INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Dispersive analyses: Perspectives and Discussion

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Workshop on Non Leptonic B decays

MITP, January 16, 2019

Vector and Scalar Form Factors

- When studying non-leptonic meson decays to build amplitudes we require building blocks:
  - ππ
  - Kπ

  $\Rightarrow$ ChPT + dispersion relations

- ππ scalar FF and Vector FF: $s=\left(p_{\pi^+}+p_{\pi^-}\right)^2$

$$\boxed{\left\langle \pi^+\pi^- \right| \frac{1}{2}\left(\bar{\mathrm{d}}\gamma^\mu \mathrm{d} - \bar{\mathrm{u}}\gamma^\mu \mathrm{u}\right) \left|0\right\rangle = F_\pi(s)\left(p_{\pi^+}-p_{\pi^-}\right)^\mu}$$

and

$$\boxed{\left\langle \pi^+\pi^- \right| \mathrm{m}_u\bar{u}u - m_d\bar{d}d\left|0\right\rangle \equiv \Gamma_\pi(s)}$$

- Kπ scalar and Vector FFs:

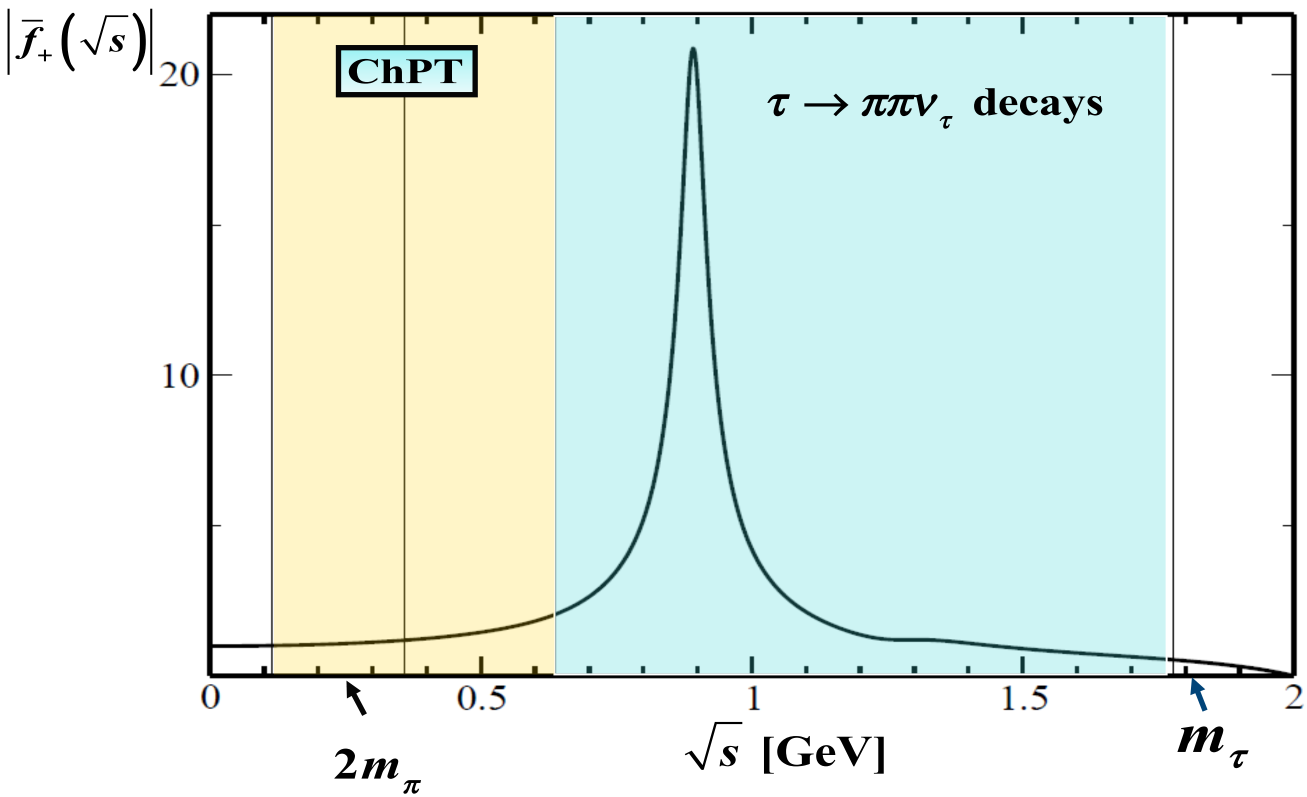
$$\boxed{\left\langle \mathrm{K}\pi \right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left|0\right\rangle = \left[\left(p_K-p_\pi\right)_\mu + \frac{\Delta_{K\pi}}{s}\left(p_K+p_\pi\right)_\mu\right]\underbrace{f_+(s)}_{\text{vector}} - \frac{\Delta_{K\pi}}{s}\left(p_K+p_\pi\right)_\mu \underbrace{f_0(s)}_{\text{scalar}}}$$

with $s=q^2=(p_K+p_\pi)^2$,

$$\Delta_{K\pi}=\left(M_K^2-M_\pi^2\right)$$

Vector and Scalar Pion Form Factors

- At low energy ⇨ Use of ChPT
- For intermediate energy region : dispersive techniques

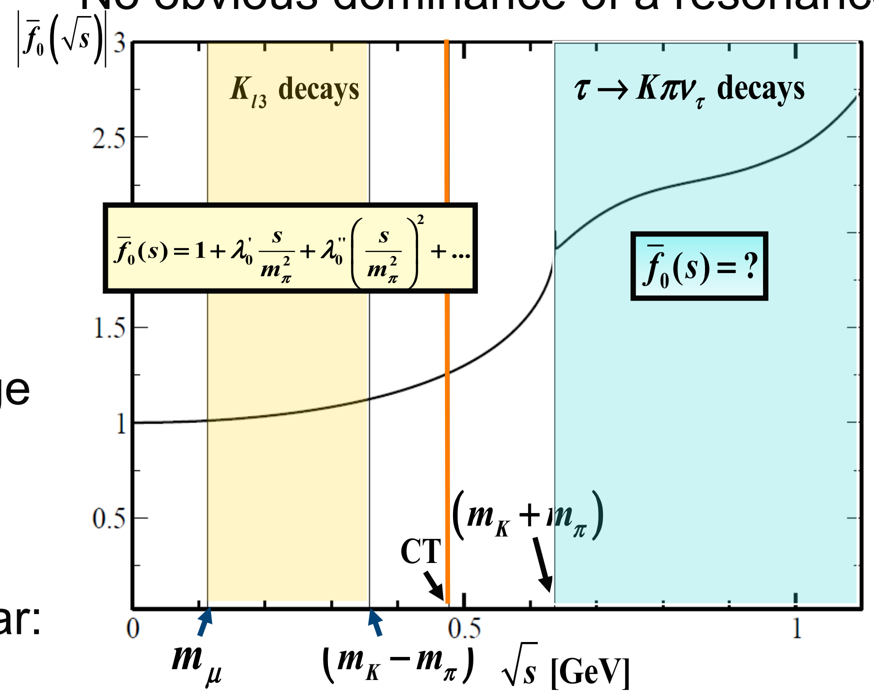


On the interest of using Dispersion Relations

- If E > 1 GeV: ChPT not valid anymore to describe dynamics of the process
  ⇨ Resonances appear :
  - For ππ: *I=1*: ρ(770), ρ(1450), ρ(1700), …, *I=0*: "σ(~500)", f_0(980),…
  - For Kπ: *I=1*: K*(892), K*(1410), K*(1680), …, *I=0*: "κ(~800)", …

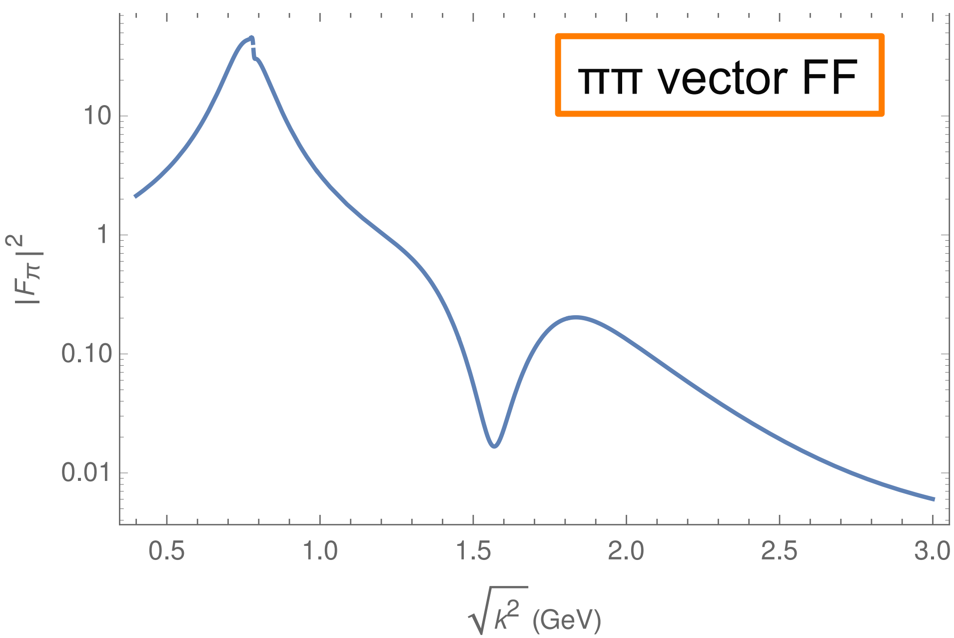
- With Dispersion Relation:
  - no need for making assumptions of a dominance of resonances
    ⇨ directly given by the parametrization,
    phase shifts taken as inputs
  - Parametrization valid in a large range of energy:
    ⇨ analyse several processes simultanously where the same quantity: FFs, amplitude appear:
    Ex: K_{l3} decays, $\tau \to K\pi\nu_\tau$

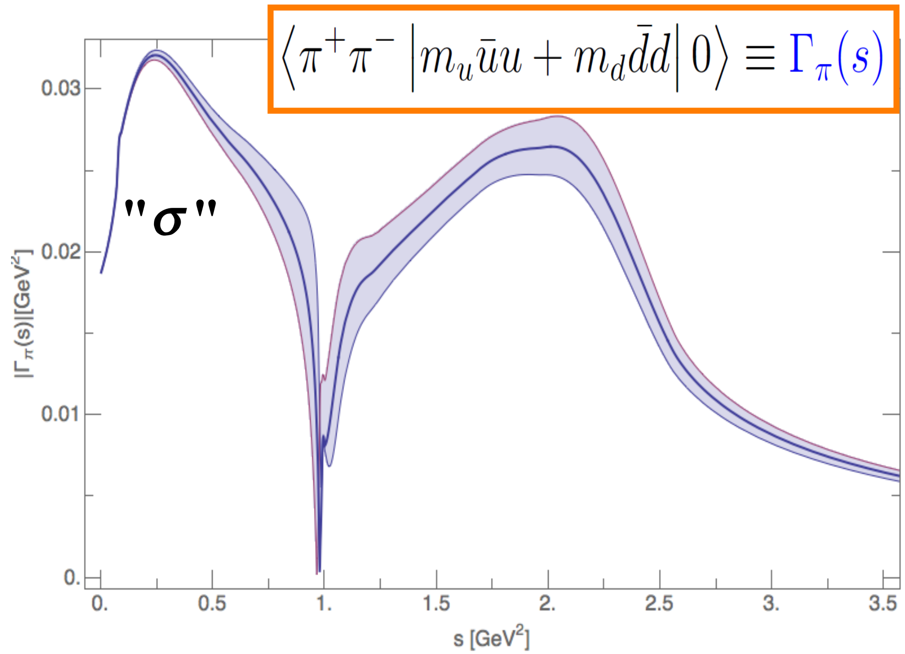
Kπ scalar form factor:
No obvious dominance of a resonance



ππ Vector and Scalar Form Factors

Pich & Portoles'01
Schneider, Kubis, Niecknig'12
Gómez-Dumm & Roig'13
Celis, Cirigliano, E.P.'14, etc

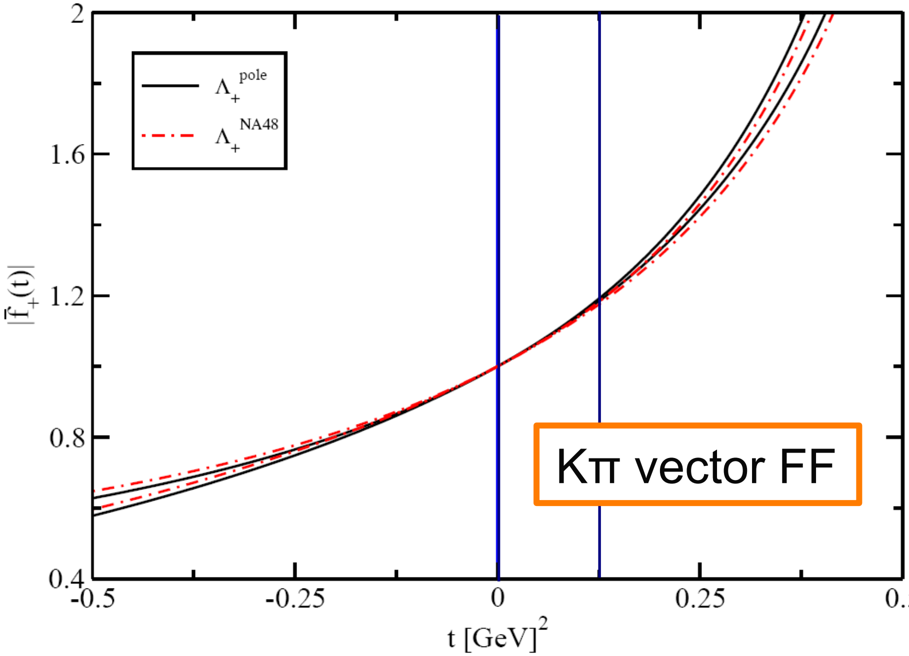




Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub, Dreiner, Hanart, Kubis, Meissner'13
Celis, Cirigliano, E.P.'14

See also *Oller & Oset'98*
Lahde & Meissner'06

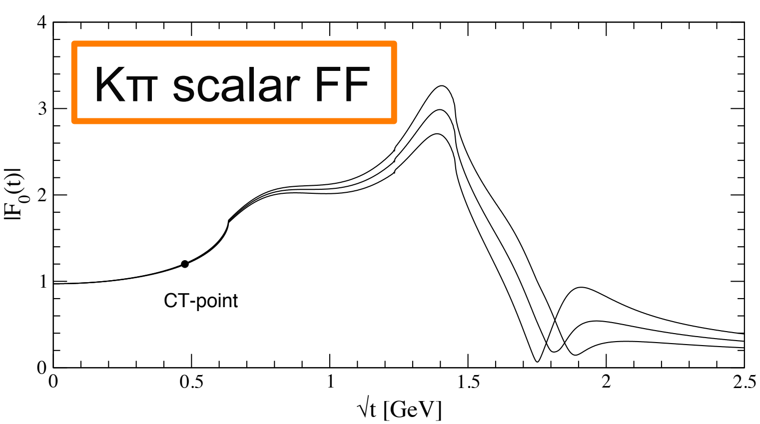
Kπ Vector and Scalar Form Factors

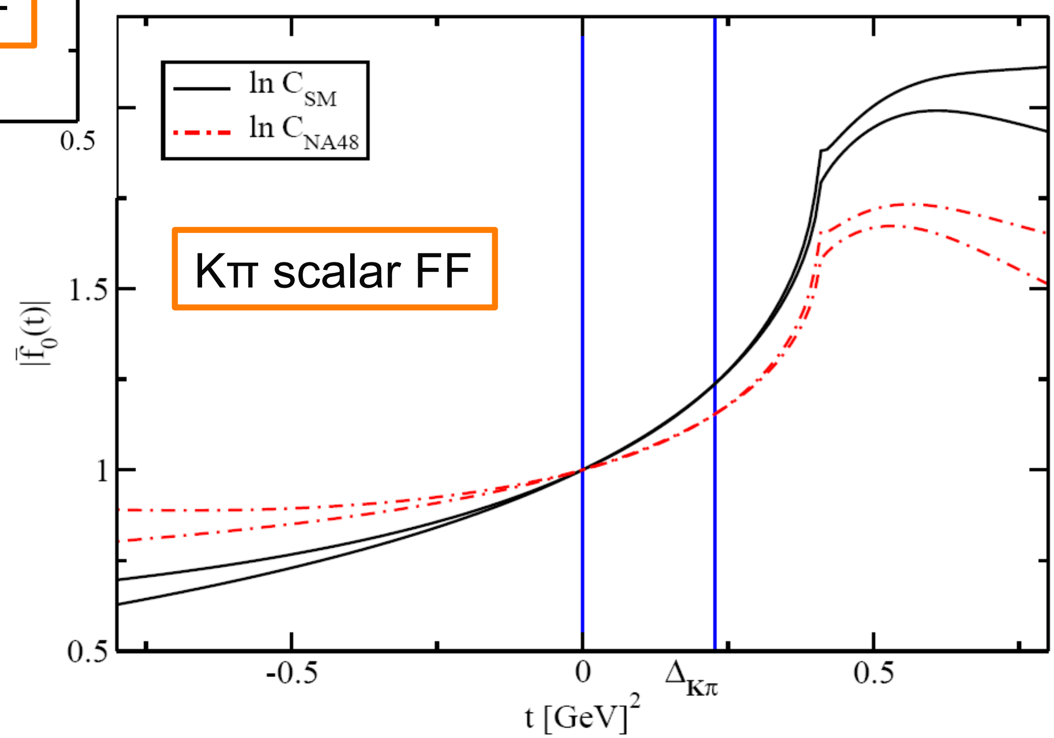


$$\bar{f}_{0,+}(t) = \frac{f_{0,+}(t)}{f_+(0)}$$

Jamin, Oller, Pich'02,'06
Jamin, Pich, Portoles'08
Boito, Escribano, Jamin'08'10
Moussallam'08

Bernard et al.'06,'10
Bernard, Boito, E.P.'11
Bernard'14

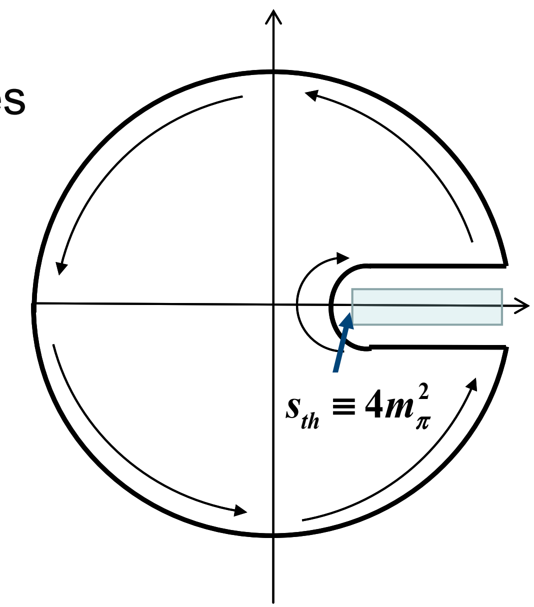




Dispersion relations: challenges

- At low energy ⇨ Use of ChPT

- For intermediate energy region : dispersive techniques



$$\boxed{F(s) = P_{n-1}(s) + \frac{s^n}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^n} \frac{\mathrm{Im}\left[F(s')\right]}{\left(s' - s - i\varepsilon\right)}}$$

polynomial

- Imaginary part known from unitarity and data:

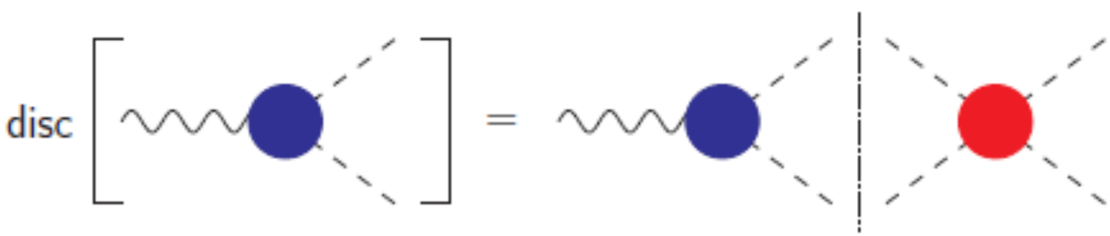
$$\boxed{\frac{1}{2i} disc\ F_{\pi\pi}(s) = \mathrm{Im}\, F_{\pi\pi}(s) = \sum_n F_{\pi\pi \to n} \left(\mathrm{T}_{n \to \pi\pi}\right)^*}$$

n = *all possible* intermediate states

Dispersion relations: challenges

- Unitarity ⇨ the discontinuity of the form factor is known

$$\boxed{\frac{1}{2i}disc\ F_{\pi\pi}(s) = \text{Im}\,F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*}$$



- The higher energy one goes $s=(p_1+p_2)^2$ more states one needs to include

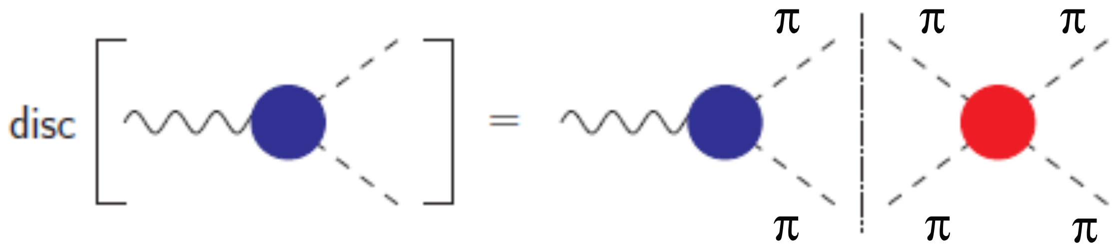
⇨ Only one channel n = ππ $\left(s < 16\, m_\pi^2\right)$

Dispersion relations: challenges

- Unitarity ⇒ determine the discontinuity of the form factor

$$\frac{1}{2i} disc\, F_{\pi\pi}(s) = \mathrm{Im}\, F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n} \left(\mathrm{T}_{n\to\pi\pi}\right)^*$$

- Only one channel n = ππ

⇒ 

$$\frac{1}{2i} disc\, F_I(s) = \mathrm{Im}\, F_I(s) = F_I(s)\ \sin\delta_I(s) e^{-i\delta_I(s)}$$

ππ scattering phase
known from experiment

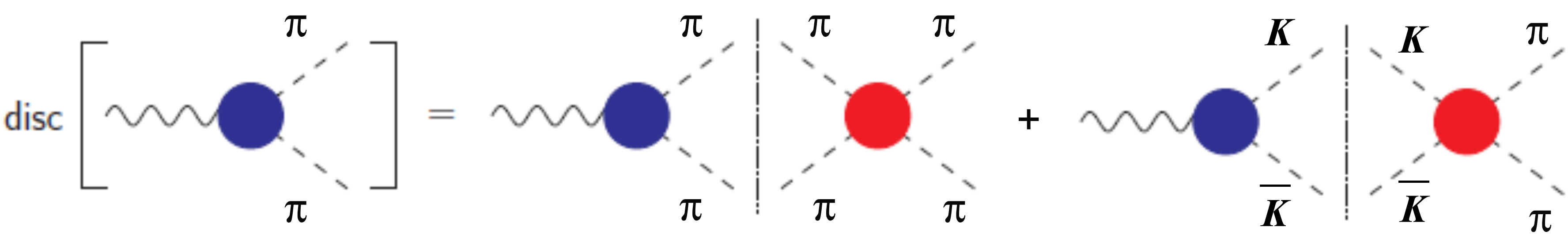
Watson's theorem

Going beyond one channel

- Unitarity ⇨ determine the discontinuity of the form factor

$$\boxed{\frac{1}{2i}\, disc\, F_{\pi\pi}(s) = \mathrm{Im}\, F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*}$$

- In practice when √s < ~1.4 GeV : 2 channels in the scalar case



⇨ $$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$n = \pi\pi, K\bar{K}$

Scattering matrix:

$$\begin{pmatrix} \pi\pi\to\pi\pi, & \pi\pi\to K\bar{K} \\ K\bar{K}\to\pi\pi, & K\bar{K}\to K\bar{K} \end{pmatrix}$$

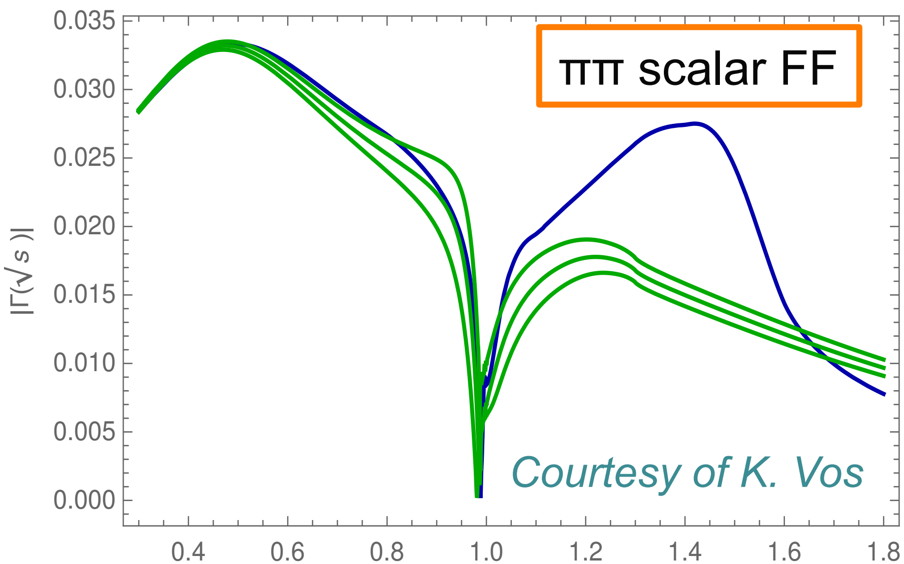
Going beyond one channel

- Unitarity ⇒ determine the discontinuity of the form factor

$$\boxed{\frac{1}{2i}\,disc\; F_{\pi\pi}(s) = \mathrm{Im}\, F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*}$$

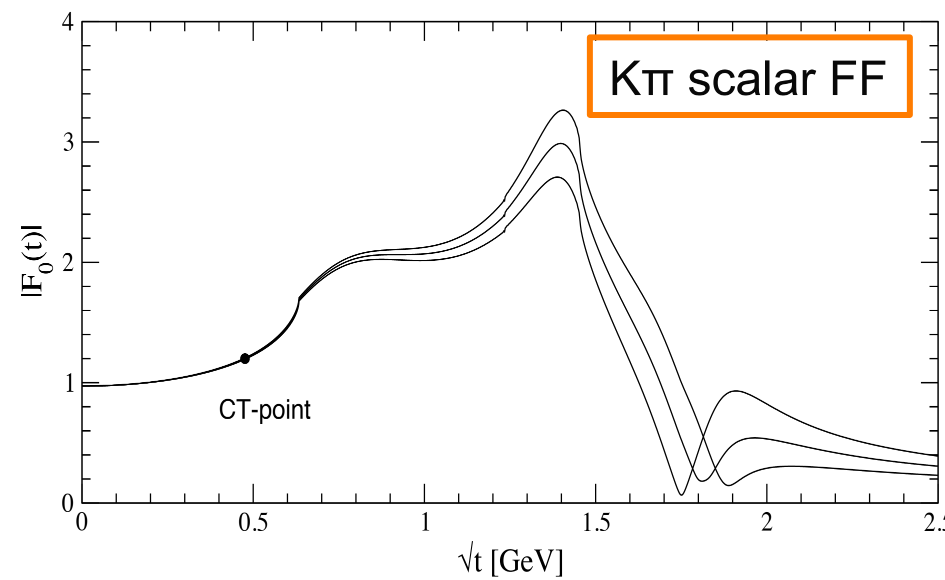
- In practice when √s < ~1.4 GeV : 2 channels in the scalar case

ππ and KK coupled channel



Daub, Hanhart, Kubis'13
Celis, Cirigliano, E.P.'14

Kπ, Kη' coupled channel



Jamin, Oller, Pich'02,'06

Going in the inelastic region

- Unitarity ⇨ determine the discontinuity of the form factor

$$\boxed{\frac{1}{2i}\,disc\ F_{\pi\pi}(s) = \mathrm{Im}\,F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*}$$

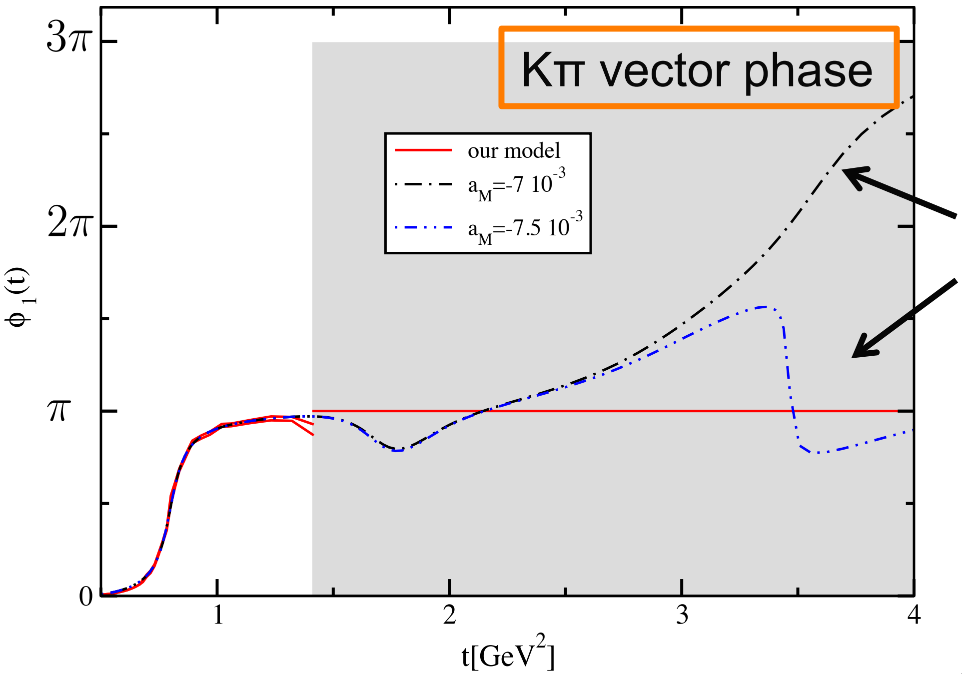
- In practice when √s < ~1.4 GeV : 2 channels in the scalar case

- For B → 3π we need to know the pion FFs up to √s ~2.5 GeV

- Challenge: how do we go beyond? Many channels open: σσ, ρρ (2π), nπ, etc

⇨ Need to rely on models, see talks by *B. Kubis*, *B. Loiseau and P.C. Magalhães*

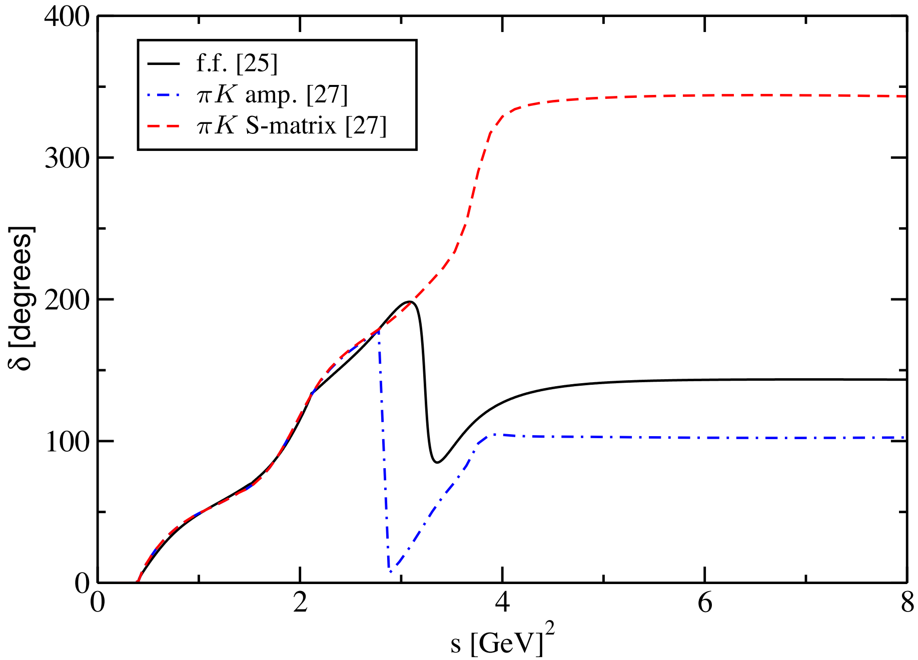
Going in the inelastic region



Bernard, Oertel, Passemar, Stern'06,'10

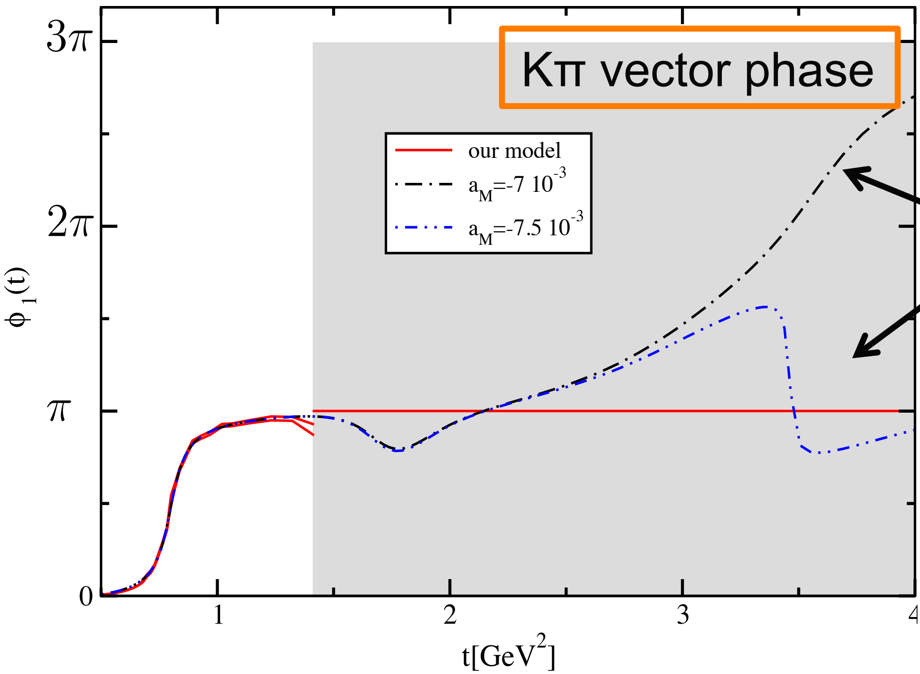
Moussallam'08

Kπ scalar phase



[25] Jamin, Oller, Pich'02,'06
[27] Buettiker, Descotes, Moussallam'03

Going in the inelastic region



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Moussallam'08

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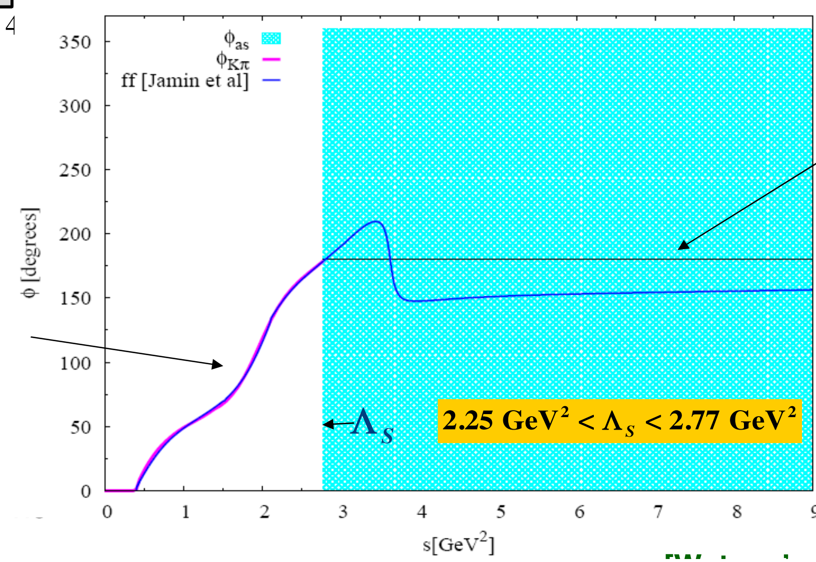


Asymptotic value

$\phi(s) \underset{s\to-\infty}{\longrightarrow} \pi$

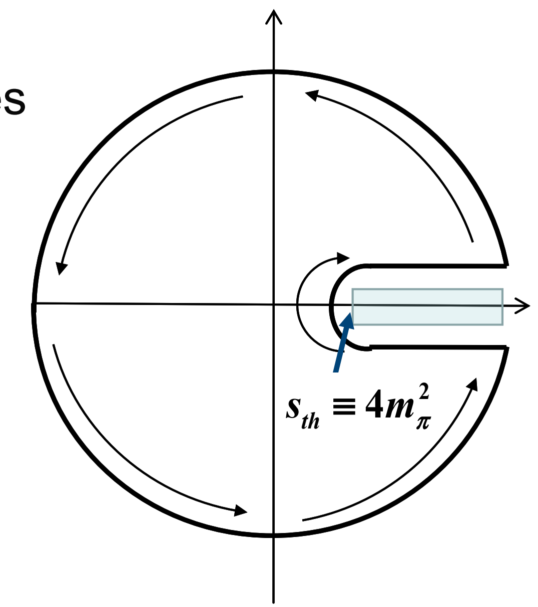
$\bar{f}_0(s) = \mathcal{O}(1/s$

Buettiker, Descotes, Moussallam'03

Kπ scattering phase

Uncertainties

- At low energy ⇨ Use of ChPT
- For intermediate energy region : dispersive techniques



$$F(s) = P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\mathrm{Im}\left[F(s')\right]}{\left(s' - s - i\varepsilon\right)}$$

polynomial

- Imaginary part known from unitarity and data:

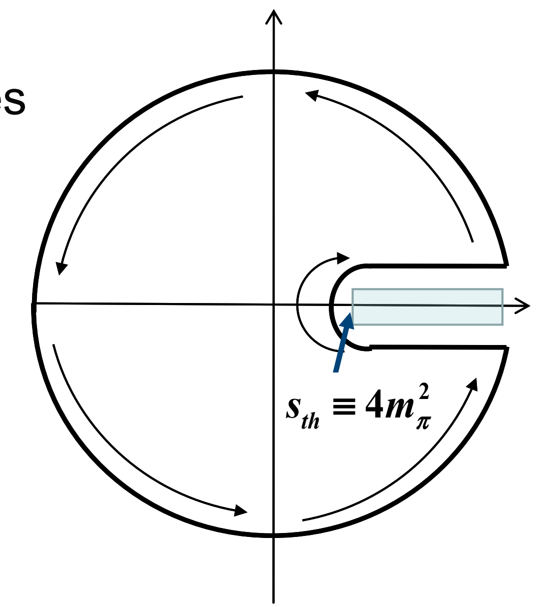
$$\frac{1}{2i} disc\ F_{\pi\pi}(s) = \mathrm{Im}\, F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*$$

- One does not go to infinity ⇨ cut the integral at some point and make some assumptions on high energy behaviour: uncertainty needs to be quantified

Uncertainties

- At low energy ⇨ Use of ChPT

- For intermediate energy region : dispersive techniques



$$F(s) = P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^n}\frac{\mathrm{Im}\left[F(s')\right]}{(s'-s-i\varepsilon)}$$

polynomial

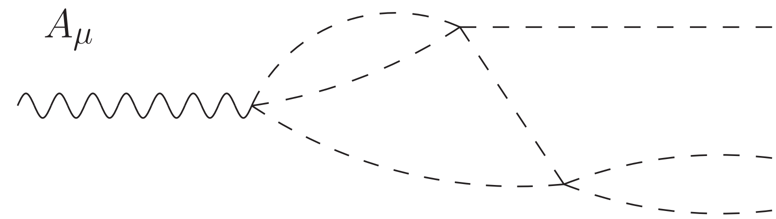
- Imaginary part known from unitarity and data:

$$\frac{1}{2i}disc\ F_{\pi\pi}(s) = \mathrm{Im}\,F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*$$

- One does not go to infinity ⇨ cut the integral at some point and make some assumptions on high energy behaviour: uncertainty needs to be quantified

Building amplitudes starting from low energies

- Following the example of η → 3π
  use $\tau \to \pi\pi\pi\nu_\tau$ for 3π amplitude



- Analytical continuation of the amplitude and decomposition

$$\boxed{\mathcal{A}_\lambda^{ijmn}(s,t,u) \propto \sum_I \sum_l \sqrt{2l+1}\, a^l_{I,\lambda}(s)\, d^l_{\lambda 0}(\theta_\pi)\, P_I^{ijmn}}$$

Lorenz, E.P. in progress

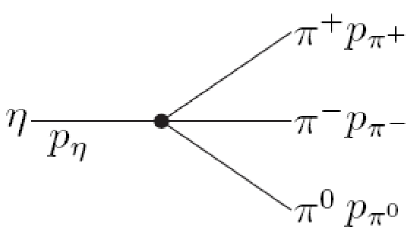
Wigner function

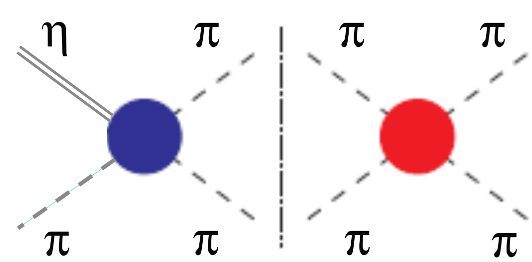
Isospin projection

$$-\sin\theta/\sqrt{l(l+1)}P_l'(\cos\theta)$$

Khuri-Treiman formalism

- Idea: Use crossing



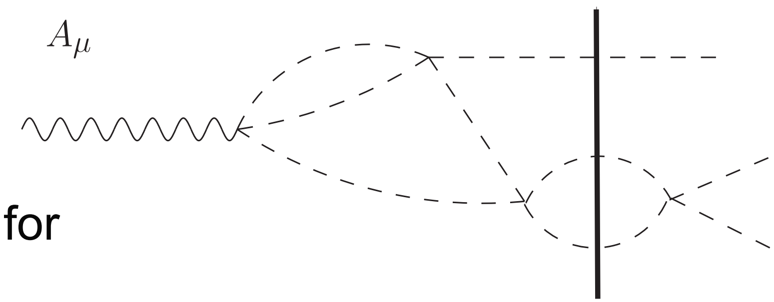


Building amplitudes starting from low energies

Lorenz, E.P. in progress

- Unitarity : $\text{Disc}\ a_{Il}^{right}(s) \equiv \text{Disc}\ a_{Il}(s)$

$$\boxed{\text{Disc}\ a_{Il}(s) = \rho(s) t_l^*(s) \left(a_{Il}^{right}(s) + a_{Il}^{left}(s) \right)}$$



- Analyticity : Write a dispersion relation for

$$\boxed{a_{Il}^{right}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Disc}\ a_{Il}^{right}(s')}{s' - s}}$$

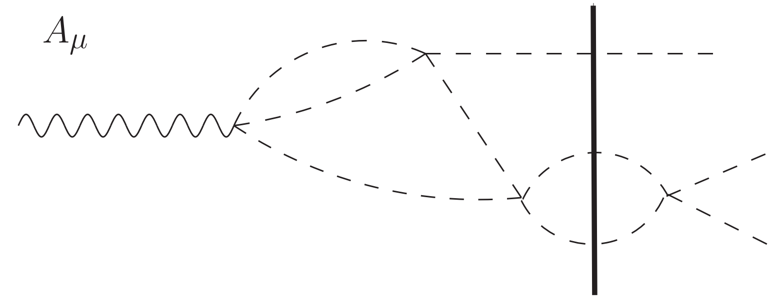
Khuri-Treiman formalism

- Solution: Inhomogeneous Omnes solution

$$\boxed{a_{Il}^{right}(s) = \Omega_{Il}(s) \left(\sum_i^{n-1} c_i s^i + \frac{s^n}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'^n} \frac{\rho(s') t_l^*(s')}{\Omega_{Il}^*(s')} \frac{a_{Il}^{left}(s')}{(s' - s)} \right)}$$

Building amplitudes starting from low energies

- Solution:



$$a_{Il}^{right}(s) = \Omega_{Il}(s)\left(\sum_i^{n-1} c_i s^i + \frac{s^n}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'^n}\frac{\rho(s')t_l^*(s')}{\Omega_{Il}^*(s')}\frac{a_{Il}^{left}(s')}{(s'-s)}\right)$$

- With

$$a_{Il}^{left}(s) \propto \sum_{I',l'}(2l'+1)\int_{-1}^{+1} dz_s(1-z_s^2)P_l'(z_s)\left(P_{l'}'(z_t)C_{st}^{II'}\,a_{I'l'}^{right}(t(s,z_s)) + P_{l'}'(z_u)C_{su}^{II'}\,a_{I'l'}^{right}(u(s,z_s))\right)$$

- Solve by an iterative procedure

Lorenz, E.P. in progress

Building amplitudes starting from low energies

- Can we apply this method to heavy mesons?

- It has been done for $D \to K\pi\pi$

  Charged channel: *Niecknig, Kubis'15,'17*
  Neutral channel: *Kou, Moussallam, Moskalets in progress*

- Several questions:
  - Inclusion on D waves for reconstruction theorem

    ➡ Reconstruction theorem proven in the case of $K,\eta \to 3\pi$, K_{l4} with truncation after S and P waves

    D waves included « by hand »

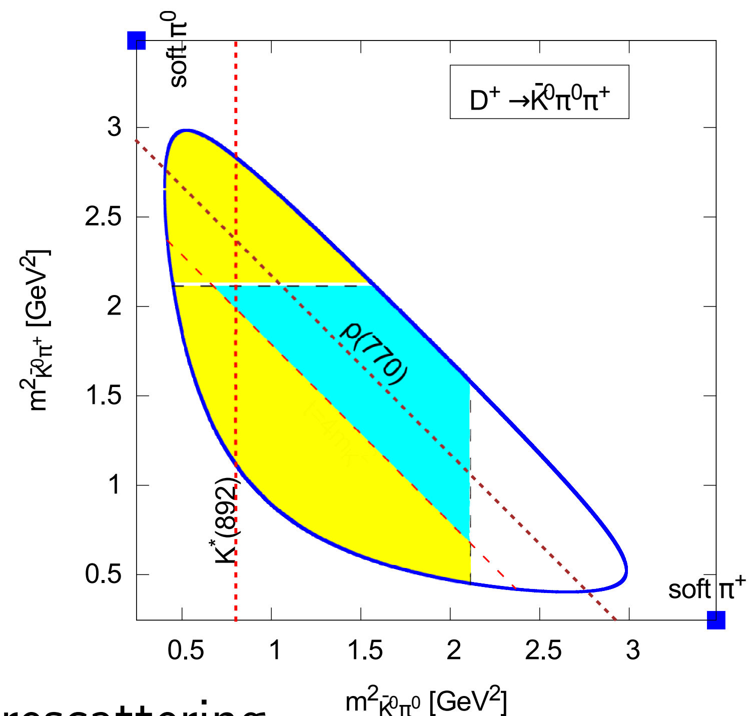
  - Method valid in the elastic region ➡ only part of the Dalitz plot is described.

  - how to include inelastic channels?
  - Many parameters to determine in the subtraction polynomial

- How to match with other approaches?
- See other approaches ➡ series of works and talks by *B. Loiseau, B. El Bennich, P.C. Magalhães*

Building amplitudes starting from low energies

B. Moussallam@
TRR110 Workshop18



Elastic rescattering

Inelasticity small

Inelasticity large $\longrightarrow$ coupled channels

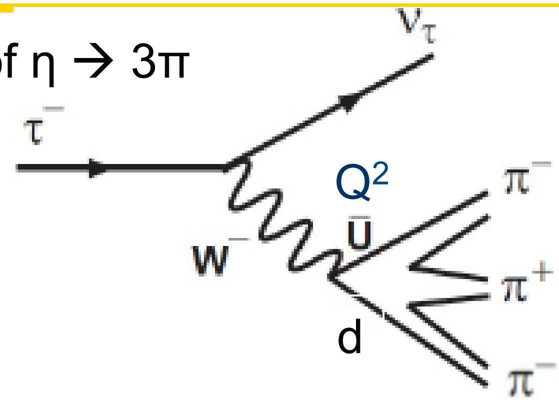
Back-up

Building amplitudes starting from low energies

Lorenz, E.P. in progress

Following the example of $\eta \to 3\pi$

- $\tau \to \pi\pi\pi\nu_\tau$



$$s = \left(p_{\pi^-} + p_{\pi_1^+}\right)^2, \quad t = \left(p_{\pi_1^+} + p_{\pi_2^+}\right)^2,$$

$$u = \left(p_{\pi^+} + p_{\pi^+}\right)^2$$

$$s + t + u = Q^2 + 3M_{\pi^\pm}^2$$

- 3-body: form factors function of one variable q^2=s ⇨ amplitude function of s and cosθ or t & u and Q^2
  Structure functions W_X *Kühn, Mirkes'92*

$$\boxed{\mathcal{M} \propto L_\mu H^\mu} \quad \text{with} \quad \boxed{H_\mu = \langle \pi\pi\pi | V_\mu - A_\mu | 0 \rangle}$$

H_μ: restricted to axial vector current A_μ by G-parity

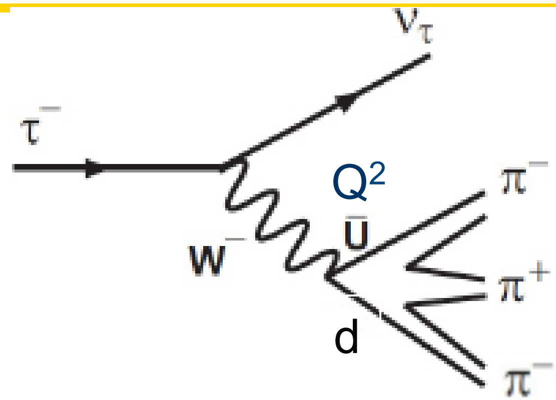
- Consider *helicity amplitudes* $\mathcal{A}_\lambda = \epsilon_\mu(\lambda) H^\mu$ simple partial wave expan.

Polarization vector of final state system with helicity $\lambda = \pm, 0, t$

Building amplitudes starting from low energies

Lorenz, E.P. in progress

- $\tau \to \pi\pi\pi\nu_\tau$



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- 3-body: form factors function of one variable q^2=s ⇨ amplitude function of s and cosθ or t & u and Q^2
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$$\boxed{\mathcal{M} \propto L_\mu H^\mu} \quad \text{with} \quad \boxed{H_\mu = \langle \pi\pi\pi | V_\mu - A_\mu | 0 \rangle}$$

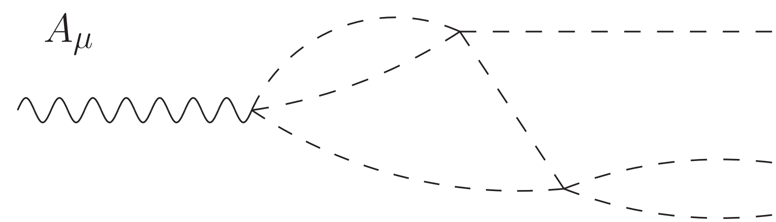
H_μ: restricted to axial vector current A_μ by G-parity

- Consider *helicity amplitudes* $\mathcal{A}_\lambda = \epsilon_\mu(\lambda) H^\mu$ simple partial wave expan.

- W_X: linear combinations of $H^{\lambda\lambda'} = \mathcal{A}_\lambda \mathcal{A}^\dagger_{\lambda'}$

Building amplitudes starting from low energies

- Analytical continuation of the amplitude and decomposition:



$$\boxed{\mathcal{A}_\lambda^{ijmn}(s,t,u) \propto \sum_I \sum_l \sqrt{2l+1}\, a^l_{I,\lambda}(s)\, d^l_{\lambda 0}(\theta_\pi)\, P_I^{ijmn}}$$

Lorenz, E.P. in progress

Wigner function Isospin projection

$$-\sin\theta/\sqrt{l(l+1)}P_l'(\cos\theta)$$

Khuri-Treiman formalism

- Bose symmetry: I+l = even

- For the transverse amplitude, P and D-waves dominating:

$$\mathscr{A}_+^{3311}(s,t,u) \propto \sum_{l=0}^{l_{max}} \sum_I (2l+1) \left[d^l_{10}(\theta_s) \left(\frac{K(s)}{4s}\right)^{l-1} P_I^{3311} a^{\text{right}}_{+,Il}(s) + d^l_{10}(\theta_t) \left(\frac{K(t)}{4t}\right)^{l-1} P_I^{3131} a^{\text{right}}_{+,Il}(t) + d^l_{10}(\theta_u) \left(\frac{K(u)}{4u}\right)^{l-1} P_I^{1331} a^{\text{right}}_{+,Il}(u) \right],$$

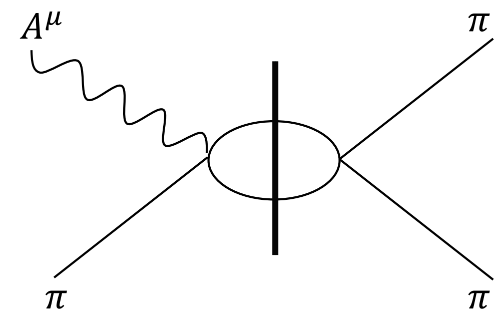
$$K(s) = \frac{t-u}{\cos\theta_s} = \sqrt{\lambda(s, M_\pi^2, M_\pi^2)}\sqrt{\lambda(s, Q^2, M_\pi^2)}.$$

Building amplitudes starting from low energies

Lorenz, E.P. in progress

- Unitarity : $\text{Disc } a_{Il}^{right}(s) \equiv \text{Disc } a_{Il}(s)$

$$\boxed{\text{Disc } a_{Il}(s) = \rho(s) t_l^*(s) \left(a_{Il}^{right}(s) + \cancel{a_{Il}^{left}(s)} \right)}$$



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$$\boxed{a_{Il}^{right}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Disc } a_{Il}^{right}(s')}{s' - s}}$$

- Neglecting left-hand cut: Omnes solution

$\pi\pi$ phase shift

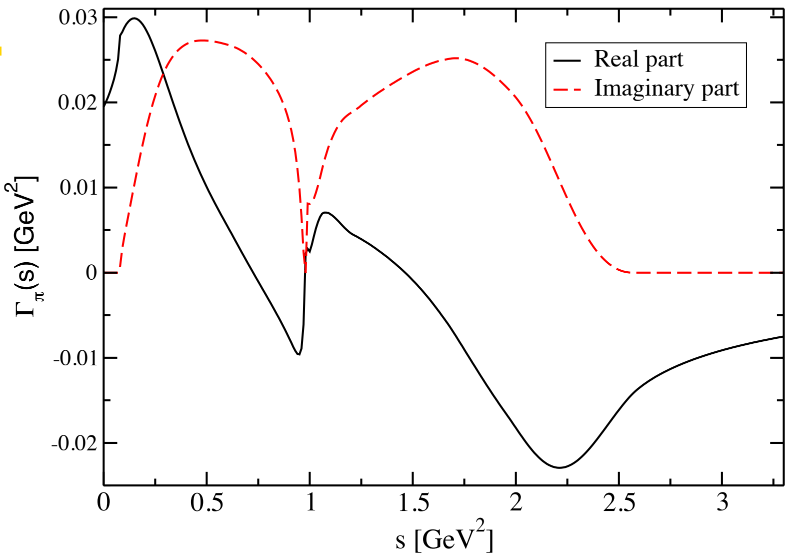
$$\boxed{a_I^{l\,\text{right}}(s) = \Omega_I^l(s) * G(s), \quad \Omega_I^l = \exp\left(\frac{s}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'} \frac{\delta_I^l(s')}{s' - s} \right)}$$

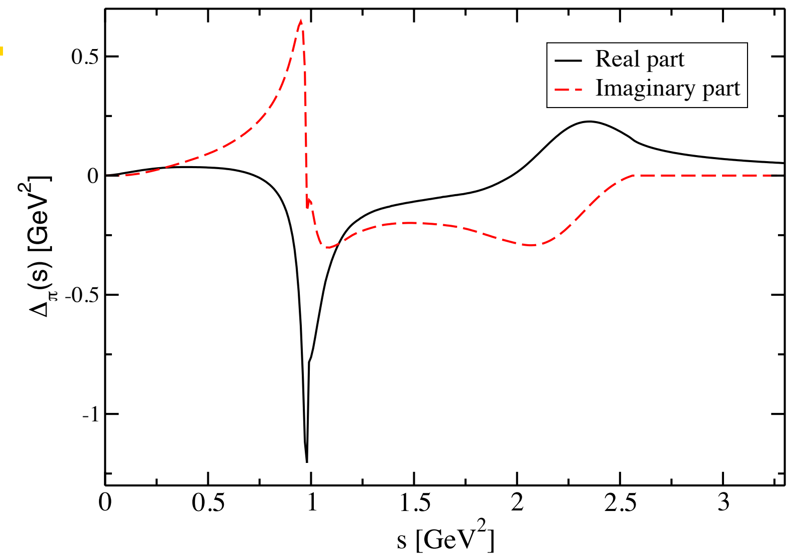
T matrix parametrization

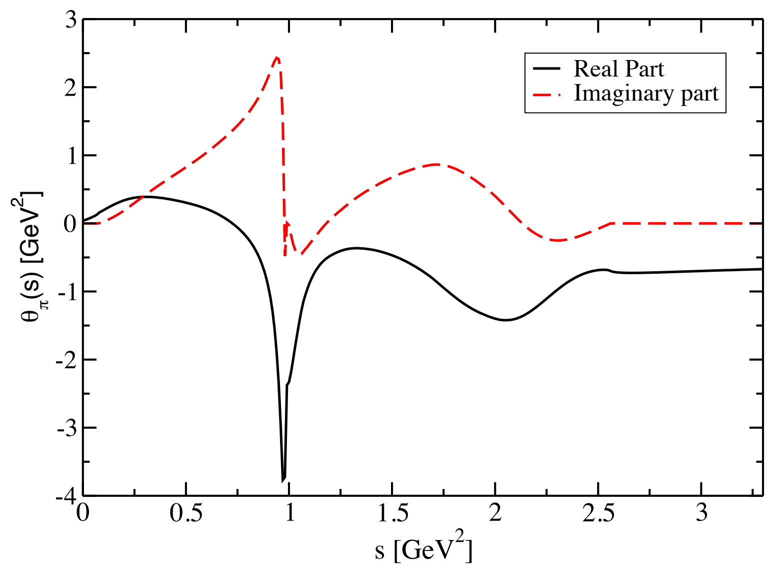
$$S_{mn} = \delta_{mn} + 2i\,\sqrt{\sigma_m \sigma_n}\,T_{mn}$$

$$S = \begin{pmatrix} \cos\gamma\ e^{2i\delta_\pi} & i\ \sin\gamma\ e^{i(\delta_\pi+\delta_K)} \\ i\ \sin\gamma\ e^{i(\delta_\pi+\delta_K)} & \cos\gamma\ e^{2i\delta_K} \end{pmatrix}$$

- Inelasticity: $\eta_0^0 \equiv \cos\,\gamma$.
- $\delta_\pi(s)$: ππ S wave phase shift
- $\delta_K(s)$: KK S wave phase shift


Real part
Imaginary part
Γ_π(s) [GeV^2]
s [GeV^2]


Real part
Imaginary part
Δ_π(s) [GeV^2]
s [GeV^2]


Real Part
Imaginary part
θ_π(s) [GeV^2]
s [GeV^2]

Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

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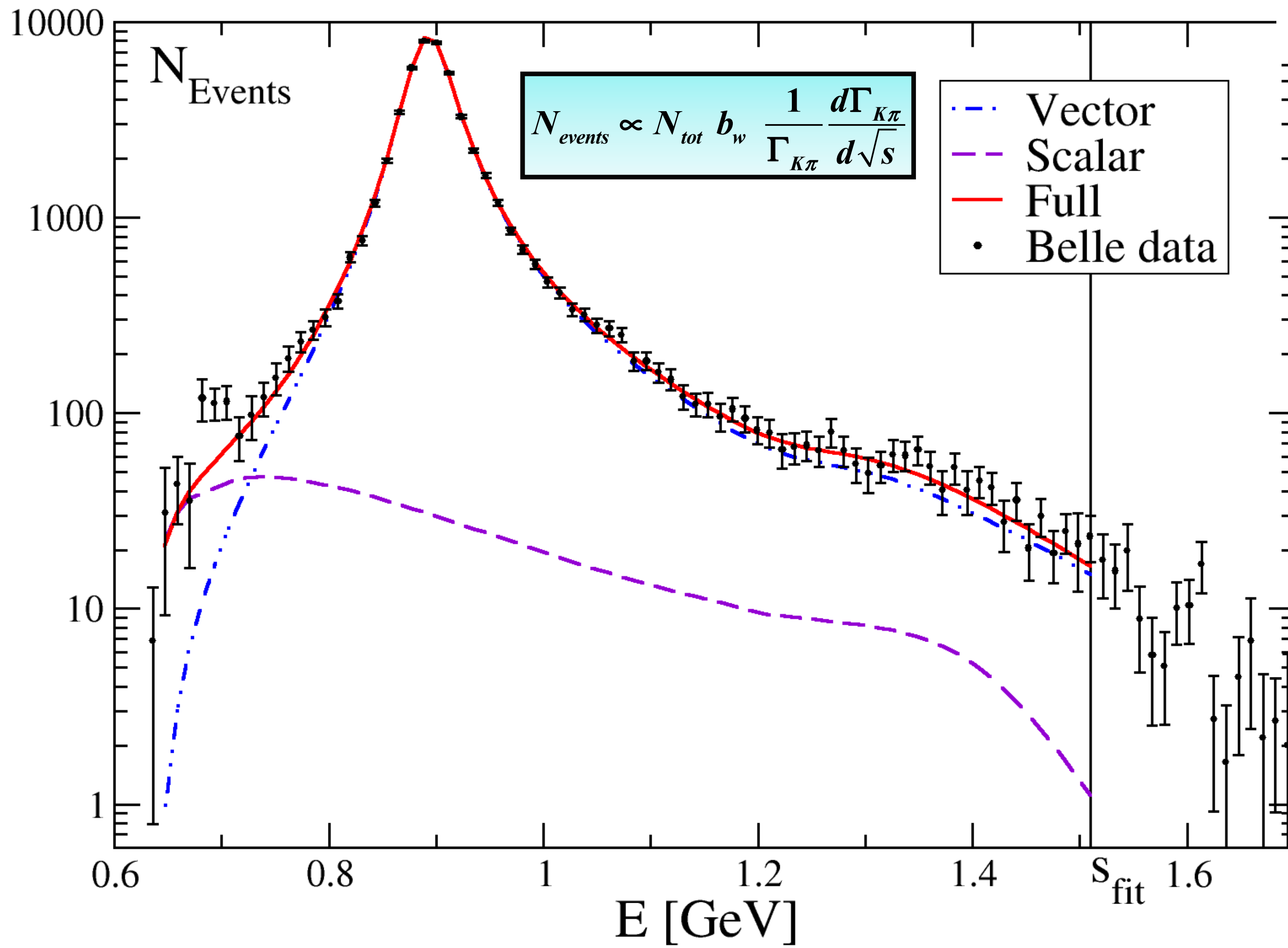
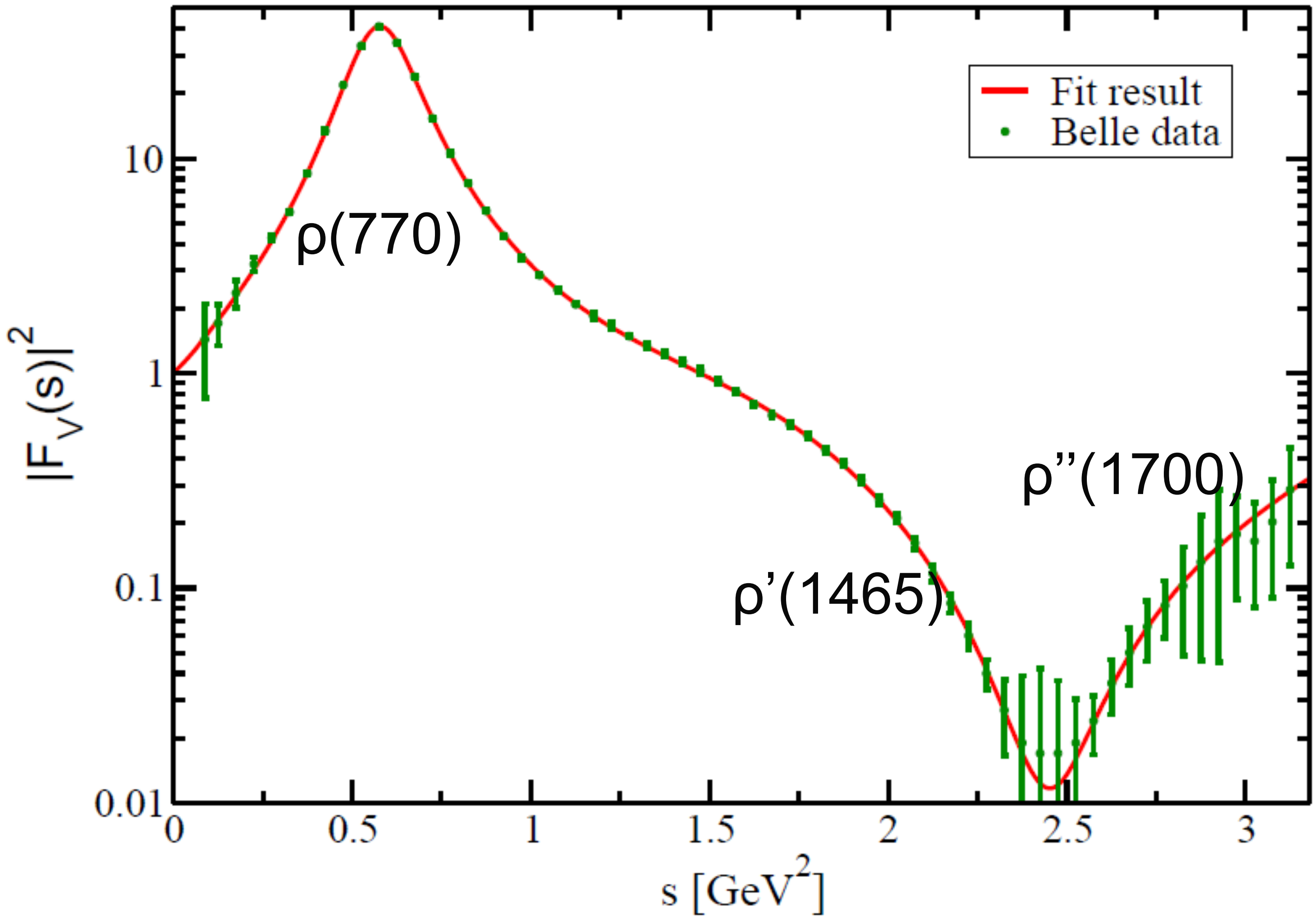


Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

$\tau \to K\pi\nu_\tau$