THEORETICAL MODELS FOR ELECTRON AND NEUTRINO SCATTERING OFF NUCLEI

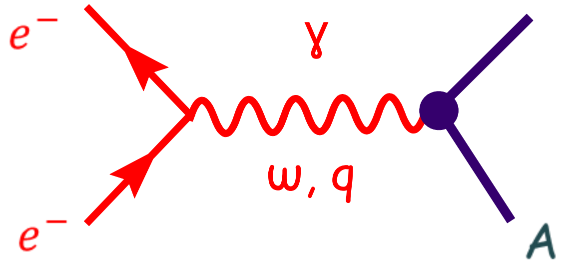
Carlotta Giusti
Università and INFN, Pavia

Workshop on Electromagnetic
Observables for Low_energy Nuclear Physics
Mainz 1-3 October 2018

ELECTRON SCATTERING

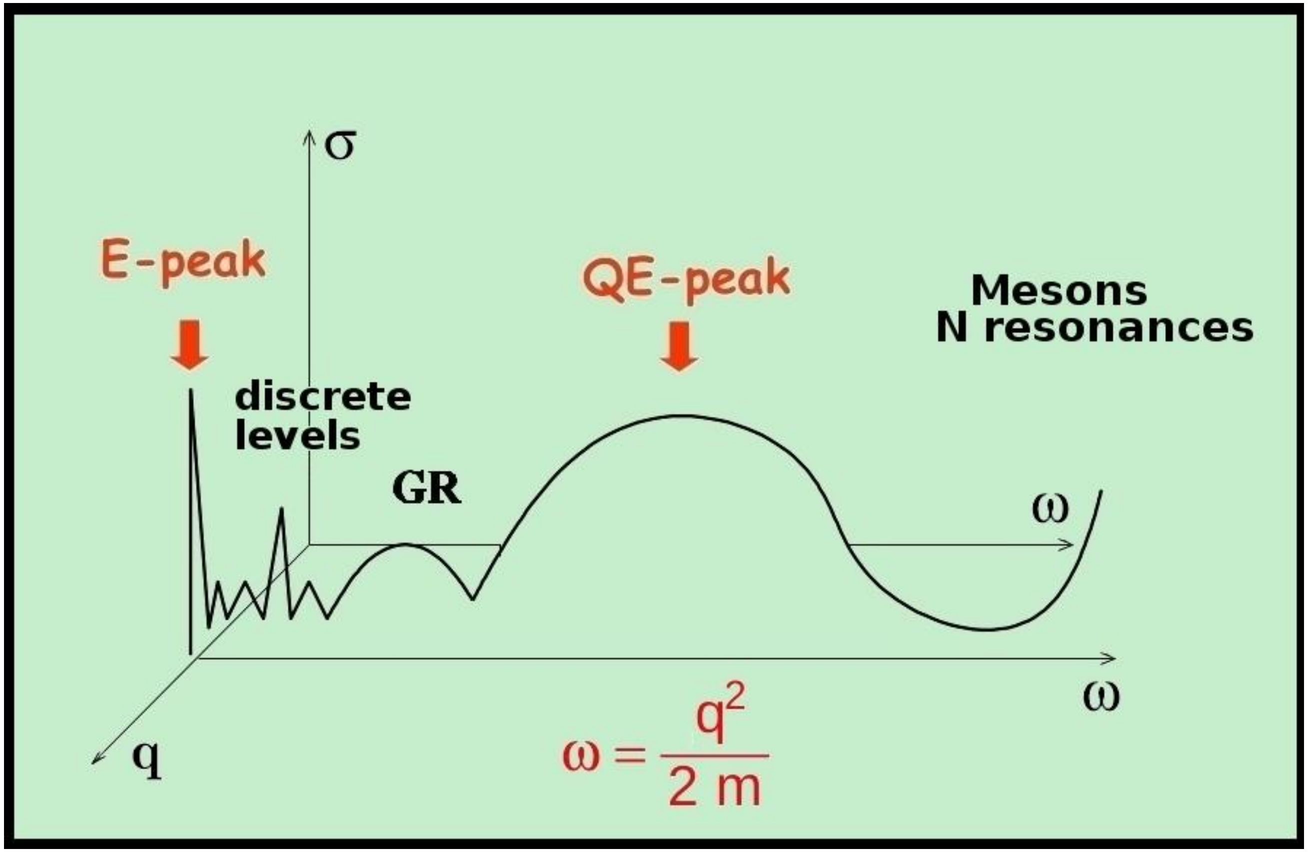
- powerful tool to investigate nuclear structure and dynamics
- predominantly EM interaction, QED, weak compared with nuclear int.
- BA one-photon exchange approx

OPEA



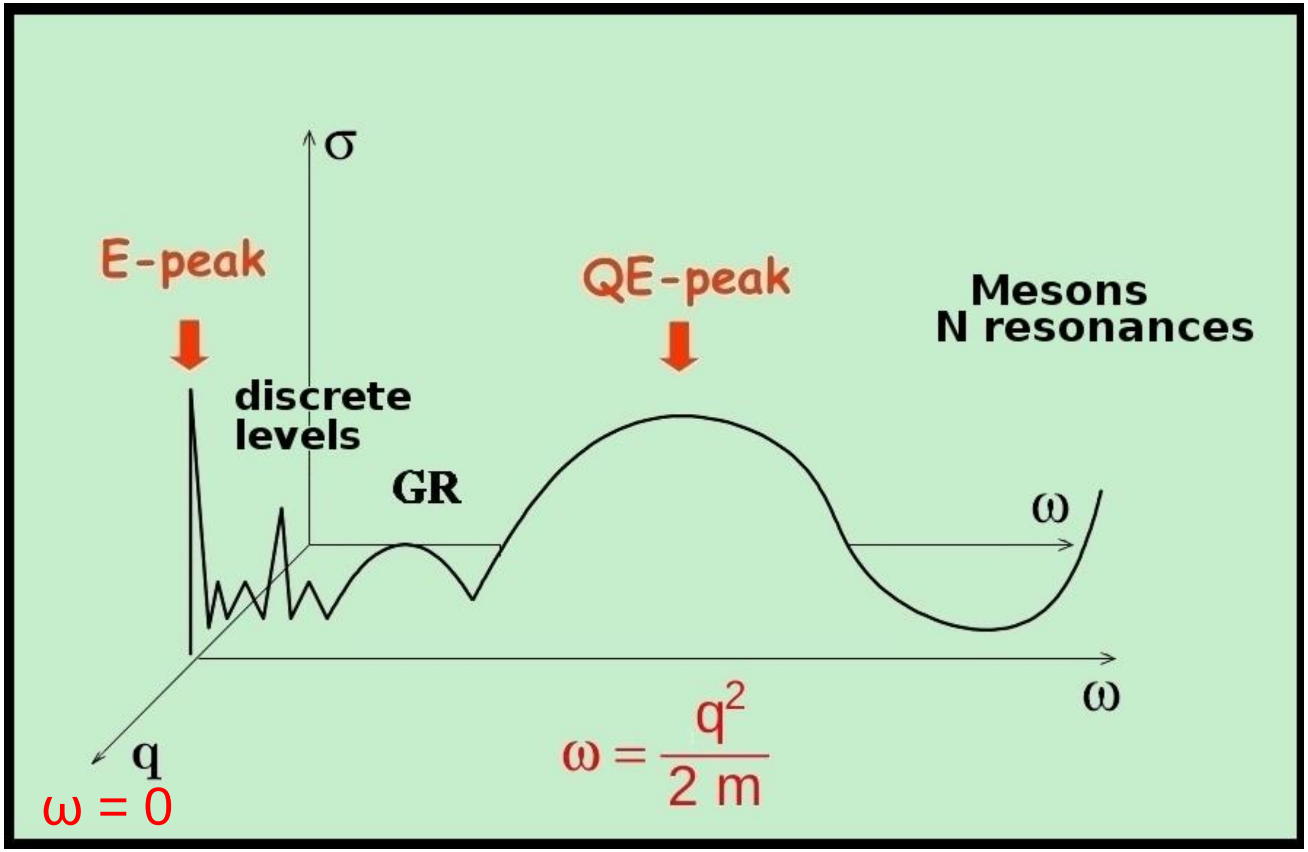
- photon can explore the whole target volume
- independently vary (ω, q) : it is possible to map the nuclear response as a function of its excitation energy with a spatial resolution that can be adjusted to the scale of the processes that need to be studied

NUCLEAR RESPONSE



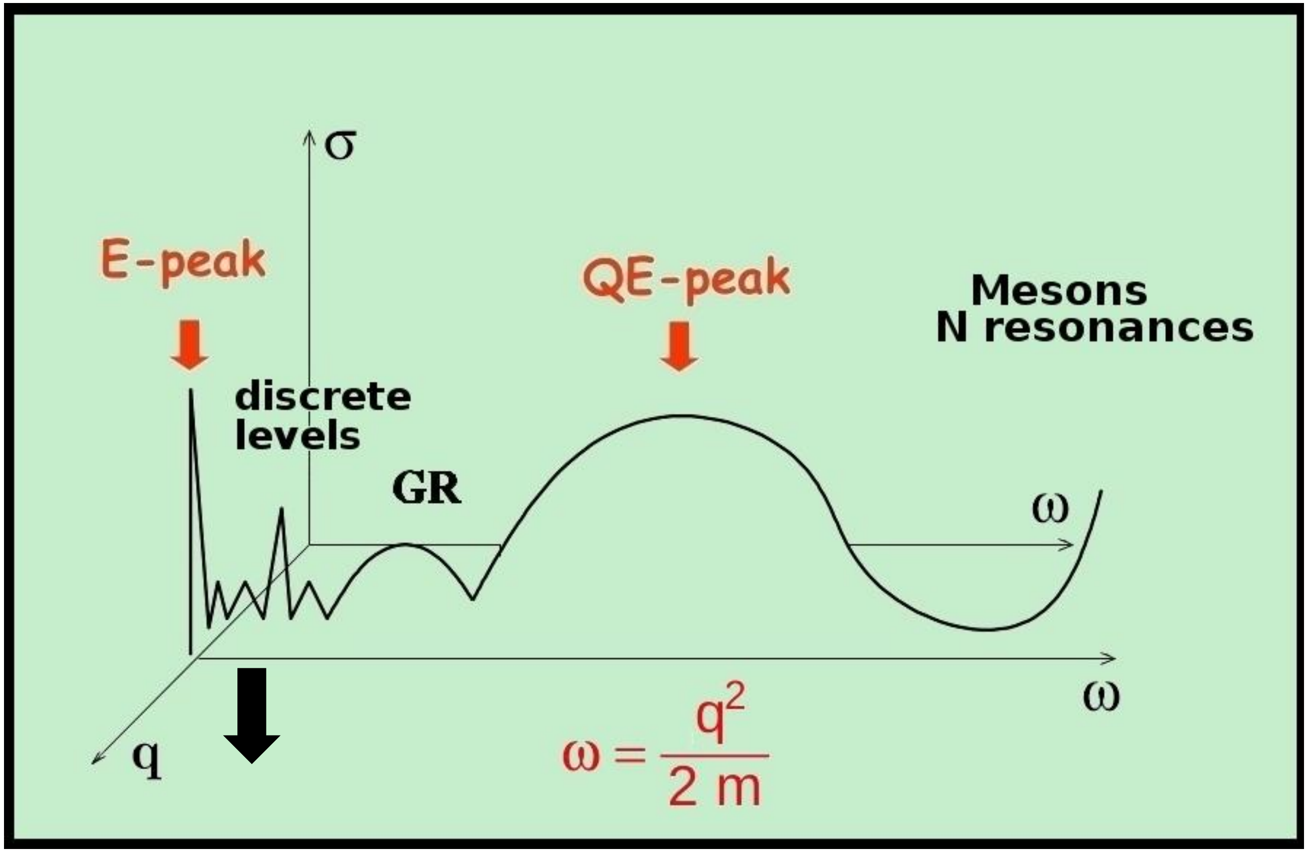
$$\omega = \frac{q^2}{2\,m}$$

NUCLEAR RESPONSE



g.s. properties
charge densities, current distr.
charge radii

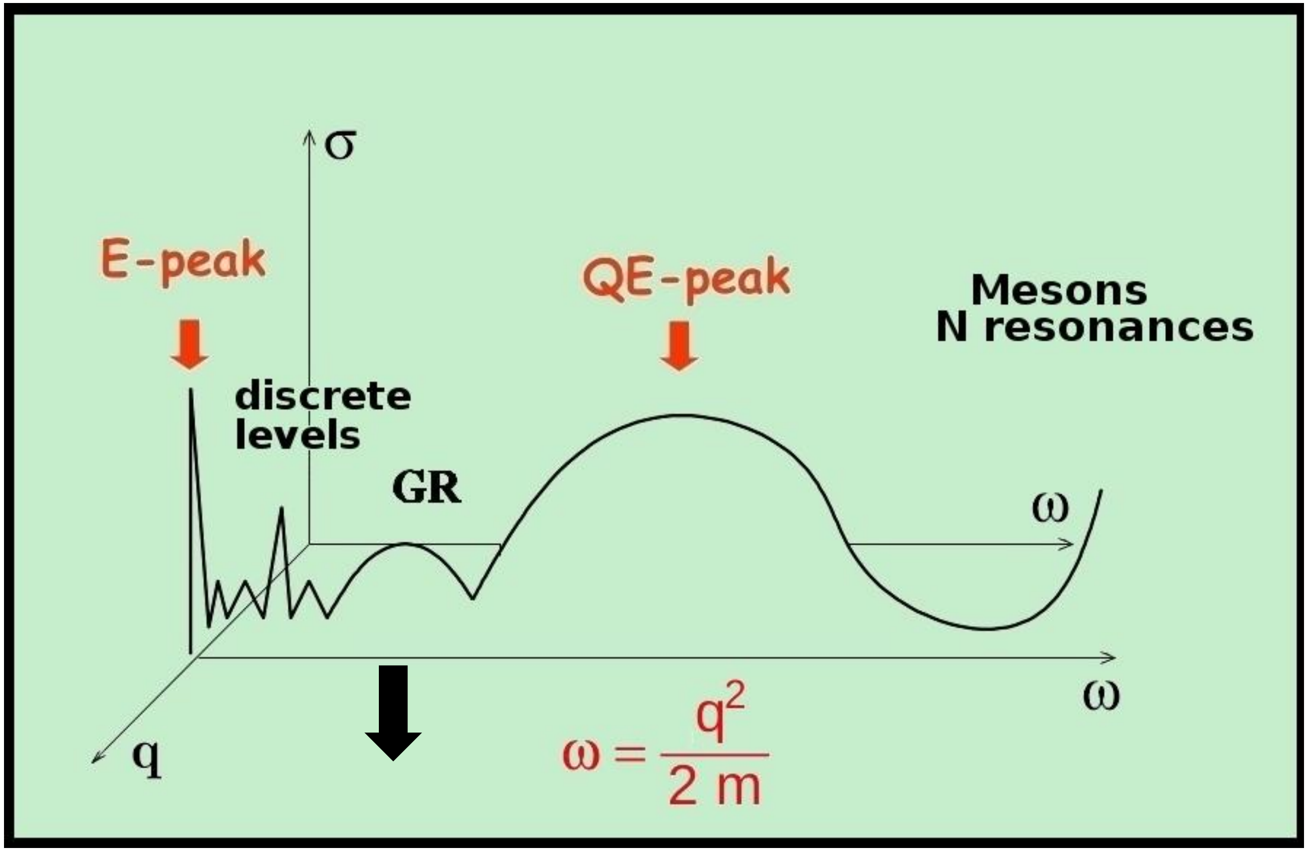
NUCLEAR RESPONSE



$$\omega = \frac{q^2}{2\,m}$$

inelastic scattering
discrete excited states

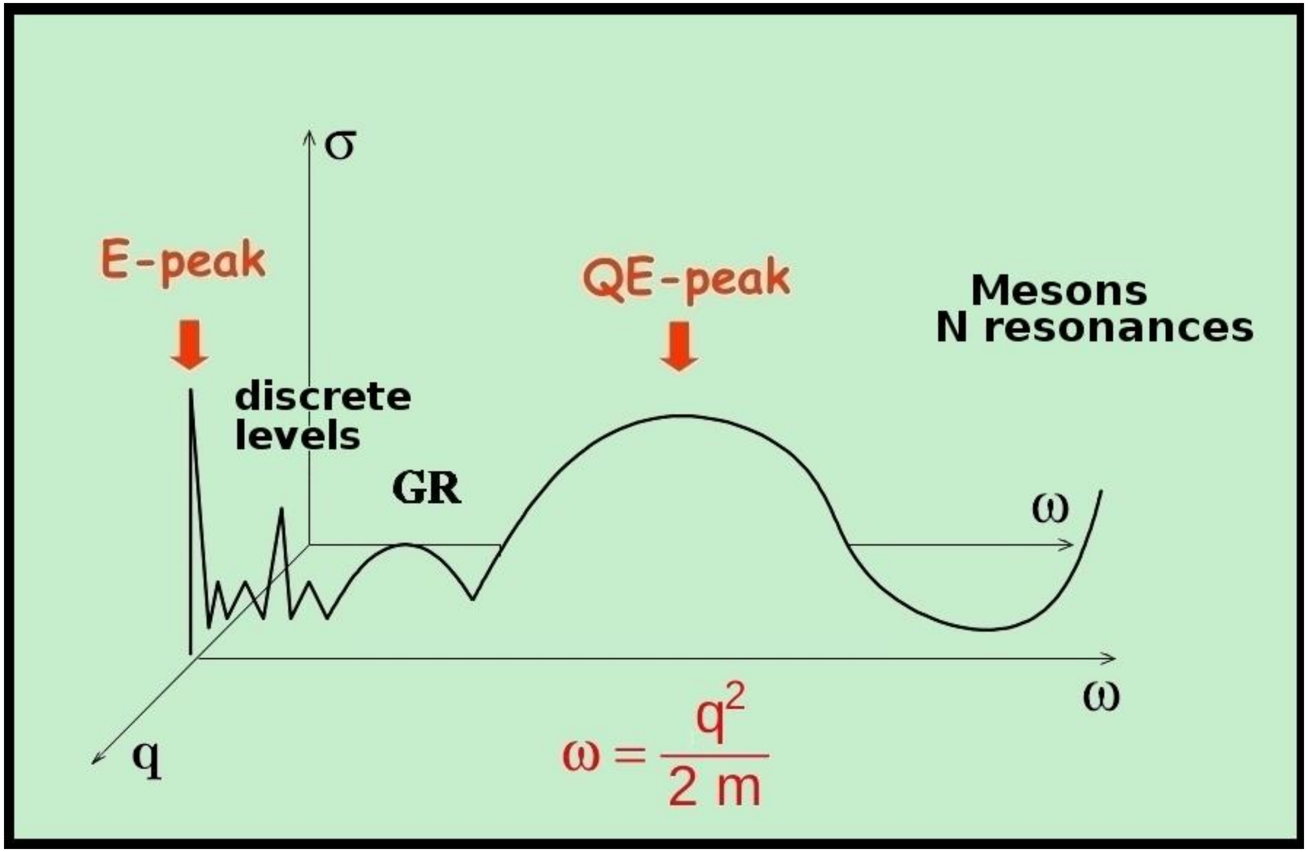
NUCLEAR RESPONSE



$$\omega = \frac{q^2}{2\,m}$$

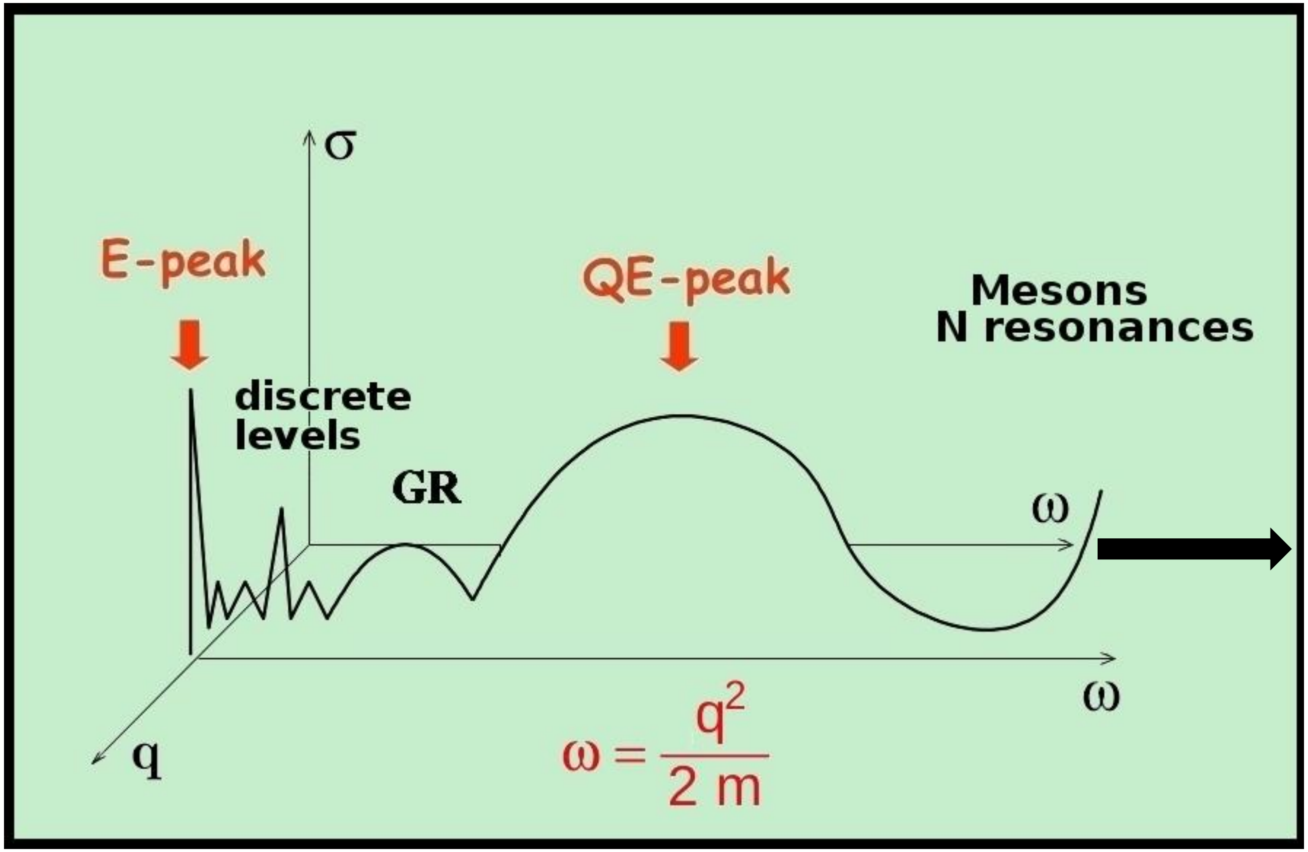
beyond particle emission threshold:
GR collective excitations
electric and magnetic giant multipole resonances

NUCLEAR RESPONSE



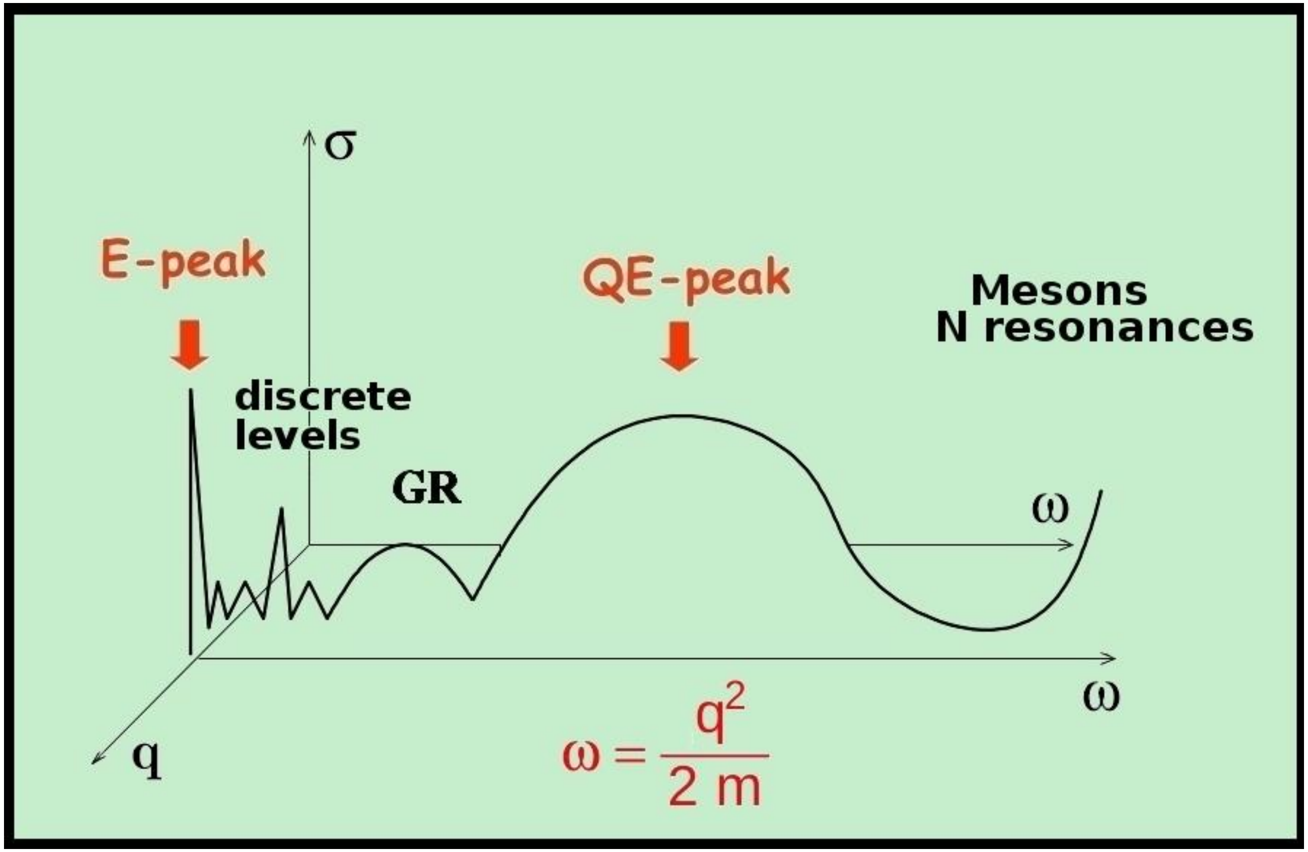
quasi-free process
one-nucleon knockout
s.p. properties, energy and mom. distr.

NUCLEAR RESPONSE



Δ, N^*, nucleon resonances, mesons, deep inelastic scattering…….

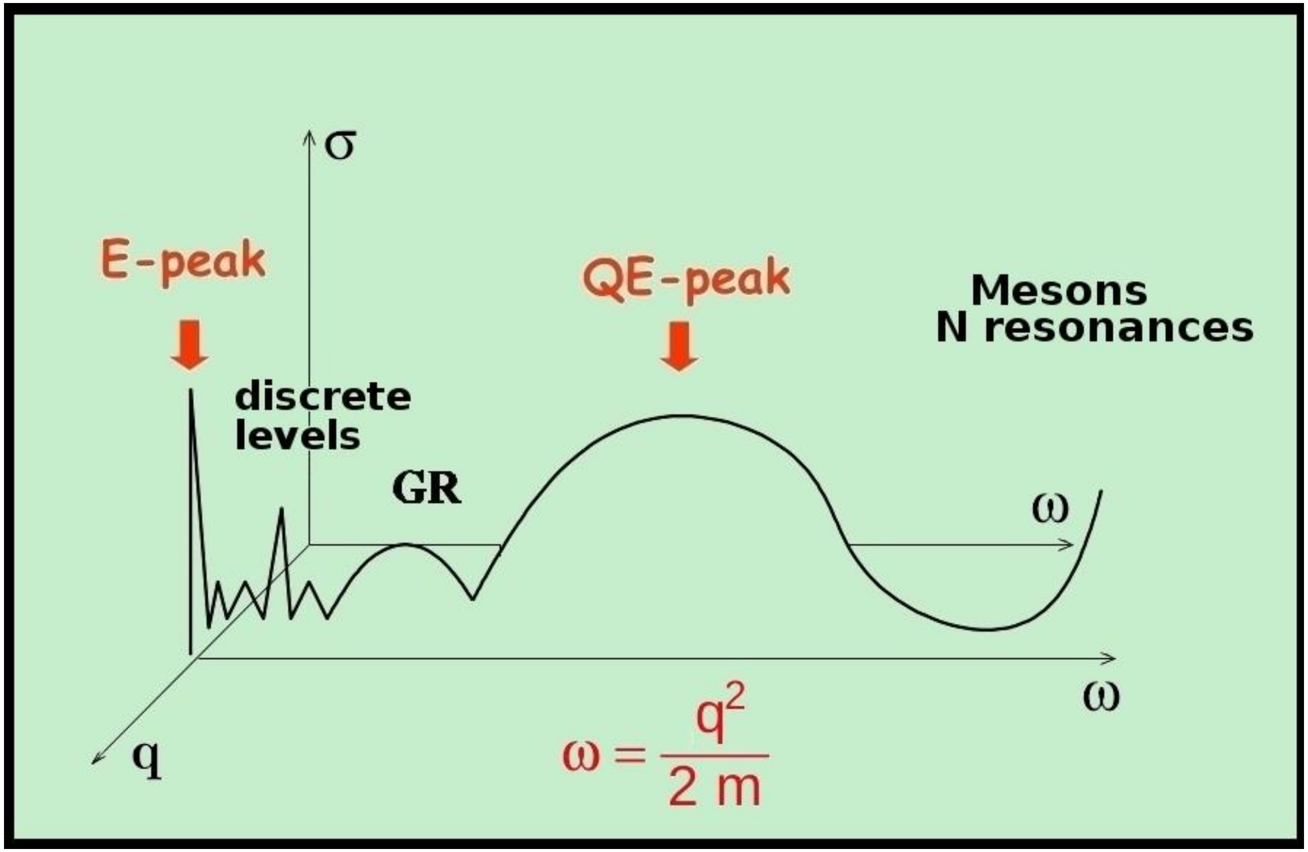
NUCLEAR RESPONSE



models for exclusive and inclusive QE electron scattering

NUCLEAR RESPONSE

$e^- \rightarrow \nu$



models for exclusive and inclusive QE electron scattering extended to neutrino scattering

from e-nucleus to ν-nucleus scattering

- extension of formalism straightforward
- in ν experiments nuclei used as neutrino detectors, nuclear effects in ν-nucleus interactions must be well under control: exploit work done for electron scattering
- electron scattering first necessary test of a nuclear model
- motivation for new dedicated electron scattering experiments
- exploit the selectivity of electron scattering to select suitable kinematics conditions where specific nuclear effects can be investigated

e-nucleus and ν-nucleus scattering

- electron scattering :

  beam energy known, ω and q determined

- neutrino scattering:

  beam energy not known, ω and q not determined, flux averaged c.s.

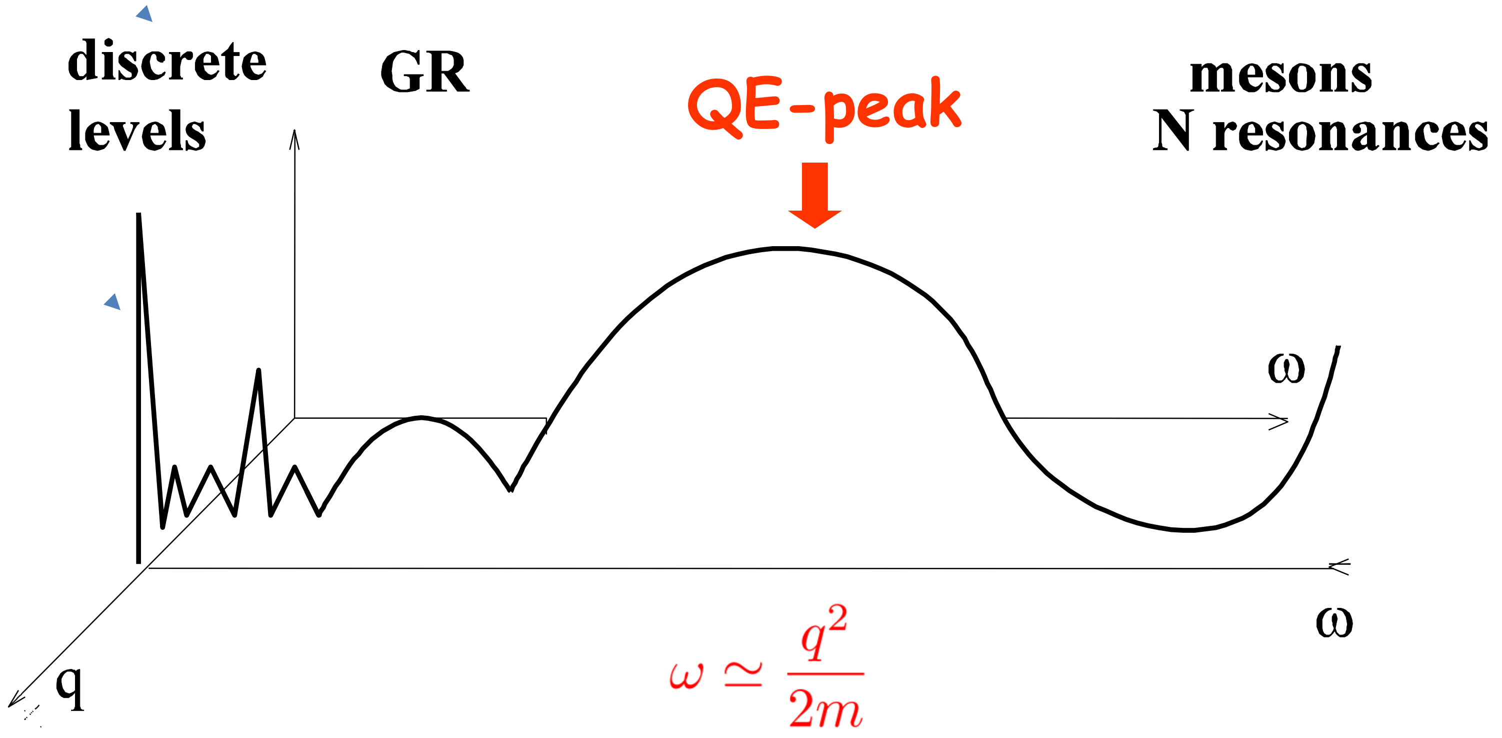
  calculations over the energy range relevant for the neutrino flux,

  broader kinematic region, not only QE, different nuclear effects

  can be included and intertwined in exp. c.s.

Electron scattering experiments in suitable kinematics to study specific nuclear effects

QE ELECTRON SCATTERING



$$\omega \simeq \frac{q^2}{2m}$$

QE-peak dominated by one-nucleon knockout

QE

$$e + A \Longrightarrow e' + N + (A - 1)$$

(e,e′p)

QE

$$e + A \Longrightarrow \boxed{e' + N} + (A-1)$$

1NKO

both e′ and N detected (e,e′p)

(A-1) discrete eigenstate

exclusive (e,e′p)

proton-hole states

properties of bound protons

s.p. aspects of nuclear structure

validity and limitation of IPSM

nuclear correlations

EXCLUSIVE

(e,e'p)

QE

(e,e')

$$e + A \Longrightarrow \boxed{e' + N} + (A-1)$$

$$e + A \Longrightarrow \boxed{e'} + N + (A-1)$$

1NKO

both e' and N detected (e,e'p)

(A-1) discrete eigenstate

exclusive (e,e'p)

proton-hole states

properties of bound protons

s.p. aspects of nuclear structure

validity and limitation of IPSM

nuclear correlations

EXCLUSIVE

only e' detected

all final states included

discrete and continuum spectrum

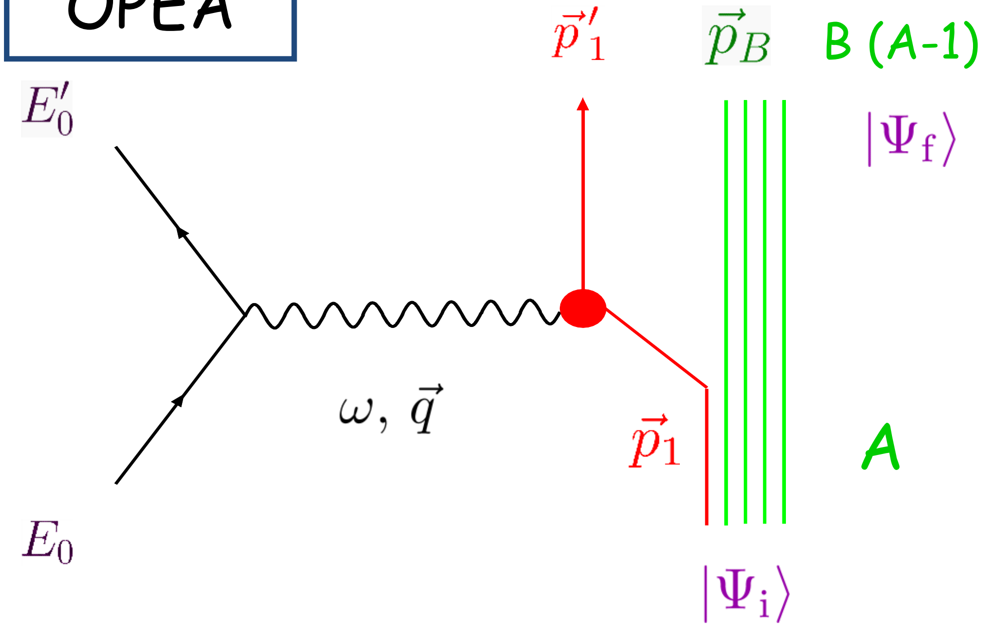
less specific information more closely related to the dynamics of initial nuclear g.s.

width of QE peak direct measurement of average mom. of nucleons in nuclei, shape depends on the energy and momentum distribution of the bound nucleons

INCLUSIVE

OPEA

(e,e'p)



$$E_\mathrm{m} = \omega - \frac{{p'_1}^2}{2m} - \frac{{p_B}^2}{2m(A-1)} = W_B^* - W_A$$

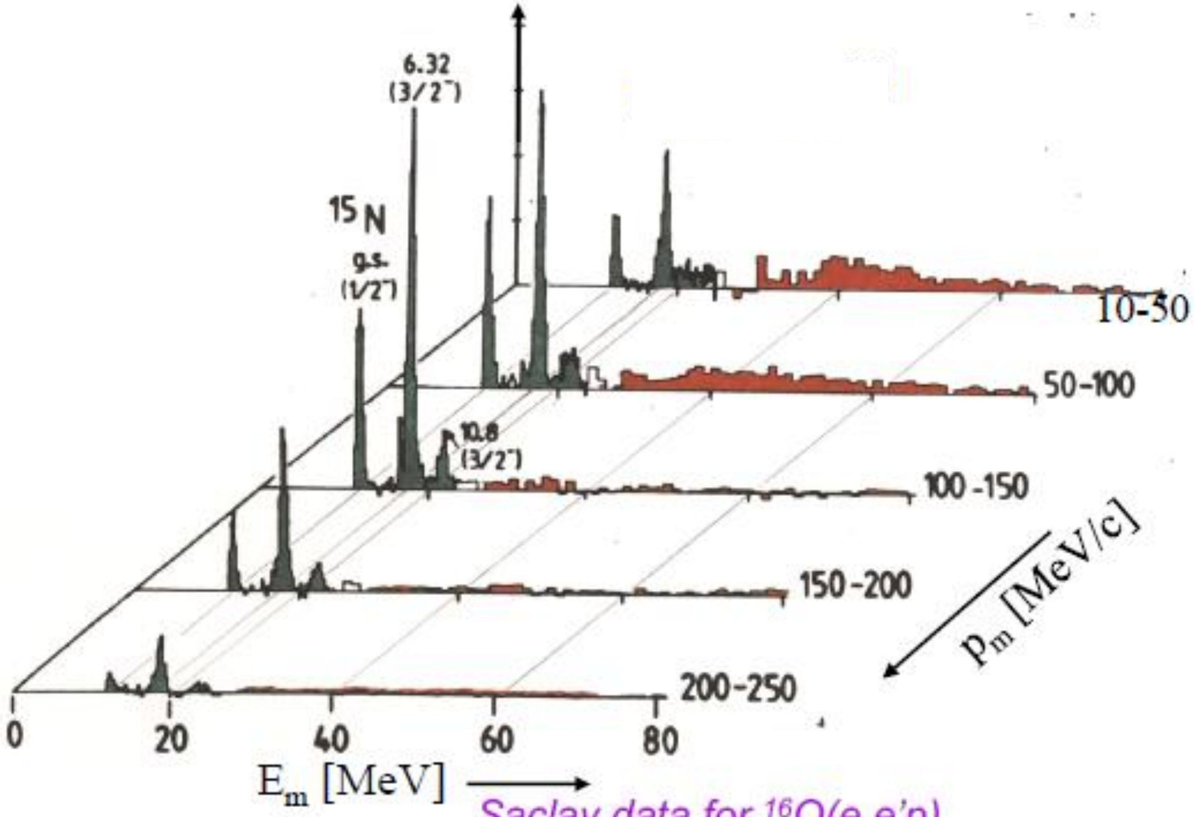
missing energy

$$\vec{p}_\mathrm{m} = \vec{q} - \vec{p'}_1 = -\vec{p}_1 = \vec{p}_B$$

missing momentum

Experimental data: E_m and p_m distributions

$^{16}\mathrm{O}(e,e'p)$



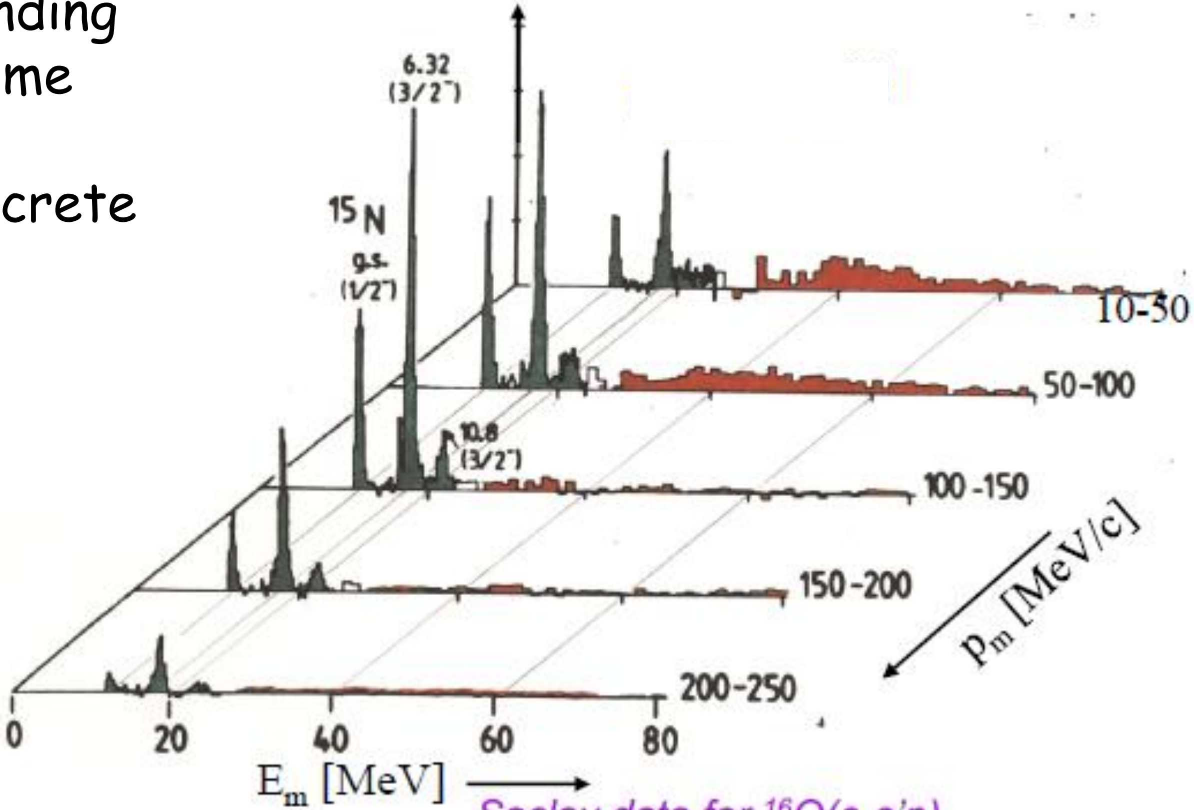
Saclay data for ^{16}O(e,e'p)

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Experimental data: E_m and p_m distributions

$^{16}\mathrm{O}(e,e'p)$

For E_m corresponding to a peak we assume that the residual nucleus is in a discrete eigenstate



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

E_m

exclusive reaction

ONE-HOLE SPECTRAL FUNCTION

$$S(\vec{p_1},\vec{\bar{p}_1};E_m) = \langle \Psi_{\rm i} | a^{+}_{\vec{\bar{p}_1}} \delta(E_m - H) a_{\vec{p_1}} | \Psi_{\rm i} \rangle$$

$$\vec{p}_1 = \vec{\bar{p}}_1$$

joint probability of removing from the target a nucleon p_1
leaving the residual nucleus in a state with energy E_m

E_m

exclusive reaction

ONE-HOLE SPECTRAL FUNCTION

$$S(\vec{p_1},\vec{\bar{p}_1};E_m) = \langle \Psi_{\rm i} | a^{+}_{\vec{\bar{p}}_1} \delta(E_m - H) a_{\vec{p_1}} | \Psi_{\rm i} \rangle$$

$\boxed{\vec{p}_1 = \vec{\bar{p}}_1}$

joint probability of removing from the target a nucleon p_1
leaving the residual nucleus in a state with energy E_m

$$\int S(\vec{p_1},\vec{\bar{p}_1};E_m)\mathrm{d}E_m = \boxed{\rho(\vec{p_1},\vec{\bar{p}_1})}$$

inclusive reaction : one-body density

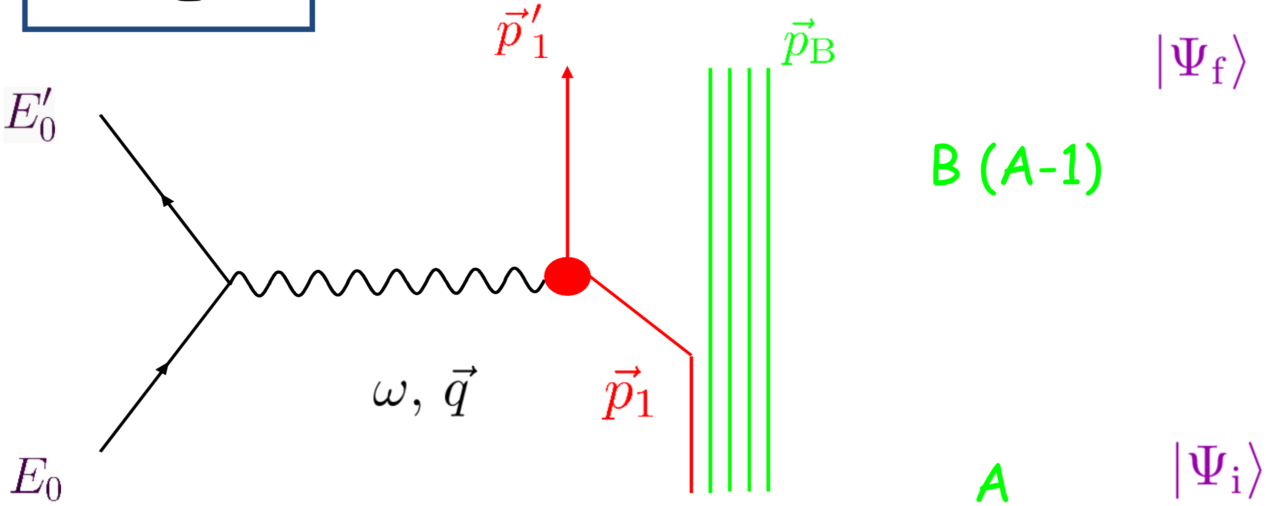
$\boxed{\vec{p}_1 = \vec{\bar{p}}_1}$ ⇒ $\rho(\vec{p_1},\vec{\bar{p}_1}) = \boxed{F(\vec{p_1})}$

MOMENTUM DISTRIBUTION

$$F(\vec{p_1}) = \int |\Psi_{\rm i}(\vec{p_1},\vec{p_2},....,\vec{p_A}|^2 \mathrm{d}\vec{p_2}....\mathrm{d}\vec{p_A}$$

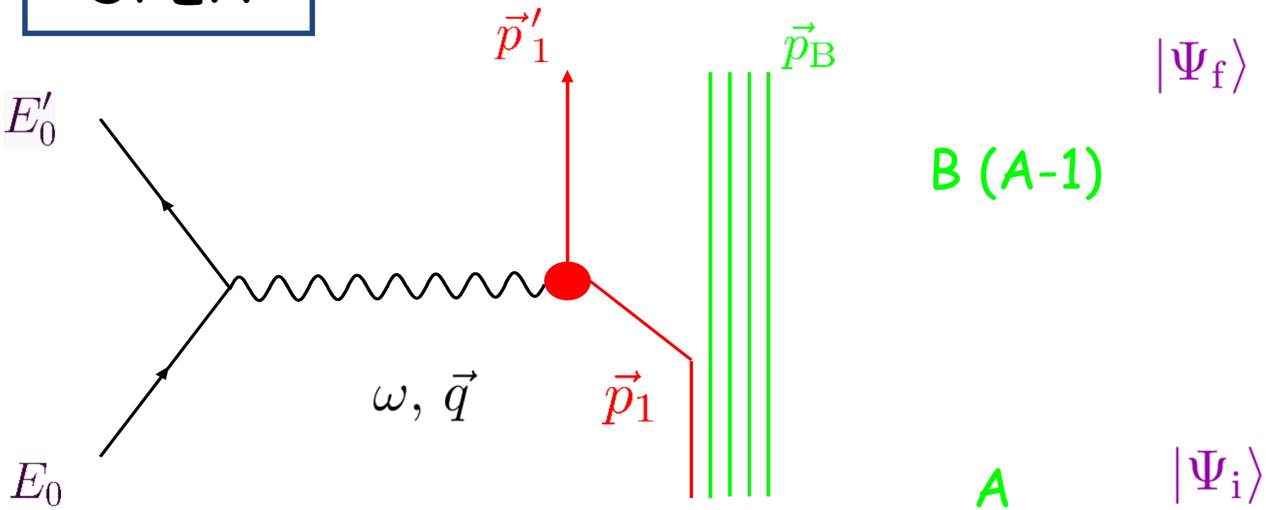
probability of finding in the target
a nucleon with momentum p_1

OPEA



$$\sigma = K L^{\mu\nu}\ W_{\mu\nu}$$

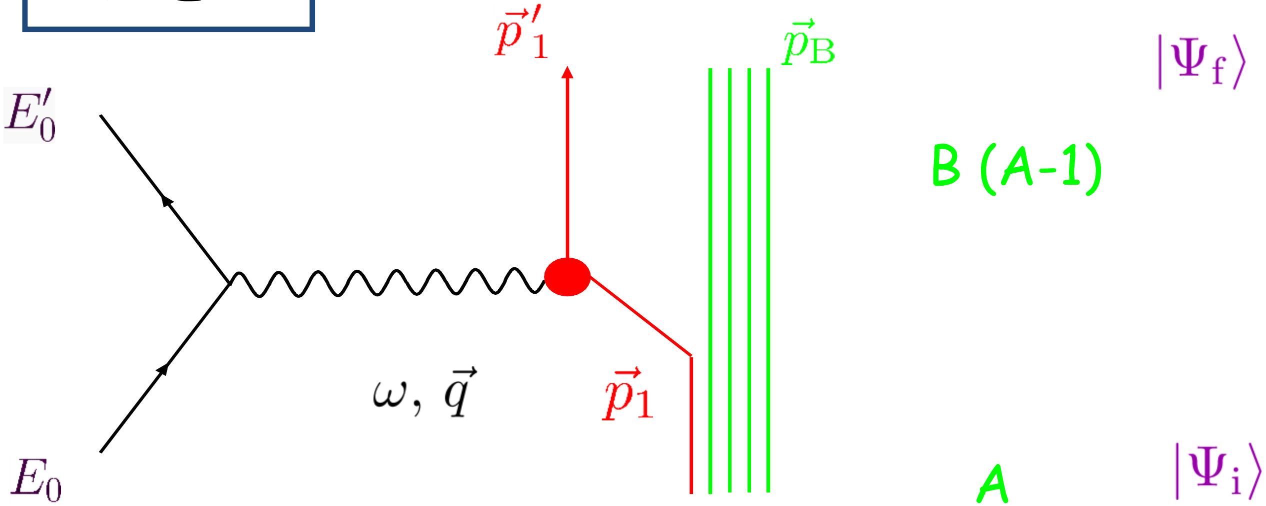
OPEA



$$\sigma = K L^{\mu\nu} \; W_{\mu\nu}$$

lepton tensor

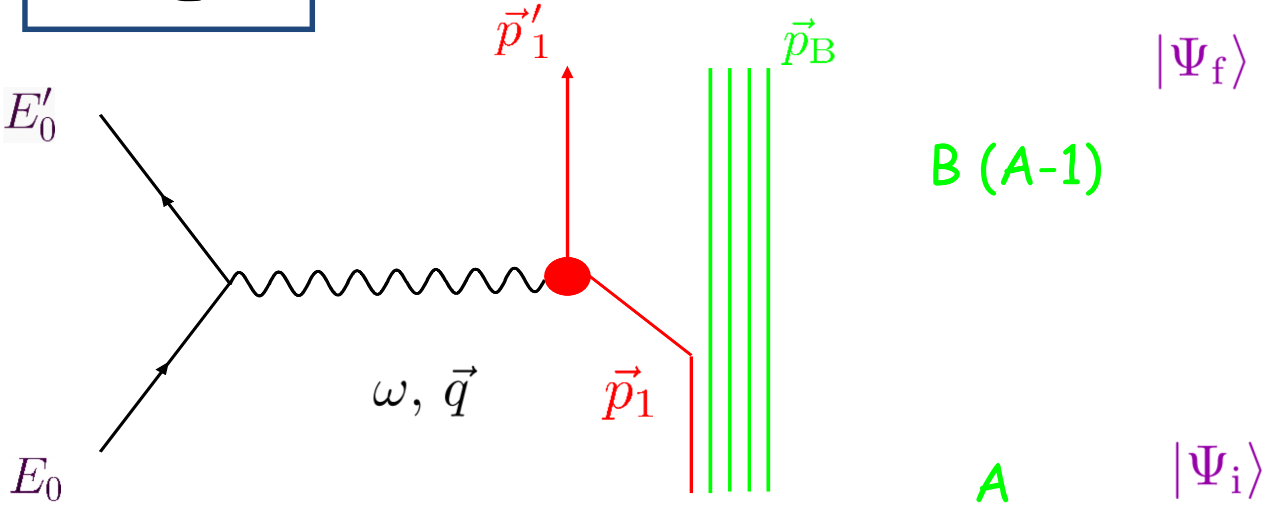
OPEA



$$\sigma = K L^{\mu\nu}\ W_{\mu\nu}$$

hadron tensor

OPEA



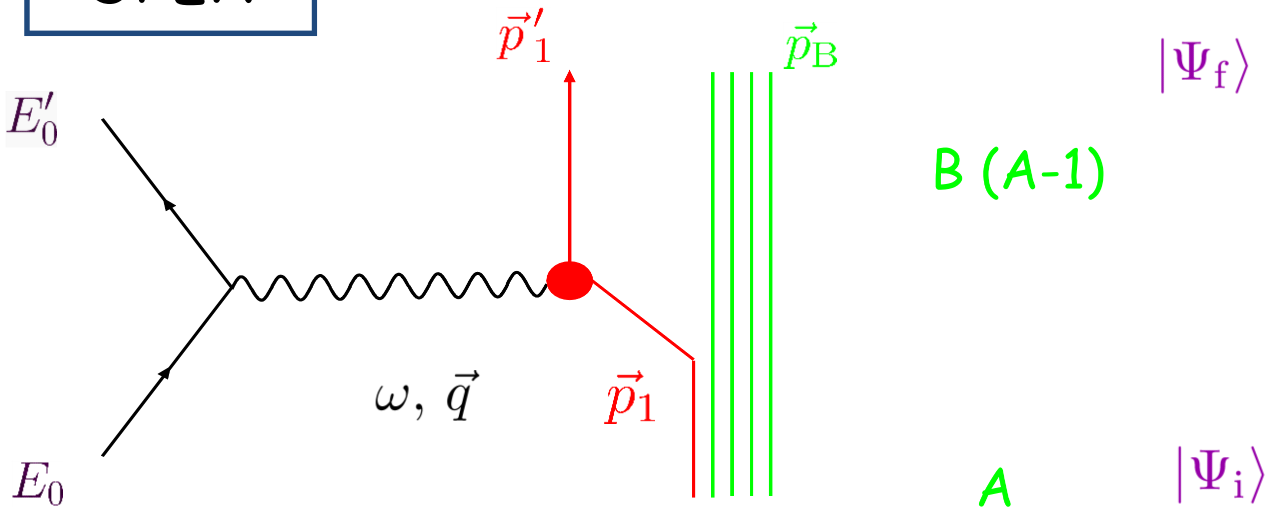
$$\sigma = K L^{\mu\nu}\ W_{\mu\nu}$$

hadron tensor

$$W^{\mu\nu} = \overline{\sum_{\rm i,f}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\rm i} - E_{\rm f})$$

$$J^{\mu}(\vec{q}) = \int {\rm e}^{{\rm i}\vec{q}\cdot\vec{r}} \langle \Psi_{\rm f} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\rm i} \rangle {\rm d}\vec{r}$$

OPEA



$$\sigma = K L^{\mu\nu}\ W_{\mu\nu}$$

$$\downarrow$$

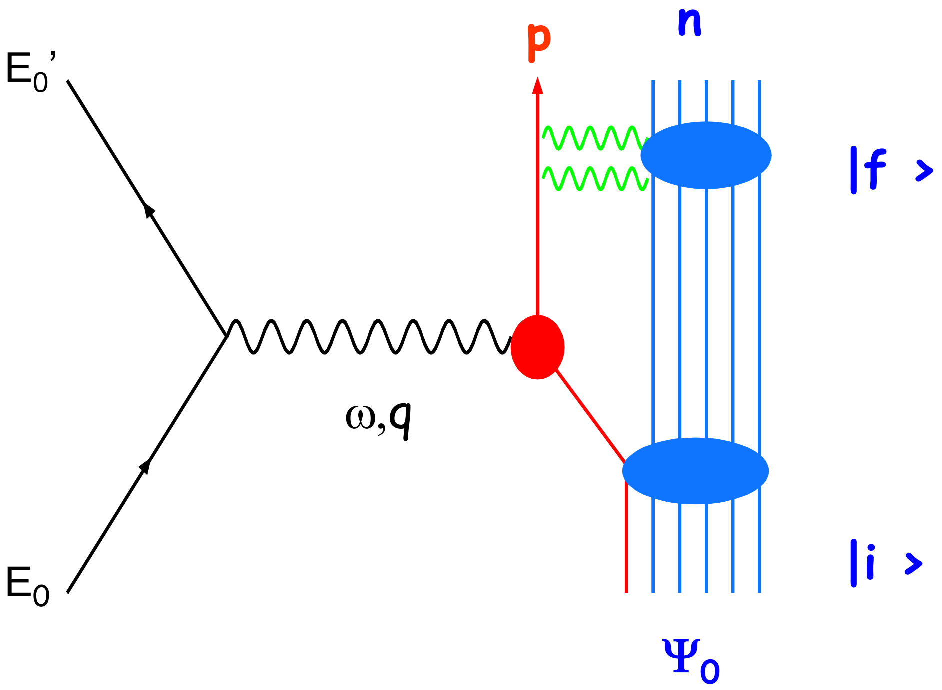
hadron tensor

$$W^{\mu\nu} = \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\mathrm{i}} - E_{\mathrm{f}})$$

$$J^{\mu}(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \boxed{\langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\mathrm{i}} \rangle} \mathrm{d}\vec{r}$$

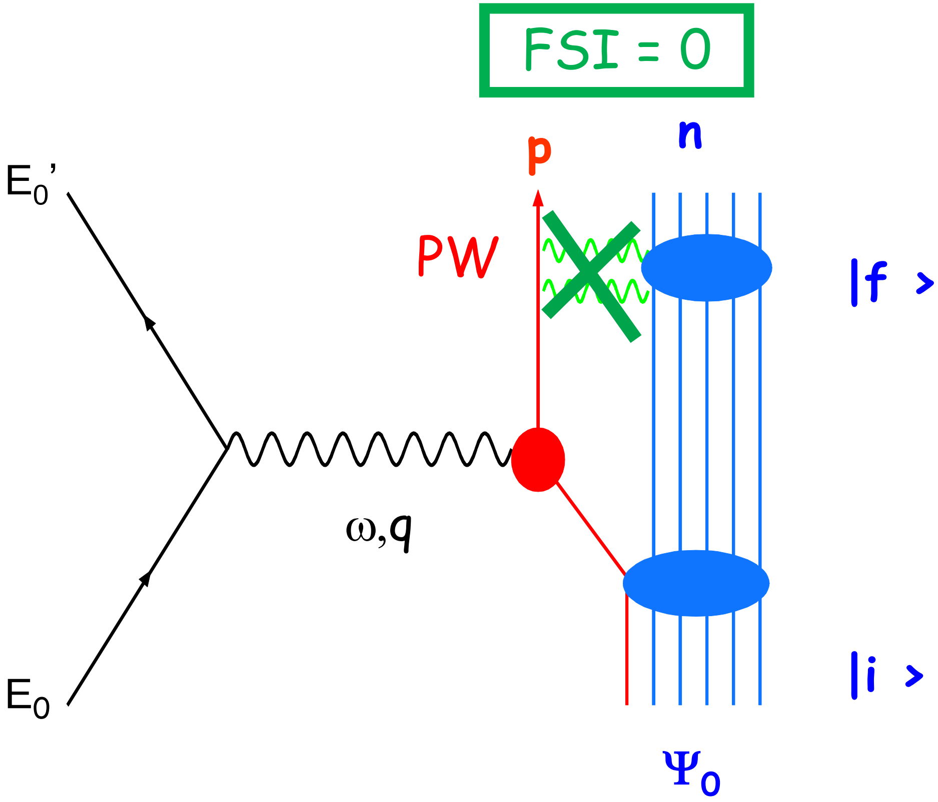
(e,e'p)

- exclusive reaction n
- DKO mechanism: the probe interacts through a one-body current with one nucleon which is then emitted the remaining nucleons are spectators
- impulse approximation IA



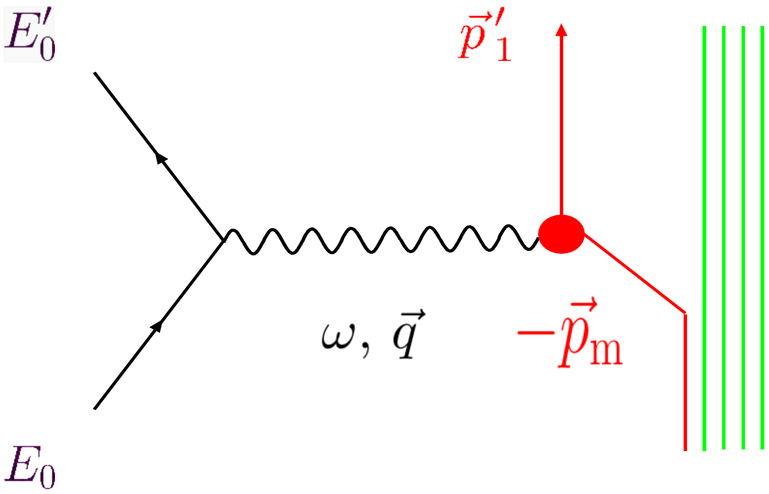
(e,e'p)

- exclusive reaction n
- DKO mechanism: the probe interacts through a one-body current with one nucleon which is then emitted the remaining nucleons are spectators
- impulse approximation IA



FSI=0

PW



PLANE-WAVE IMPULSE APPROXIMATION

PWIA

factorized cross section

$$\sigma = K\ \sigma_{\rm ep}\ \ S(E_{\rm m}, -\vec{p}_{\rm m})$$

spectral function

$$S(E_{\rm m}, -\vec{p}_{\rm m}) = \sum_n\ \lambda_n(E_{\rm m})\ \ |\phi_n(-\vec{p}_{\rm m})|^2$$

spectroscopic factor

overlap function

$$S(E_{\rm m}, -\vec{p}_{\rm m}) = \sum_n \lambda_n(E_{\rm m}) \ |\phi_n(-\vec{p}_{\rm m})|^2$$

spectroscopic factor

overlap function

For each E_m the mom. dependence of the SF is given by the mom. distr. of the quasi-hole states n produced in the target nucleus at that energy and described by the normalized OF

The norm of the OF, the spectroscopic factor gives the probability that n is a pure hole state in the target.

IPSM

ϕ_n s.p. SM state

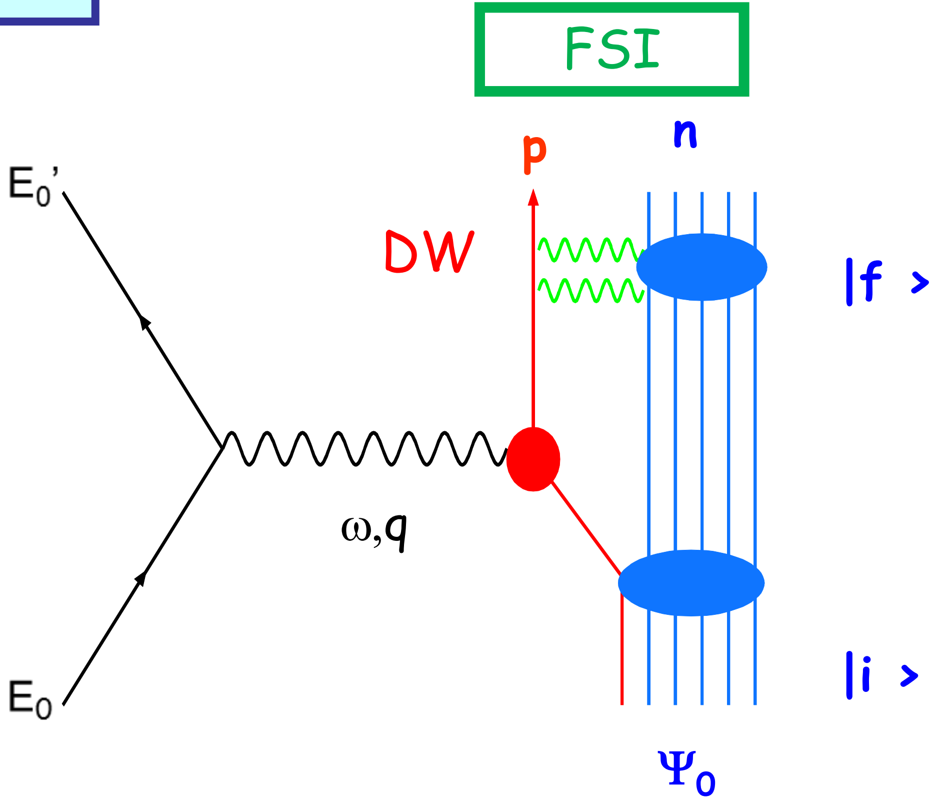
λ_n 1 occupied SM states

0 empty SM states

There are correlations and the strength of the quasi-hole state is fragmented over a set of s.p. states $0 \leq \lambda_n \leq 1$

DWIA (e,e'p)

- exclusive reaction n
- DKO IA
- FSI DWIA
- unfactorized c.s.
- non diagonal SF



$$\boxed{\langle f \mid J^{\mu}(\boldsymbol{q}) \mid i \rangle \Rightarrow \lambda_n^{1/2} \, \langle \chi_{\boldsymbol{p}}^{(-)} \mid j^{\mu}(\boldsymbol{q}) \mid \phi_n \rangle}$$

Direct knockout DWIA (e,e′p)

$$\lambda_n^{1/2}\ \langle\chi^{(-)} \mid j^{\mu} \mid \phi_n\rangle$$

- j^{μ} one-body nuclear current
- $\chi^{(-)}$ s.p. scattering w.f. $H^{+}(\omega+E_m)$
- ϕ_n s.p. bound state overlap function $H(-E_m)$
- λ_n spectroscopic factor
- $\chi^{(-)}$ and ϕ consistently derived as eigenfunctions of a Feshbach optical model Hamiltonian

DWIA-RDWIA calculations

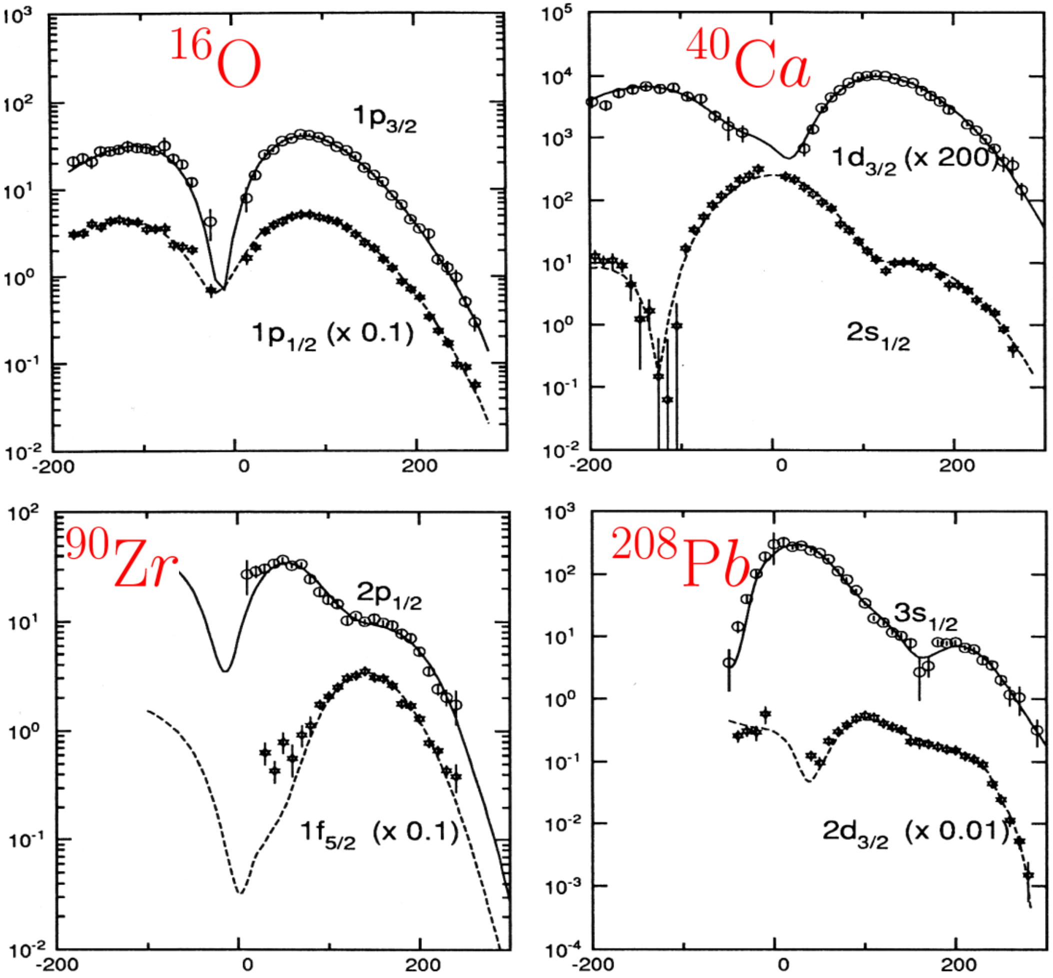
- phenomenological ingredients usually adopted
- $\chi^{(-)}$ phenomenological optical potential
- ϕ_n phenomenological s.p. wave functions WS, HF MF (some calculations including correlations are available)
- nonrelativistic (DWIA) relativistic (RDWIA) ingredients
- λ_n extracted in comparison with data: reduction factor applied to the calculated c.s. to reproduce the magnitude of the experimental c.s.

DWIA-RDWIA calculations

- phenomenological ingredients usually adopted
- $\chi^{(-)}$ phenomenological optical potential
- ϕ_n phenomenological s.p. wave functions WS, HF MF (some calculations including correlations are available)
- nonrelativistic (DWIA) relativistic (RDWIA) ingredients
- λ_n extracted in comparison with data: reduction factor applied to the calculated c.s. to reproduce the magnitude of the experimental c.s.

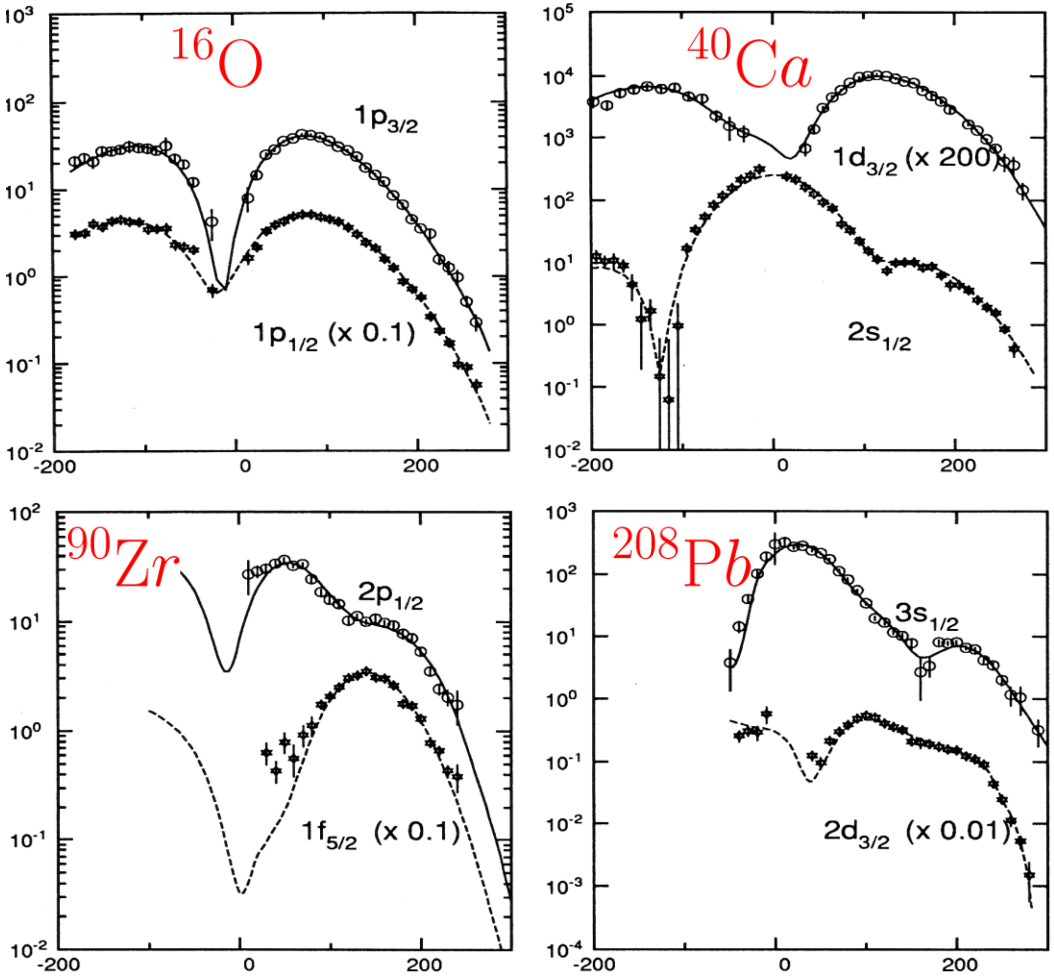
DWIA and RDWIA: excellent description of (e,e′p) data

Experimental data: p_m distributions



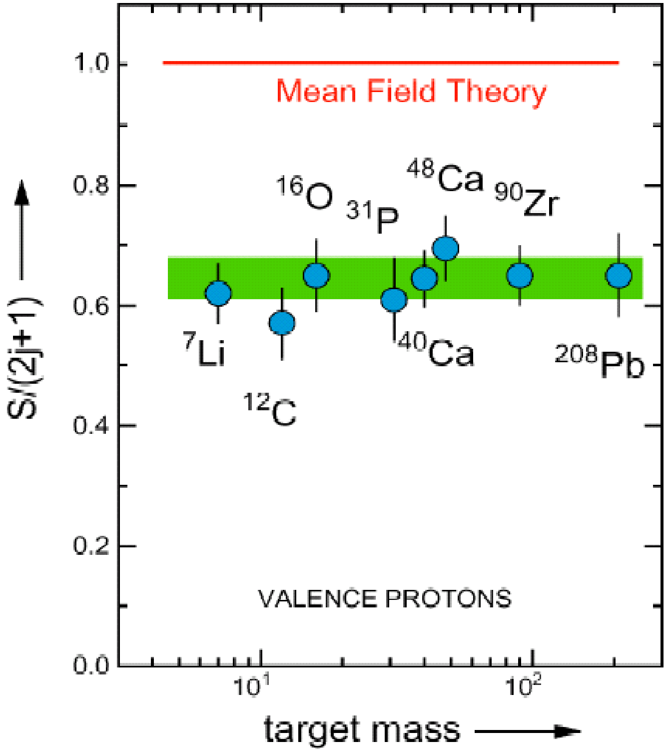
NIKHEF data & CDWIA calculations

Experimental data: p_m distributions



reduction factors applied:
spectroscopic factors

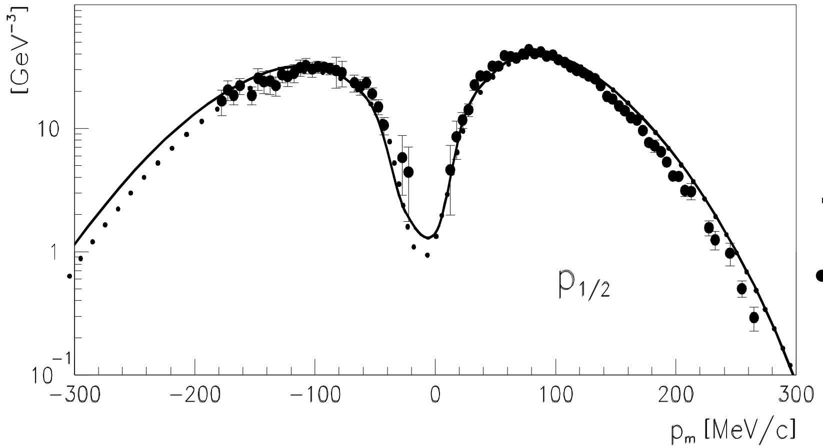
0.6 - 0.7



NIKHEF data & CDWIA calculations

Relativistic RDWIA

^{16}O(e,e'p)



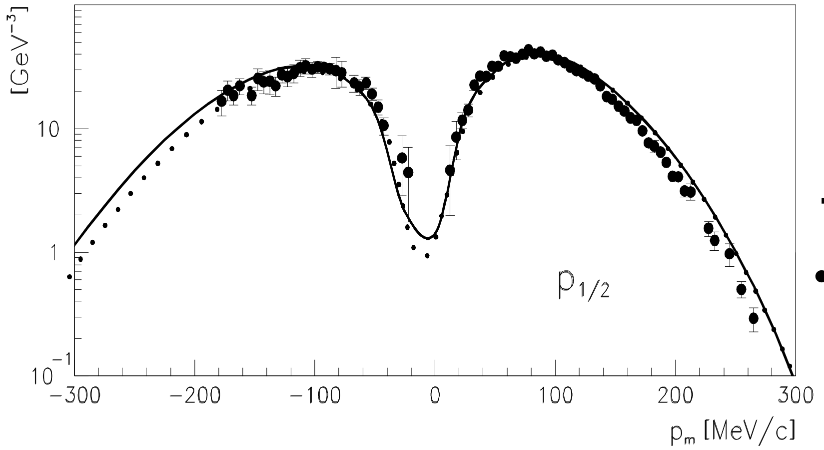
NIKHEF parallel kin E_0 = 520 MeV T_p = 90 MeV

— rel RDWIA

• • • • • • nonrel DWIA

Relativistic RDWIA

$^{16}O(e,e'p)$



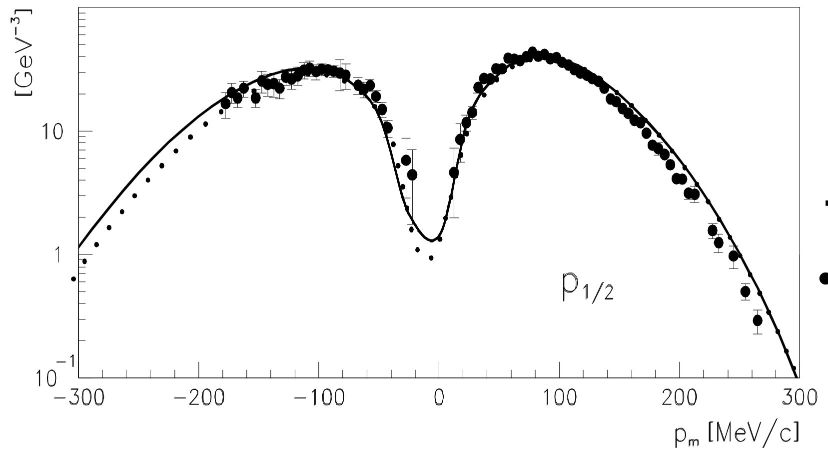
NIKHEF parallel kin $E_0 = 520$ MeV $T_p = 90$ MeV

—— rel RDWIA $\lambda_n = 0.7$

•••••• nonrel DWIA $\lambda_n = 0.65$

Relativistic RDWIA

^{16}O(e,e′p)

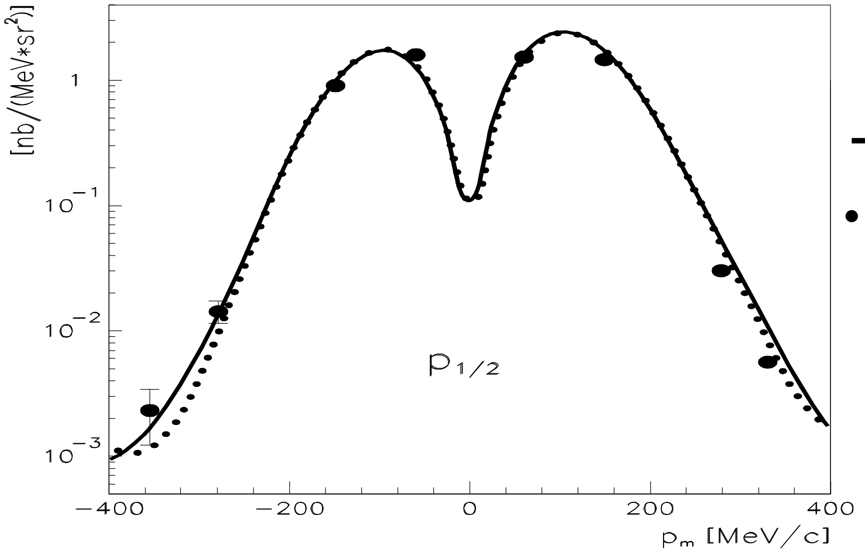


NIKHEF parallel kin E_0 = 520 MeV T_p = 90 MeV

——— rel RDWIA λ_n = 0.7

•••••• nonrel DWIA λ_n =0.65

JLab (ω,q) const kin E_0 = 2445 MeV ω =439 MeV T_p= 435 MeV

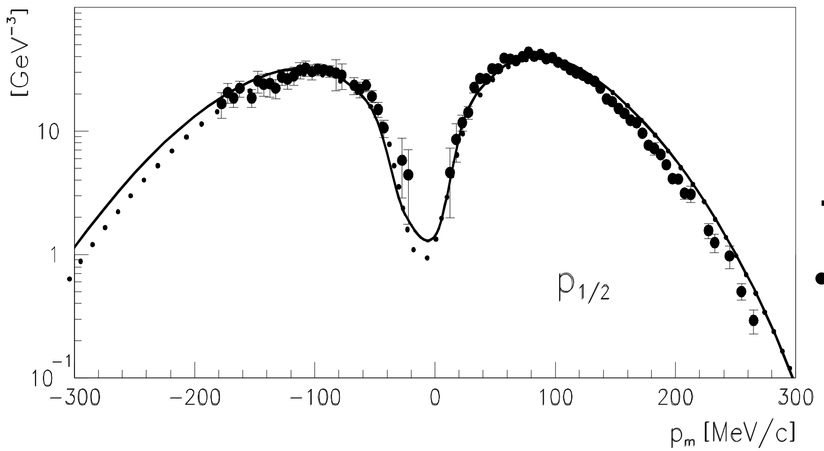


——— RDWIA diff opt.pot. λ_n = 0.7

••••••

Relativistic RDWIA

$^{16}O(e,e'p)$

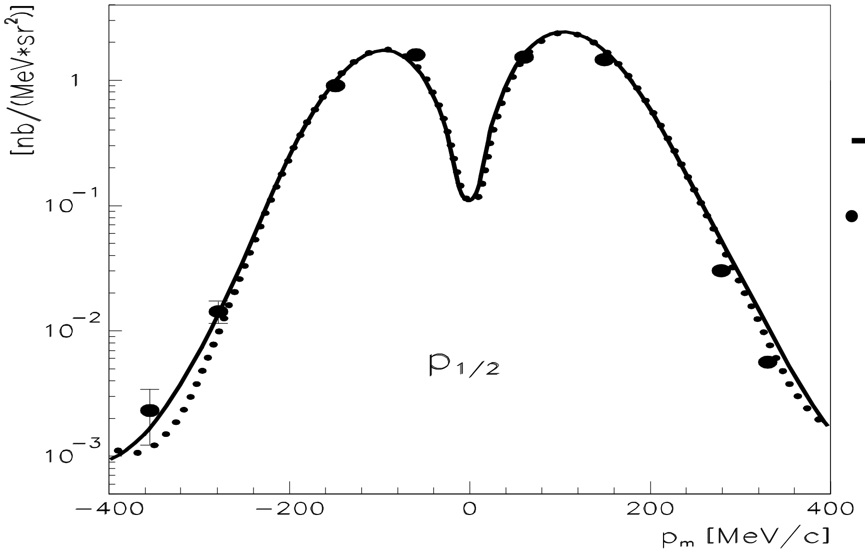


NIKHEF parallel kin E_0 = 520 MeV T_p = 90 MeV

—— rel RDWIA λ_n = 0.7

•••••• nonrel DWIA λ_n =0.65

JLab (ω,q) const kin E_0 = 2445 MeV ω =439 MeV T_p= 435 MeV



—— RDWIA diff opt.pot. λ_n = 0.7

••••••

(e,e′p)

- DWIA-RDWIA: DWIA with relativistic corrections cannot account for all effects of relativity
- bound and scattering states should be obtained from a microscopic many-body calculations. Recent microscopic calculations of the spectral function and optical potential within a NR framework
- Experiments on nuclei of interest for neutrino experiments very useful
- Different kinematics to test theoretical models and investigate contributions sensitive to the kin. conditions
- Polarisation experiments give access to information not available from unpolarised c.s. measurements

QE e-nucleus scattering

$$e + A \Longrightarrow e' + N + (A-1)$$

- only e' detected inclusive (e,e')

QE e-nucleus scattering

$$e + A \Longrightarrow \enclose{circle}{e'} + N + (A-1)$$

- only e' detected inclusive (e,e')

CCQE ν-nucleus scattering

$$\nu_l(\bar{\nu}_l) + A \Longrightarrow l^-(l^+) + N + (A-1)$$

QE e-nucleus scattering

$$e + A \Longrightarrow \boxed{e'} + N + (A - 1)$$

- only e' detected inclusive (e,e')

CCQE ν-nucleus scattering

$$\nu_l(\bar{\nu}_l) + A \Longrightarrow \boxed{l^-(l^+)} + N + (A - 1)$$

- only final lepton detected inclusive CC

QE e-nucleus scattering

$$e + A \Longrightarrow \boxed{e'} + N + (A - 1)$$

- only e' detected inclusive (e,e')

CCQE ν-nucleus scattering

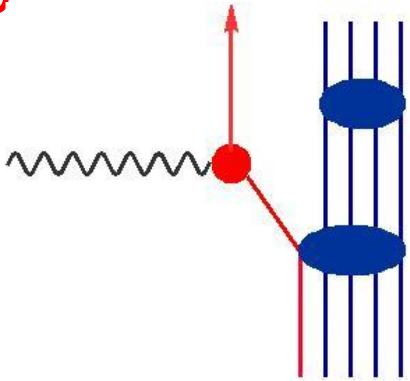
$$\nu_l(\bar{\nu}_l) + A \Longrightarrow \boxed{l^-(l^+)} + N + (A - 1)$$

- only final lepton detected inclusive CC
- same model as for inclusive (e,e')

IMPULSE APPROXIMATION

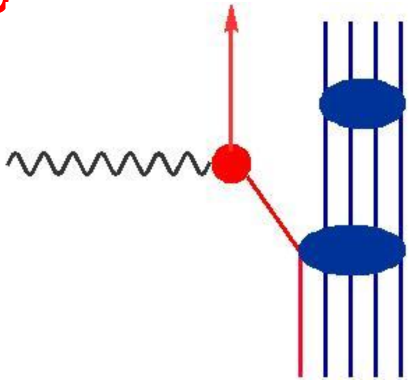
IMPUSE APPROXIMATION

- EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO



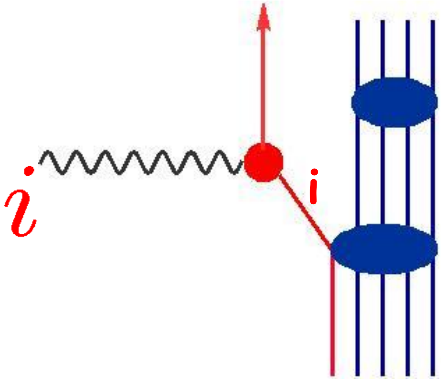
IMPULSE APPROXIMATION

- EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO



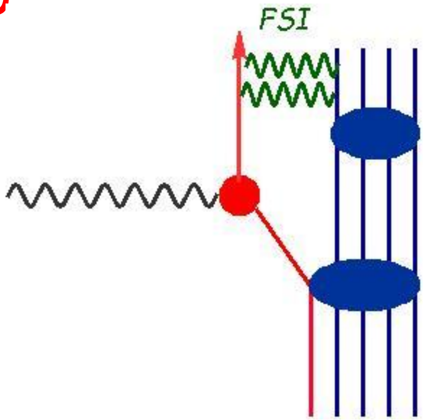
- INCLUSIVE SCATTERING: c.s given by the sum of integrated direct 1NKO over all the nucleons

$\sum_i$



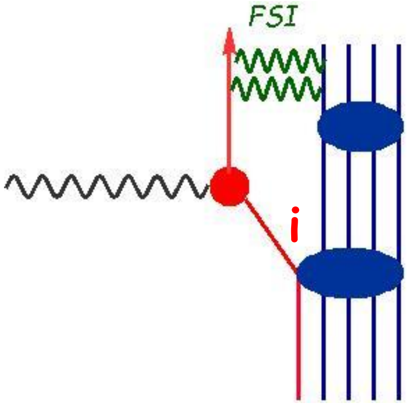
IMPULSE APPROXIMATION

- EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO



- INCLUSIVE SCATTERING: c.s given by the sum of integrated direct 1NKO over all the nucleons

$\sum_i$



FINAL-STATE INTERACTION between the emitted nucleon and the residual nucleus

EXCLUSIVE SCATTERING: FSI

DWIA

FSI described by a complex OP, the imaginary part gives a reduction of the calculated c.s. which is essential to reproduce (e,e’p) data

EXCLUSIVE SCATTERING: FSI

DWIA

FSI described by a complex OP, the imaginary part gives a reduction of the calculated c.s. which is essential to reproduce (e,e′p) data

INCLUSIVE SCATTERING: FSI

DWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part conceptually wrong because the flux is not conserved

INCLUSIVE SCATTERING: RGF

Green's Function Model (GF or RGF)

FSI are accounted for by the complex energy dependent OP: the formalism translates the flux lost toward inelastic channels, represented by the Im part of the OP, into the strength observed in inclusive reactions.

The OP is responsible for the redistribution of the flux in all the final-state channels and in the sum over all the channels the flux is conserved.

The OP becomes a powerful tool to include important inelastic contributions not included in other models based on the IA

FSI for the inclusive scattering : Green's Function Model

- with suitable approximations (basically related to the IA) the components of the inclusive response can be written in terms of the s.p. optical model Green's function
- the explicit calculation of the s.p. GF can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of the eigenfunctions of the non Herm optical potential V and V^+
- matrix elements similar to RDWIA
- scattering states eigenfunctions of V and V^+ (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved
- in each channel flux is lost towards other channels and flux is gained due to the flux in the other channels just toward the considered channel

Relativistic Green's Function Model

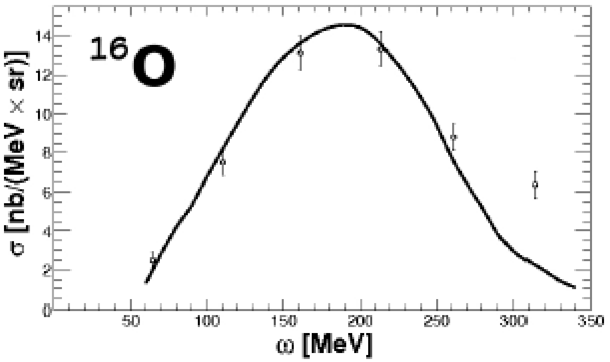
- the imaginary part of the OP includes inelastic channels, contributions beyond 1NKO (rescattering, multi-nucleon, non-nucleonic contributions…) not included in usual models based in the IA

- energy dependence of the OP reflects the different contribution of the different inelastic channels open at different energies, results sensitive to the kinematic conditions

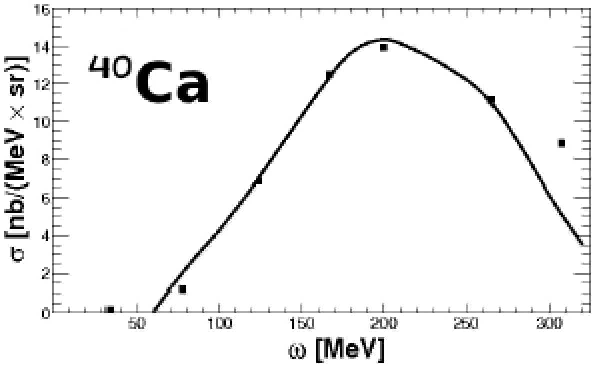
- inelastic channels more important in neutrino scattering

(e,e')

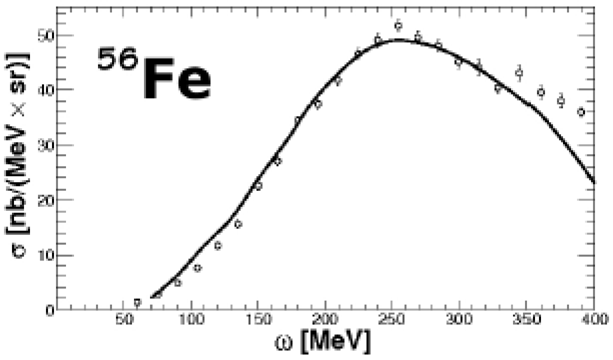
RGF



E_0 = 1080 MeV ϑ = 32°

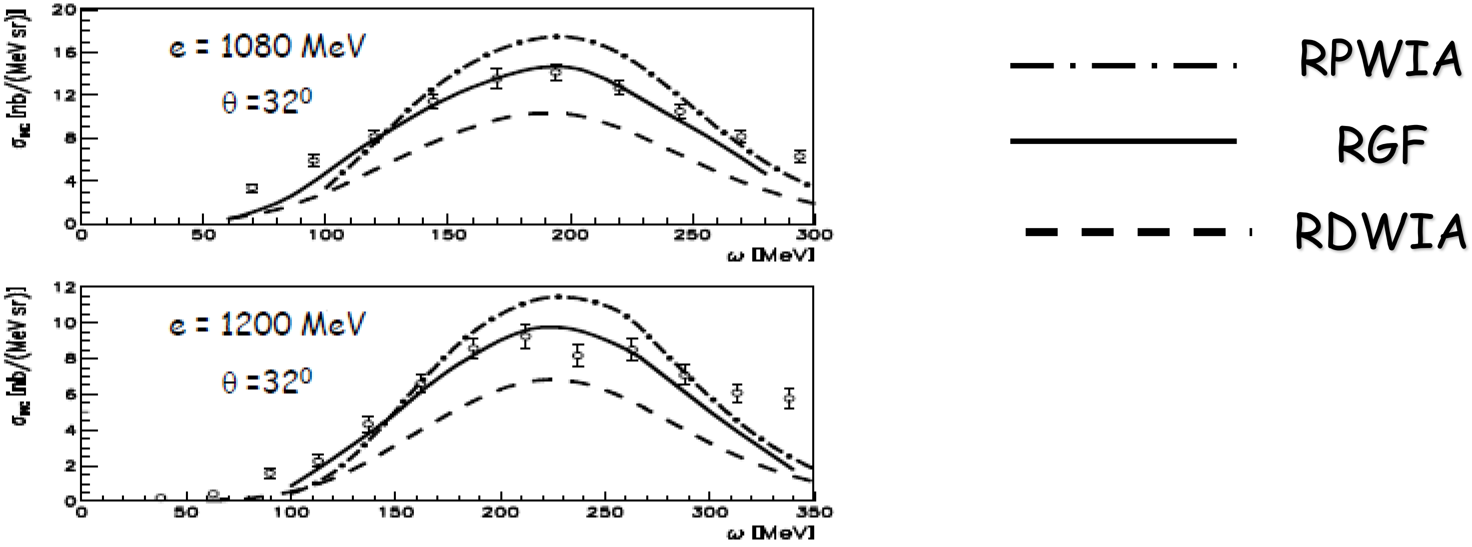


E_0 = 841 MeV ϑ = 45.5°



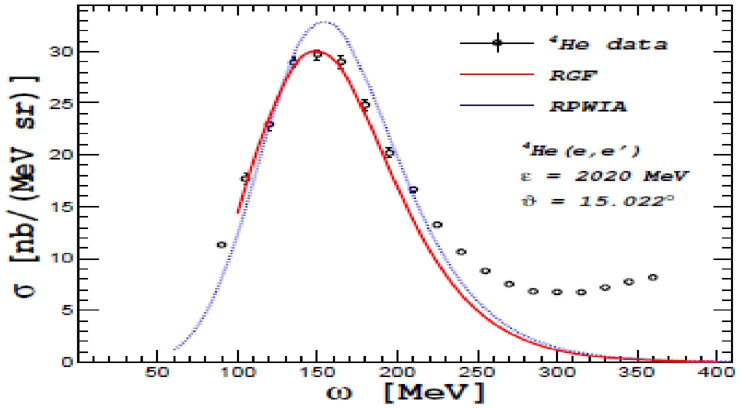
E_0 = 2020 MeV ϑ = 20°

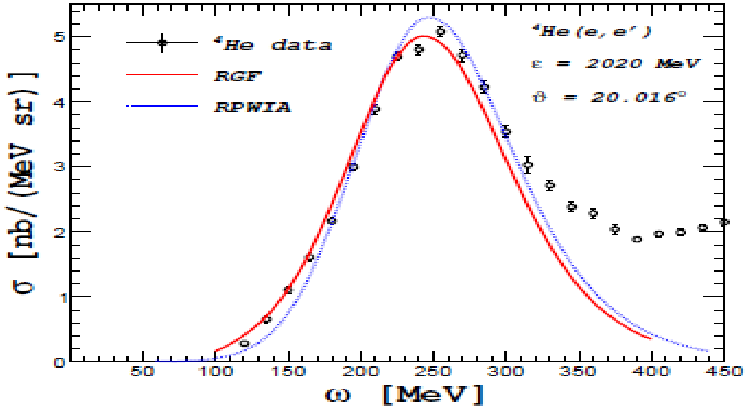
$^{16}O(e,e')$

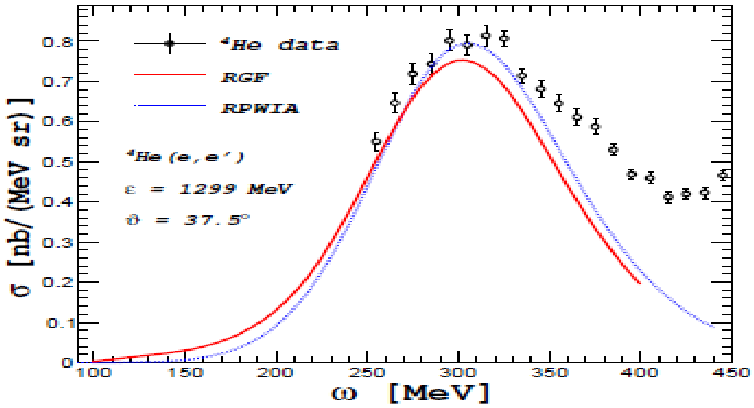


data from Frascati NPA 602 405 (1996)

^{4}He(e,e')





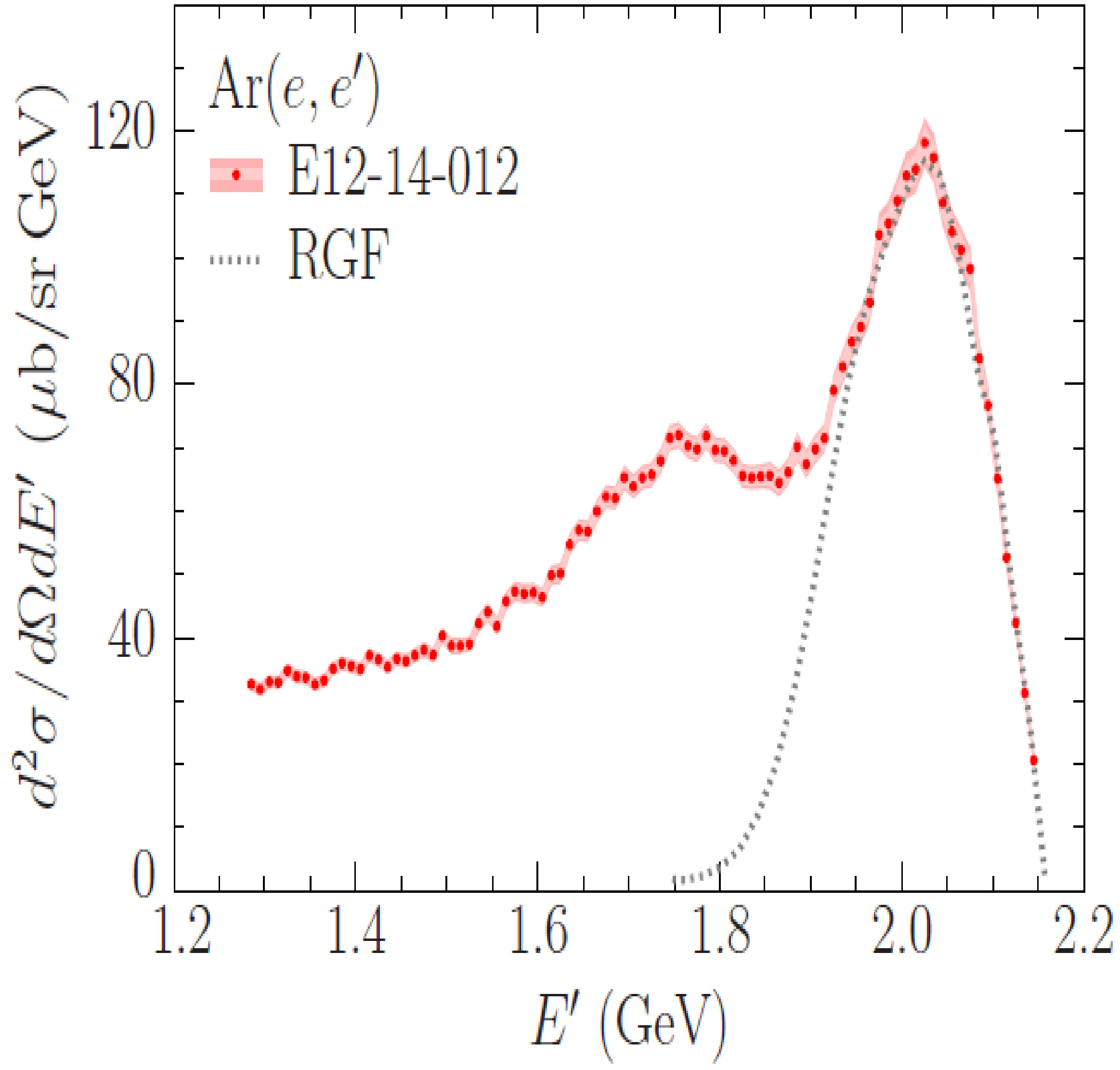


RGF

RPWIA

^{40}Ar(e,e')

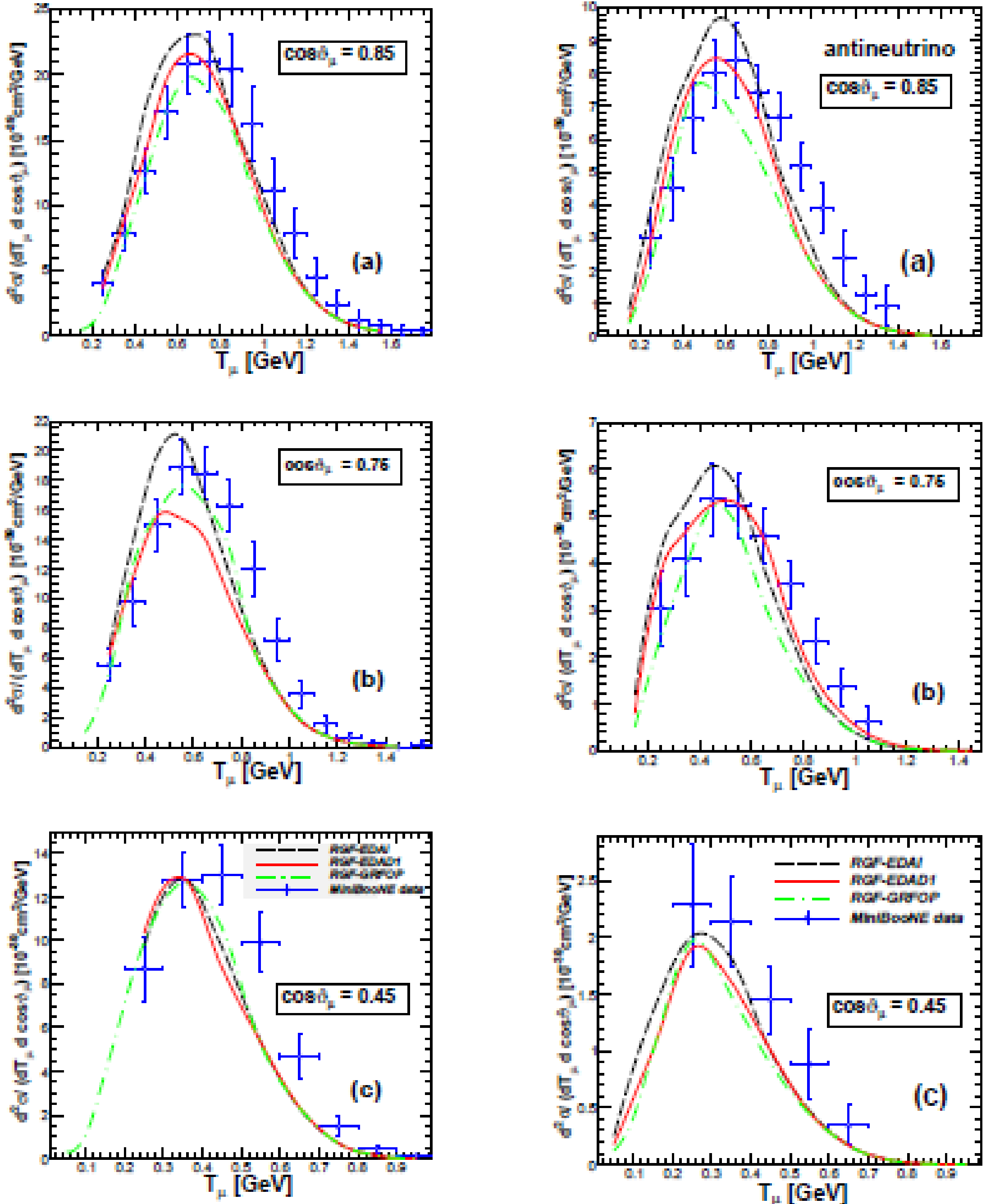
new data from JLab



H. Dai et al. in preparation

$^{12}C(\nu_\mu, \mu^-)$ $^{12}C(\bar{\nu}_\mu, \mu^+)$

MiniBooNe CCQE data



RGF-EDAI

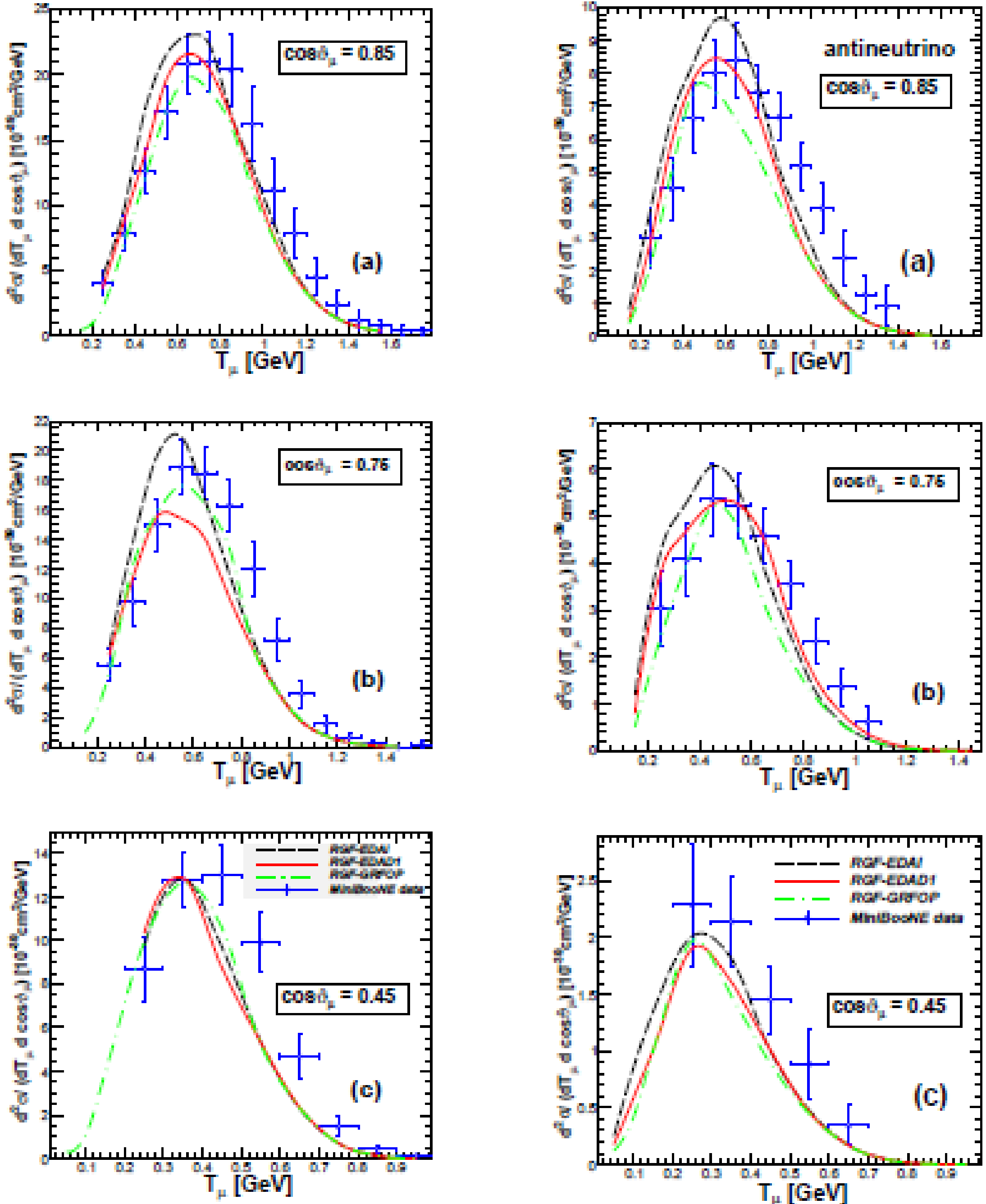
RGF-EDAD1

RGF-GRFOP

M.V. Ivanov et al. PRC 94 014608 (2016)

MiniBooNe CCQE data

$^{12}C(\nu_\mu, \mu^-)$ $^{12}C(\bar{\nu}_\mu, \mu^+)$



different ROPs available for the calculations, with different Im parts

RGF-EDAI

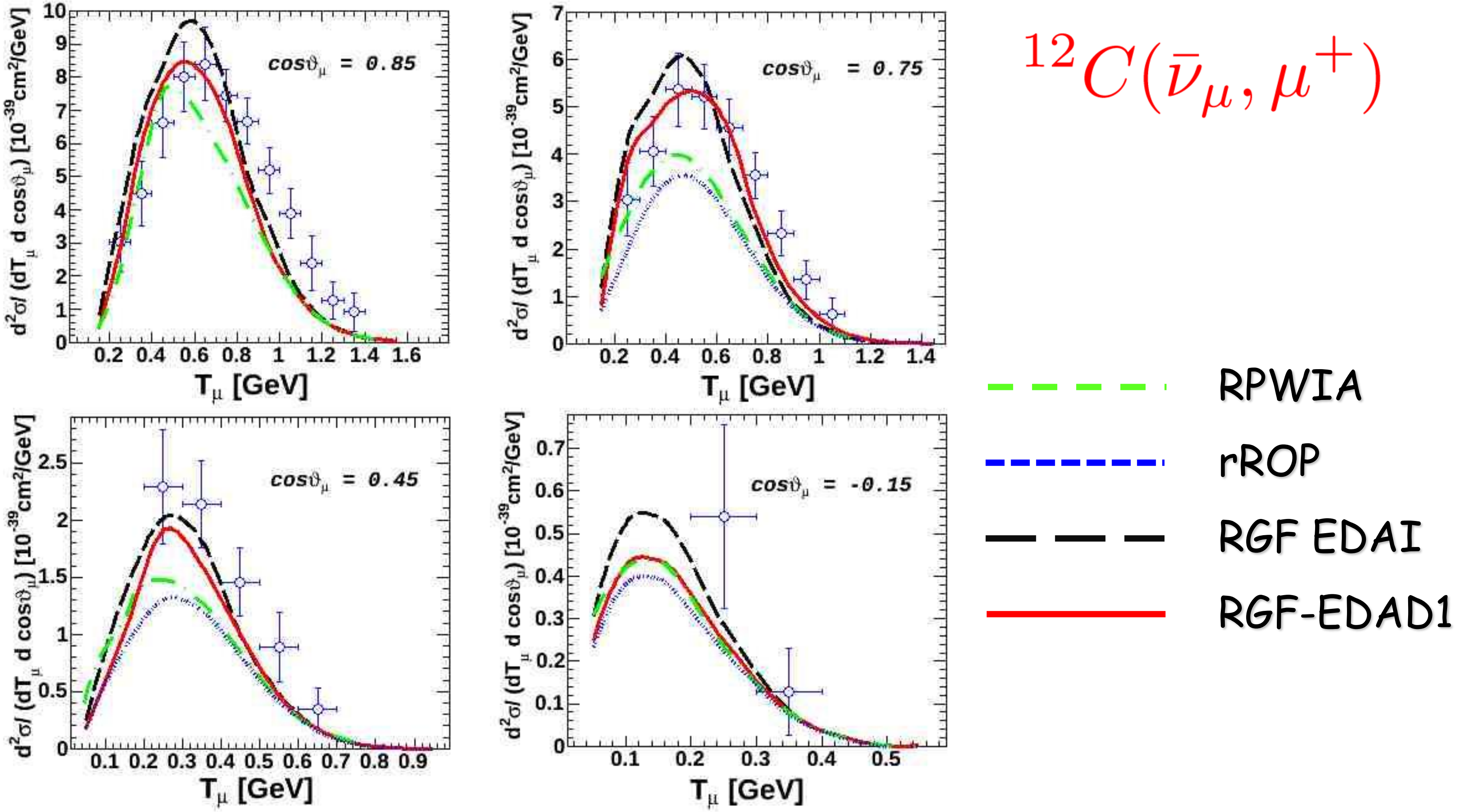
RGF-EDAD1

RGF-GRFOP

M.V. Ivanov et al. PRC 94 014608 (2016)

Comparison with MiniBooNe CCQE data

$^{12}C(\bar{\nu}_\mu, \mu^+)$



A. Meucci and C. Giusti PRD 85 (2012) 093002

CONCLUSIONS

- RGF: in many cases good agreement with (e,e′), CCQE (and NCE) data
- RGF: more theoretical OP would improve the theoretical content of the model
- RGF: MEC non included, a new consistent model required
- comparison of different theoretical models: helpful to test the models and keep all nuclear effects under control, the role of a specific effect or contribution depends on the model
- new (e,e′) experiments: nuclei of interest for neutrino experiments and in different kinematics useful to test the theoretical models