QCD factorisation for three-body B decays

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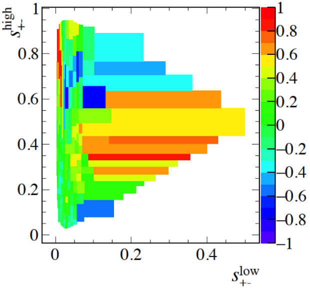
Three-body nonleptonic B-decays

- Three-body nonleptonic B-decays provide another fertile testing-ground to study CP violation
- Events populate a Dalitz plot, wealth of data
- Also local CP asymmetries are possible to study



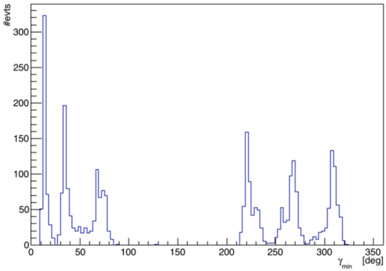
Three-body nonleptonic B-decays in flavour SU(3)

- Extracting CKM angle γ with charmless three-body B decays

  [Bertholet,Ben-Haim,Bhattacharya,Charles,London'18]

  - Use SU(3) flavour symmetry to relate $B \to K\pi\pi$ and $B \to KK\bar{K}$ amplitudes
  - Find six solutions

- Left: Use four modes and SU(3) limit. Right: Use five modes and free $\alpha_{SU(3)}$



- When averaged over entire Dalitz plane, the effect of SU(3) breaking is only at the percent level.

- Nonleptonic three-body B-decays using flavour SU(3) or one of its SU(2) subgroups have been extensively studied, including local and integrated symmetry breaking effects

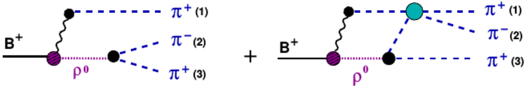
  [Gronau,Rosner,Bhattacharya,Imbeault,London,Engelhard,Nir,Raz,Charles,Descotes-Genon,Ocariz,Pérez Pérez, . . .]

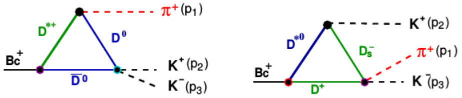
  - See talks and discussion on Monday

- Rescattering effects in $B^+ \to \pi^+\pi^-\pi^+$ and $B_c \to K^+K^-\pi^+$ decays

  [Bediaga,Frederico,Magalhães,'15+]

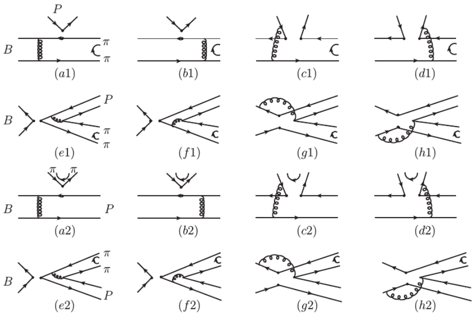
  - See talks and discussion on Thursday





Three-body nonleptonic B-decays in pQCD

- Study of quasi-two-body decays $B_{(s)} \to P\rho \to P\pi\pi$ and $B_{(s)} \to K^* h \to K\pi\, h$ in the isobar model in perturbative QCD approach [Y. Li,A.-J. Ma,W.-F. Wang,Z.-J. Xiao'16,'18]



- Focuses on P-wave $\pi\pi$ and $K\pi$ resonant contributions
- Amplitude $B \to h_1 h_2 h_3$ is factorised via

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2}^{\mathrm{P-wave}} \otimes \Phi_{h_3}$$

Three-body nonleptonic *B*-decays in pQCD

- P-wave contributions are parameterized into the time-like vector FFs involved in the kaon-pion DAs. [Y. Li,A.-J. Ma,W.-F. Wang,Z.-J. Xiao'16,'18]

$$\Phi_{K\pi}^{P\text{-wave}} = \frac{1}{\sqrt{2N_c}}[\not{p}\Phi_{v\nu=-}^{I=\frac{1}{2}}(z,\zeta,\omega^2) + \omega\Phi_s^{I=\frac{1}{2}}(z,\zeta,\omega^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{\omega(2\zeta-1)}\Phi_{t\nu=+}^{I=\frac{1}{2}}(z,\zeta,\omega^2)]\,.$$

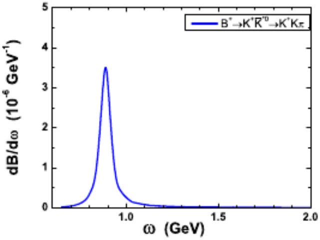
$$\Phi_{v\nu=-}^{I=\frac{1}{2}} = \phi_0 = \frac{3F_{K\pi}(s)}{\sqrt{2N_c}}x(1-x)\left[1 + a_{1K^*}^{||}3(2x-1) + a_{2K^*}^{||}\frac{3}{2}(5(2x-1)^2-1)\right]P_1(2\zeta-1)\,,$$

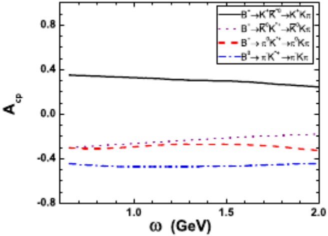
$$\Phi_s^{I=\frac{1}{2}} = \phi_s = \frac{3F_s(s)}{2\sqrt{2N_c}}(1-2x)P_1(2\zeta-1)\,,$$

$$\Phi_{t\nu=+}^{I=\frac{1}{2}} = \phi_t = \frac{3F_t(s)}{2\sqrt{2N_c}}(2x-1)^2P_1(2\zeta-1)\,,$$

$$F_{K\pi}(s) = \frac{m_{K^*}^2}{m_{K^*}^2 - s - im_{K^*}\Gamma(s)},$$

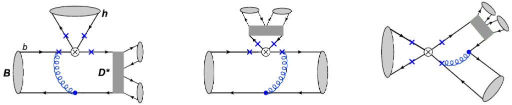
$$s = \omega^2 = m^2(K\pi).$$





Three-body nonleptonic B-decays in pQCD

- Study of the quasi-two-body decays $B \to D^* h \to D\pi h$ with $h = (\pi, K)$ in pQCD with virtual contributions from off-shell D^*. [W.-F. Wang,J. Chai'18]



- Factorisation of amplitude

$$\mathcal{A} = \phi_B \otimes H \otimes \phi_h \otimes \phi_{D\pi}$$

- $D\pi$ distribution amplitude

$$\phi_{D\pi}(z,b,s) = \frac{F_{D\pi}(s)}{2\sqrt{2N_c}} 6z(1-z)\left[1 + a_{D\pi}(1-2z)\right] \exp\left(-\omega_{D\pi}^2 b^2/2\right)$$

- Results

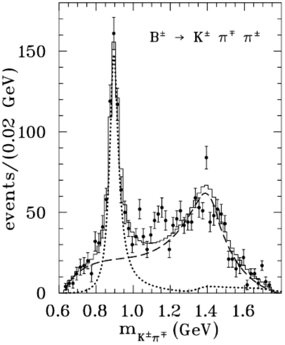
TABLE II: The PQCD predictions of the virtual contributions from D^* state in the $D\pi$ invariant mass region $\sqrt{s} > 2.1$ GeV for the $B \to D^* h \to D\pi h$ decays.

Mode	Unit	Branching fraction
$\bar{B}^0 \to D^{*+}\pi^- \to D^0\pi^+\pi^-$	(10^{-4})	$0.87^{+0.43}_{-0.27}(\omega_B)^{+0.08}_{-0.07}(f_{D^*})^{+0.08}_{-0.06}(a_{D\pi}) \pm 0.03(A)^{+0.02}_{-0.01}(\omega_{D\pi})$
$\bar{B}^0 \to D^{*+}K^- \to D^0\pi^+K^-$	(10^{-5})	$0.72^{+0.35}_{-0.22}(\omega_B)^{+0.07}_{-0.06}(f_{D^*}) \pm 0.06(a_{D\pi}) \pm 0.03(A) \pm 0.02(\omega_{D\pi})$
$B^- \to D^{*0}\pi^- \to D^+\pi^-\pi^-$	(10^{-4})	$1.91^{+0.86}_{-0.59}(\omega_B)^{+0.17}_{-0.16}(f_{D^*})^{+0.12}_{-0.10}(a_{D\pi}) \pm 0.07(A)^{+0.04}_{-0.05}(\omega_{D\pi})$
$B^- \to D^{*0}K^- \to D^+\pi^-K^-$	(10^{-5})	$1.48^{+0.65}_{-0.46}(\omega_B) \pm 0.13(f_{D^*})^{+0.09}_{-0.08}(a_{D\pi}) \pm 0.05(A)^{+0.02}_{-0.03}(\omega_{D\pi})$

- Amplitudes consist of a QCDF-term plus a phenomenological contribution added to the QCD penguin amplitudes. [El-Bennich,Furman,Kaminski,Lesniak,Loiseau,Moussallam'09]

$$\mathcal{M}_S^- \equiv \langle \pi^- \; (K^-\pi^+)_S | H_{eff} | B^- \rangle = \frac{G_F}{\sqrt{2}} (M_B^2 - m_\pi^2) \frac{m_K^2 - m_\pi^2}{q^2} f_0^{B^-\pi^-}(q^2) \; f_0^{K^-\pi^+}(q^2)$$
$$\times \left\{ \lambda_u \left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c \right) \right.$$
$$\left. - \frac{2q^2}{(m_b - m_d)(m_s - m_d)} \left[\lambda_u \left(a_6^u(S) - \frac{a_8^u(S)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(S) - \frac{a_8^c(S)}{2} + c_6^c \right) \right] \right\}.$$

- $c_{4,6}^{u,c}$ are fitted to reproduce the $K\pi$ effective mass and helicity angle distributions, the $B \to K^*\pi$ BR and CP asymmetries
- Dashed/solid: S/P-wave contribution



Three-body nonleptonic B-decays in QCDF

- Study of charmless three-body B decays based on the framework of quasi two-body factorization. [Cheng,Chua,Soni'07; Cheng,Chua,Zhang'13,'16]
  - Nonresonant contributions are treated in HMχPT
- Partial wave $\pi\pi$ final state interactions and CP violation in $B \to 3\pi$ decays [Dedonder,Loiseau,Furman,Kaminski,Lesniak'10]
  - FSI described by pion non-strange scalar and vector FF for S and P wave, and by relativistic Breit-Wigner for D wave.
- Introduce parametrizations of three-body amplitudes alternative to the isobar model, based on a quasi-two-body factorization [Boito,Dedonder,El-Bennich,Escribano,Kaminski,Lesniak,Loiseau'17]
  - Two-body hadronic FSI are taken into account in terms of unitary S- and P-wave $\pi\pi, \pi K, KK$ FFs.
  - FFs determined from analyticity, unitarity, low-energy behaviour predicted by effective theories of QCD . . .

Three-body nonleptonic B-decays in QCDF

- Study of CP violation in $B^- \to K^-\pi^+\pi^-$ and $B^- \to \pi^-\pi^+\pi^-$ modes and BRs of $B^- \to K^-\sigma$ and $B^- \to \pi^-\sigma$ within QCDF [Qi,Guo,Z.-Y. Wang,C. Wang,Zhang'18]
- Non-resonant part treated in HMχPT plus additional exp-factor.

$$A_{NR} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^s \Big[\langle \pi^+\pi^- | (\bar{u}b)_{V-A} | B^- \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] + \langle \pi^- | \bar{d}b | B^- \rangle \langle K^-\pi^+ | \bar{s}d | 0 \rangle (-2a_6^p + 2a_8^p) \Big].$$

$$A^{\rm HMChPT}_{\rm current-ind} \equiv \langle \pi^+(p_1)\pi^-(p_2) | (\bar{u}b)_{V-A} | B^- \rangle \langle K^-(p_3) | (\bar{s}u)_{V-A} | 0 \rangle = -\frac{f_\pi}{2} [2m_3^2 r + (m_B^2 - s_{12} - m_3^2)\omega_+ + (s_{23} - s_{13} - m_2^2 + m_1^2)\omega_-],$$

$$A_{\rm current-ind} = A^{\rm HMChPT}_{\rm current-ind} e^{-\alpha_{\rm NR} p_B \cdot (p_1+p_2)} e^{i\phi_{12}},$$

- Resonant part: Use Bugg factor for σ propagator, Breit-Wigner for others

$$\sum_R A_R = A_\sigma + A_{\rho,\omega} + \sum_i A_{(K^*)^i} + A_{K_0^*} + A_{K_2^*}$$

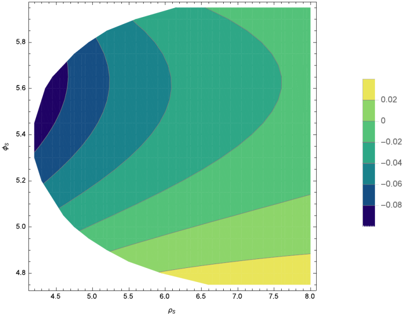
$$R_\sigma(s) = M\Gamma_1(s) \Big/ \left[M^2 - s - g_1^2(s) \frac{s - s_A}{M^2 - s_A} z(s) - iM\Gamma_{\rm tot}(s) \right]$$

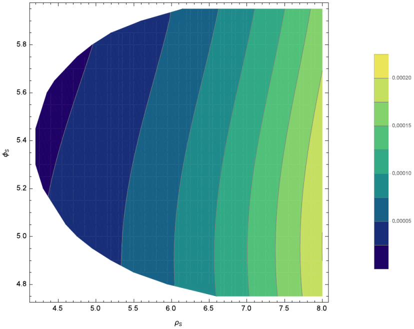
- Total amplitude is sum of resonant and non-resonant: $A = \sum_R A_R + A_{NR}$

Three-body nonleptonic B-decays in QCDF

- $\mathcal{A}_{CP}(B^- \to K^- \sigma)$ (left) and $10^5 \mathcal{B}_{CP}(B^- \to K^- \sigma)$ (right) as function of the weak annihilation parameters ρ_S and Φ_S

  [Qi,Guo,Z.-Y. Wang,C. Wang,Zhang'18]





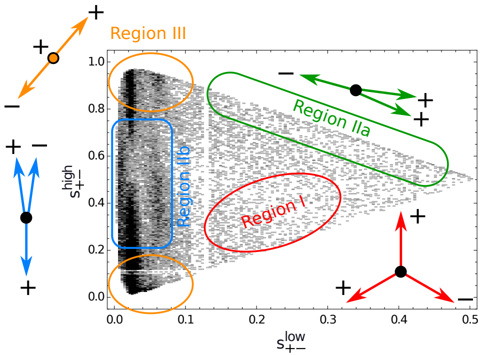
- Can accommodate exptl. data, but large $\rho_S \in [4.2, 8]$ required.

Three-body nonleptonic *B*-decays in QCDF

- Focus on $B^+ \to \pi^+ \pi^- \pi^+$ in a factorisation approach

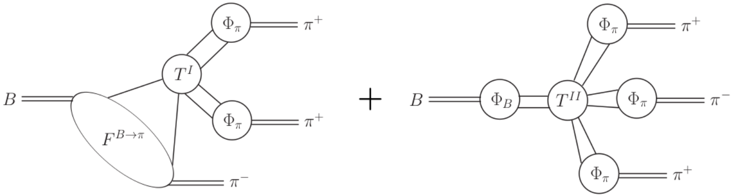
[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15]

  - Identify different regions in Dalitz plot
  - Each region obeys its own factorisation formula



Central region

[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15]



- Factorisation formula

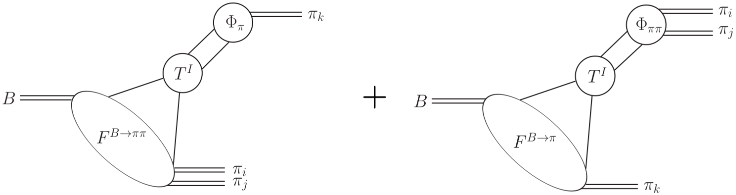
$$\langle\pi^+\pi^+\pi^-|\mathcal{Q}_i|B^+\rangle_c = T_i^I \otimes F^{B\to\pi} \otimes \Phi_\pi \otimes \Phi_\pi + T_i^{II} \otimes \Phi_B \otimes \Phi_\pi \otimes \Phi_\pi \otimes \Phi_\pi$$

- At tree-level all convolutions are finite
- $1/m_b^2$ and α_s suppressed compared to two-body case

Edges of Dalitz plot

[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15; Klein,Mannel,Virto,Vos'17]

- Features of the edges
  - Three-body decays resemble two-body ones
  - Resonances close to the edges



- Factorisation formula

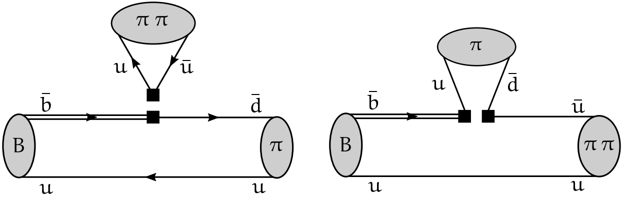
$$\langle\pi^+\pi^+\pi^-|\mathcal{Q}_i|B\rangle_e = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^+\pi^-} + T_i^I \otimes F^{B\to\pi^+\pi^-} \otimes \Phi_{\pi^+}$$

- Always an improvement over quasi-two-body decays, reduces to $B \to \rho\pi$ for ρ dominance and zero-width approximation

Edges of Dalitz plot

[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15; Klein,Mannel,Virto,Vos'17]

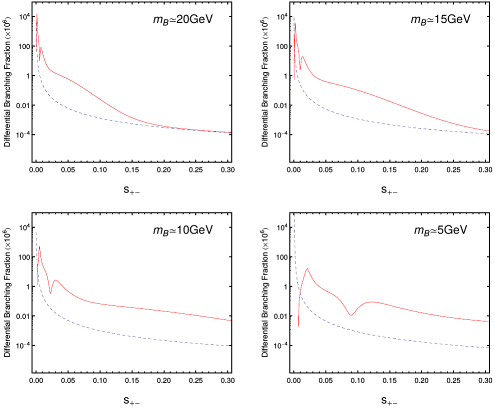
- Leading contributions to hard kernels



- Same operators as in two-body case, different final state
- New nonperturbative input (source of strong phases):
  2πLCDA, $B \to \pi\pi$ form factor

Matching the two regions

- For large enough m_B the two regions should be well separated [Kränkl,Mannel,Virto'15]



Full 2πLCDA (red) and perturbative contribution (dashed)

- For realistic values of m_B
  - not enough phase space to reach a perturbative regime in the center
  - Dalitz plot completely dominated by the edges
  - No part of the Dalitz plot is really center-like

New nonperturbative input

- New nonperturbative input from data or model [Klein,Mannel,Virto,Vos'17]
- 2πLCDA [Polyakov'99]

$$\phi^q_{\pi\pi}(u,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iu(k^+_{12}x^-)} \left\langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^-n_-)\not{n}_+ q(0)|0\right\rangle$$

$$s = (k_1+k_2)^2,\ \zeta = k_1/s$$

- Both isoscalar ($I=0$) and isovector ($I=1$) contribute
- At leading order only normalization needed

$$\int du\ \phi^{I=1}_{\pi\pi}(u,\zeta,s) = (2\zeta-1)F_\pi(s) \qquad \int du\ \phi^{I=0}_{\pi\pi}(u,\zeta,s) = 0$$

- Time-like pion formfactor $F_\pi(s)$ from $e^+e^- \to \pi\pi(\gamma)$ data

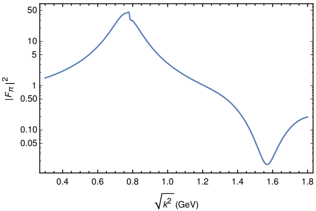
$$\left\langle \pi^+(k_1)\pi^-(k_2)|\bar{q}\gamma_\mu q|0\right\rangle = F_\pi(k^2)(k_1-k_2)_\mu \qquad k^2 = s = (k_1+k_2)^2$$

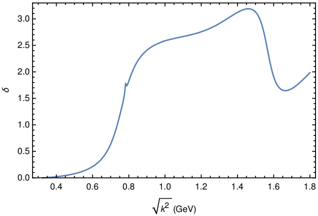
Time-like pion FF

- Pion vector form factor $F_\pi(k^2) = |F_\pi| e^{i\delta}$ in the time-like region

  [Shekhovtsova,Przedzinski,Roig,Was'12; Hanhart'12]

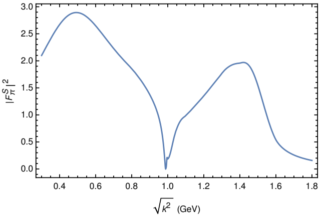
  - Magnitude well constrained, phase not, adds to model dependence

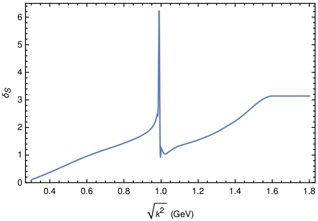




- Also isoscalar part $F_\pi^S(k^2) = \left\langle \pi^+(k_1)\pi^-(k_2) | m_u \bar{u}u + m_d \bar{d}d | 0 \right\rangle / m_\pi^2$ needed

  [Celis,Cirigliano,Passemar'13; Daub,Hanhart,Kubis'15]





New nonperturbative input

- $B \to \pi\pi$ form factor [Klein,Mannel,Virto,Vos'17]
  - Was studied in $B \to \pi\pi\,\ell\,\nu$ decays [Faller,Feldmann,Khodjamirian,Mannel,van Dyk'13; Böer,Feldmann,van Dyk'16]
- For $B^+ \to \pi^+\pi^-\pi^+$ only vector form factor relevant

$$k_{3\mu}\left\langle \pi^+(k_1)\pi^-(k_2)|\bar{b}\gamma^\mu\gamma^5 u|B^+(p)\right\rangle = i\, m_\pi\, F_t(s,\zeta)$$

- Both isoscalar (S-wave) and isovector (P-wave) contributions

$$F_t = F_t^{I=0} + F_t^{I=1}$$

  - Isovector $F_t^{I=1}$ part studied with QCD Light-Cone Sum Rules [Hambrock,Khodjamirian'15; Cheng,Khodjamirian,Virto'17]
  - Assumption / model for $F_t^{I=1}$ [Klein,Mannel,Virto,Vos'17]
    - Decay $B \to \pi\pi$ proceeds only resonantly through $B \to \rho \to \pi\pi$
  - Model for $F_t^{I=0}$. Fit β and ϕ from data [Klein,Mannel,Virto,Vos'17]

$$F_t^{I=0}(q^2) = \frac{m_B^2}{m_\pi f_\pi}\beta e^{i\phi} F_\pi^S(q^2)$$

[Celis,Cirigliano,Passemar'13; Daub,Hanhart,Kubis'15]

$B \to \pi\pi\pi$ decay amplitude

[Kränkl,Mannel,Virto'15; Klein,Mannel,Virto,Vos'17]

- At leading order, leading twist

$$\mathcal{A}_{s_\pm^{\rm low}<<1} = \frac{G_F}{\sqrt{2}} m_B^2 \Bigg[f_\pi \frac{m_\pi}{m_B^2} (\lambda_u (a_1 + a_4^u) + \lambda_c a_4^c) F_t(s_\pm^{\rm low}, \zeta) + (\lambda_u (a_2 - a_4^u) - \lambda_c a_4^c)(2\zeta - 1) F_\pi(s_\pm^{\rm low}) f_0(s_\pm^{\rm low}) \Bigg] ,$$

- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
- $\lambda_u = |\lambda_u| e^{i\gamma}$ weak phase

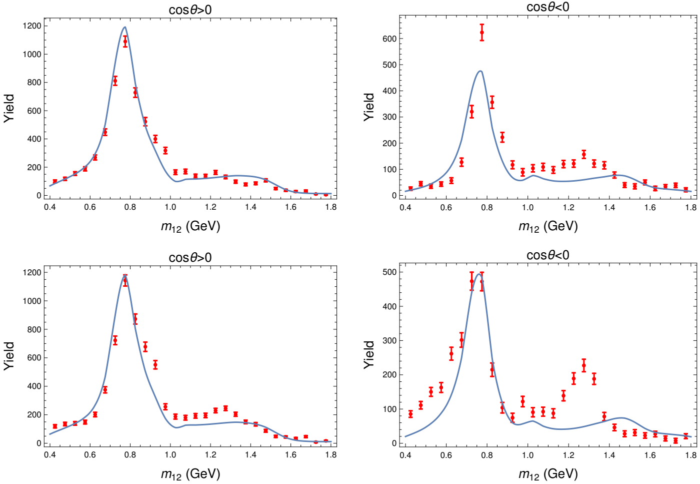
4 inputs that can be obtained from data

- $B \to \pi$ form factor f_0
- Single pion DA gives the pion decay constant f_π
- $B \to \pi\pi$ form factor F_t
- 2π LCDA gives F_π

Dalitz projections

[Klein,Mannel,Virto,Vos'17]

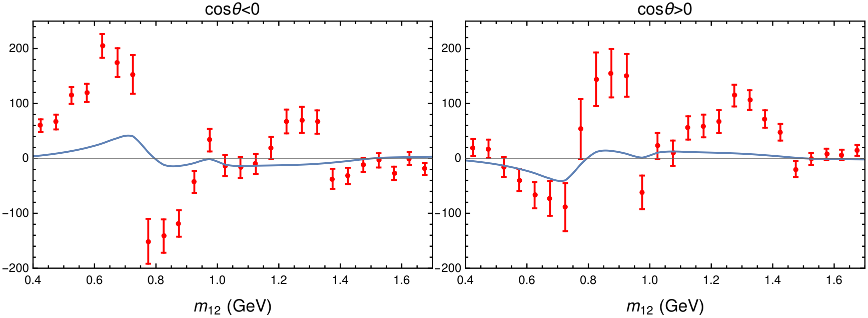
- Best fit values: $\beta = 0.18, \quad \phi = 18^\circ$



- Upper/lower row: $B^\pm$ decay. Fit describes data rather poorly

[Klein,Mannel,Virto,Vos'17]

- Difference between B^+ and B^- yield, fit vs. data



- CP violation seen in model much smaller than in data

Outlook

- To Do / Possible improvements
  - Study 2π LCDA using $\bar{B}^0 \to D^+(\pi^-\pi^0)$ [Virto,Vos,TH, to be resurrected at MITP]
  - Apply to all $B \to hhh$
  - perform $SU(3)$ analysis $\to$ combine with QCDF
  - Include $\mathcal{O}(\alpha_s)$ corrections and higher-twist corrections
  - refinements in the modelling of the form factors
  - ...