Theory of (quasi-) two-body non-leptonic B decays

M. Beneke (TU München)

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Outline

- Motivation
- Flavour amplitudes
- Theory for hadronic matrix elements
- Tree-dominated decays
- Penguin-dominated decays
- Polarization

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Charmless hadronic B decays – motivation

Decay amplitude governed by three factors:

$$\mathcal{A}(\bar{B} \to f) = \lambda_u^{(D)} A_f^u + \lambda_c^{(D)} A_f^c = \sum_i \left[\, \lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED} \,\right]_i$$

- C – Wilson coefficient of tree operators ($C \sim 1$) and loop-suppressed penguin ($C \sim 0.1$).

- $\lambda_p^{(D)} \equiv V_{pb}V_{pD}^*$ – CKM factors
  $\lambda_u^{(d)} \sim \lambda_c^{(d)} \sim \lambda^3$ for $b \to d$ transitions: penguin is sub-dominant.
  $\lambda_c^{(s)} \sim \lambda^2 \gg \lambda_u^{(s)} \sim \lambda^4$ for $b \to s$ transitions: penguin dominates despite loop-suppression.

- $\langle f|\mathcal{O}|\bar{B}\rangle$ – "Hadronic matrix element" depends on spin and parity of final state, and whether reached only through annihilation. CP violation depends on rescattering phases.

Large number of different final states.
Good place to look for direct CP violation.
Fundamental physics and challenging QCD dynamics.

Amplitudes

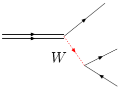
Flavour amplitudes

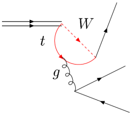
Operators in the effective weak interaction Lagrangian
Scale: $\mu \geq m_b$

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \Big(C_1 \mathcal{O}_1^p + C_2 \mathcal{O}_2^p$$
$$+ \sum_{i=(\rm EW)pen,\, mag} C_i \mathcal{O}_i \Big)$$

$$\Rightarrow \qquad \langle f | \mathcal{O} | \bar{B} \rangle_{\rm QCD+QED}$$

$$\mathcal{O}_{1,2}^p = (\bar{p}\Gamma b)\,(\bar{D}\Gamma' p)$$
$$\mathcal{O}_{i,\rm QCD\,pen} = (\bar{D}\Gamma b) \sum_{q=u,d,s,c,b} (\bar{q}\Gamma' q)$$



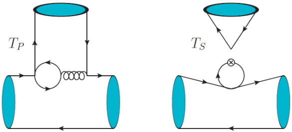


Flavour (u,d,s) flow amplitudes
Scale: $\mu \sim \Lambda$

Assigns flavours of an operator to the flavour quantum numbers of the initial and final state.
E.g. $\mathcal{O}_{i,\rm QCD\,pen}$

$$P^{u,c} \sim \alpha_4^{u,c}(M_1 M_2) \sum_{q=u,d,s} [\bar{q}_s q][\bar{q} D]$$
$$S^{u,c} \sim \alpha_3^{u,c}(M_1 M_2) \sum_{q=u,d,s} [\bar{q}_s D][\bar{q} q]$$

with $[\ldots][\ldots]$ referring to $M_1 M_2$

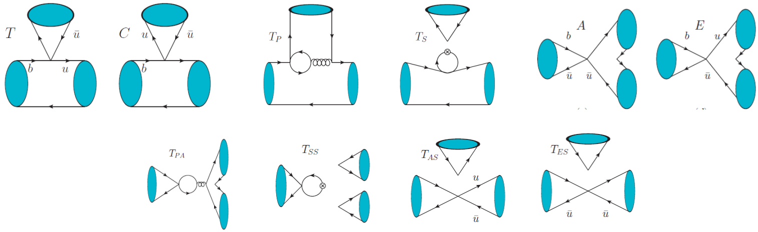


[source: He, Wei, 1803.04227]

Three parameterizations

- Flavour SU(3) irreducible matrix elements
- Topological amplitudes (often with flavour SU(3) or SU(2))

$$T, C, P, P_{\mathrm{EW}}, S, E, A, \ldots$$



- Flavour amplitudes in factorization

$$\alpha_1(M_1M_2), \ldots, \beta_1^p(M_1M_2), \ldots$$

Same number of amplitudes if nonet symmetry is assumed for pseudoscalar/vector light meson multiplet.

Flavour amplitudes in factorization [MB, Neubert, hep-ph/0308039]

$$\sum_{p=u,c} A_{M_1M_2} \Big\{ \boldsymbol{B}\boldsymbol{M}_1 \left(\alpha_1 \boldsymbol{U}_p + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_2 \boldsymbol{\Lambda}_p$$

$$+ \boldsymbol{B}\boldsymbol{M}_1\boldsymbol{\Lambda}_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \boldsymbol{U}_p + \alpha_3^p + \alpha_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_2\right]$$

$$+ \boldsymbol{B}\left(\beta_2 \boldsymbol{U}_p + \beta_3^p + \beta_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_1\boldsymbol{M}_2\boldsymbol{\Lambda}_p$$

$$+ \boldsymbol{B}\boldsymbol{\Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_1 \boldsymbol{U}_p + \beta_4^p + b_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_1\boldsymbol{M}_2\right]$$

$$+ \boldsymbol{B}\left(\beta_{S2} \boldsymbol{U}_p + \beta_{S3}^p + \beta_{S3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_1\boldsymbol{\Lambda}_p \cdot \mathrm{Tr}\boldsymbol{M}_2$$

$$+ \boldsymbol{B}\boldsymbol{\Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \boldsymbol{U}_p + \beta_{S4}^p + b_{S4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M}_1\right] \cdot \mathrm{Tr}\boldsymbol{M}_2 \Big\}$$

$$\boldsymbol{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta_s' \end{pmatrix}$$

$$\boldsymbol{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_q}{\sqrt{2}} + \frac{\phi_q}{\sqrt{2}} & \rho^- & K^{*-} \\ \rho^+ & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_q}{\sqrt{2}} + \frac{\phi_q}{\sqrt{2}} & \bar{K}^{*0} \\ K^{*+} & K^{*0} & \omega_s + \phi_s \end{pmatrix}$$

$$\boldsymbol{B} = (B^-, \bar{B}^0, \bar{B}_s) \qquad \boldsymbol{\Lambda}_p = \begin{pmatrix} 0 \\ \lambda_p^{(d)} \\ \lambda_p^{(s)} \end{pmatrix} \qquad \boldsymbol{U}_p = \begin{pmatrix} \delta_{pu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{\boldsymbol{Q}} = \frac{3}{2}\boldsymbol{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

- 30 amplitude parameters (18 u + 12 c)
  $\to$ 24 (12 u + 12 c) [Eliminate U_p or Q for $p = u$]
  $\to$ 20 [Absorb β_3^p, β_{S3}^p into α_4^p, α_3^p]
  $\to$ 18 (9 u + 9 c) [β_2 and $\beta_{4,\mathrm{EW}}^c$ not independent] [He, Wei, 2018]
- SU(3) need not be assumed, then parameters depend on specific M_1M_2.

SU(3) flavour symmetry

SU(3) [Zeppenfeld, 1981; Savage, Wise, 1989] light flavour symmetries provide amplitude relations. Usually leading order only in $m_s \ll \Lambda_{\rm QCD}$

If octet and singlet is combined into nonet M^i_j, using that the Hamiltonian contains only the $\bar{\mathbf{3}}$, **6** and **15** rep, for $p = u$ (same for $p = c$) [Hsia, Chang, He, 1512.09223]

18 parameters

$$
\begin{aligned}
T = {} & A^T_{\bar 3} B_i H(\bar 3)^i (\bar M^k_l \bar M^l_k) + C^T_{\bar 3} B_i \bar M^i_k \bar M^k_j H(\bar 3)^j \\
& + \tilde A^T_6 B_i H(6)^{ij}_k \bar M^l_j \bar M^k_l + \tilde C^T_6 B_i \bar M^i_j H(6)^{jk}_l \bar M^l_k \\
& + A^T_{\overline{15}} B_i H(\overline{15})^{ij}_k \bar M^l_j \bar M^k_l + C^T_{\overline{15}} B_i \bar M^i_j H(\overline{15})^{jk}_l \bar M^l_k \\
& + B^T_{\bar 3} B_i H(\bar 3)^i \bar M^j_j \bar M^k_k + \tilde B^T_6 B_i H(6)^{ij}_k \bar M^k_j \bar M^l_l \\
& + B^T_{\overline{15}} B_i H(\overline{15})^{ij}_k \bar M^k_j \bar M^l_l + D^T_{\bar 3} B_i \bar M^i_j H(\bar 3)^j \bar M^l_l \, ,
\end{aligned}
$$

- Same number, 18. Including SU(3) breaking does not require modification of the flavour flow amplitudes. But they depend on $M_1 M_2$.
- Some simplifications, since perturbation Haniltonian $m_s \bar{s} s$ has definite SU(3) properties [Grinstein, Lebed, 1996].
- SU(3) a good working assumption presently – after accounting for large SU(3) breaking in form factors and decay constant (as found in factorization)
  But: SU(3) fits should be prepared to account for leading SU(3) breaking when measurements become very precise.

Amplitude dictionary

$p=u$	topological	$p=c$	topological
α_1	T	–	–
α_2	C	–	–
$\alpha_3^u+\beta_{S3}^u$	S^u	$\alpha_3^c+\beta_{S3}^c$	S
$\alpha_4^u+\beta_3^u$	P^u	$\alpha_4^c+\beta_3^c$	P
–	–	$\alpha_{3,\mathrm{EW}}^c$	P_{EW}
–	–	$\alpha_{4,\mathrm{EW}}^c$	P_{EW}^C
β_1	E	$\beta_{3,\mathrm{EW}}^c$	$\ldots$
β_2	A	$\beta_{4,\mathrm{EW}}^c$	$\ldots$
β_4^u	$\ldots$	β_4^c	PA
β_{S1}	$\ldots$	$\beta_{S3,\mathrm{EW}}^c$	$\ldots$
β_{S2}	$\ldots$	$\beta_{S4,\mathrm{EW}}^c$	$\ldots$
β_{S4}^u	$\ldots$	β_{S4}^c	$\ldots$

- For PV final states two sets, for VV three, one for each helicity amplitude
- In factorization all β's are power-suppressed.

Amplitude magnitudes, rough estimates

$p = u$		topological	$p = c$		topological
α_1	1	T	–	–	–
α_2	0.25	C	–	–	–
$\alpha_3^u + \beta_{S3}^u$	0.006	S^u	$\alpha_3^c + \beta_{S3}^c$	0.006	S
$\alpha_4^u + \beta_3^u$	0.1	P^u	$\alpha_4^c + \beta_3^c$	0.1	P
–	–	–	$\alpha_{3,\mathrm{EW}}^c$	0.01	P_{EW}
–	–	–	$\alpha_{4,\mathrm{EW}}^c$	0.001	P_{EW}^C
β_1	0.03	E	$\beta_{3,\mathrm{EW}}^c$	0.0002	…
β_2	0.01	A	$\beta_{4,\mathrm{EW}}^c$	0.0001	…
β_4^u	0.002	…	β_4^c	0.002	PA
β_{S1}	n/a	…	$\beta_{S3,\mathrm{EW}}^c$	n/a	…
β_{S2}	n/a	…	$\beta_{S4,\mathrm{EW}}^c$	n/a	…
β_{S4}^u	n/a	…	β_{S4}^c	n/a	…

- n/a = not available = negligible?
- For PP
- From QCDF, not data fits

Theory of hadronic matrix elements based on the $1/m_b$ expansion

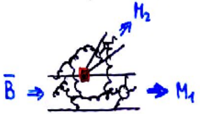
Hadronic matrix elements from QCD factorization [BBNS, 1999-2001]

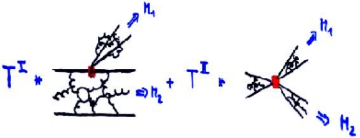
Heavy quark limit: $m_b \gg \Lambda_{\rm QCD}$
Large-energy limit: $E_M \approx m_b/2 \gg \Lambda_{\rm QCD}$
Scales: m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, $(M_{\rm EW}, \Lambda_{\rm NP})$

$\mu \approx m_b$

↓

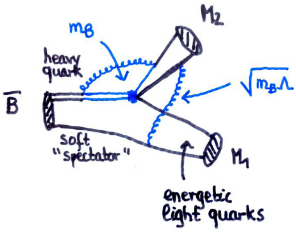
$\mu \approx 1$ GeV

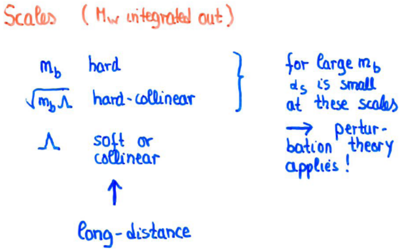




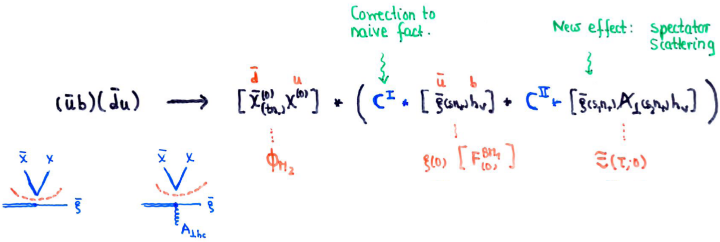
- Reduces $\langle M_1 M_2|\mathcal{O}|B\rangle$ to simpler $\langle M|\mathcal{O}|B\rangle$ (form factors), $\langle 0|\mathcal{O}|B\rangle$, $\langle M|\mathcal{O}|0\rangle$ (decay constants and distribution amplitudes).
- Calculation from first principles, but limited accuracy by $\Lambda_{\rm QCD}/m_b$ corrections.

Scales and factorization, SCET_I matching





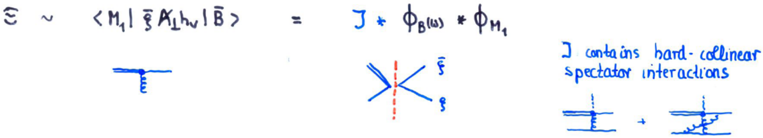
Λ/m_b and Λ/E expansion → SCET+HQET (for non-leptonics [Chay, Kim (2002); MB, Feldmann (2003); Bauer et al. (2003); MB, Jäger (2005), review 1501.07374])



$\text{SCET}_{\text{I}} \to \text{SCET}_{\text{II}}$ matching

- M_2 factorizes from the $B \to M_1$ transition below m_b and above the hard-collinear scale $\sqrt{m_b \Lambda}$.
- Strong phases in SCET_{I} matching coefficients only.
- Valid up to $1/m_b$ corrections.

Integrate out the hard-collinear scale



$$\Xi_{B\pi}(\tau, E) = \frac{m_B}{4m_b} \int_0^\infty d\omega \int_0^1 dv\, J(\tau; v, \ln(E\omega/\mu^2))\, \hat{f}_B \phi_{B+}(\omega) f_\pi \phi_\pi(v).$$

- SCET_{II} factorization of $\bar{\xi} h_v$ into LCDAs is not known, therefore need form factors as input.

QCD factorization

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{BM_1}(0) \int_0^1 du\, T_i^I(u) \Phi_{M_2}(u)$$
$$+ \int_0^\infty d\omega \int_0^1 dudv\, T_i^{II}(\omega, u, v)\, \Phi_B(\omega) \Phi_{M_1}(v) \Phi_{M_2}(u)$$
$$= F^{BM_1}\, T_i^{\mathrm{I}} \star \Phi_{M_2} + \Phi_B \star [H_i^{\mathrm{II}} \star J^{\mathrm{II}}] \star \Phi_{M_1} \star \Phi_{M_2}$$

- Rigorous at leading power in $1/m_b$
- Strong phases are $\delta \sim \mathcal{O}(\alpha_s(m_b), \Lambda/m_b)$. $\mathrm{SCET_I}$ matching coefficients only.
- Some special power-suppressed effects might be factorizable, but no factorization in general is known for $1/m_b$ corrections. In particular, all weak annihilation topologies are $1/m_b$
- Compare

$$\left| 1 + \alpha_s + c \frac{\Lambda}{m_b} \right|^2 \to 20\% \qquad \left| c \frac{\Lambda}{m_b} \right|^2 \to \text{factor of } 4$$

when c is misestimated by a factor of 2 ($c = 1 \to 2$)

PQCD [Keum, Li, Sanda (2000); Lü, Ukai, Zhang (2000)]

Main conceptual difference: $B \to M_1$ form factor ($\bar{\xi}\Gamma h_v$ SCET_I operator) assumed to factorize in LCDAs after self-consistent regularization of endpoint divergences by transverse-momentum-dependent Sudakov resummation

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \;=\; F^{BM_1}\, T_i^{\text{I}} \star \Phi_{M_2} + \Phi_B \star [H_i^{\text{II}} \star J^{\text{II}}] \star \Phi_{M_1} \star \Phi_{M_2}$$
$$\to \phi_B \star [T_i^{\text{PQCD}} \star J^{\text{PQCD}}] \star \phi_{M_1} \star \phi_{M_2}.$$

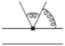
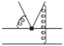
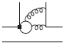
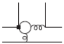
- QCDF/SCET analyses disagree with this statement [Descotes-Genon, Sachrajda, hep-ph/0109260; Lange, Neubert (2003)]
- $1/m_b$ power counting of contributions not clear. No NLO calculation has ever been performed, IR singularities appear [Li, Mishima, 0901.1272 and (2014)]
- With $k_\perp$ non-zero tree diagrams can have imaginary parts.
  Main difference for phenomenology is a phase in the tree annihilation amplitude.
  Apart from conceptual issues, question about long-distance sensitivity

$\to$ Afternoon discussion

Status of NNLO QCD factorization calculations

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h, \mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h, \mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I, \mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 in progress	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

from G. Bell [FPCP 2010]

Missing NNLO penguin amplitude partially computed (tree operator matrix elements) [Bell, MB, Huber, Li, 2015], penguin operator matrix elements still in progress.

For PP, PV and longitudinal polarization amplitude of VV.

QCDF analyses at NLO

Analyses of complete sets of final states

- PP, PV
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237
- VV
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237
- AP, AV, AA
  Cheng, Yang, 0709.0137, 0805.0329
- SP, SV
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403
- TP, TV
  Cheng, Yang, 1010.3309

Based on NLO hard-scattering functions.
Well-established successes and problems.

Similar analyses exist in PQCD.

QCDF analyses at NLO

Analyses of complete sets of final states

- PP, PV
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- SP, SV
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403
- TP, TV
  Cheng, Yang, 1010.3309

Following: amplitudes and phenomenology with NNLO results (except polarization)

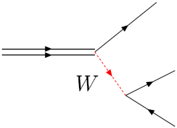
SU(3), LCSR – Other talks

Based on NLO hard-scattering functions.
Well-established successes and problems.

Similar analyses exist in PQCD.

Tree-dominated modes

$[T \sim \alpha_1, C \sim \alpha_2]$

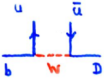


2-loop: Bell, 0705.3127, 0902.1915; Bell, Pilipp, 0910.1016; MB, Huber, Li, 0911.3655 + 1-loop spectator-scattering (MB, Jäger, 2005)

Colour-allowed vs. colour-suppressed

$$H_{\rm eff} = C_1\,[\bar u_i b_i]_{\rm V-A}[\bar d_j u_j]_{\rm V-A} + C_2\,[\bar u_i b_j]_{\rm V-A}[\bar d_j u_i]_{\rm V-A}$$
$$= (C_1 + \frac{C_2}{N_c})\,[\bar u b]_{\rm V-A}[\bar d u]_{\rm V-A} + 2C_2\,[\bar u T^A b]_{\rm V-A}[\bar d T^A u]_{\rm V-A}$$
$$= (C_2 + \frac{C_1}{N_c})\,[\bar d b]_{\rm V-A}[\bar u u]_{\rm V-A} + 2C_1\,[\bar d T^A b]_{\rm V-A}[\bar u T^A u]_{\rm V-A}$$

$$C_1(xm_b) \sim 1.1 \qquad C_2(xm_b) \sim -0.3\ldots-0.1$$



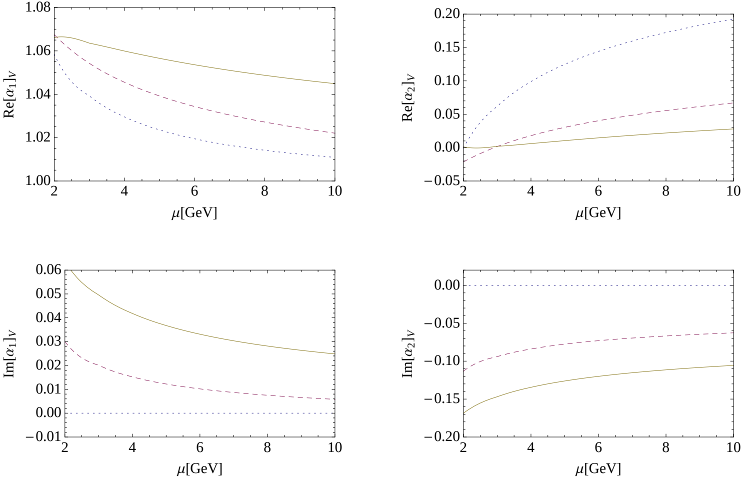
- Colour-allowed and colour-suppressed (topological) “tree” amplitude

$$T \sim \alpha_1(\pi\pi) \propto f_\pi\Phi_\pi\left[(C_1 + \frac{C_2}{N_c} + \alpha_s 2C_2)\,f_+^{B\pi}(0) + \alpha_s 2C_2\,\frac{f_B f_\pi\,\Phi_\pi}{m_B\lambda_B}\right]$$
$$C \sim \alpha_2(\pi\pi) \propto f_\pi\Phi_\pi\left[(C_2 + \frac{C_1}{N_c} + \alpha_s 2C_1)\,f_+^{B\pi}(0) + \alpha_s 2C_1\,\frac{f_B f_\pi\,\Phi_\pi}{m_B\lambda_B}\right]$$

In effect, (N)NLO is (N)LO for the colour-suppressed tree amplitude.

$$C \propto \alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\rm NLO} + \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.123]_{\rm LOsp} + [0.072]_{\rm tw3}\right\}$$

Size of the 2-loop vertex correction



Numerical result (tree amplitudes)

$$
\begin{aligned}
T \equiv a_1(\pi\pi) &= 1.009 + [0.023 + 0.010i]_{\text{NLO}} + [0.026 + 0.028i]_{\text{NNLO}} \\
&\quad - \left[\frac{r_{\text{sp}}}{0.485}\right] \left\{[0.015]_{\text{LOsp}} + [0.037 + 0.029i]_{\text{NLOsp}} + [0.009]_{\text{tw3}}\right\} \\
&= 1.00 + 0.01i \quad \rightarrow \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
\end{aligned}
$$

$$
r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0) \lambda_B}
$$

$$
\begin{aligned}
C \equiv a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\
&\quad + \left[\frac{r_{\text{sp}}}{0.485}\right] \left\{[0.123]_{\text{LOsp}} + [0.053 + 0.054i]_{\text{NLOsp}} + [0.072]_{\text{tw3}}\right\} \\
&= 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
\end{aligned}
$$

- Sizeable correction to imaginary part (phases), but cancellation between vertex and spectator-scattering.
- The colour-suppressed amplitudes are dominated by spectator-scattering. [But $\arg\,(C/T_{\pi\pi}) \lesssim 15^\circ$.]
- Qualitative understanding why colour-suppressed decay modes ($\pi^0\pi^0, \ldots$) can be large. Allows $|C/T|_{\pi\pi} \approx 0.7$, if λ_B is small.
  However, does not solve problems related to C (see below)

Branching fractions (tree-dominated decays) [MB, Huber, Li, 2009]

	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43\,^{+0.06}_{-0.06}\,^{+1.45}_{-0.84}$	$(\star)$	$5.82\,^{+0.07}_{-0.06}\,^{+1.42}_{-1.35}$	$(\star)$	5.48 ± 0.35
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37\,^{+0.86}_{-0.69}\,^{+1.22}_{-0.97}$	$(\star)$	$5.70\,^{+0.70}_{-0.55}\,^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.21
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33\,^{+0.11}_{-0.08}\,^{+0.42}_{-0.17}$		$0.63\,^{+0.12}_{-0.10}\,^{+0.64}_{-0.42}$		1.53 ± 0.26
$B^- \to \pi^-\rho^0$	$8.68\,^{+0.42}_{-0.41}\,^{+2.71}_{-1.56}$	$(\star\star)$	$9.84\,^{+0.41}_{-0.40}\,^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38\,^{+0.90}_{-0.77}\,^{+2.18}_{-1.41}$	$(\star)$	$12.13\,^{+0.85}_{-0.73}\,^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80\,^{+0.62}_{-0.56}\,^{+1.76}_{-2.10}$	$(\star)$	$13.76\,^{+0.49}_{-0.44}\,^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28\,^{+0.39}_{-0.39}\,^{+1.37}_{-1.42}$	$(\star\star)$	$8.14\,^{+0.34}_{-0.33}\,^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08\,^{+0.27}_{-0.19}\,^{+3.82}_{-3.50}$	$(\dagger)$	$21.90\,^{+0.20}_{-0.12}\,^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52\,^{+0.04}_{-0.03}\,^{+1.11}_{-0.43}$		$1.49\,^{+0.07}_{-0.07}\,^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho^-_L\,\rho^0_L$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\,\rho^-_L$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

First error γ, $|V_{cb}|$. $|V_{ub}|$ uncertainty *not* included. Second error from hadronic inputs.
Brackets ($\star$, $\star\star$, $\dagger$): form factor uncertainty not included.

Factorization test

$$\frac{\Gamma(B^- \to \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\, |V_{ud}|^2\, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From exclusive semi-leptonic data [HFAG 2014]
  $|V_{ub}| f_+(0) = (9.23 \pm 0.24) \times 10^{-4}$
  equivalent to

$$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\rm exp} = 1.27 \pm 0.04$$

- to be compared to $[\lambda_B = 350\,\text{MeV}]$

$$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\rm th} = 1.24^{+0.16}_{-0.10}$$

Leading uncertainties: λ_B (B LCDA), α_2^π (pion LCDA), power corrections.

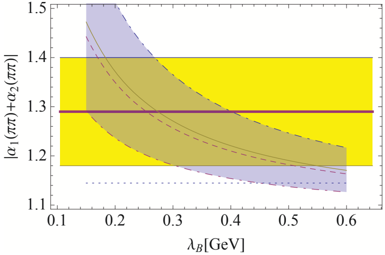


Figure from BHL2009 with obsolete data (yellow band), $|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\rm exp} = 1.29 \pm 0.11$.

Colour-suppressed tree can be large only if it also has a large relative phase.

Tree amplitudes – summary

- NNLO corrections individually sizeable, but ultimately not large due to cancellation.
- Colour-allowed modes well described by factorization
- Less so the purely colour-suppressed ones.
  - ⋆ C/T can be large but still need large phase for πK puzzle and other CP asymmetries (below).
    Also found in SU(3) fits.
  - ⋆ Apparent π, ρ non-universality.
  - ⋆ In some applications of PQCD [Li, Mishima (2009)], QCDF [Cheng, Chua, 2009], and in FAT (factorization-assisted topological amplitude approach) [Lü et al. (2016)] a complex parameter for power corrections to $C \sim \alpha_2$ is introduced.

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Challenges

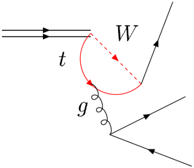
- Determine λ_B precisely to remove main parameter uncertainty at LP.
- Power corrections to the SCET matching of the colour-octet operators

$$2C_1\,[\bar{d}T^A b]_{\mathrm{V-A}}[\bar{u}T^A u]_{\mathrm{V-A}},\;\; 2C_2\,[\bar{u}T^A b]_{\mathrm{V-A}}[\bar{d}T^A u]_{\mathrm{V-A}}$$

[cf. BBNS, hep-ph/0006124; MB, Vernazza, 0810.3575]
Affects both C and T, more C due to $C_1 \gg |C_2|$.

QCD penguin-dominated modes

$[P^{u,c} \sim \alpha_4^{u,c} \sim \lambda_{u,c}^{(D)} \sum_q [\bar{q}_s q][\bar{q}D]]$

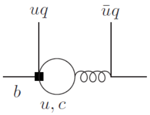


2-loop: Bell, MB, Huber, Li, 1507.03700 + 1-loop spectator-scattering (MB, Jäger, 2006)

Penguin amplitudes

- Magnitude of $\alpha_4^c + \beta_3^c$ controls BR of most $b \to s$ modes.
- Interference of QCD penguin is main source of CP violation.

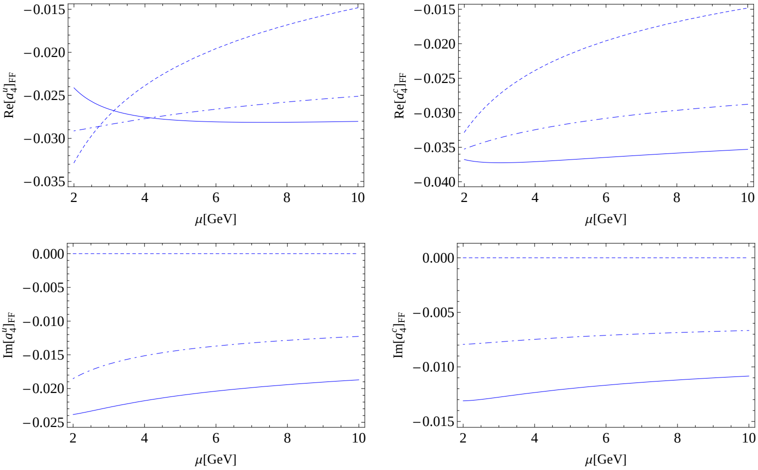
$$\left[\frac{P^c}{T}\right]_{\pi\pi}, \quad \left[\frac{T}{P^c}\right]_{\pi K}, \quad \left[\frac{P^u}{P^c}\right]_{\phi K}$$



- Two amplitudes $P^{u,c} \sim \hat{\alpha}_4^{u,c}$. Dominant contribution beyond tree-level from tree operators $\mathcal{O}_{1,2}^p$.
- Very little known (experimentally) for singlet penguin $S^{u,c} \sim \alpha_3^{u,c} + \beta_{S3}^{u,c} \sim \lambda_{u,c}^{(D)} \sum_q [\bar{q}q][\bar{q}_s D]$. ($B \to \pi\phi$ in the absence of $\omega - \phi$ mixing.)

  Small in QCDF (see above). The $\eta^{(\prime)} K^{(*)}$ branching fractions arise mainly due to inference of the P^c amplitudes for PP, PV. [Lipkin (1991, 1998); MB, Neubert (2002)]

Size of 2-loop penguin vertex correction [$C_{1,2}$ only] [Bell, MB, Huber, Li, 1507.03700]



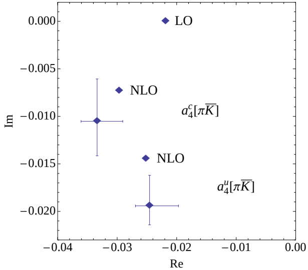
Numerical result (penguin amplitudes)

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1} - [0.32 + 0.71i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right] \left\{[0.13]_{LO} + [0.14 + 0.12i]_{HV} - [0.01 - 0.05i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i$$

$$r_{sp} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1} - [0.77 + 0.50i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right] \left\{[0.13]_{LO} + [0.14 + 0.12i]_{HV} + [0.01 + 0.03i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i$$

- Two-loop is 40% (15%) of the imaginary (real) part of $a_4^u(\pi\bar{K})$, and 50% (25%) in the case of $a_4^c(\pi\bar{K})$.
- Spectator-scattering not relevant.



$$\hat{P}^p \sim \alpha_4^p(M_1M_2) = a_4^p(M_1M_2) + \{1,-1,0\} \times r_\chi^{M_2} a_6^p(M_1M_2) + \underbrace{\beta_3^p(M_1M_2)}_{\approx 0.03}$$

$$\mathrm{PP} \sim \underbrace{a_4}_{\mathrm{V}\mp\mathrm{A}} + \underbrace{r_\chi a_6}_{\mathrm{S+P}}$$

$$\mathrm{PV} \sim a_4 \approx \frac{\mathrm{PP}}{3}$$

$$\mathrm{VP} \sim a_4 - r_\chi a_6 \sim -\mathrm{PV}$$

$$\mathrm{VV} \sim a_4 \sim \mathrm{PV}$$

Large NNLO correction to a_4^p diluted by important/dominant power-suppressed effects.

Use $B \to M_1^+ M_2^-$ (Br and A_{CP}) to determine P_c/T.
Small phases ($\to$ CP asymmetries)

$$\hat{P}^p \sim \alpha_4^p(M_1M_2) = a_4^p(M_1M_2) + \{1,-1,0\} \times r_\chi^{M_2} a_6^p(M_1M_2) + \underbrace{\beta_3^p(M_1M_2)}_{\approx 0.03}$$

$$\text{PP} \sim \underbrace{a_4}_{V\mp A} + \underbrace{r_\chi a_6}_{S+P}$$

$$\text{PV} \sim a_4 \approx \frac{\text{PP}}{3}$$

$$\text{VP} \sim a_4 - r_\chi a_6 \sim -\text{PV}$$

$$\text{VV} \sim a_4 \sim \text{PV}$$

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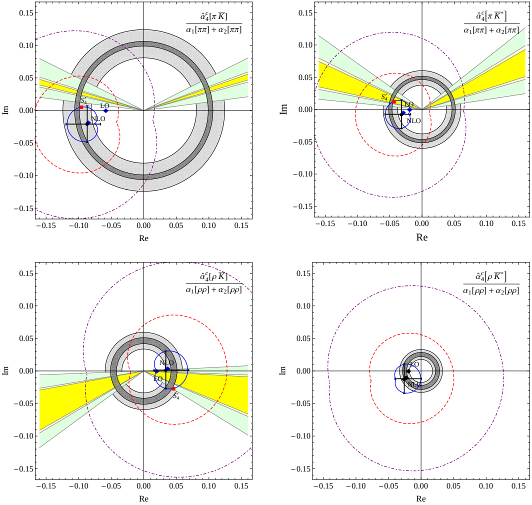


The $B \to \pi K$ system and its PV, VP, VV variants

$$\mathcal{A}_{B^- \to \pi^- \bar{K}^0} = \lambda_c^{(s)} [P_c - \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [P_u - \frac{1}{3} P_u^{C,EW}]$$

$$\sqrt{2}\, \mathcal{A}_{B^- \to \pi^0 K^-} = \lambda_c^{(s)} [P_c + P_c^{EW} + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + C + P_u + P_u^{EW} + \frac{2}{3} P_u^{C,EW}]$$

$$\mathcal{A}_{\bar{B}^0 \to \pi^+ K^-} = \lambda_c^{(s)} [P_c + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + P_u + \frac{2}{3} P_u^{C,EW}]$$

$$\sqrt{2}\, \mathcal{A}_{\bar{B}^0 \to \pi^0 \bar{K}^0} = \lambda_c^{(s)} [-P_c + P_c^{EW} + \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [C - P_u + P_u^{EW} + \frac{1}{3} P_u^{C,EW}]$$

Ratios with little dependence on γ, but sensitive to electroweak penguins.
CP asymmetry differences and sum rules.

$$R_{00} = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0)}{\Gamma(B^- \to \pi^- \bar{K}^0)} = |1 - r_{\rm EW}|^2 + 2 \cos\gamma \,{\rm Re}\, r_C + \ldots$$

$$R_L = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0) + 2\Gamma(B^- \to \pi^0 K^-)}{\Gamma(B^- \to \pi^- \bar{K}^0) + \Gamma(\bar{B}^0 \to \pi^+ K^-)} = 1 + |r_{\rm EW}|^2 - \cos\gamma \,{\rm Re}(r_T\, r_{\rm EW}^*) + \ldots$$

$$\delta A_{\rm CP} = A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2 \sin\gamma \Big({\rm Im}(r_C) - {\rm Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

theory: $r_{\rm EW} \approx 0.12 - 0.01i$, $r_C \approx 0.03[\times 2?] - 0.02i$, $r_T \approx 0.18 - 0.02i$

$$r_{\rm EW} = \frac{3}{2}\, R_{\pi K}\, \frac{\alpha^c_{3,\rm EW}(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)} \qquad r_C = -R_{\pi K} \left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{\alpha_2(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)} \qquad r_T = -\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{\alpha_1(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)}$$

where $R_{\pi K} = (f_\pi/f_K)\cdot(F_0^{B\to K}/F_0^{B\to\pi}) \approx 1.$

- Direct CP asymmetry difference

$$\delta A_{\rm CP} = A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\Big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

theory: $r_{\rm EW} \approx 0.12 - 0.01i, \qquad r_C \approx 0.03[\times 2?] - 0.02i, \qquad r_T \approx 0.18 - 0.02i$

	theory	data
$\delta A_{\rm CP}$	0.03 ± 0.03	0.122 ± 0.022

- Hadronic explanation needs large C *and* large phase. Phase in P_c does not work (T/P interderence). Problem since 2003.

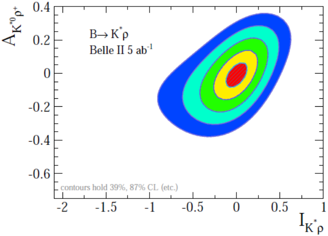
Explore this for πK^*, ρK, ρK^*! Larger effects expected and different signs, since P_c is strongly dependent on V or P.

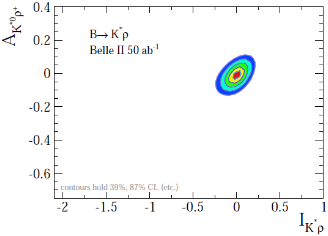
B2TIP report: Explore πK^*, ρK, ρK^* at BELLE-II
Larger effects expected and different signs, since P_c is strongly dependent on V or P.
Can shed light on possible non-universality and large phase of C
Electroweak penguins

Isospin sum rule:

$$I_{\pi K} = A_{\rm CP}(\pi^+ K^-) + \frac{\bar\Gamma_{\pi^-\bar K^0}}{\bar\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^-\bar K^0) - \frac{2\bar\Gamma_{\pi^0 K^-}}{\bar\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\bar\Gamma_{\pi^0\bar K^0}}{\bar\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0\bar K^0)$$
$$\propto 2\sin\gamma\,{\rm Im}(r_{\rm EW}(r_T + 2r_C)) + \ldots$$

	theory	data
$I_{\pi K}$	0.005 ± 0.01	-0.14 ± 0.11





Time-dependent CP asymmetry ΔS in $b \to s$

$$\Delta S_f = -\eta_f S_f - \sin(2\beta) = \frac{2\,\mathrm{Re}(d_f)\cos(2\beta)\sin\gamma + |d_f|^2\,(\sin(2\beta+2\gamma) - \sin(2\beta))}{1 + 2\,\mathrm{Re}(d_f)\cos\gamma + |d_f|^2}$$

$$d_f = \epsilon_{\rm KM}\hat{d}_f \quad \text{with} \quad \epsilon_{\rm KM} = \left|\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*}\right| \sim 0.025$$

$$\pi K_S \quad \hat{d}_f \sim \frac{[-P^u] + [C]}{[-P^c]} \qquad \rho K_S \quad \hat{d}_f \sim \frac{[P^u] - [C]}{[P^c]}$$

$$\eta' K_S \quad \hat{d}_f \sim \frac{[-P^u] - [C]}{[-P^c]} \qquad \phi K_S \quad \hat{d}_f \sim \frac{[-P^u]}{[-P^c]}$$

$$\eta K_S \quad \hat{d}_f \sim \frac{[P^u] + [C]}{[P^c]} \qquad \omega K_S \quad \hat{d}_f \sim \frac{[P^u] + [C]}{[P^c]}$$

[Quantities in square brackets have positive real part.]

$P^c(\pi K) \sim 2P^c(\rho K) \sim 0.4P^c(\eta' K) \sim 2.3P^c(\eta K) \sim 1.3P^c(\phi K) \sim 2.3P^c(\omega K)$

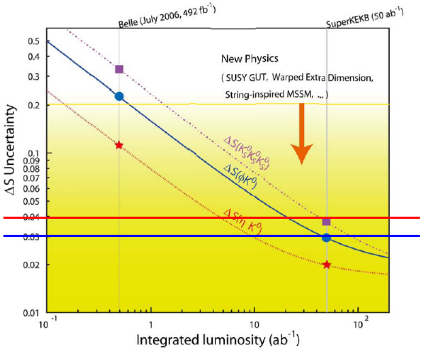
- $\epsilon_{\rm KM}\,|P^u/P^c| \approx 0.02$ are roughly independent of f
- Influence of C determines the difference between the different modes $\Longrightarrow$ need to know C well, here *real* part.

Precision matters for ΔS in $b \to s$

Mode	ΔS_f (Theory)	ΔS_f [Range*]
ϕK_S	$0.02^{+0.01}_{-0.01}$	$[+0.01, 0.05]$
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	$[+0.00, 0.03]$
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	$[+0.03, 0.13]$
$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	$[-0.29, 0.01]$
ηK_S	$0.10^{+0.11}_{-0.07}$	$[-0.76, 0.27]$
ωK_S	$0.13^{+0.08}_{-0.08}$	$[+0.02, 0.21]$

[MB; Cheng, Chua, Soni; Buchalla, Hiller, Nir, Raz; 2005]

- $\eta' K_S$ (red) has colour-suppressed tree contamination. Conservatively increase uncertainty.
- ϕK_S (blue) optimal. Theoretical uncertainty becomes limiting only with $\approx 50\,\mathrm{ab}^{-1}$ at SuperKEKB.



Penguin amplitudes – summary

- Calculations: NNLO corrections individually sizeable, but ultimately not large due to dilution by power-suppressed effects.
  C_{3-6} and C_{8g} [Kim, Yoon, 1107.1601] still missing.
- Structure of α_4 penguin amplitudes (P vs V) and generic size of CPV in agreement with factorization.
  But there is a non-negligible deficit, compatible with a $1/m_b$ correction.

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  But there is a non-negligible deficit, compatible with a $1/m_b$ correction.

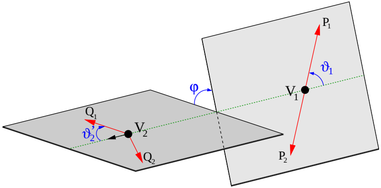
Challenges

- Calculations: Determine NNLO correction to the NLP scalar penguin amplitude a_6 to complete the short-distance prediction.

- How combine QCDF + SU(3) + phenomenological parameterization of power corrections? Can we understand better the deficit? – Charm penguins, annihilation or generic power corrections?

- There are more penguin amplitudes + penguin annihilation ...
  What can be learnt from BELLE-II data? Need to go to BR $\sim 10^{-7}$.

Polarization



Complete VV analysis in QCDF: MB, Rohrer, Yang, hep-ph/0612290

VV helicity amplitudes and angular distribution

- Vector-vector – three helicity amplitudes [→ five observables]

$$\mathcal{A}_0,\ \mathcal{A}_-,\ \mathcal{A}_+$$

$$\frac{d\Gamma_{B\to V_1V_2\to\ldots}}{d\cos\vartheta_1 d\cos\vartheta_2 d\varphi} \propto |\mathcal{A}_0|^2\cos^2\vartheta_1\cos^2\vartheta_2 + \frac{1}{4}\sin^2\vartheta_1\sin^2\vartheta_2\left(|\mathcal{A}_+|^2+|\mathcal{A}_-|^2\right)$$
$$-\cos\vartheta_1\sin\vartheta_1\cos\vartheta_2\sin\vartheta_2\left[\Re\left(\mathrm{e}^{-i\varphi}\mathcal{A}_0\mathcal{A}_+^*\right)+\Re\left(\mathrm{e}^{+i\varphi}\mathcal{A}_0\mathcal{A}_-^*\right)\right]$$
$$+\frac{1}{2}\sin^2\vartheta_1\sin^2\vartheta_2\,\Re\left(\mathrm{e}^{2i\varphi}\mathcal{A}_+\mathcal{A}_-^*\right),$$

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- Vector-vector – three helicity amplitudes [→ five observables]

$$\mathcal{A}_0,\ \mathcal{A}_-,\ \mathcal{A}_+$$

$$\frac{d\Gamma_{B\to V_1V_2\to\ldots}}{d\cos\vartheta_1 d\cos\vartheta_2 d\varphi} \propto |\mathcal{A}_0|^2\cos^2\vartheta_1\cos^2\vartheta_2 + \frac{1}{4}\sin^2\vartheta_1\sin^2\vartheta_2\left(|\mathcal{A}_+|^2+|\mathcal{A}_-|^2\right)$$
$$-\cos\vartheta_1\sin\vartheta_1\cos\vartheta_2\sin\vartheta_2\left[\Re\left(e^{-i\varphi}\mathcal{A}_0\mathcal{A}_+^*\right)+\Re\left(e^{+i\varphi}\mathcal{A}_0\mathcal{A}_-^*\right)\right]$$
$$+\frac{1}{2}\sin^2\vartheta_1\sin^2\vartheta_2\,\Re\left(e^{2i\varphi}\mathcal{A}_+\mathcal{A}_-^*\right),$$

- Parametric hierarchy

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda}{m_b} : \left(\frac{\Lambda}{m_b}\right)^2$$

due to $V-A$ weak interaction and helicity conservation of high-energy QCD (m_b/Λ).
$[\bar\chi\chi][\bar\xi h_v]$ operators:

$$\underbrace{\bar\chi \not{n}_-(1\mp\gamma_5)\chi}_{\mathcal{A}_0,\ \text{LP}},\ \underbrace{\bar\chi\not{n}_-\gamma_\perp^\mu(1\mp\gamma_5)\chi}_{\mathcal{A}_\pm\ \text{LP, not V}-\text{A}},\ \underbrace{\bar\chi\not{D}_\perp(1\mp\gamma_5)\chi}_{\mathcal{A}_-\ \text{NLP}}$$

- Transverse amplitudes are power corrections. No factorization theorem!

VV transverse helicity amplitudes

- Form factor term factorizable at NLO
- Hard-spectator scattering factorization-violating for $\mathcal{A}_-$

$$-\frac{2 f_B f_{V_1}^{\perp}}{m_B m_b F_-^{B\to V_1}(0)} \frac{m_b}{\lambda_B} \int_0^1 dx dy \, \frac{\phi_1^{\perp}(x) \phi_{b2}(y)}{\bar{x}^2 y} \qquad \phi^{\perp}(x) \to 6x\bar{x}, \quad \phi_b(x) \to 3x^2$$

In practice relevant only for the colour-suppressed tree amplitude.

- Transverse penguin annihilation $\mathcal{O}(1)$ numerically for transverse penguin amplitude

$$P^h = A^h_{V_1 V_2} \left[\alpha_4^h + \beta_3^h \right]$$

$$A^-_{V_1 V_2} \alpha_4^- \ll A^0_{V_1 V_2} \alpha_4^0 \quad \text{but} \quad A^-_{V_1 V_2} \beta_3^- \approx A^0_{V_1 V_2} \beta_3^0$$

$$\frac{P^-}{P^0} \approx \frac{A^-_{\rho K^*}}{A^0_{\rho K^*}} \, \frac{\alpha_4^{c-} + \beta_3^{c-}}{\alpha_4^{c,0}} \approx \frac{0.05 + [-0.04, 0.10]}{0.12}$$

$\Rightarrow$ Theoretically [Kagan, 2004; Rohrer, 2004; MB, Yang, Rohrer, 2006] expect and empirically find

$$\mathcal{A}_0 \gg \mathcal{A}_- \gg \mathcal{A}_+ \quad \text{tree decays} \qquad \mathcal{A}_0 \approx \mathcal{A}_- \gg \mathcal{A}_+ \quad \text{penguin decays}$$

VV transverse helicity amplitudes

- Fit P_h^c to data (ϕK^*). T_h, $P_{h,\mathrm{EW}}^c$ from theory.

	Observable	QCDF [4]	pQCD [11]	This work
$f^0_{\rho K^*}$	CP average	$0.22^{+0.03+0.53}_{-0.03-0.14}$	$0.65^{+0.03+0.03}_{-0.03-0.04}$	$0.164 \pm 0.015 \pm 0.022$
	CP asymmetry	$-0.30^{+0.11+0.61}_{-0.11-0.49}$	$0.0364^{+0.0120}_{-0.0107}$	$-0.62 \pm 0.09 \pm 0.09$
$f^\perp_{\rho K^*}$	CP average	$0.39^{+0.02+0.27}_{-0.02-0.07}$	$0.169\ ^{+0.027}_{-0.018}$	$0.401 \pm 0.016 \pm 0.037$
	CP asymmetry	–	$-0.0771^{+0.0197}_{-0.0186}$	$0.050 \pm 0.039 \pm 0.015$
$\delta^{\parallel-0}_{\rho K^*}$	CP average [rad]	$-0.7\ ^{+0.1+1.1}_{-0.1-0.8}$	$-1.61\ ^{+0.02}_{-3.06}$	$-0.77 \pm 0.09 \pm 0.06$
	CP difference [rad]	$0.30^{+0.09+0.38}_{-0.09-0.33}$	$-0.001^{+0.017}_{-0.018}$	$-0.109 \pm 0.085 \pm 0.034$
$\delta^{\parallel-\perp}_{\rho K^*}$	CP average [rad]	$\equiv \pi$	$3.15\ ^{+0.02}_{-4.30}$	$3.160 \pm 0.035 \pm 0.044$
	CP difference [rad]	$\equiv 0$	$-0.003^{+0.025}_{-0.024}$	$0.014 \pm 0.035 \pm 0.026$

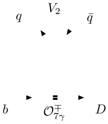
[LHCb 1812.07008, note $f^0_{\rho K*}$ was 0.57 ± 0.12 from BaBar in 2006.]

- This seems to work, but not a pure QCDF result.
  The only VV observable that can be calculated at leading power from first principles is the longitudinal branching fraction and its CP asymmetry

Transverse polarization and electromagnetic dipole operators

Parametric hierarchy is violated by electromagnetic interactions [MB, Rohrer, Yang, 2005]

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ \qquad 1 : \frac{\Lambda}{m_b} : \frac{\Lambda^2}{m_b^2} \qquad \Rightarrow \qquad 1 : \frac{m_b}{\Lambda} : 1$$



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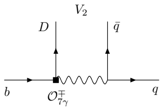
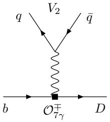




γ always off-shell, $q^2 \sim m_b^2$ $\qquad$ γ nearly on-shell, $q^2 = m_{V_2}^2 \sim \Lambda^2$

- V_2 longitudinal $\Rightarrow$ photon propagator is cancelled $\Rightarrow$ effective local four-quark interaction
- for V_2 transverse no cancellation $\Rightarrow$ local $b \to D\gamma$ transition followed by long-distance $\gamma \to V_2$ transition, enhanced by large photon propagator

New operator in SCET_I (leading in heavy quark power counting)

$$\left[\bar{\xi} W \gamma_{\perp\mu}(1 \mp \gamma_5) h_v\right](0) \left[W_\gamma^\dagger i D_{\gamma\perp}^\mu W_\gamma\right](0),$$

Consider electromagnetic dipole operators including both chiralities

$$Q_{7\gamma}^{\mp} = -\frac{e\bar{m}_b}{8\pi^2}\,\bar{D}\sigma_{\mu\nu}(1\pm\gamma_5)F^{\mu\nu}b,$$

Electroweak penguin coefficients

$$P_{\mp}^{\rm EW}(V_1V_2) = C_7 + C_9 + \frac{1}{N_c}(C_8+C_{10}) \;\mp\; \underbrace{\frac{2\alpha_{\rm em}}{3\pi}\,C_{7\gamma,\rm eff}^{\mp}R_{\mp}\,\frac{m_B\bar{m}_b}{m_{V_2}^2}}_{\text{dipole operator contribution}} \;+\ldots$$

with R_- a form factor ratio that equals 1 in the heavy-quark limit and $R_+ \sim m_b/\Lambda$.

Leading QCD penguin to EW penguin amplitude (in some units)

$$P_-(\rho K^*) \approx -1 \qquad\qquad P_-^{\rm EW}(\rho K^*) \approx -0.3 + 0.7\ \text{[dipole]}$$

For positive helicity $\quad 0.7 \to 0.7 \times (10-20) \times \dfrac{C_{7\gamma}^{+}}{C_{7\gamma,\rm eff}^{-}}$

The $B \to \rho K^*$ system

$$
\begin{aligned}
\mathcal{A}_h(\rho^- \bar{K}^{*0}) &= P_h \\
\sqrt{2}\,\mathcal{A}_h(\rho^0 K^{*-}) &= [P_h + P_h^{EW}] + e^{-i\gamma}\,[T_h + C_h] \\
\mathcal{A}_h(\rho^+ K^{*-}) &= P_h + e^{-i\gamma}\,T_h \\
-\sqrt{2}\,\mathcal{A}_h(\rho^0 \bar{K}^{*0}) &= [P_h - P_h^{EW}] + e^{-i\gamma}\,[-C_h],
\end{aligned}
$$

T_h, C_h CKM suppressed.

Consider CP-averaged negative helicity decay rate ratio ($p_h^{EW} = P_h^{EW}/P_h$)

$$
R \equiv \frac{\bar{\Gamma}_-(\rho^0 \bar{K}^{*-})}{\bar{\Gamma}_-(\rho^0 \bar{K}^{*0})} = \left| \frac{1 + p_-^{EW}}{1 - p_-^{EW}} \right|^2 + \Delta = \begin{cases} 3.0 \pm 0.7 \\ 0.6 \pm 0.1 \;\; \text{without dipole operator} \end{cases}
$$

(Fit P_- to data, use QCDF for the other amplitudes)

The $B \to \rho K^*$ system

$$\begin{aligned}
\mathcal{A}_h(\rho^- \bar{K}^{*0}) &= P_h \\
\sqrt{2}\,\mathcal{A}_h(\rho^0 K^{*-}) &= [P_h + P_h^{EW}] + e^{-i\gamma}\,[T_h + C_h] \\
\mathcal{A}_h(\rho^+ K^{*-}) &= P_h + e^{-i\gamma}\,T_h \\
-\sqrt{2}\,\mathcal{A}_h(\rho^0 \bar{K}^{*0}) &= [P_h - P_h^{EW}] + e^{-i\gamma}\,[-C_h],
\end{aligned}$$

T_h, C_h CKM suppressed.

Consider CP-averaged negative helicity decay rate ratio ($p_h^{EW} = P_h^{EW}/P_h$)

$$R \equiv \frac{\bar{\Gamma}_-(\rho^0 \bar{K}^{*-})}{\bar{\Gamma}_-(\rho^0 \bar{K}^{*0})} = \left|\frac{1 + p_-^{EW}}{1 - p_-^{EW}}\right|^2 + \Delta = \begin{cases} 3.0 \pm 0.7 \\ 0.6 \pm 0.1 \;\; \text{without dipole operator} \end{cases}$$

(Fit P_- to data, use QCDF for the other amplitudes)

- $Q_{7\gamma}^+$ contribution to $\bar{\mathcal{A}}_+$ is suppressed only by $C_{7\gamma}^+/C_{7\gamma}^-$, while other contributions have additional Λ/m_b suppression $\Rightarrow$ Sensitivity to $C_{7\gamma}^+ \approx 0.1$ may be possible (or better?).
- An alternative to studies of photon polarization in $B \to K^\star\gamma$. Here the ρ meson (decay) acts as the *polarization analyzer.*

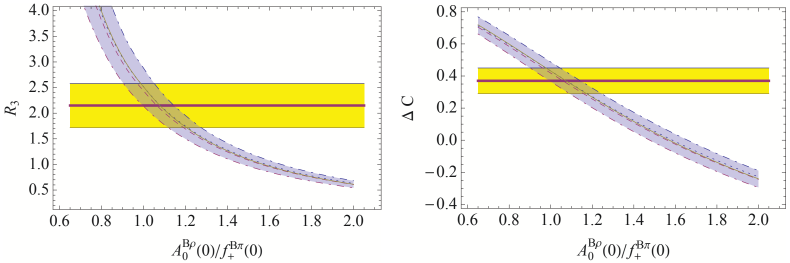
Wishlist

- Theoretical
  - NNLO for penguins inclduding a_6
  - λ_B from theory
  - Factorization techniques for electromagnetic effects.
  - Factorization + SU(3) [+ breaking]
  - Analyze non-factorizable power-correction to colour-suppressed tree (LCSR for NLP SCET operator?)
  - Revisit annihilation amplitude estimates
- Experimental
  - Accurate πK^*, ρK measurements, all channels, and ρK^* including angular analysis ($\rightarrow C, P, P_{\text{EW}}$)
  - Modes that probe new amplitudes (S, weak annihilation)
  - Triple product asymmetries
  - λ_B from $B \rightarrow \gamma \ell \nu$

Back-up slides

Charged $\pi^{\mp}\rho^{\pm}$ modes

$$R_3 = \frac{\Gamma(\bar{B}_0 \to \pi^+\rho^-)}{\Gamma(\bar{B}_0 \to \pi^-\rho^+)} \qquad \Delta C = \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]$$



Both depend mainly on $f_\pi A_0^{B\to\rho}(0)/(f_\rho f_+^{B\to\pi}(0)) \times \alpha_1(\rho\pi)/\alpha_1(\pi\rho) = R\, e^{i\delta_T}$. E.g.,

$$\Delta C = \frac{1-R^2}{1+R^2} + \frac{4R^2}{(1+R^2)^2}\,(a\cos\delta_a + b\cos\delta_b)\cos\gamma + \ldots$$

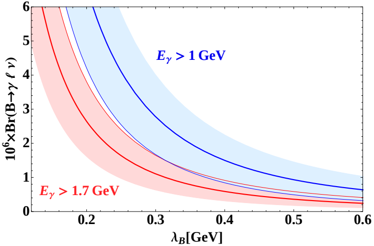
Slightly smaller but compatible with $B \to \rho$ to $B \to \pi$ form factor ratio from QCD sum rules (≈ 1.2).

λ_B from $B \to \gamma\ell\nu$

$$iF_{\text{stat}}(\mu)\Phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\, \langle 0|(\bar{q}_s Y_s)(tn_-) \not{n}_- \gamma_5 (Y_s^\dagger h_v)(0)|\bar{B}_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \Phi_{B+}(\omega,\mu)$$

$\Gamma(B \to \gamma\ell\nu) \propto 1/\lambda_B^2$. Dominant parametric dependence.



[MB, Rohrwild, 2011]

- NLL + tree-level $1/m_b$ [MB, Rohrwild, 2011]
- QCD sum rule estimate of power-suppressed soft form factor [Braun, Khodjamirian, 2012] and at NLO [MB, Braun, Ji, Wei, (2018); Y. Wang, (2016, 2018)]
- BELLE [1504.05831] gives $\lambda_B > 217\,\text{MeV}$ (with caveats), superseded by [1810.12976], $\lambda_B > 240\,\text{MeV}$. Promising for BELLE-II

Direct CP asymmetries

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13}_{-0.14}{}^{+0.21}_{-0.19}$	$0.77^{+0.14}_{-0.15}{}^{+0.23}_{-0.22}$	$0.10^{+0.02}_{-0.02}{}^{+1.24}_{-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42^{+1.77}_{-1.76}{}^{+1.87}_{-1.88}$	$10.18^{+1.91}_{-1.90}{}^{+2.03}_{-2.62}$	$-1.17^{+0.22}_{-0.22}{}^{+20.00}_{-6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25^{+1.36}_{-1.36}{}^{+2.13}_{-2.58}$	$8.08^{+1.52}_{-1.51}{}^{+2.52}_{-2.65}$	$-3.23^{+0.61}_{-0.61}{}^{+19.17}_{-3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83}_{-0.77}{}^{+1.48}_{-2.23}$	$-4.33^{+0.84}_{-0.78}{}^{+3.29}_{-2.32}$	$-1.41^{+0.27}_{-0.25}{}^{+5.54}_{-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40}_{-0.40}{}^{+1.39}_{-0.74}$	$2.10^{+0.39}_{-0.39}{}^{+1.40}_{-2.86}$	$2.07^{+0.39}_{-0.39}{}^{+2.76}_{-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21}_{-0.22}{}^{+0.55}_{-0.84}$	$-0.88^{+0.16}_{-0.17}{}^{+1.31}_{-0.91}$	$-0.48^{+0.09}_{-0.09}{}^{+1.09}_{-1.15}$	-14 ± 11
$\pi^-\bar{K}^{*0}$	$1.36^{+0.25}_{-0.26}{}^{+0.60}_{-0.47}$	$1.49^{+0.27}_{-0.29}{}^{+0.69}_{-0.56}$	$0.27^{+0.05}_{-0.05}{}^{+3.18}_{-0.67}$	-3.8 ± 4.2
$\pi^0 K^{*-}$	$13.85^{+2.40}_{-2.70}{}^{+5.84}_{-5.86}$	$18.16^{+3.11}_{-3.52}{}^{+7.79}_{-10.57}$	$-15.81^{+3.01}_{-2.83}{}^{+69.35}_{-15.39}$	-6 ± 24
$\pi^+ K^{*-}$	$11.18^{+2.00}_{-2.15}{}^{+9.75}_{-10.62}$	$19.70^{+3.37}_{-3.80}{}^{+10.54}_{-11.42}$	$-23.07^{+4.35}_{-4.05}{}^{+86.20}_{-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33}_{-3.00}{}^{+7.59}_{-12.57}$	$-15.11^{+2.93}_{-2.65}{}^{+12.34}_{-10.64}$	$2.16^{+0.39}_{-0.42}{}^{+17.53}_{-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72}_{-0.67}{}^{+5.44}_{-4.30}$	$-1.54^{+0.45}_{-0.58}{}^{+4.60}_{-9.19}$	$7.26^{+1.21}_{-1.34}{}^{+12.78}_{-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38}_{-1.28}{}^{+3.38}_{-5.35}$	$-3.45^{+0.67}_{-0.59}{}^{+9.48}_{-4.95}$	$-1.02^{+0.19}_{-0.18}{}^{+4.32}_{-7.86}$	-5 ± 45

Direct CP asymmetries

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13+0.21}_{-0.14-0.19}$	$0.77^{+0.14+0.23}_{-0.15-0.22}$	$0.10^{+0.02+1.24}_{-0.02-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42^{+1.77+1.87}_{-1.76-1.88}$	$10.18^{+1.91+2.03}_{-1.90-2.62}$	$-1.17^{+0.22+20.00}_{-0.22-6.62}$	4.0 ± 2.1
π^+K^-	$7.25^{+1.36+2.13}_{-1.36-2.58}$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.17}_{-0.61-3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83+1.48}_{-0.77-2.23}$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40+1.39}_{-0.40-0.74}$	$2.10^{+0.39+1.40}_{-0.39-2.86}$	$2.07^{+0.39+2.76}_{-0.39-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21+0.55}_{-0.22-0.84}$	$-0.88^{+0.16+1.31}_{-0.17-0.91}$	$-0.48^{+0.09+1.09}_{-0.09-1.15}$	-14 ± 11
$\pi^-\bar{K}^{*0}$	$1.36^{+0.25+0.60}_{-0.26-0.47}$	$1.49^{+0.27+0.69}_{-0.29-0.56}$	$0.27^{+0.05+3.18}_{-0.05-0.67}$	-3.8 ± 4.2
π^0K^{*-}	$13.85^{+2.40+5.84}_{-2.70-5.86}$	$18.16^{+3.11+7.79}_{-3.52-10.57}$	$-15.81^{+3.01+69.35}_{-2.83-15.39}$	-6 ± 24
π^+K^{*-}	$11.18^{+2.00+9.75}_{-2.15-10.62}$	$19.70^{+3.37+10.54}_{-3.80-11.42}$	$-23.07^{+4.35+86.20}_{-4.05-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33+7.59}_{-3.00-12.57}$	−1		
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72+5.44}_{-0.67-4.30}$	–		
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38+3.38}_{-1.28-5.35}$	–		

$\rho^-\bar{K}^0$	$0.38^{+0.07+0.16}_{-0.07-0.27}$	$0.22^{+0.04+0.19}_{-0.04-0.17}$	$0.30^{+0.06+2.28}_{-0.06-2.39}$	-12 ± 17
ρ^0K^-	$-19.31^{+3.42+13.95}_{-3.61-8.96}$	$-4.17^{+0.75+19.26}_{-0.80-19.52}$	$43.73^{+7.07+44.00}_{-7.62-137.77}$	37 ± 11
ρ^+K^-	$-5.13^{+0.95+6.38}_{-0.97-4.02}$	$1.50^{+0.29+8.69}_{-0.27-10.36}$	$25.93^{+4.43+25.40}_{-4.90-75.63}$	20 ± 11
$\rho^0\bar{K}^0$	$8.63^{+1.59+2.31}_{-1.65-1.69}$	$8.99^{+1.66+3.60}_{-1.71-7.44}$	$-0.42^{+0.08+19.49}_{-0.08-8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17^{+2.80+7.98}_{-2.96-5.39}$	$-5.67^{+0.96+10.86}_{-1.01-9.79}$	$17.80^{+3.15+19.51}_{-3.01-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75^{+1.62+4.78}_{-1.66-6.48}$	$-10.84^{+1.98+11.67}_{-2.09-9.09}$	$-2.43^{+0.46+4.60}_{-0.42-19.43}$	-37 ± 37

Weak annihilation fits [in the context of QCD factorization]

Power corrections that do not factorize.
Not necessarily small numerically compared to penguin amplitude
Many distinct WA amplitudes related to the same phenomenological parameter in the simple model currently used in QCDF

$$P = a_4 + \{1, -1, 0\}\, r_\chi a_6 + \beta_3$$
$$A(\bar{B}_d \to K^- K^+) \propto \lambda_u^{(d)} (b_1 + 2b_4) + \lambda_c^{(d)} 2b_4$$
$$A(\bar{B}_s \to \pi^- \pi^+) \propto \lambda_u^{(d)} (b_1 + 2b_4) + \lambda_c^{(d)} 2b_4$$

Gobal fits [Bobeth, Gorbahn, Vickers; Chang, Sun, Yang et al.]. Size OK within factor 2 or so, less universal.

Compare

$$\left|1 + \alpha_s + c\,\frac{\Lambda}{m_b}\right|^2 \to 20\% \qquad \left|c\,\frac{\Lambda}{m_b}\right|^2 \to \text{factor of } 4$$

when c is misestimated by a factor of 2 ($c = 1 \to 2$), e.g. b_4.

- QCDF should be prepared to parameterize general power corrections. Theoretical work to be done.