_+) = (s_- - s_+)\left[\left(i_1^P + i_2^P F_1^{\bar{B}^0 K^0}(s_0)\right) F_{ud}^{[K^+K^-]^0}(s_0) + \left(i_3^P + i_4^P F_1^{\bar{B}^0 K^0}(s_0)\right) F_{1s}^{K^+K^-}(s_0)\right].$$

- For isovector $[K^+K^-]_P$ and $[K^0_S K^\mp]_P$ pairs, contributions from $\rho(770)^{0(\mp)}$, $\rho(1450)^{0(\mp)}$, $\rho(1700)^{0(\mp)}$ resonances:

$$\begin{aligned}\mathcal{A}^{\bar{0}}_{P,1}(s_0, s_-, s_+) &= (s_- - s_+)\left(i_5^P + i_6^P F_1^{\bar{B}^0 K^0}(s_0)\right) F_{ud}^{[K^+K^-]^1}(s_0) \\ &+ \left[s_0 - s_+ - \left(m^2_{B^0} - m^2_K\right)\frac{m^2_K - m^2_{K^0}}{s_-}\right] i_7^P F_1^{\bar{K}^0 K^-}(s_-) \\ &+ \left[s_0 - s_- + \left(m^2_{B^0} - m^2_K\right)\frac{m^2_K - m^2_{K^0}}{s_+}\right] i_8^P F_1^{\bar{K}^0 K^+}(s_+).\end{aligned}$$

→ Vector-isocalar form factor: $F_{ud}^{[K^+K^-]^0}(s) = F_{1u}^{K^+K^-}(s) + F_{1d}^{K^+K^-}(s)$

→ Vector-isocalar form factor: $F_{ud}^{[K^+K^-]^1}(s) = F_{1u}^{K^+K^-}(s) + F_{1d}^{K^+K^-}(s)$

- Vector form factors $F_{1q}^{K^+K^-}(s)$, $q = u, d, s$ from vector dominance, quark model assumptions, isospin symmetry in Ref. [A] C. Bruch, A. Khodjamirian, J. H. Kühn, Eur. Phys. J. **C39**, 41 (2005). See also Ref.[A] for $F_1^{\bar{K}^0 K^\mp}(s)$.

$$\langle M_1 M_2^* | O_i(\mu) | B \rangle = \langle M_1 | J_1^\nu | B \rangle \langle M_2^* | J_{2\nu} | 0 \rangle + \langle M_1 | J_3^\nu | 0 \rangle \langle M_2^* | J_{4\nu} | B \rangle$$

- $\langle M_1(p_1) | J_1^\nu | B \rangle$ $(= \langle M_1(p_1) \overline{B} | J_1^\nu | 0 \rangle)$ transition form factor: Light-front & relativistic constituent quark models - light-cone sum rules - continuum functional QCD - lattice-QCD. Semi-leptonic decays, e.g. $D^0 \to \pi^- e^+ \nu_e$
- $\langle M_2^* | J_{2\nu} | 0 \rangle \propto \langle M_3 M_4 | J_{2\nu} | 0 \rangle$: form factor, creation from a $\bar{q}q$ pair. Dispersion relations + field theory $\to$ form factor known if $M_3 M_4$ strong interaction known at all energies [G. Barton , Introduction to dispersion techniques in field theory, W. A. Benjamin, INC., New York (1965)]. Two-body data + unitarity + asymptotic QCD + chiral symmetry at low energies.
- $\langle M_1 | J_3^\nu | 0 \rangle$: weak decay constant, known from experiment, e.g. f_π, f_K. Evaluated with lattice-regularized QCD and other nonperturbative approaches.
- $\langle M_2^* | J_{4\nu} | B \rangle \propto \langle M_3 M_4 | J_{4\nu} | B \rangle$: complicated - biggest uncertainty. Semi-leptonic processes: $B^0 \to K^+ \pi^- \mu^+ \mu^-$ or $D^0 \to K^- \pi^+ \mu^+ \mu^-$. Soft-collinear effective theory: amplitude can be factorized in terms of generalized B-to-two-body form factor and two-hadron light-cone distribution amplitude. Our model: $\langle M_2^*(p_2) | J_{4\nu} | B \rangle$ related to $\langle M_2^*(p_2) [\to M_3 M_4] | J_{2\nu} | 0 \rangle$ form factor.