Results for Few-Body electromagnetic processes

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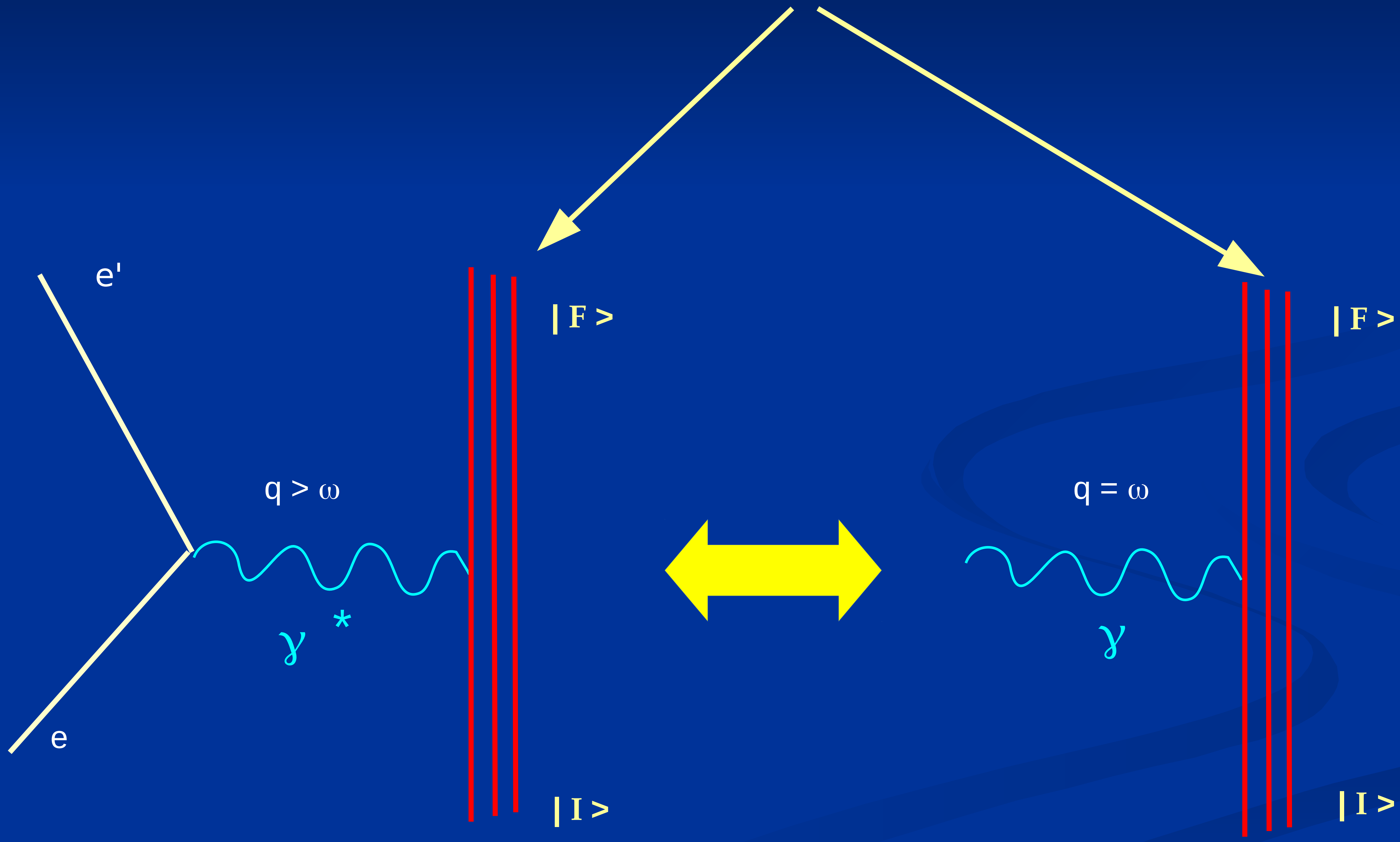
UNIVERSITÀ DEGLI STUDI DI TRENTO

INFN

SFB workshop on

"**Electromagnetic observables for low-energy nuclear physics**"

Physics of e.m. Interactions with Nuclei



What can one learn from a comparison theory-experiment?

Which questions might such a comparison answer ?

General features of e.m. interaction with nuclei

The e.m. interaction is **perturbative** compared to the nuclear **strong** interaction

$$H = \mathbf{H}_{\mathbf{N}} + \mathbf{V}_{\mathbf{em}}$$

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$$\sigma \sim |\langle F | V_{em} | I \rangle|^2$$

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- **V_{em} is the e.m interaction between the electron and the *e.m property of the relevant d.o.f.***

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- **| I > is a bound state (g.s.)**
- **| F > can be a bound or a continuum (scattering) state**

What can one learn from a comparison theory-experiment?

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Related questions one might answer:

1: Are Protons and Neutrons the relevant (effective) degrees of freedom of H_N? *"free-like"? "modified"?*

(question and answers are kinematics (scale) dependent!)

2: how do Protons and Neutrons interact? In the same way as it appears from NN scattering data? (*Off-shell part of the potential?)*

What can one learn from a comparison theory-experiment?

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Related questions one might answer:

1: **Are Protons and Neutrons the only relevant (effective) degrees of freedom also for V_{em} ? i.e. are there other currents on play ?**

2: **How do such additional currents look like? How are they connected to H_N (is charge conservation enough? Do we need more??)**

What can one learn from a comparison theory-experiment?

$$\sigma \sim |\langle F | \, V_{em} \, | I \rangle|^2$$

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Related questions one might answer:

1: given the relevant d.o.f., which is the appropriate *technique* to calculate the ground state wave function?

2: Are we able to control the possible necessary approximations, i.e. to estimate the accuracy?

3: Is the estimated accuracy compatible to the experimental one?

What can one learn from a comparison theory-experiment?

$$\sigma \sim |<F|\ V_{em}\ |I>|^{2}$$

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- | F > can be a bound or a continuum (scattering) state

Related questions one might answer:

1: given the relevant d.o.f. are we able to calculate the many body scattering state

2: Are we able to control necessary approximations, namely to estimate the accuracy?

3: how does the estimated accuracy on | F > reflects on the accuracy of the observable?

4: are experimental and theoretical accuracy compatible ?

Other general questions:

- What are the limits in q and ω of the non-relativistic framework?
- Is first order perturbation theory sufficient?
- Given the fact that the "low momentum transfer q" (ω < q, therefore "low energy" ω !) corresponds to large λ, i.e. the virtual photon probes long range properties, do we see the emergence of a collective behaviour from a pure MIcroscopic description in terms of A interacting protons and neutrons?
- When can an "experimental bump" be interpreted as a "collective behaviour" or a "quantum mechanical resonance"?

Low-energy/momentum observables

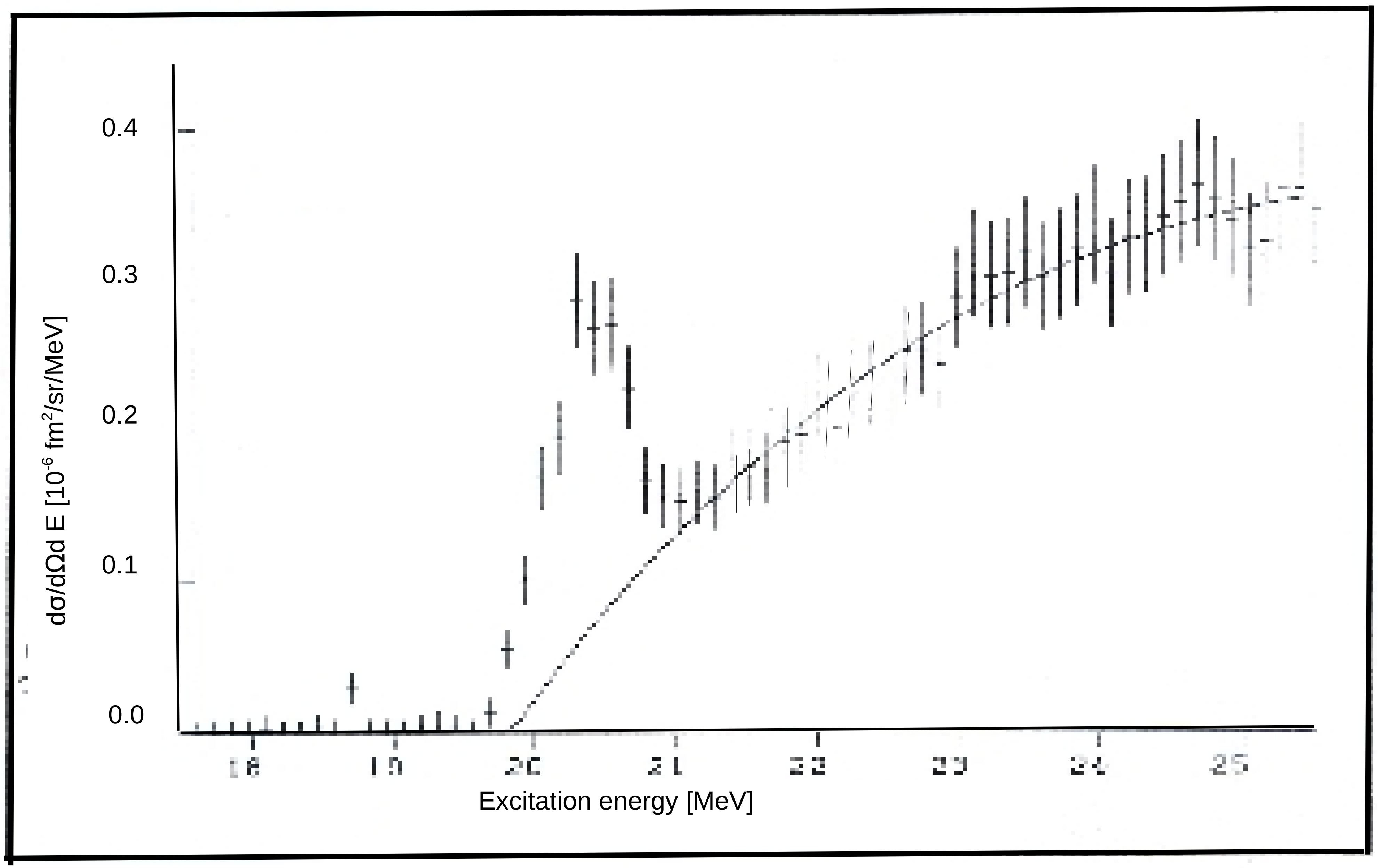
Example N.1:

0^+ "Resonance" in ^{4}He

0^+ Resonance in ^{4}He

Position at E_R = 20.1 MeV, (i.e. **above** the ^{3}H-p threshold)
Γ = 270±70 keV - first seen in hadronic [p-^{3}H, T(d,pn)T] reactions,
Strong evidence in (e,e') scattering

G. Koebschall et al./ Quasi bound state in ^{4}He - Nucl. Phys. A405, 648 (1983)



The (e,e') cross section

$$\frac{d\sigma}{d\omega d\Omega} = V_L(q,\omega)|< F | \rho | I >|^2 + V_T(q,\omega) |< F | J_T | I >|^2$$

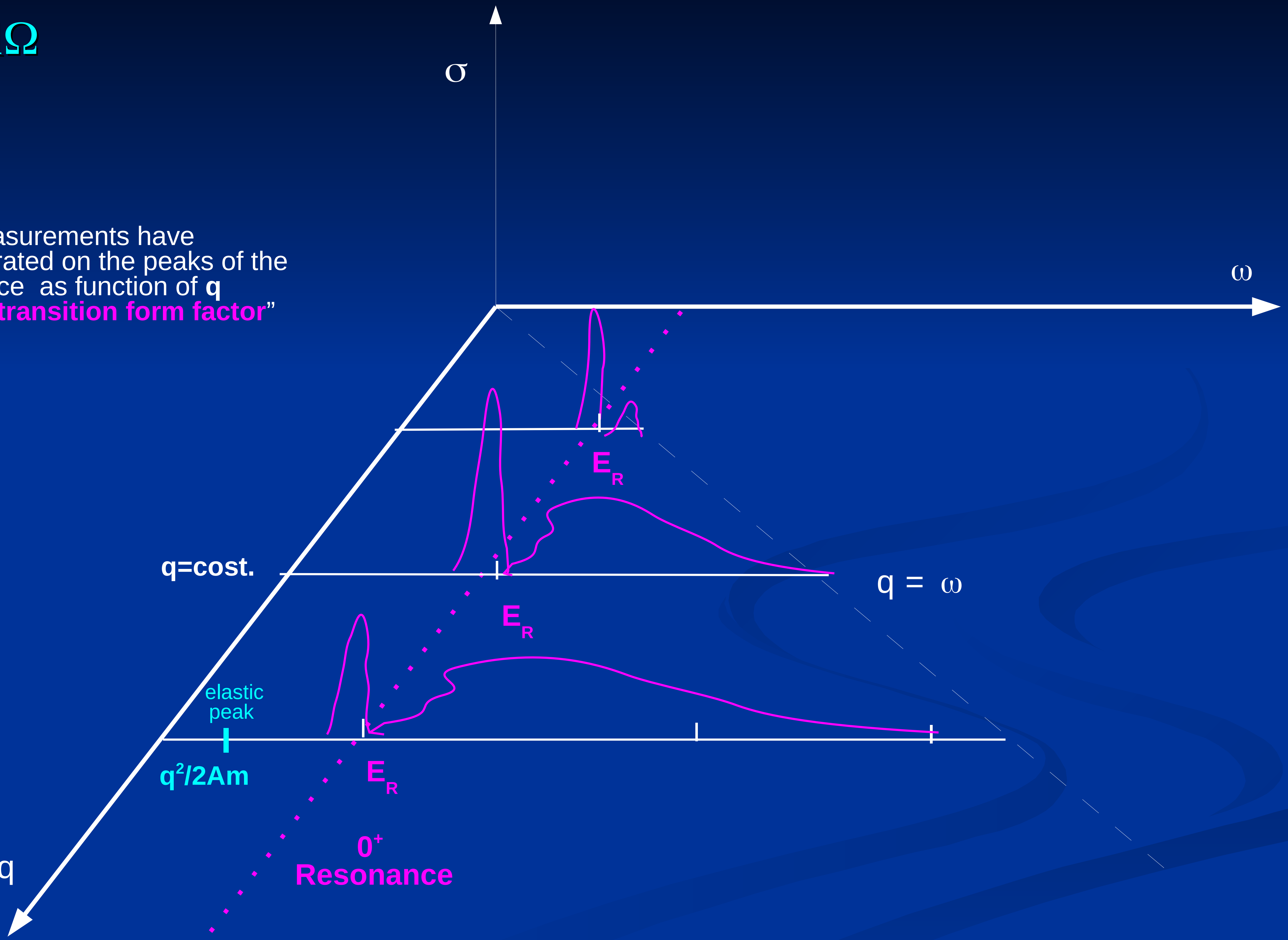
$< F |$ inclusive

$$\mathbf{S(q, \omega)} = \Sigma_n |<n| \rho |0>|^2 \delta(\omega - E_n + E_0)$$

Electron scattering

$$\frac{d\sigma}{d\omega d\Omega} = V_L(q,\omega)\,|< F \,|\, \rho \,|\, I >|^2 + V_T(q,\omega)\,|< F \,|\, J_T \,|\, I >|^2$$

The measurements have concentrated on the peaks of the resonance as function of **q**
i.e. the "**transition form factor**"



The 0^+ resonance of ^{4}He is a typical isoscalar monopole (C0) excitation

Isoscalar monopole excitation operator

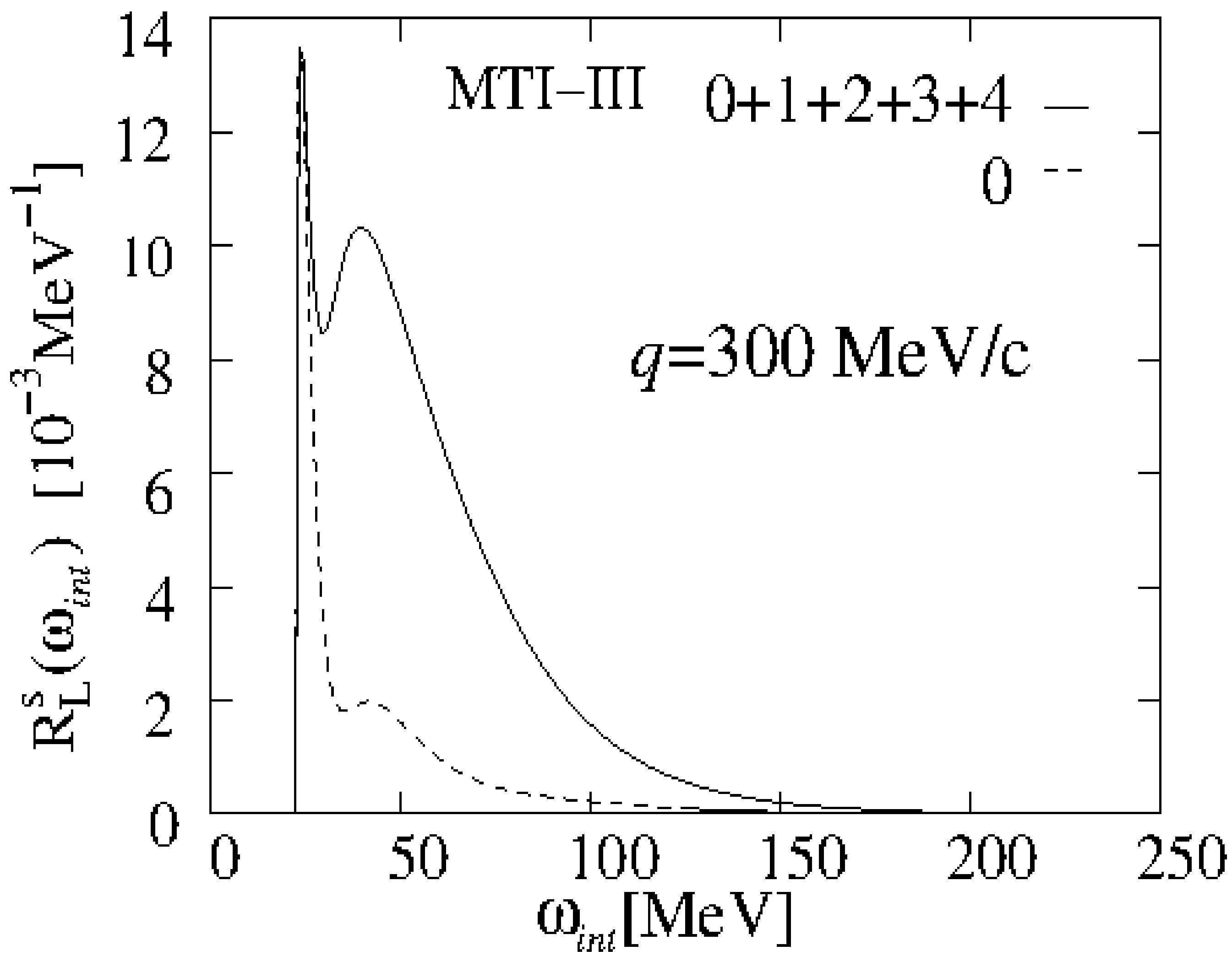
$$\frac{d\sigma}{d\omega d\Omega} = V_L(q,\omega)|\langle F|\, \rho \,|I\rangle|^2 + V_T(q,\omega)\,|\langle F|\; J_T \;|I\rangle|^2$$

$\langle F|$ inclusive

$$S_M(q,\omega) = \sum_n |\langle n|\, C0(q)\, |0\rangle|^2\, \delta(\omega - E_n + E_0)$$

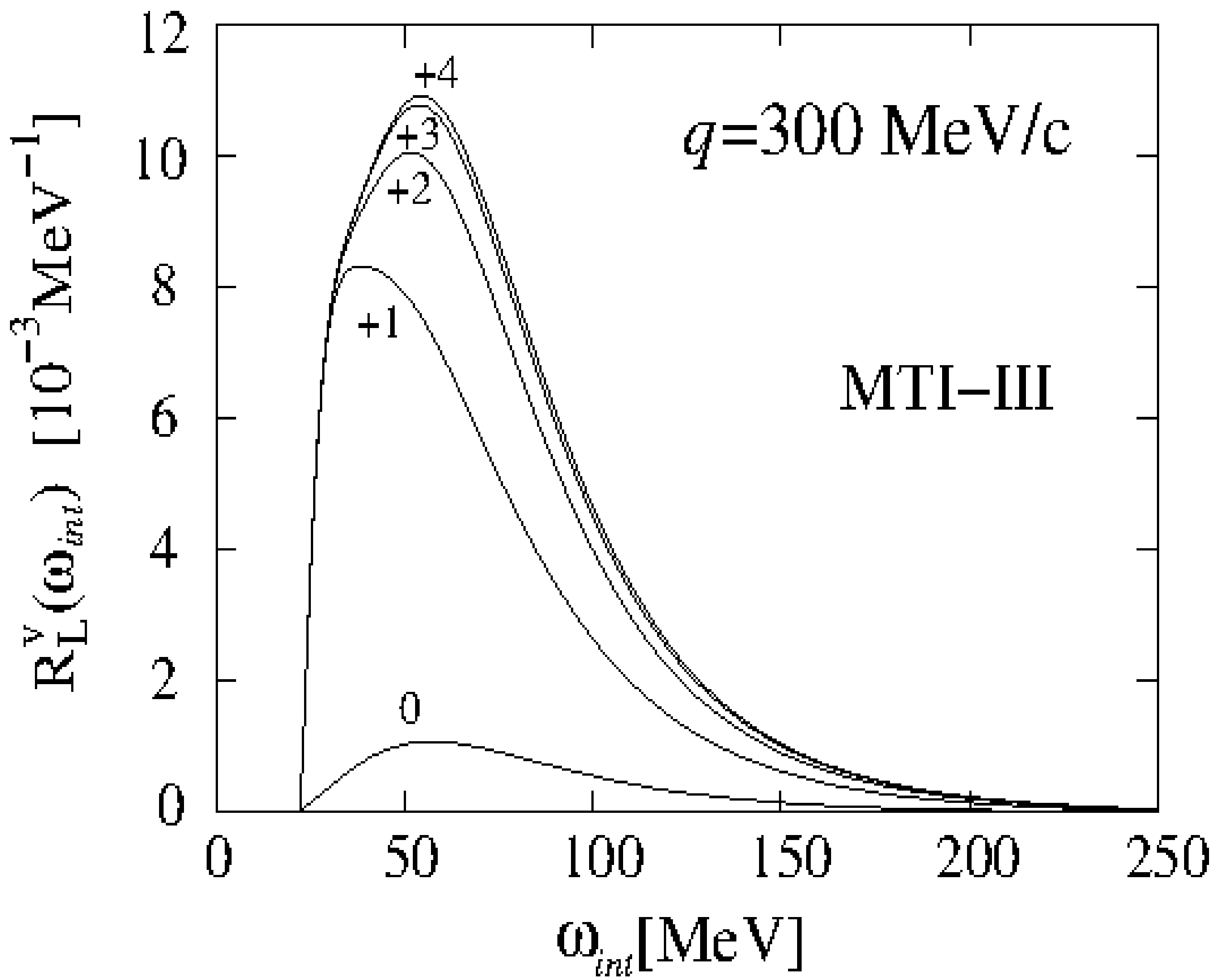
$$\sum_i^A j_0(q\, r_i)\, Y_{00}$$

$$\rho = \sum_i^A e^{i\mathbf{q}\cdot\mathbf{r}_i}\, \frac{1+\tau_i^3}{2}$$



Bacca et al. Phys.Rev.C76:014003(2007)

isoscalar

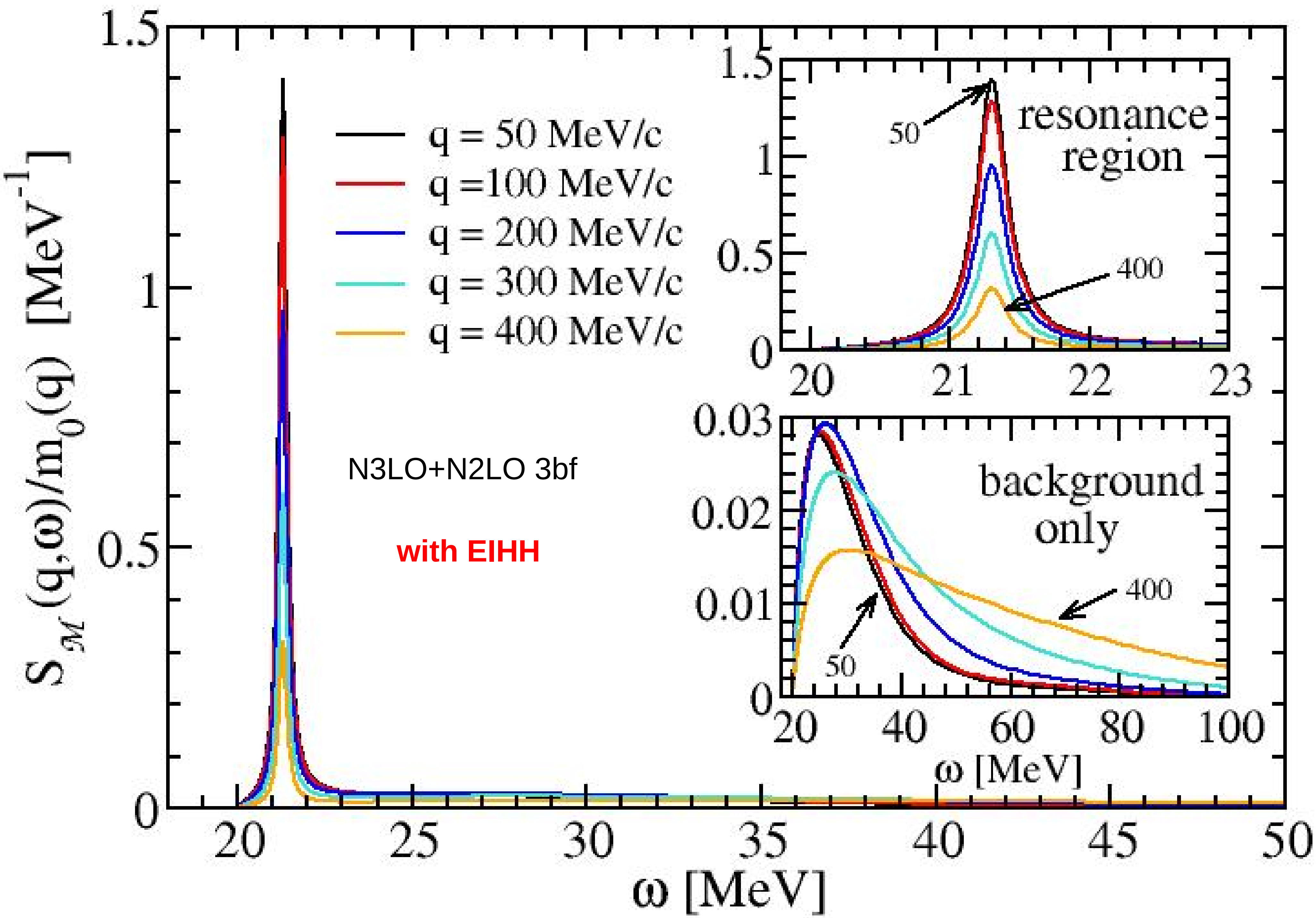


isovector

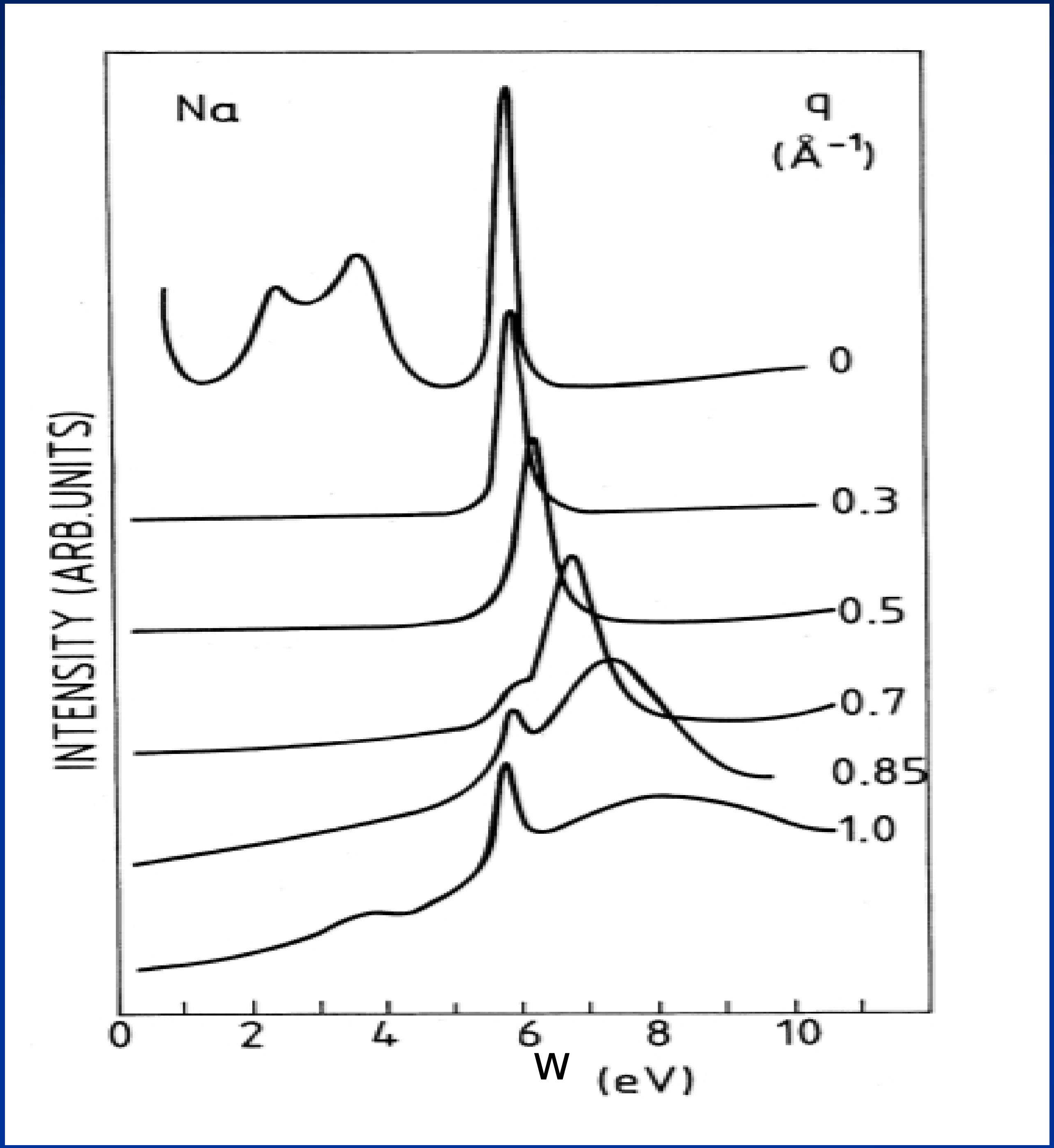
Negligible !

Monopole response as function of ω

S.Bacca et al. PRL 110 042503 (2013)



Similar behaviour in a many-body system: the plasmon



The many-body system:
valence-electron gas
in metals

The dynamical structure function
$S(q,\omega)$
presents a **prominent** peak at **low q**.

For **increasing q** the peak becomes **less and less** pronounced
on an **increasing background**

More later!

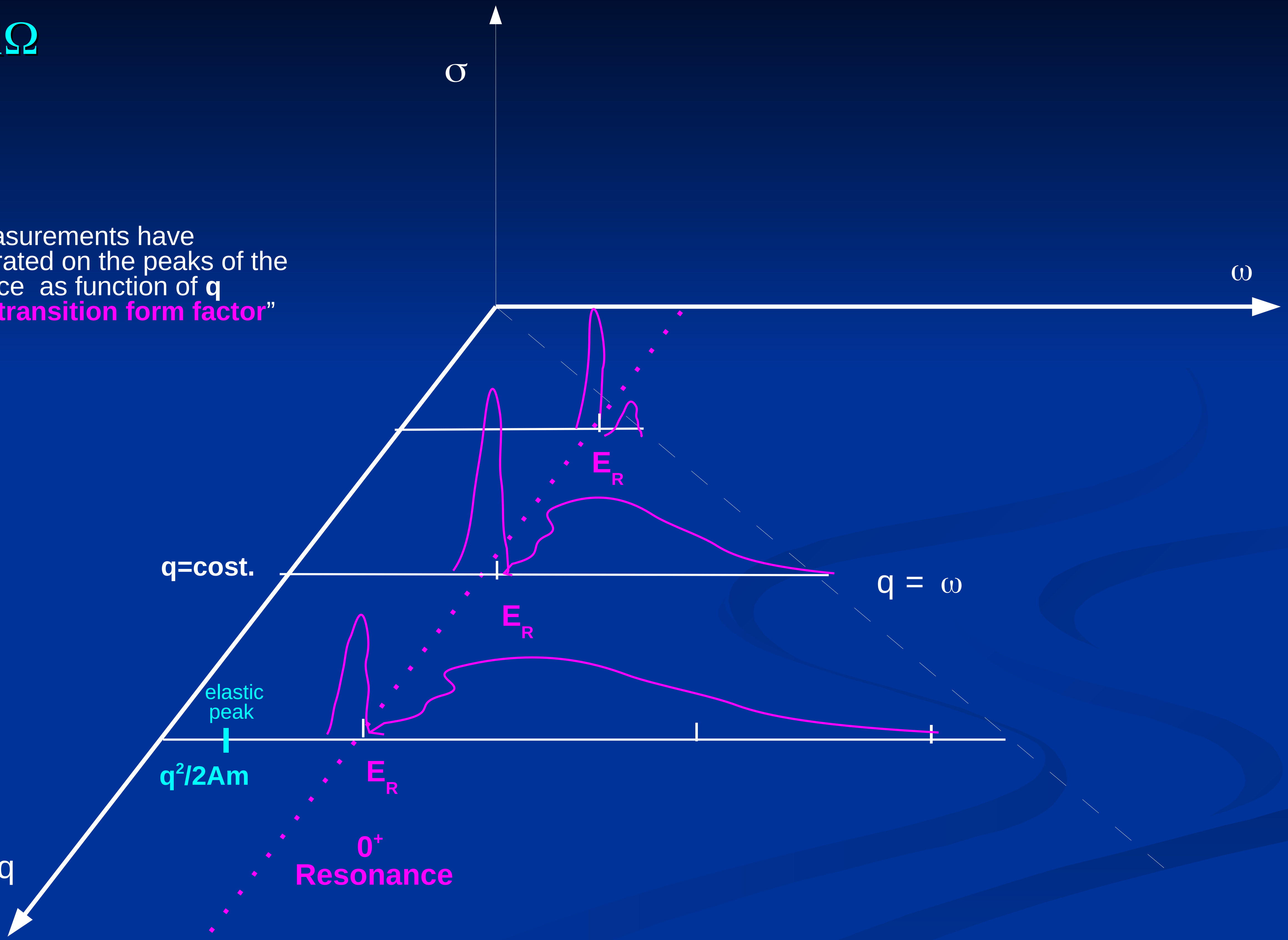
An interesting aspect of this resonance: its transition form factor as a "prism" of nuclear potentials

In S.Bacca et al. PRL 110 042503 (2013), $S_M(q,\omega)$ was calculated via the Lorentz Integral Transform **(LIT)** **m**ethod and looked in particular at the **transition form factor** or two different realistic potentials (**N3LO+N2LO 3bf, AV18 +UIX**)

Electron scattering

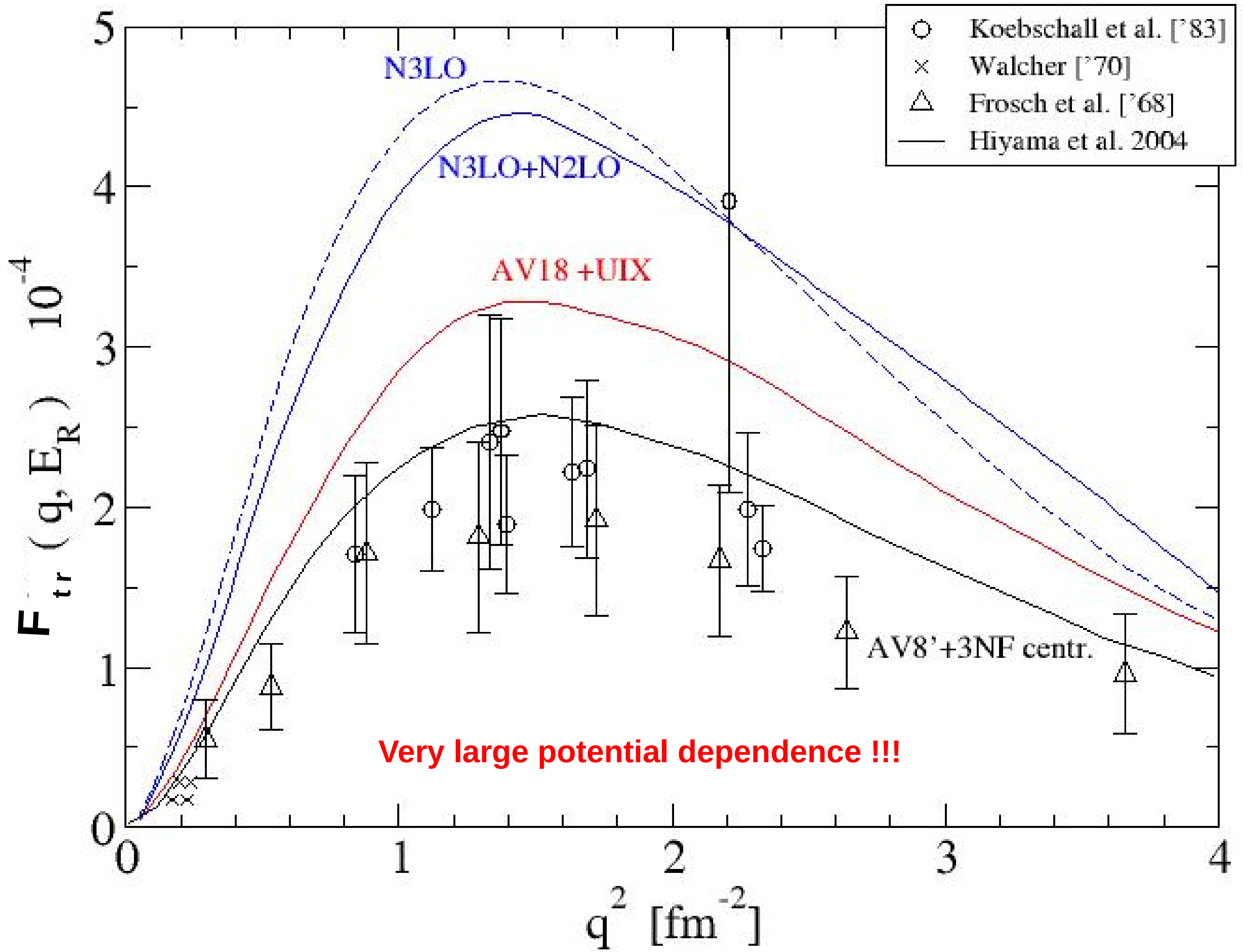
$$\frac{d\sigma}{d\omega d\Omega} = V_L(q,\omega)\,|\langle F\,|\,\rho\,|\,I\rangle|^2 + V_T(q,\omega)\,|\langle F\,|\,J_T\,|\,I\rangle|^2$$

The measurements have concentrated on the peaks of the resonance as function of **q** i.e. the "**transition form factor**"



Transition form factor$|<E_R|\ C0(q)\ |0>|^2$

S.Bacca et al. PRL 110 042503 (2013)

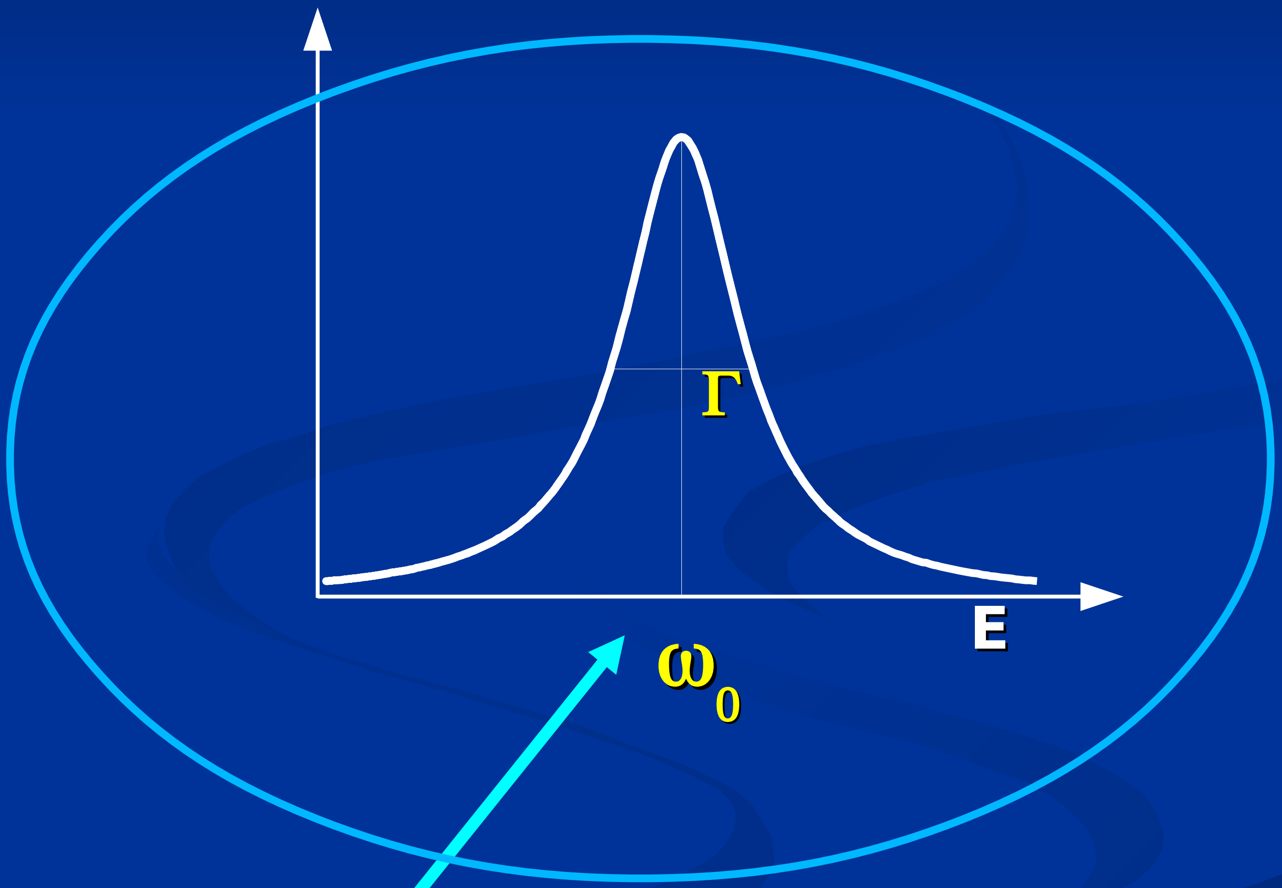


How were those results obtained:

With the integral transform method

$$S_M(q\,,\omega) = \mathrm{Im}\{ <0|\; C0^{\dagger}(q)\;\; (\omega + E_0 - H + i\,\varepsilon)^{-1}\; C0\,(q)\;\; |0> \}$$

Integral transform
with Lorentzian
kernel



$$\Lambda_L(\mathbf{q}\,,\,\omega_0\,,\,\Gamma\,) = \int d\,\omega\; S_M(q\,,\,\omega)\;\mathbf{L}\,(\omega,\;\omega_0,\;\Gamma\;)$$

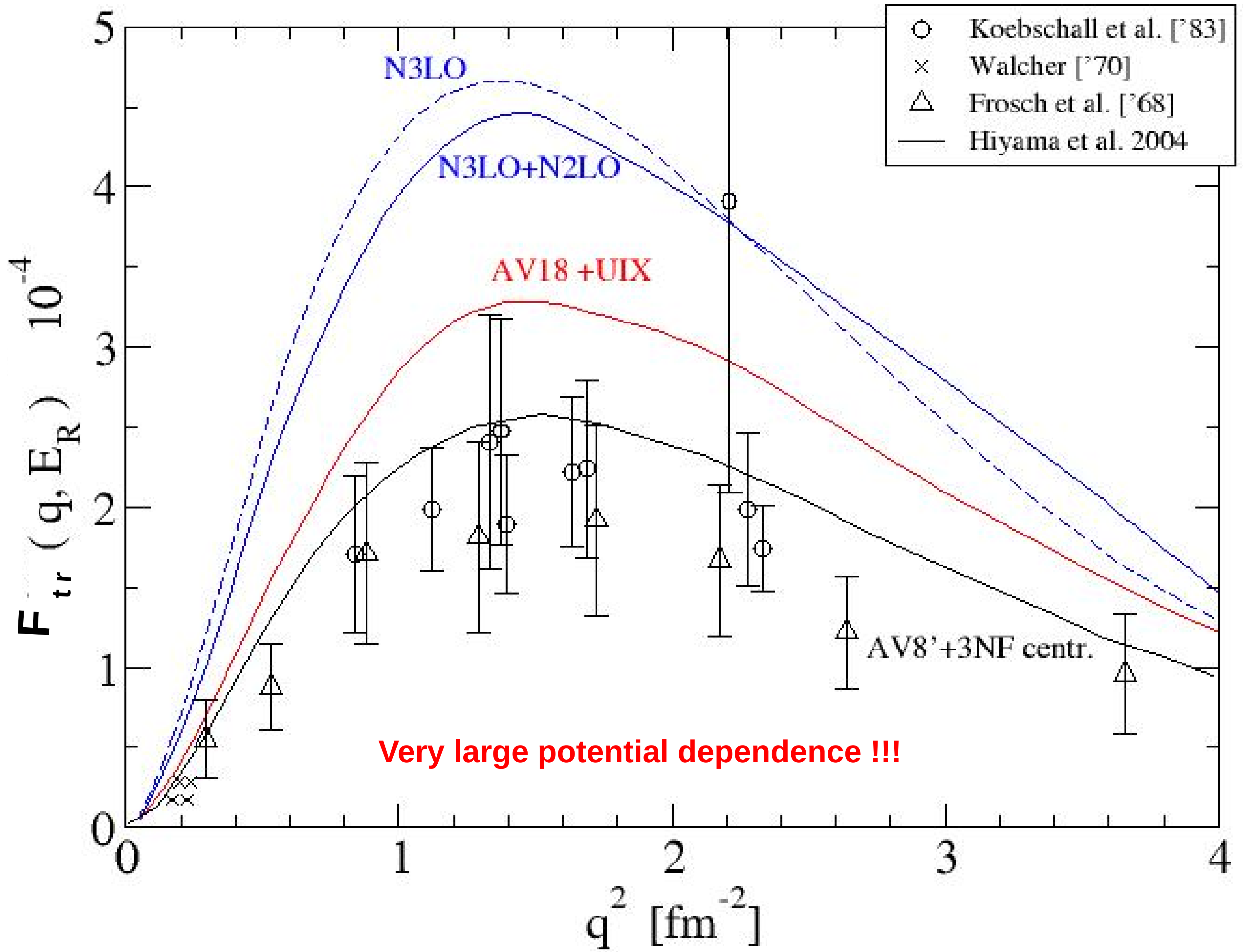
One first calculates $\Lambda_L(\mathbf{q}, \omega_0, \Gamma)$ with bound state methods and then inverts the transform

$$\Lambda_L(\mathbf{q}, \omega_0, \Gamma) = \int d\,\omega\, S_M(q, \omega)\, L(\omega, \omega_0, \Gamma)$$

The many-body scattering problem is avoided

Transition form factor $|<E_R|\, C0(q)\, |0>|^2$

S.Bacca et al. PRL 110 042503 (2013)



Example N.2:

Longitudinal $S(q,\omega)$

(all multipoles)

Longitudinal (Coulomb) excitation operator

$$\frac{d\sigma}{d\omega d\Omega} = V_L(q,\omega)|< F | \rho | I >|^2 + V_T(q,\omega) |< F | J_T | I >|^2$$

$< F |$ inclusive

$$S_L(q,\omega) = \Sigma_n |<n| \rho |0>|^2 \delta(\omega - E_n + E_0)$$

$$\sum_p^Z e^{i\,q\,.\,r_p}$$

Same procedure of calculation with integral transform:

One first calculates $\Lambda_L(\mathbf{q}, \omega_0, \Gamma)$ and then inverts the transform

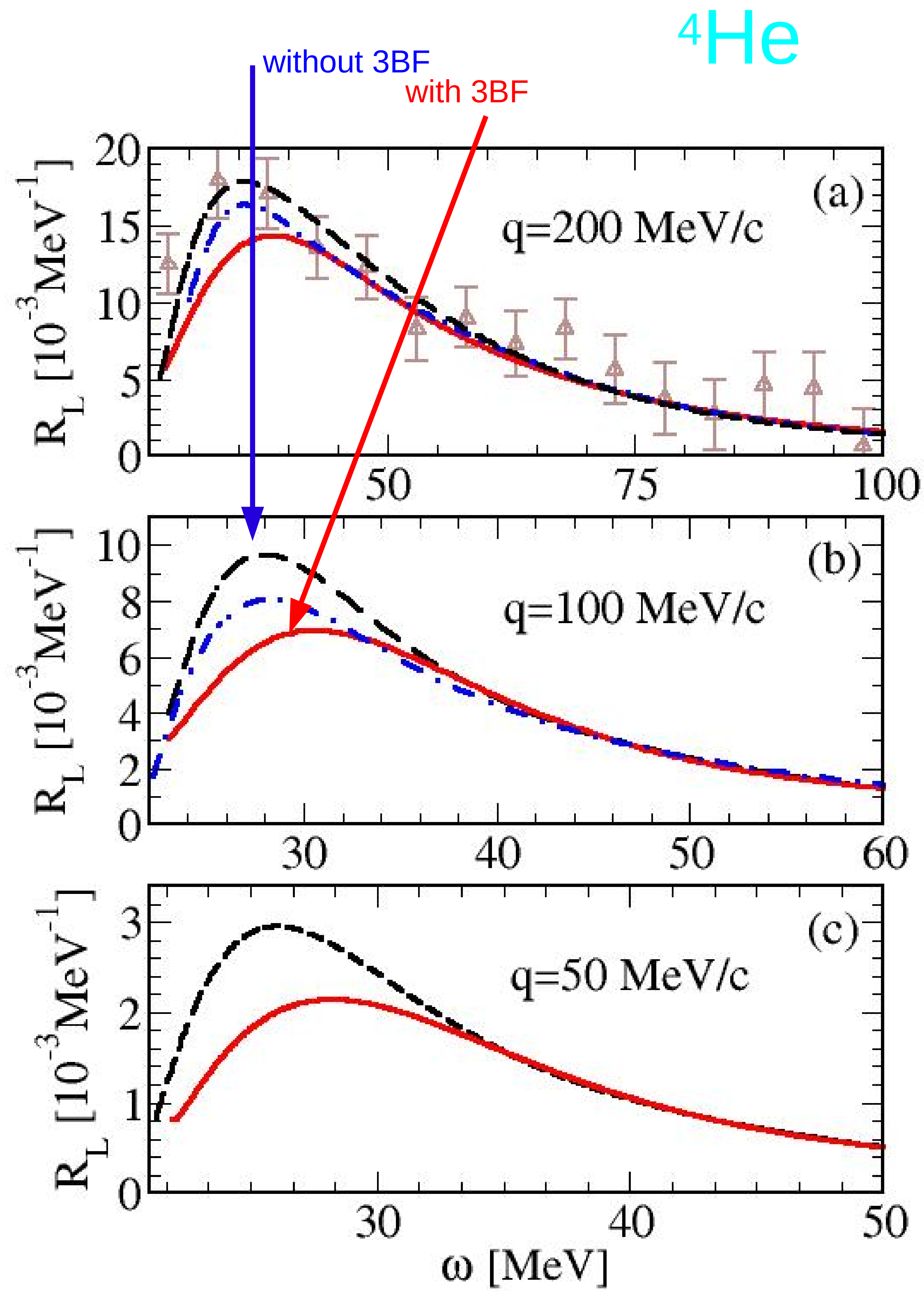
$$\Lambda_L(\mathbf{q}, \omega_0, \Gamma) = \int d\omega \, S_L(q, \omega) \, \mathbf{L}(\omega, \omega_0, \Gamma)$$

LOW q:

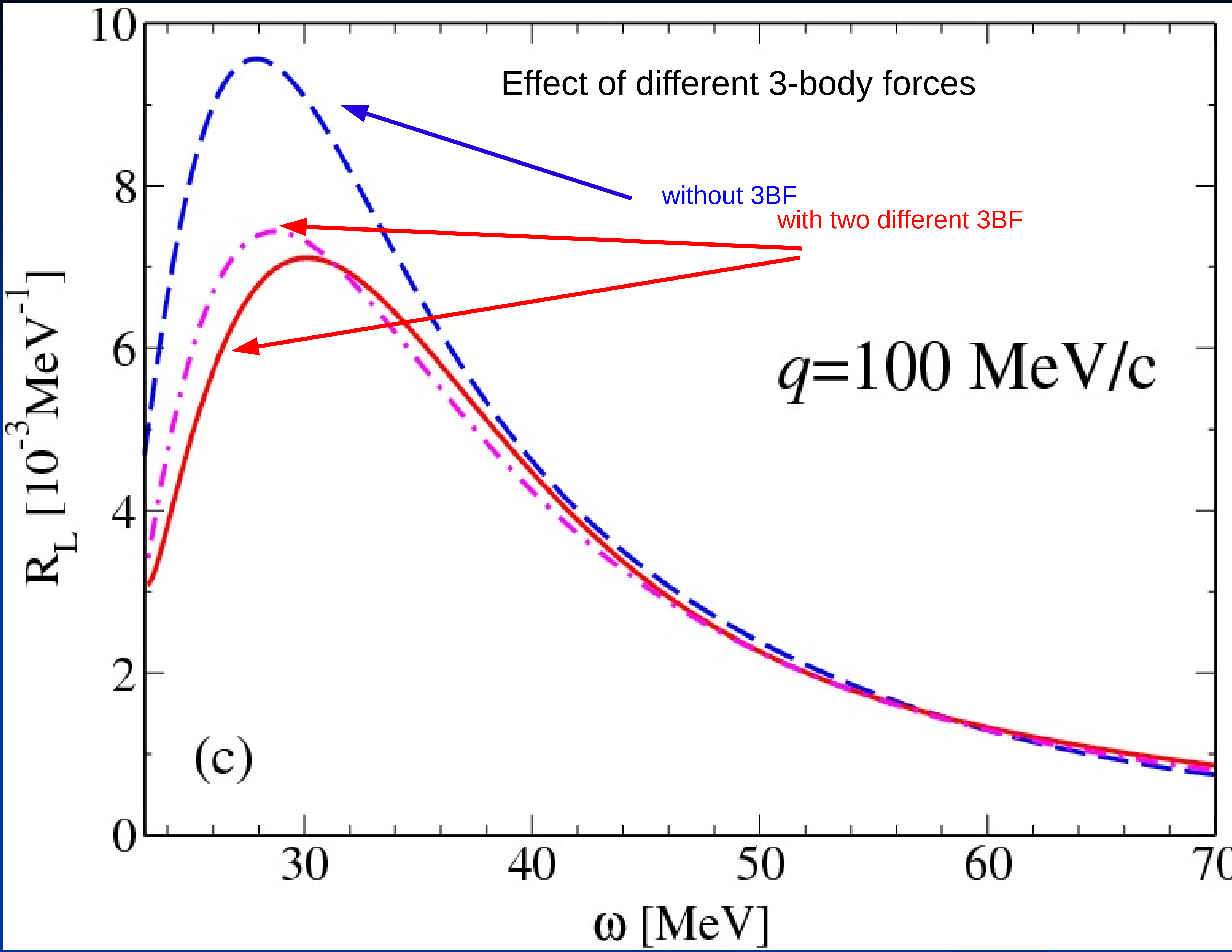
SURPRISE !

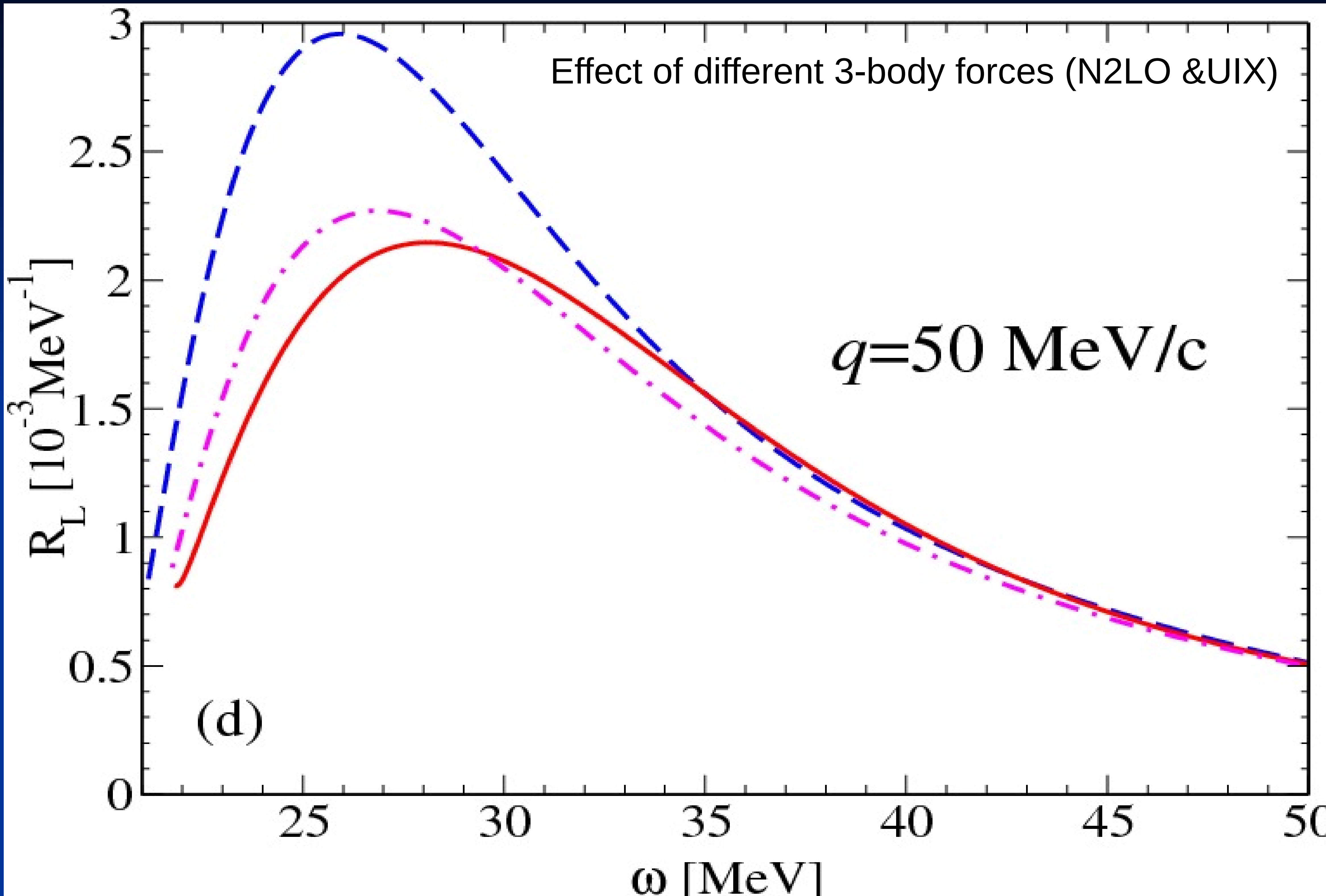
LARGE EFFECT OF 3-BODY FORCE

NO MEASUREMENTS AT LOW q !!!



S.Bacca et al., PRL 102 (2009) 162501


Effect of different 3-body forces
without 3BF
with two different 3BF
q=100 MeV/c
(c)
R_L [10^-3 MeV^-1]
ω [MeV]
0
2
4
6
8
10
30
40
50
60
70

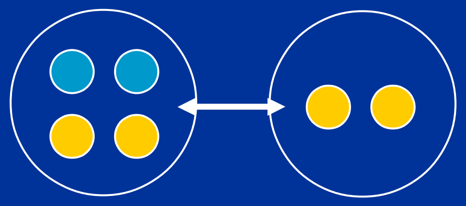

Effect of different 3-body forces (N2LO &UIX)
q=50 MeV/c
(d)
R_L [10^-3 MeV^-1]
ω [MeV]
0
0.5
1
1.5
2
2.5
3
25
30
35
40
45
50

Example N.3:

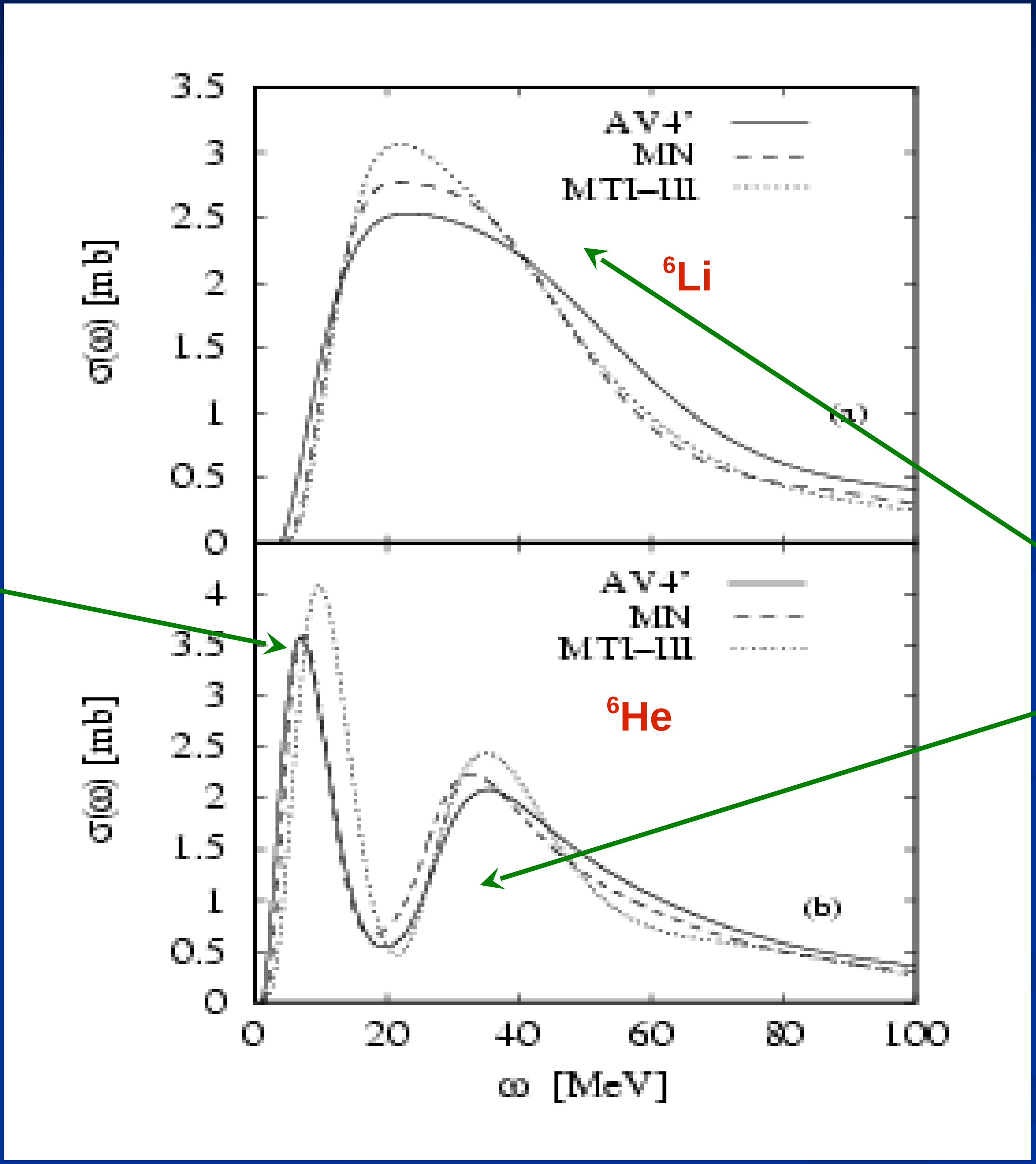
”Collettive behaviour” in 4,6 body systems

6-Body total photodisintegration

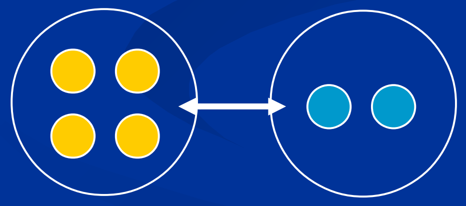
S.Bacca et al. PRL89(2002)052502



soft mode ??



Theory:
LIT+ EIHH

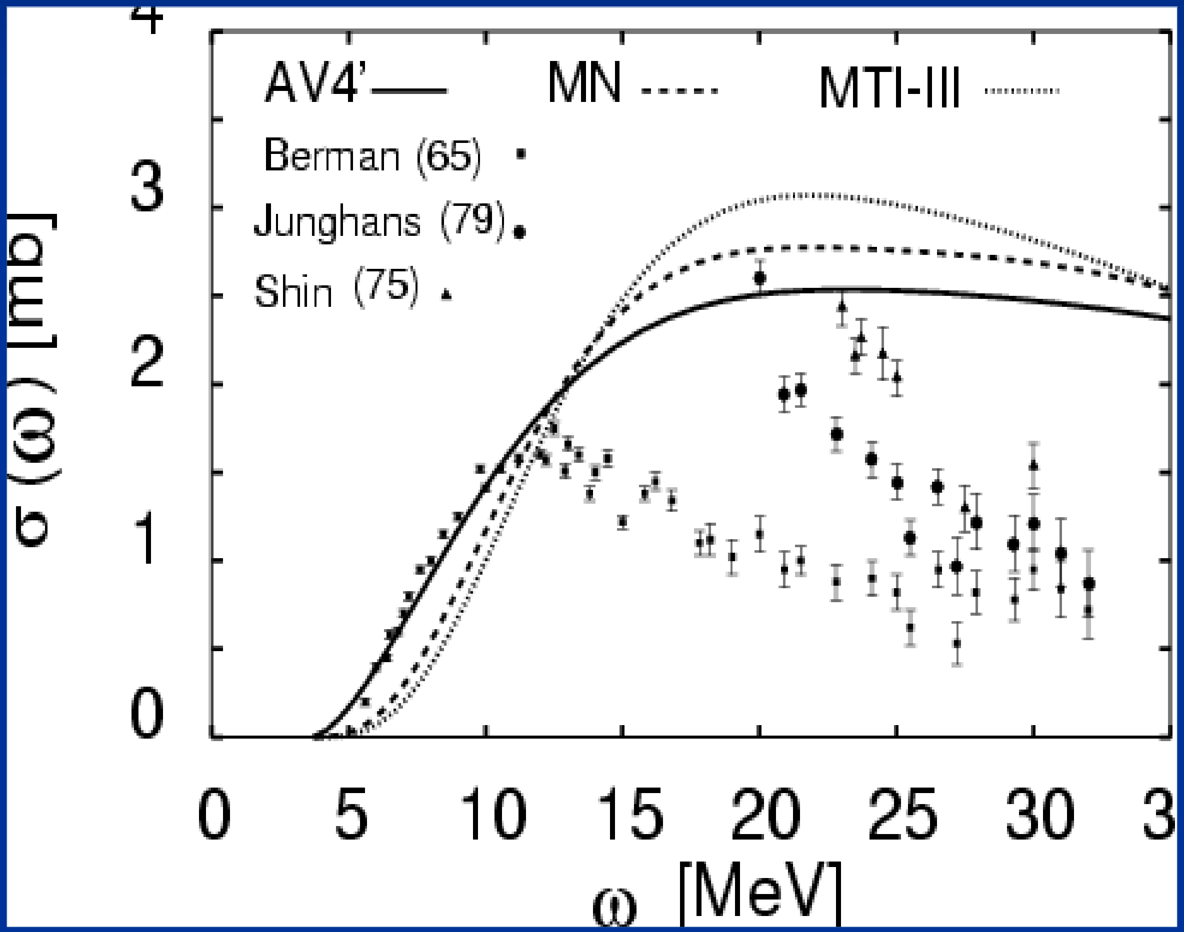


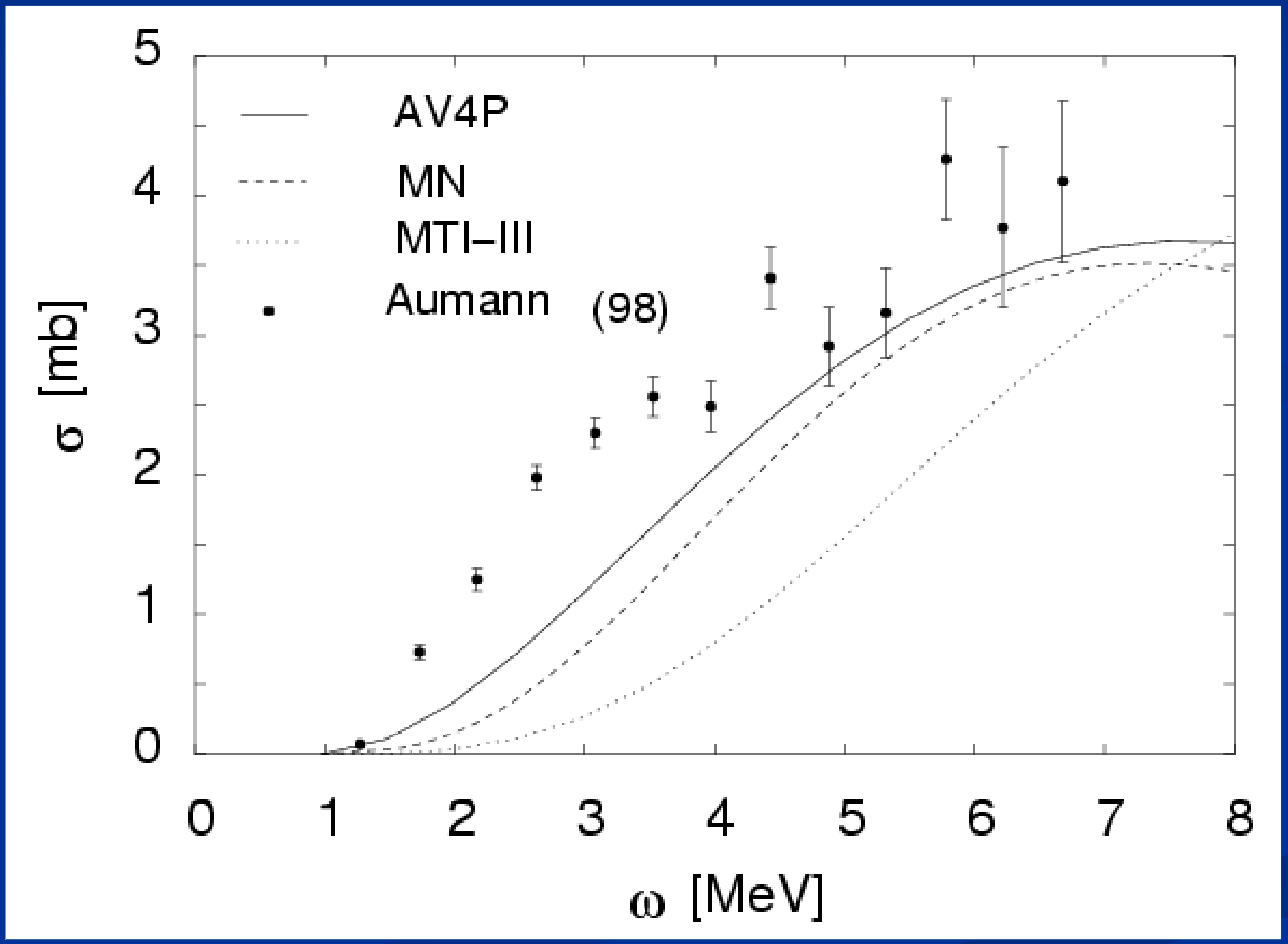
classical GT mode ??

comparison with experiment

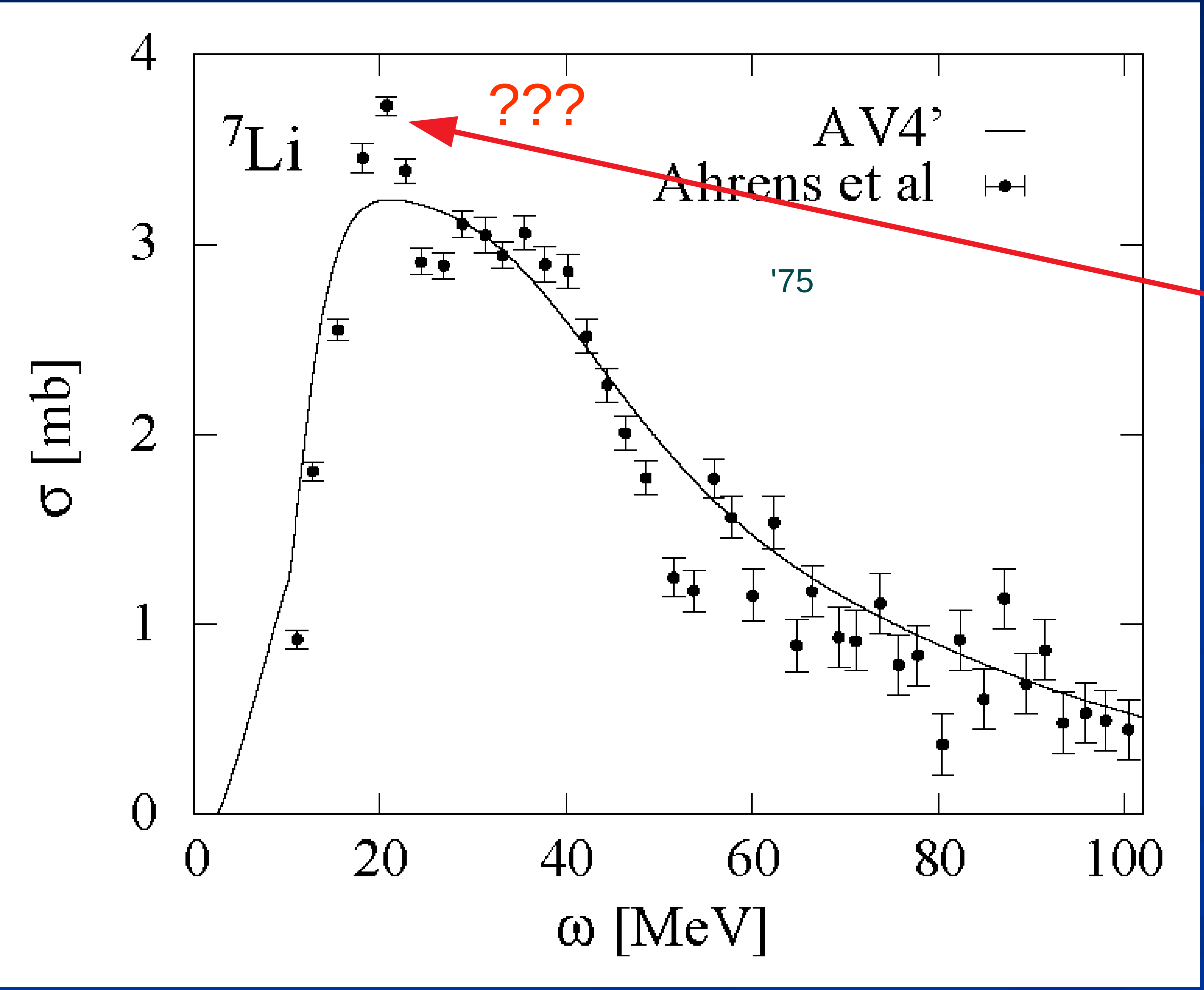
^{6}Li

^{6}He





7-Body total photodisintegration with LIT method

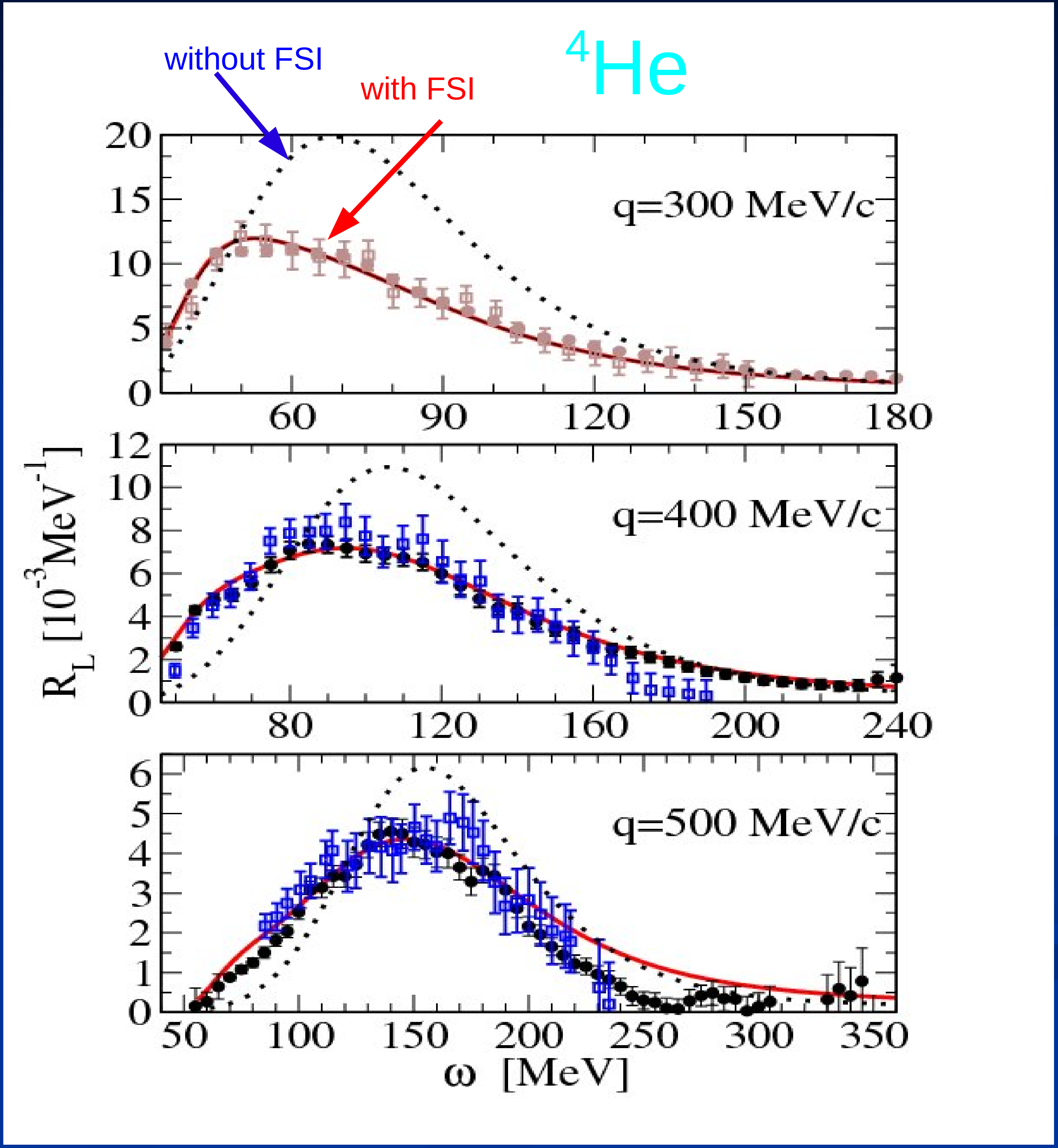


S.Bacca et al.
Phys.Lett. B603
(2004) 159-164

Higher energy/momentum observables (q.e.)

"quasi elastic" electron scattering

Role of Final State Interaction:



dotted: PWIA

full: AV18+UIX

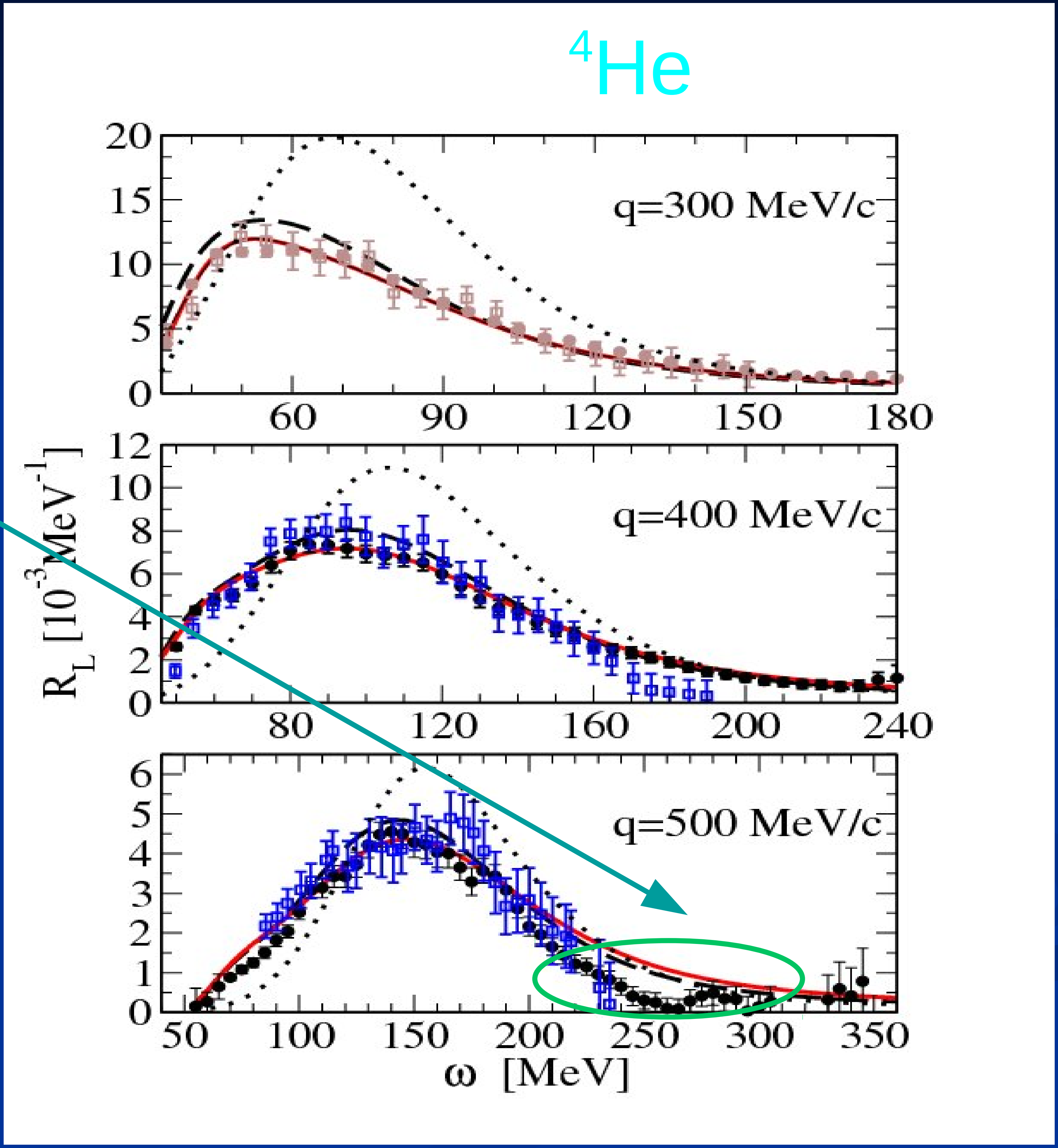
S.Bacca et al., PRL 102 (2009) 162501

“quasi elastic” electron scattering

Role of relativity???

dashed: AV18

full: AV18+UIX



S.Bacca et al.,
PRL 102 (2009) 162501

Check frame dependence

V.D. Efros et al. RC72 (2005) 011002(R)

One can perform calculation in various frames:

Laboratory: $P_T = 0$

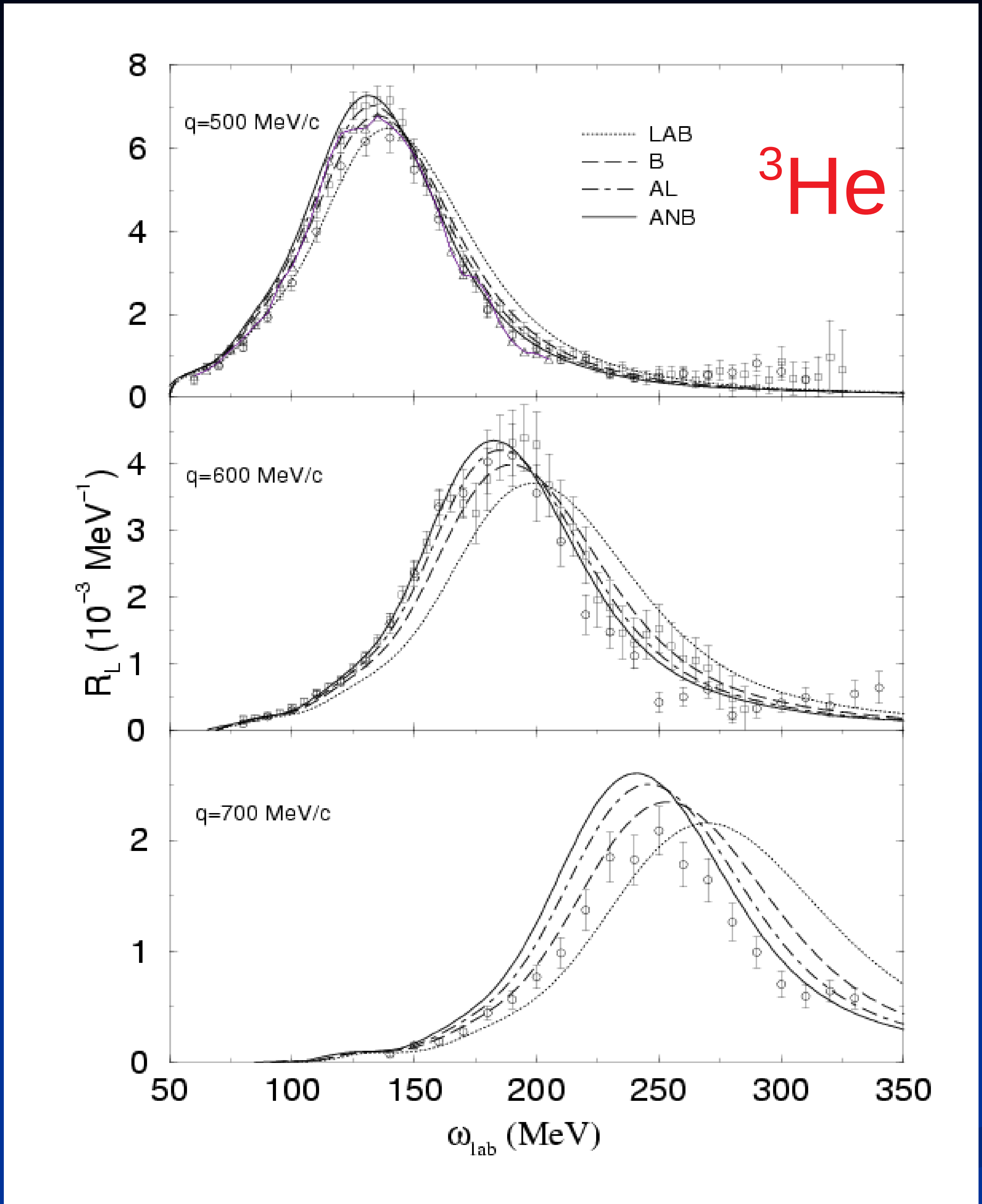
Breit: $P_T = -q/2$

Anti-Lab: $P_T = -q$

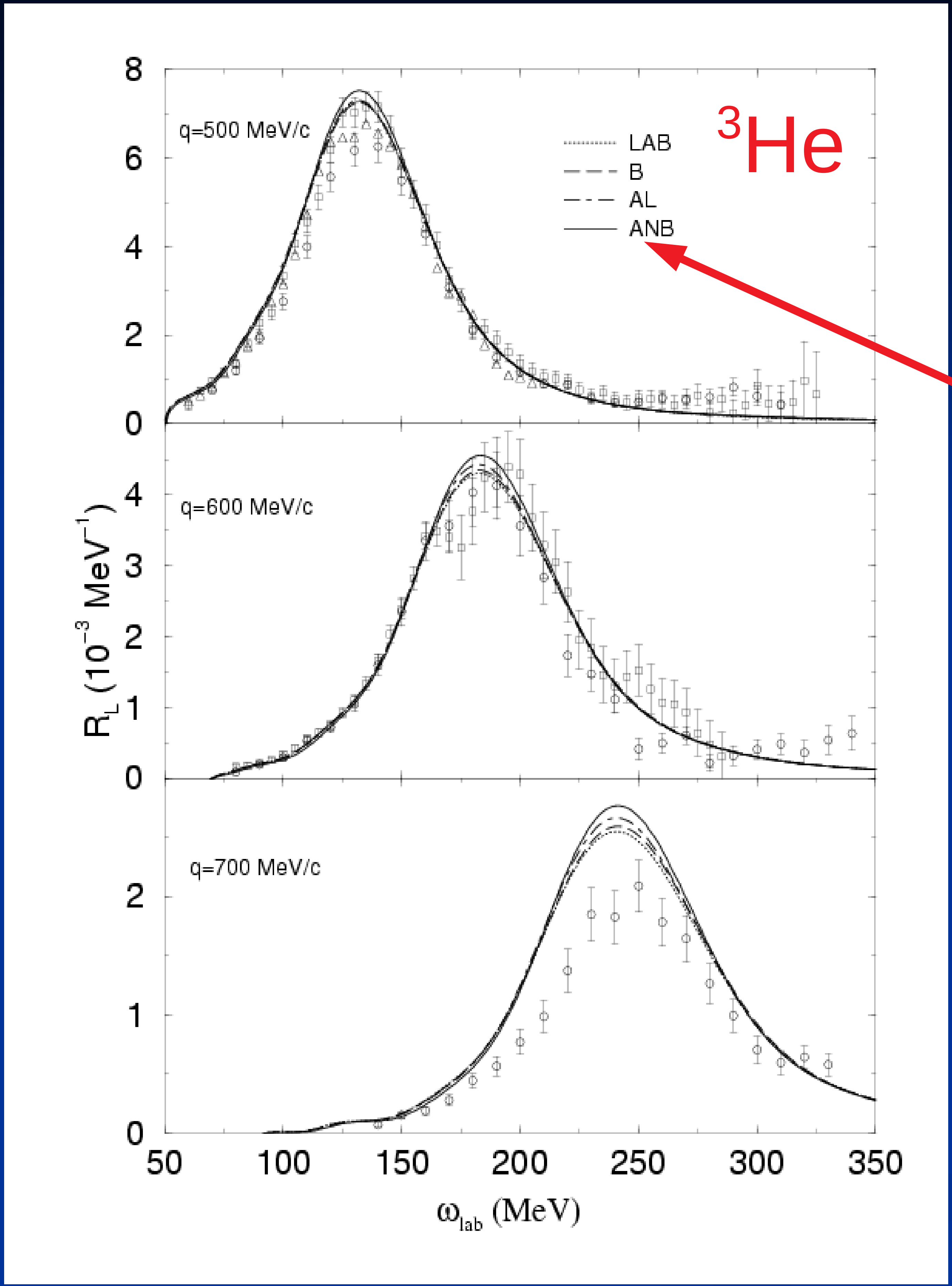
Active Nucleon Breit: $P_T = -Aq/2$

Results must be compared in the LAB frame

$$R_L(q,\omega) = \frac{q^2}{(q_{fr})^2}\,\frac{E_T^{fr}}{M_T}\,R_L^{fr}(q^{fr},\omega^{fr})$$



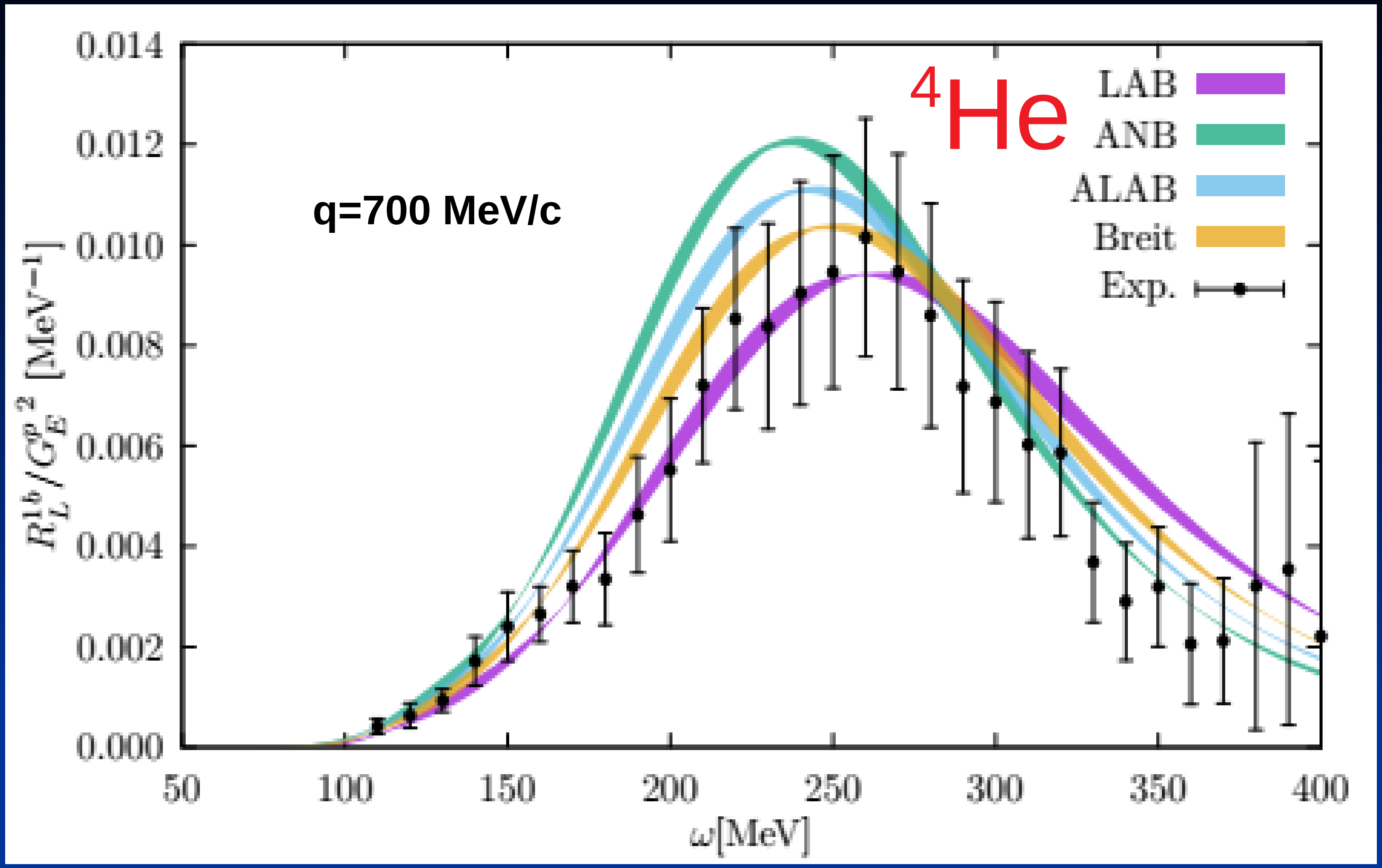
Exp: Marchand 1985, Dow 1988, Carlson 2002



Quasielastic region:
assume two-body kinematics as input of a n.r. calculation and use the correct relativistic relative momentum

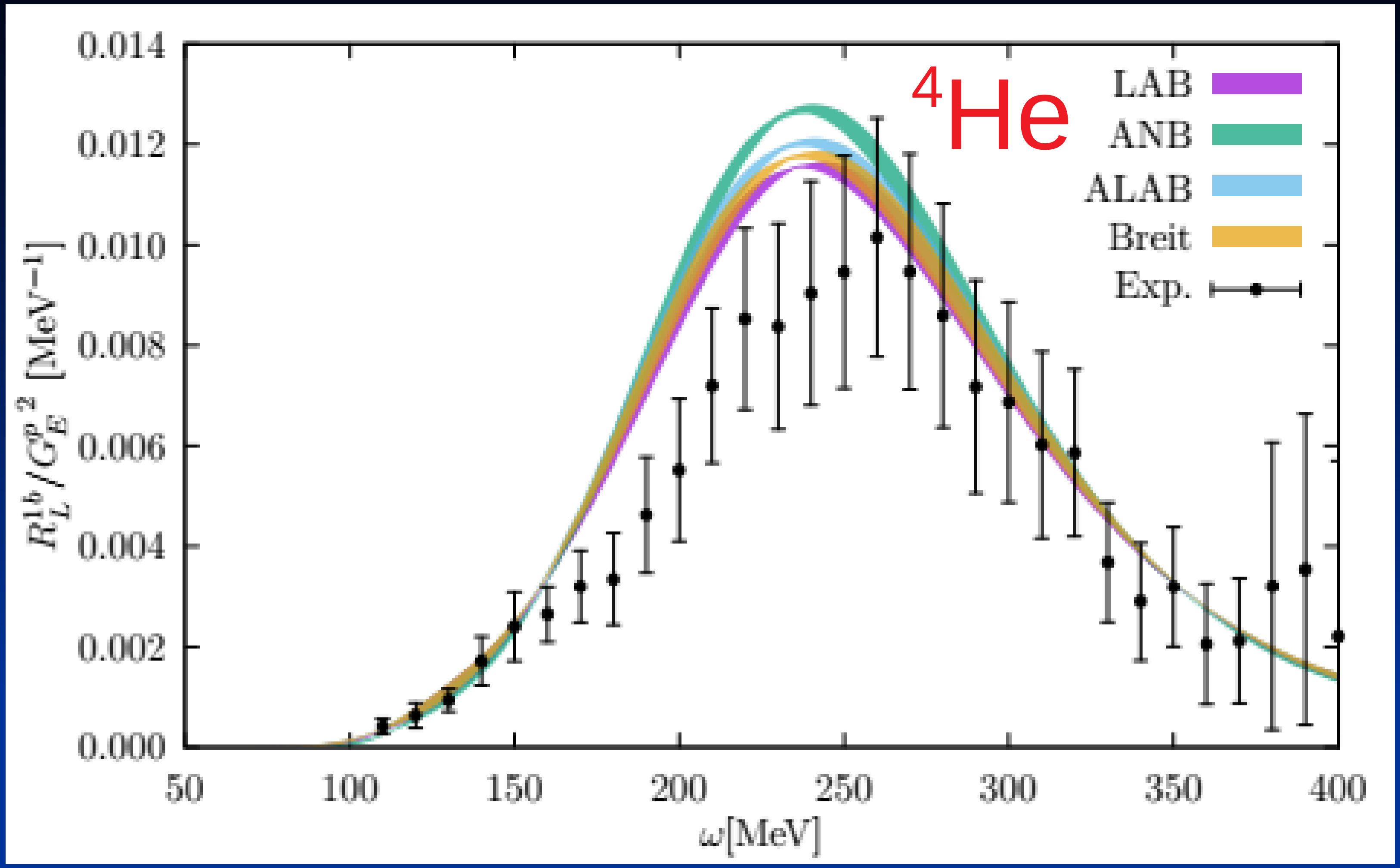
Curves collapse around the ANB result

ANB frame minimizes relativistic effects!



The same happens also in ^{4}He!

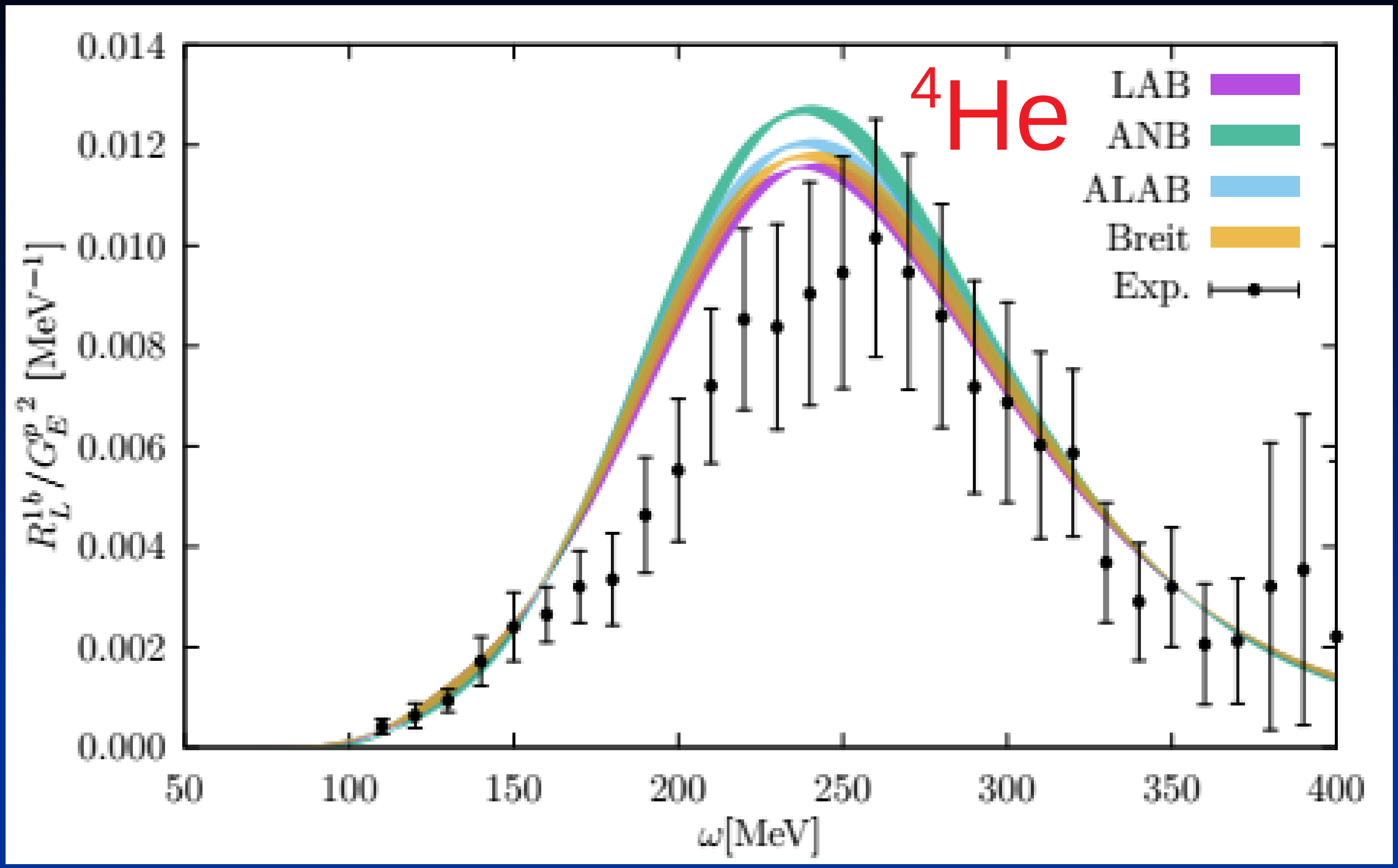
N.Rocco et al. Phys. Rev. C 97, 055501 (2018)



frame dependence much reduced!

N.Rocco et al. Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics ***[2-body break-up 1-(A-1)]***
one can treat the relativistic kinematical inputs
correctly!!



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N.Rocco et al. Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics *[2-body break-up 1-(A-1)]*
one can treat the relativistic kinematical inputs
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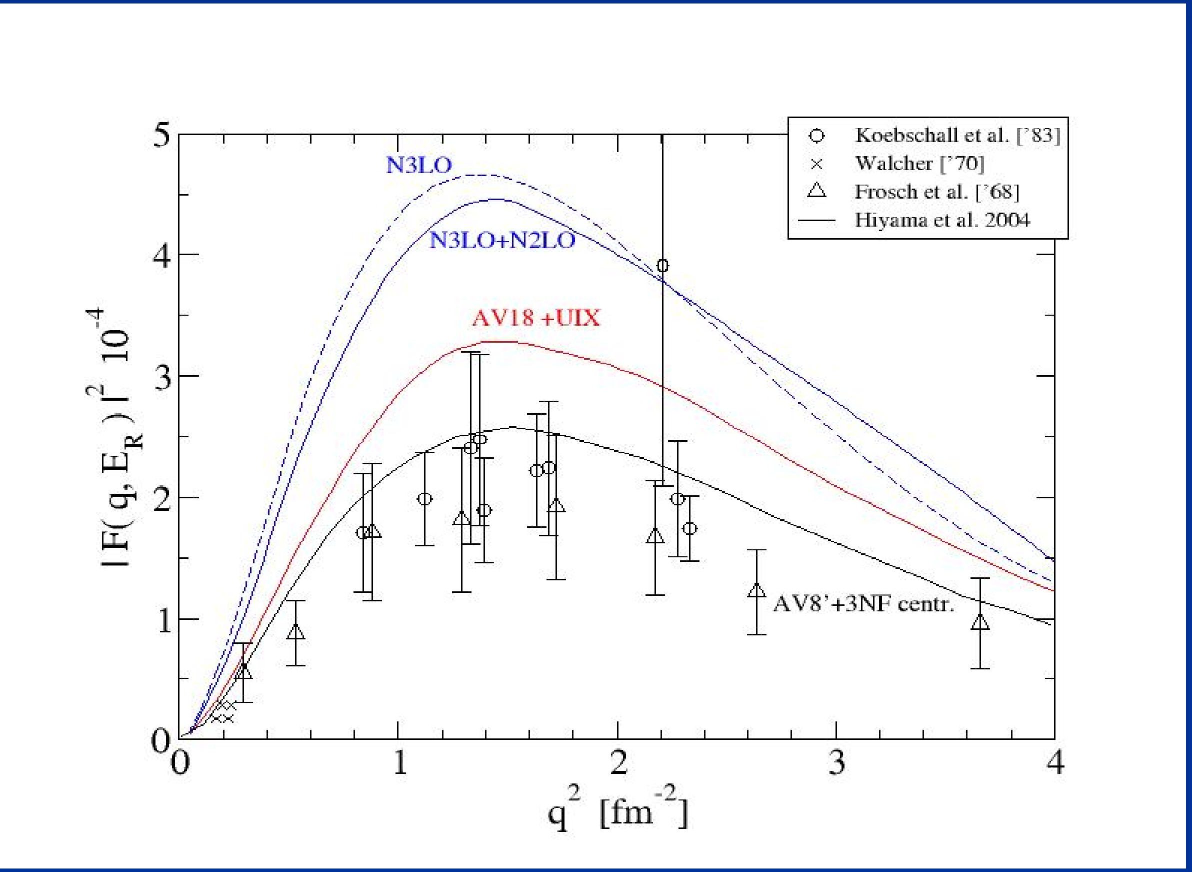
One can extend the applicability of a n.r. calculation choosing the right frame

Conclusions:

which measurements ⟷ which questions



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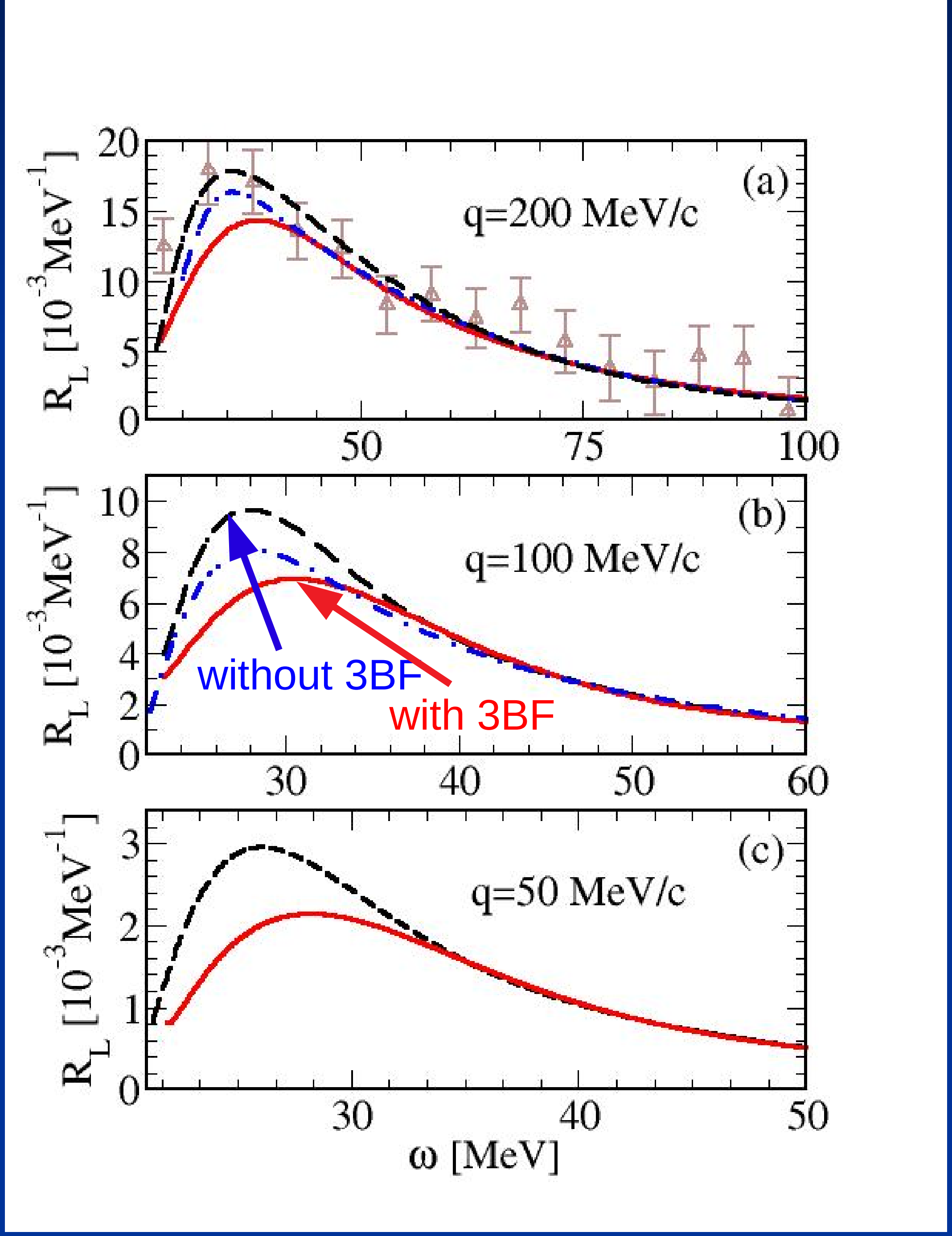

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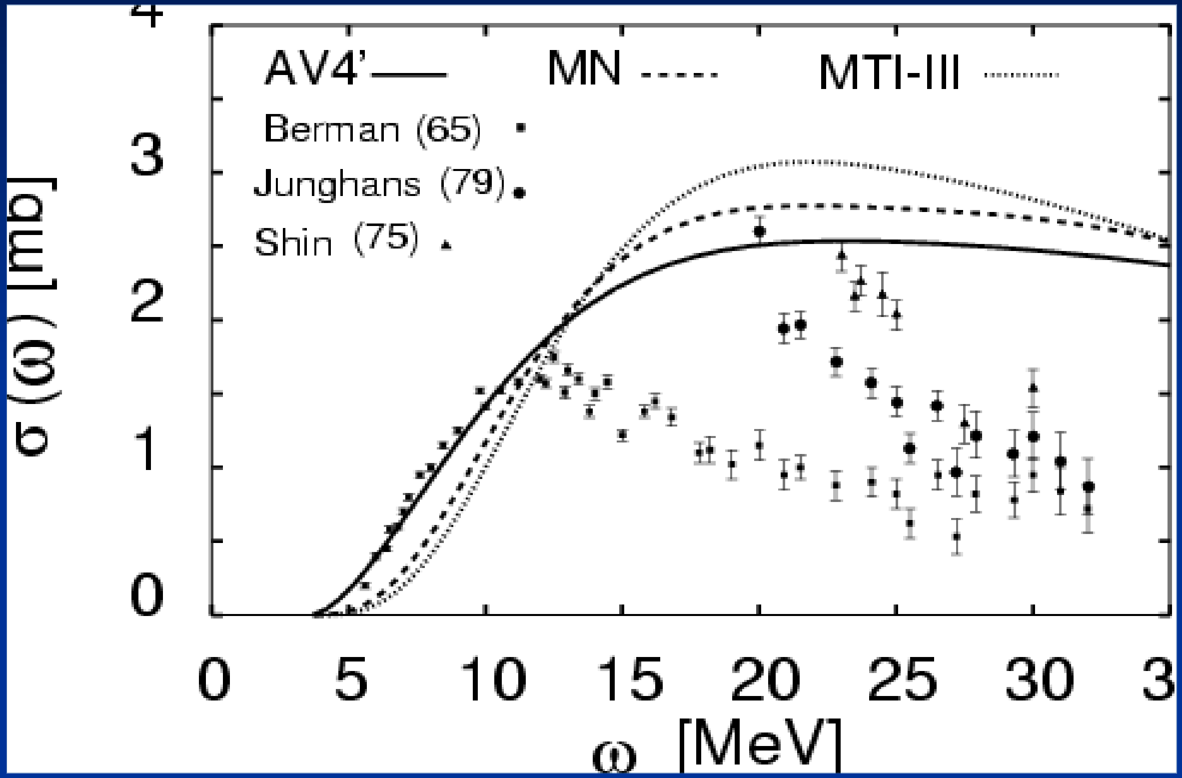

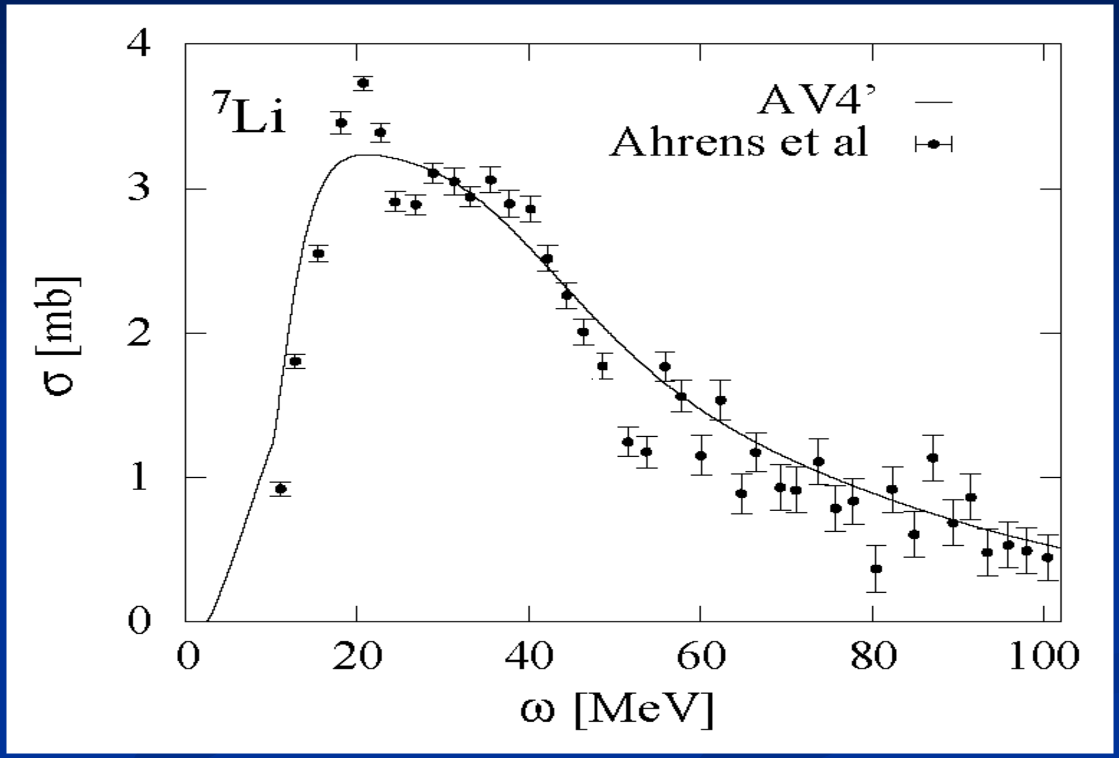

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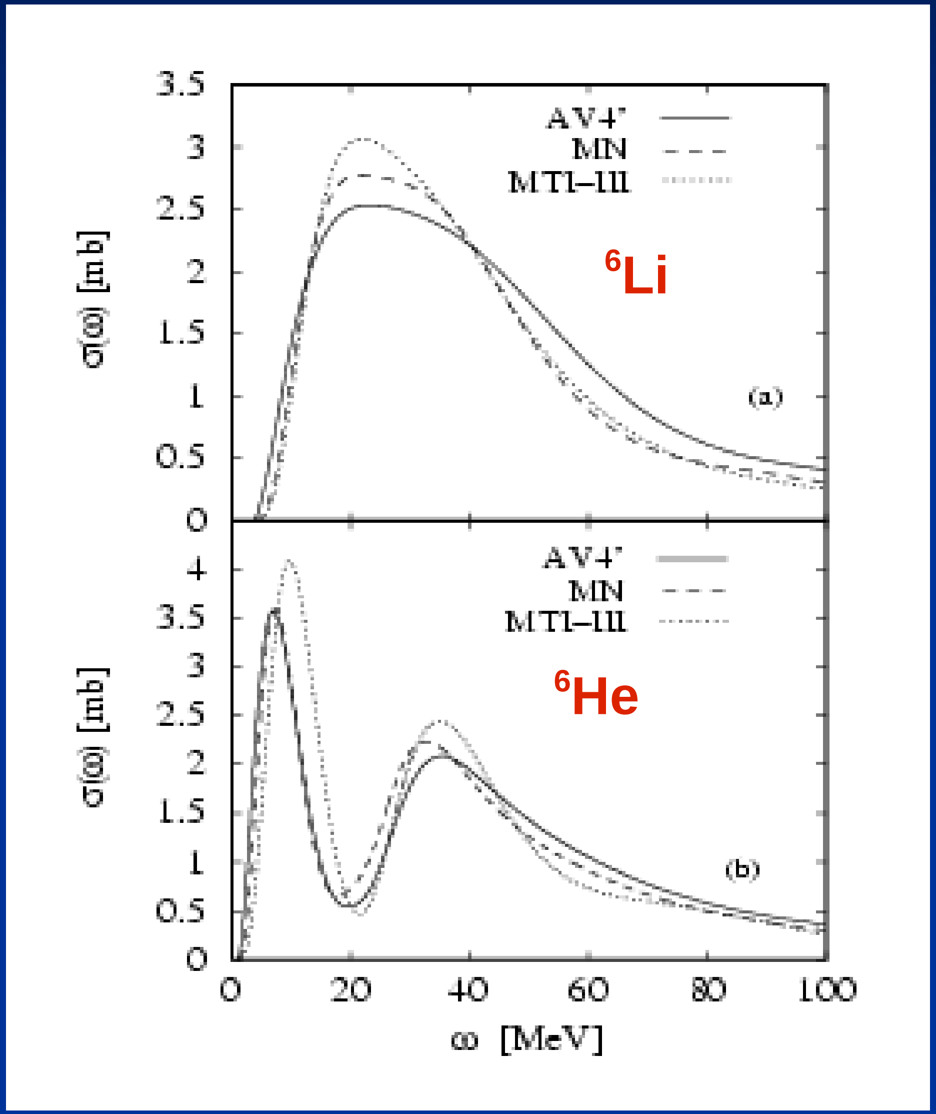

...do we see the emergence of a collective behaviour from a pure MIcroscopic description in terms of A interacting protons and neutrons?

Conclusions:

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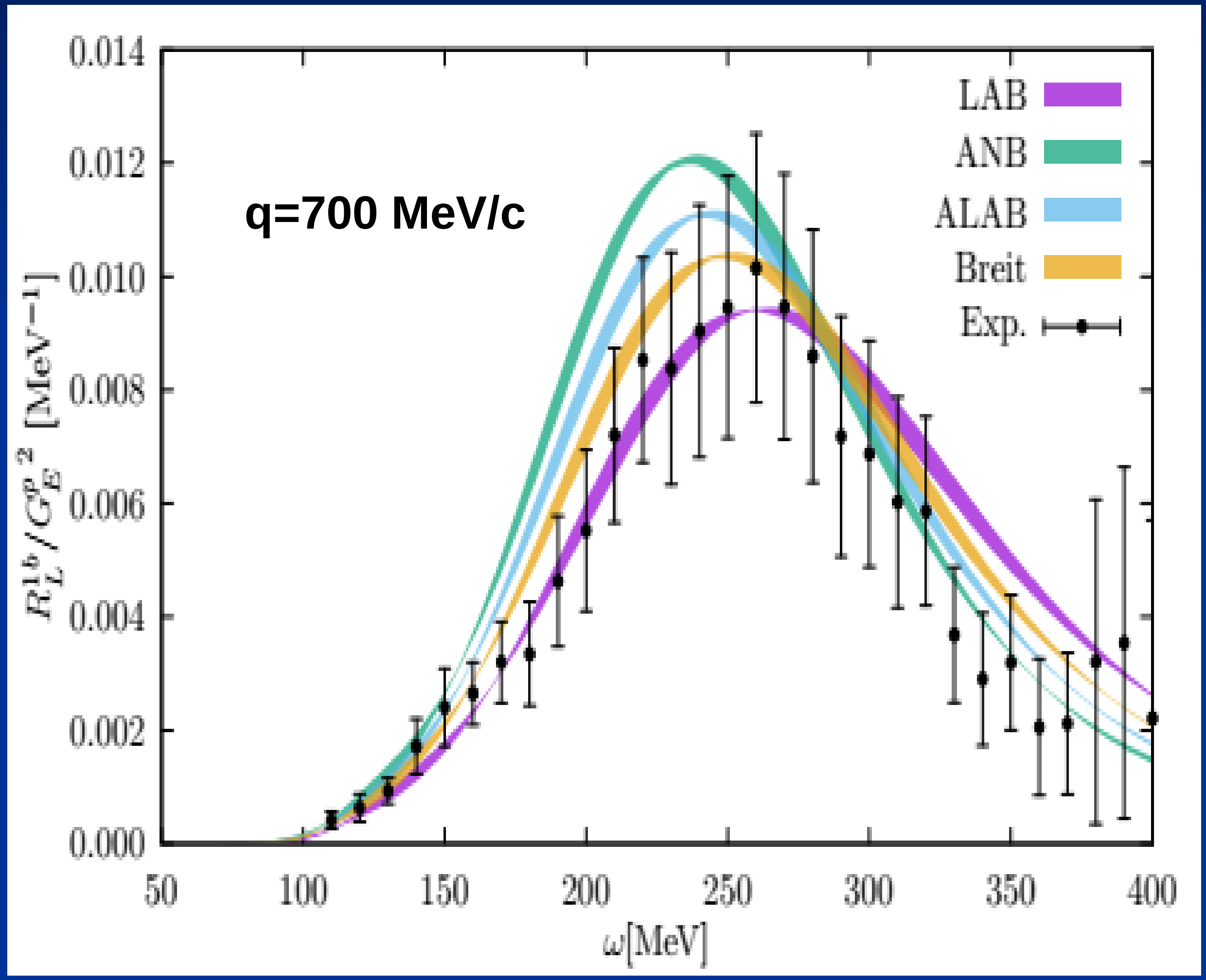
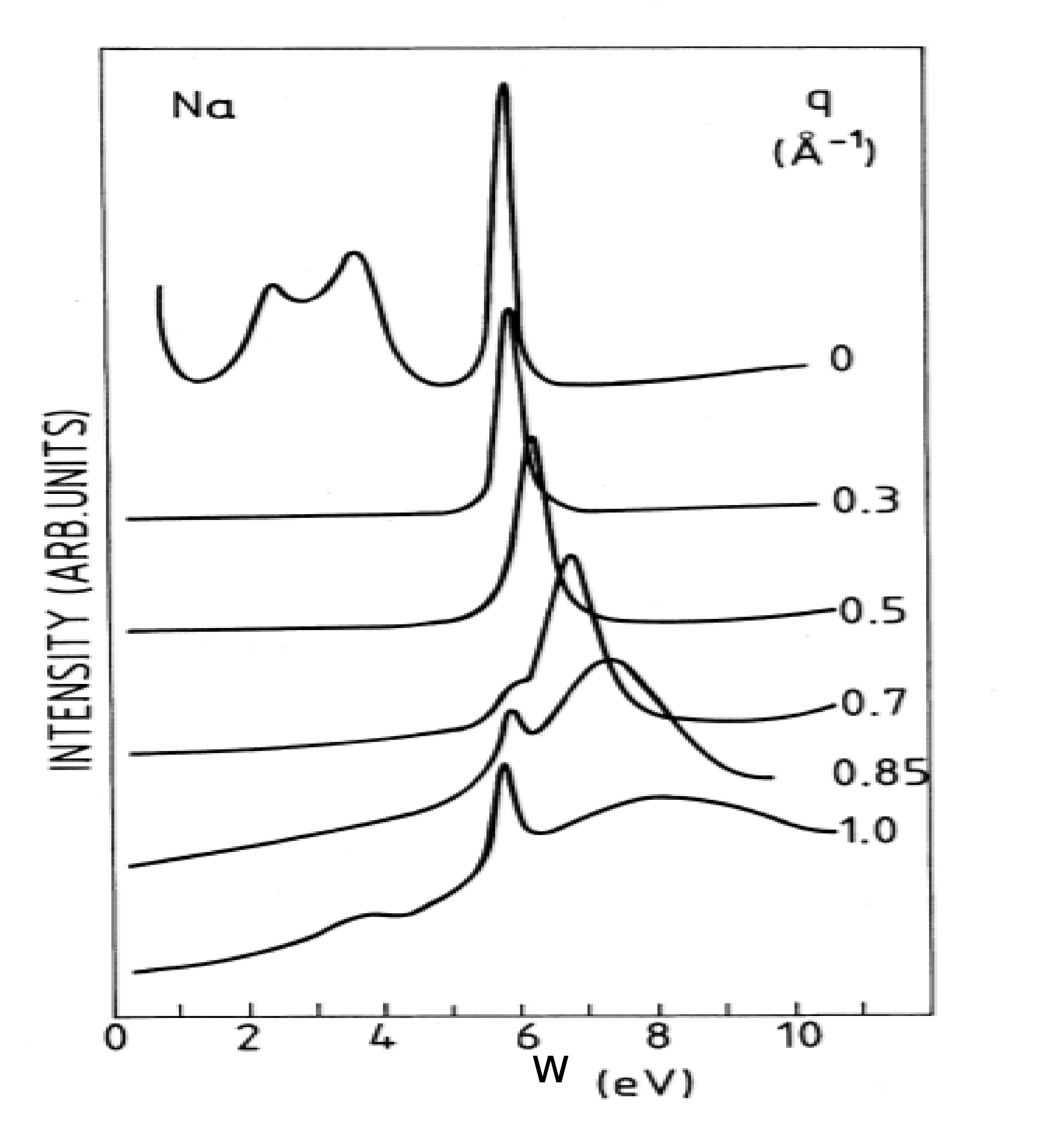


What are the limits in q and ω of the non-relativistic framework?

We need to check on (e,e') to interpret neutrino data!!

More on Collective behaviours in few-body systems

A typical example of collectivity in a many-body system: the plasmon



The many-body system: *valence-electron gas in metals*

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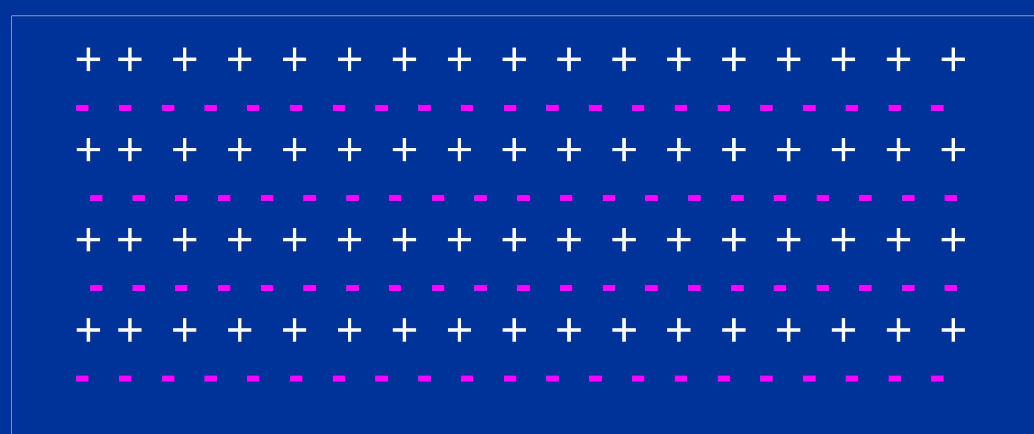
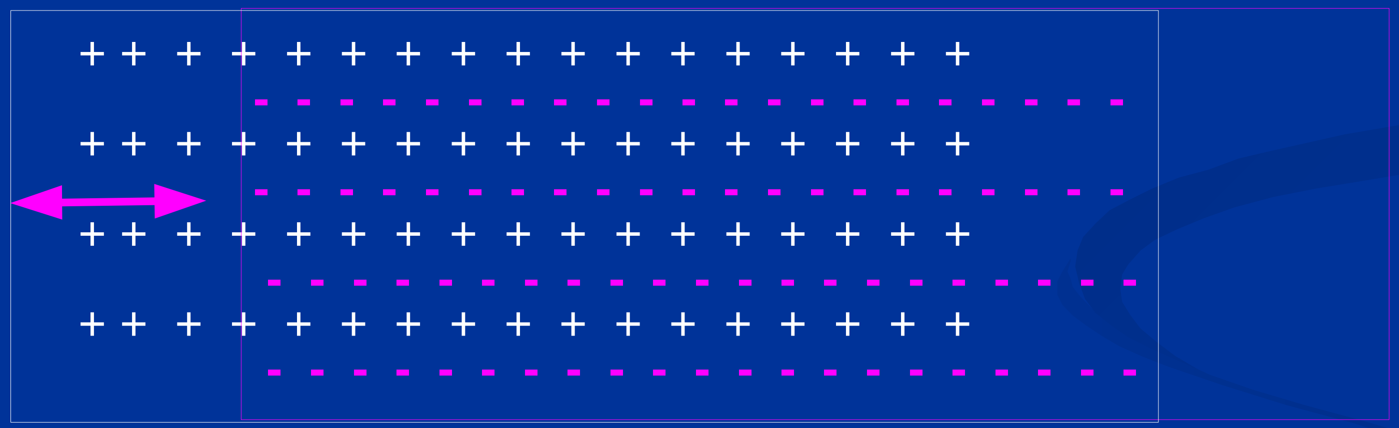
A classical interpretation of the plasmon:

a ***"collective"*** oscillation of the free electron density with respect to the fixed positive ions in a metal



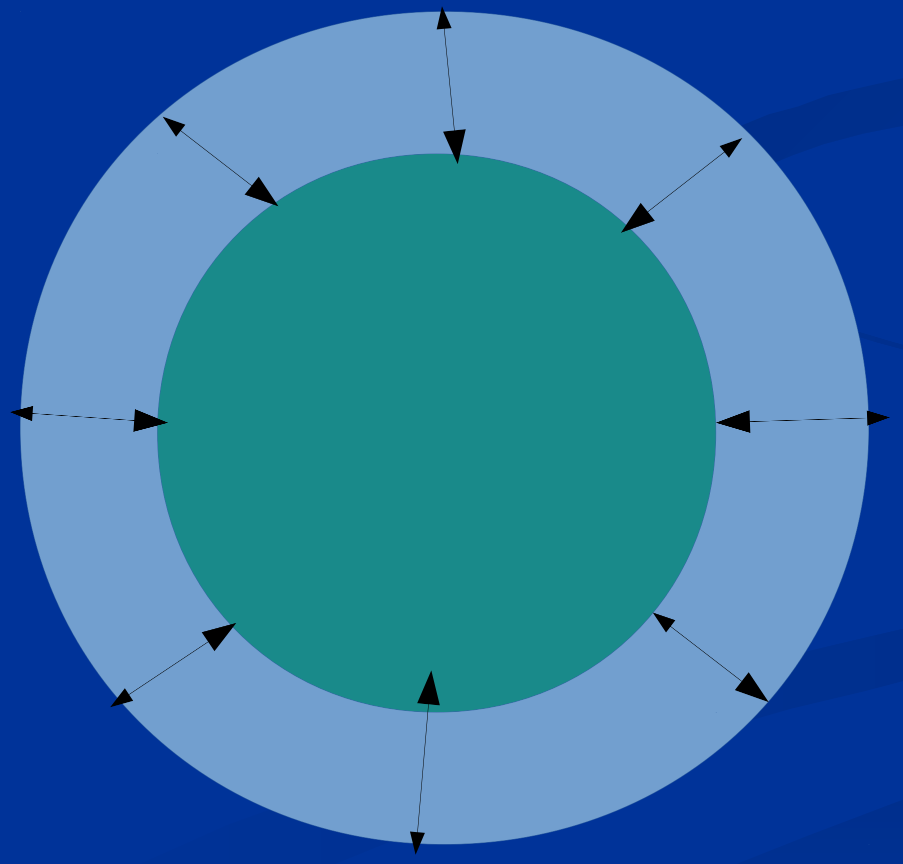
A classical interpretation of the plasmon:

a ***"collective"*** oscillation of the free electron density with respect to the fixed positive ions in a metal



Monopole Resonance

Historically, for larger systems "**GMR"** has been observed and interpreted as an harmonic collective motion of compression



Since we can calculate the whole $S_M(q,\omega)$ we can investigate whether an ***ab initio*** calculation of the ^{4}He **isoscalar monopole** strength exhibits features that are believed to characterize a **collective breathing mode**, and how they depend on different nuclear forces.

When the first measurements of the 0^+ resonance of ^{4}He appeared in 1965 (*Frosch et al.*) Werntz and Ueberall asked the interesting question:

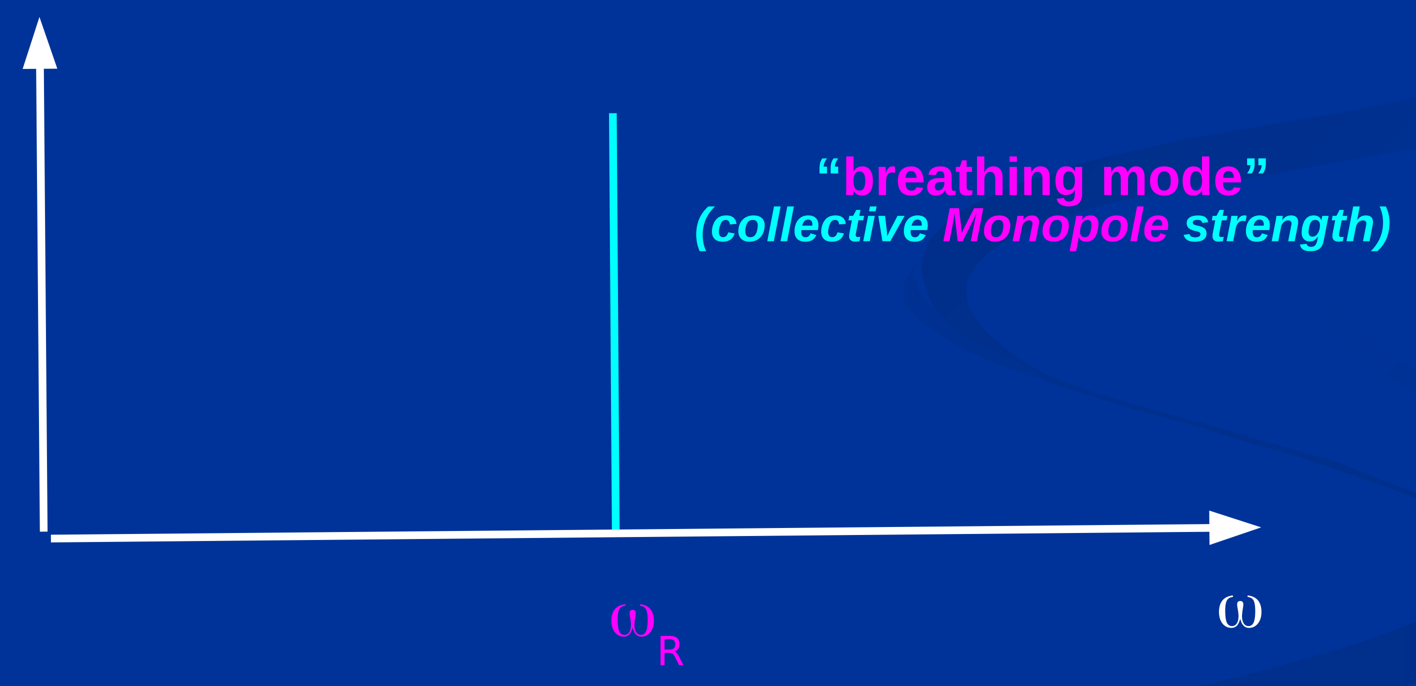
Is the 0^+ resonance of ^{4}He a *collective breathing mode*?

Their simple breathing mode model (***density scaling***) implies

a) the transition density $|\langle 0^+_R | \sum_i \delta(r-r_i) | 0\rangle|^2$ changes sign at $r = \langle r^2\rangle^{1/2}$

b) the breathing mode exhausts m_1 i.e.

$$\lim_{q \to 0} \frac{1}{q^4} S_M(q,\omega)$$



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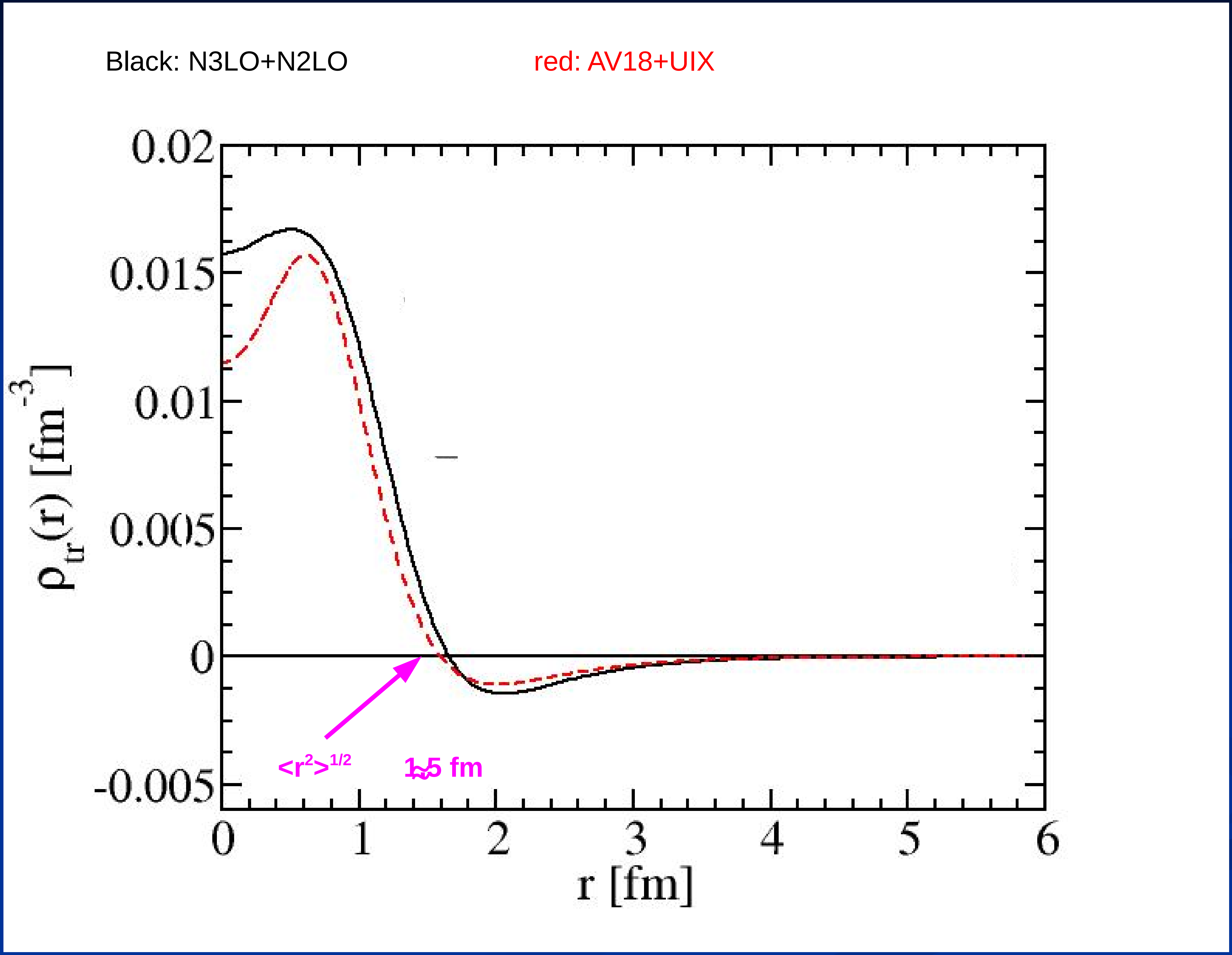

“Sum Rules provide useful yardsticks for measuring quantitatively the degree of collectivity of a given excited state”

D.Rowe in “Nuclear Collective motion” 1970

Sum Rules and Collectivity

$$m_0(q) = \int S_M(q,\omega)\, d\omega = \langle 0|\; C0(q)\;\; C0(q)\; |0\rangle$$

$$m_1(q) = \int S_M(q,\omega)\;\omega\; d\omega = \langle 0|\; C0(q)\; H\; C0(q)\; |0\rangle = \mathbf{A/2m < r^2 >}$$

$$m_{-1}(q) = \int S_M(q,\omega) / \omega\; d\omega = 2\alpha_M = \textit{Compressibility}$$

Sum Rules and Collectivity

$$m_0(q) = \int S_D(q,\omega)\, d\omega = \langle 0|\; D(q)\; D(q)\; |0\rangle$$

$$m_1(q) = \int S_D(q,\omega)\, \omega\, d\omega = \langle 0|\, D(q)\, H\, D(q)\, |0\rangle = \mathbf{NZ\,/\,(2mA)}$$

"TRK sum rule"

$$m_{-1}(q) = \int S_M(q,\omega)\,/\,\omega\, d\omega = 2\alpha_D =$$ *Polarizability*

“Sum Rules provide useful yardsticks for measuring quantitatively the degree of collectivity of a given excited state”

D.Rowe in “Nuclear Collective motion” 1970

“A typical isoscalar collective state exhausts something like 50% of $\boldsymbol{m_0}$*”*

D.Rowe in “Nuclear Collective motion” 1970

Sum rules:

q $[\frac{\mathrm{MeV}}{\mathrm{c}}]$	$\|F_{\mathcal{M}}(q)\|^2$	m_0	m_1 [MeV]	r_0 %	r_1 %
50	0.00034	0.00063	0.021	53	34
	0.00024	0.00064	0.018	38	28
100	0.0042	0.0085	0.262	50	34
	0.0031	0.0086	0.258	37	25
200	0.0248	0.0683	2.42	36	22
	0.0190	0.0710	2.48	27	16
300	0.0297	0.129	5.89	23	11
	0.0242	0.139	6.33	17	8
400	0.0154	0.126	8.43	12	4
	0.0141	0.143	9.39	10	3

N3LO+N2LO

AV18+UIX

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N3LO+N2LO

AV18+UIX

Is the 0^+ *resonance* of the α-particle a "*breathing mode*"
???

Notice that

$$S_M(q\,,\omega) = \sum\!\!\!\!\!\!\int_n |< n\,|C0\,(q)|0 >|^2\,\delta\,(\,\omega - E_n + E_0)$$

can be rewritten as

$$S_M(q\,,\omega\,) = \mathrm{Im}\{<0|\;\; C0(q)\;\;(\,\omega\;+E_0 - H + i\,\varepsilon)^{-1}\;C0\,(q)\;\;|0>\}$$

Illustration of the calculation of $\Lambda_L(\mathbf{q}, \omega_0, \Gamma)$

Notice the similarity!

$$S_M(q, \omega) = \mathrm{Im}\{\langle 0| \, C0(q) \, (\omega + E_0 - H + i\,\varepsilon)^{-1} \, C0(q) \, |0\rangle\}$$

$$\Lambda_L(\mathbf{q}, \omega_0, \Gamma) = \mathrm{Im}\{\langle 0| \, C0(q) \, (\omega_0 + E_0 - H + i\,\Gamma)^{-1} \, C0(q) \, |0\rangle\}$$

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Γ **finite!**

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Γ **finite!**

the calculation of $\Lambda_L(\mathbf{q}, \omega_0, \Gamma)$ is a bound state like problem !!!

$$\Lambda_L(\mathbf{q}, \omega_0, \Gamma) = \mathrm{Im}\{\langle \mathbf{0}| \, C0(q) \, (\omega_0 + \mathbf{E}_0 - \mathbf{H} + i\,\Gamma)^{-1} \, C0(q) \, |\mathbf{0}\rangle\}$$

Because of a finite Γ, now it is perfectly **legitimate** to solve the problem on a **square integrable** basis

$$\Lambda_L(q\, ,\, \omega_0\, ,\, \Gamma\,) = \mathrm{Im}\{ <0| \; C0(q) \; (\omega_0 + E_0 - H + i\,\Gamma)^{-1} \; C0\,(q) \; |0> \}$$

$$\Sigma_n |m><m| = I \qquad\qquad \Sigma_n |n><n| = I$$

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$$\Sigma_n |m><m| = I \qquad\qquad \Sigma_n |n><n| = I$$

By diagonalizing $\mathbf{H}_{mn}$ one obtains eigenvalues ξ_ν and eigenfunctions $|\xi_\nu\rangle$ and Λ_L becomes a sum of Lorentzians centered in ξ_ν of width Γ

$$\Lambda_L(\mathbf{q}\,,\omega_0\,,\,\Gamma\,) = \sum_\nu \frac{|<\nu\,|C0\,(q)|0>|^2}{(\,\omega_0+E_0-\xi_\nu\,)^2+\Gamma^2}$$

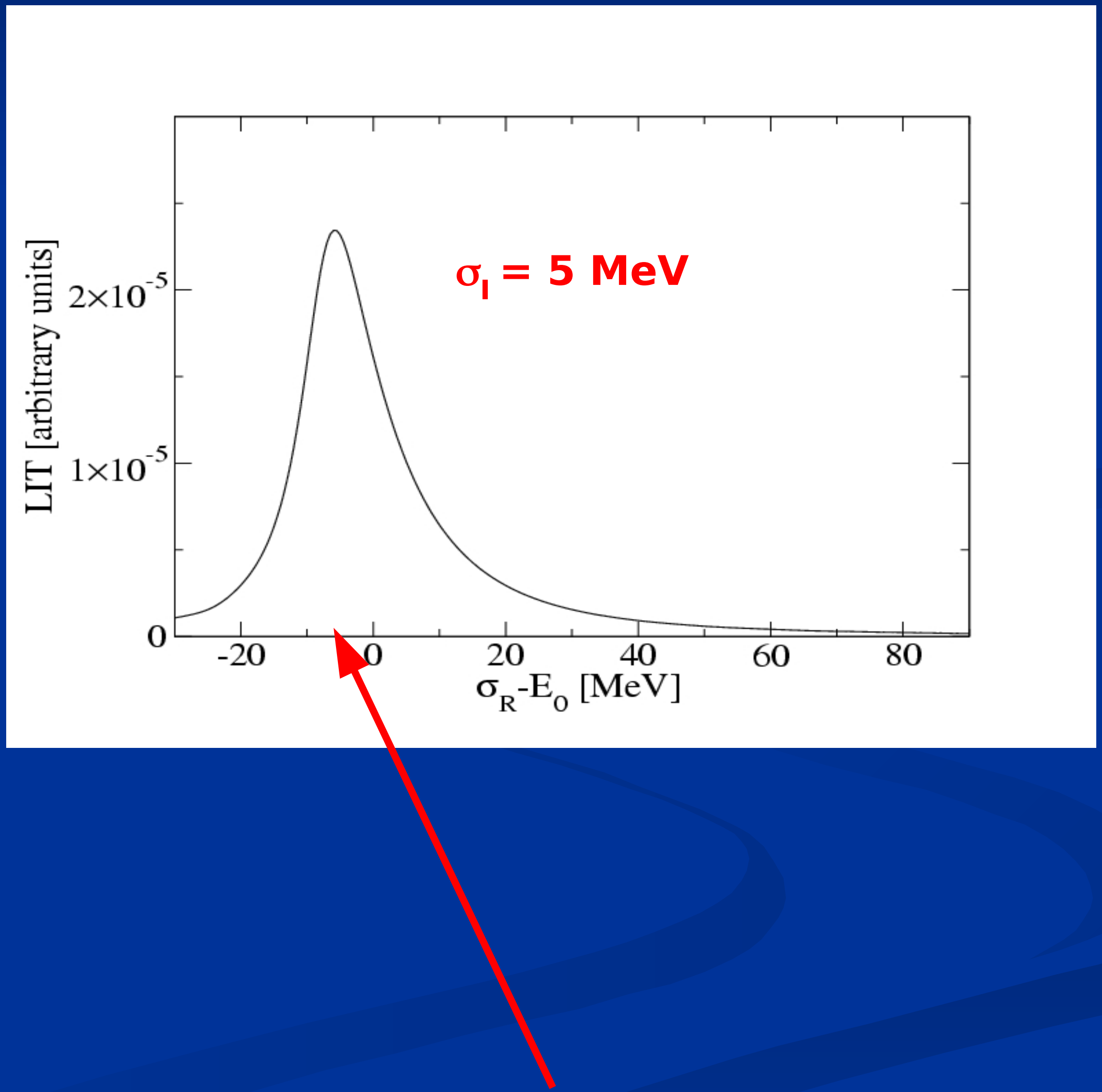
We have used the **Hyperspherical Harmonics**

basis and the Suzuki-Lee unitary transformation to speed up the convergence **(EIHH)**

As potentials we have used

both AV18+UIX and N3LO+N2LO

Some results:



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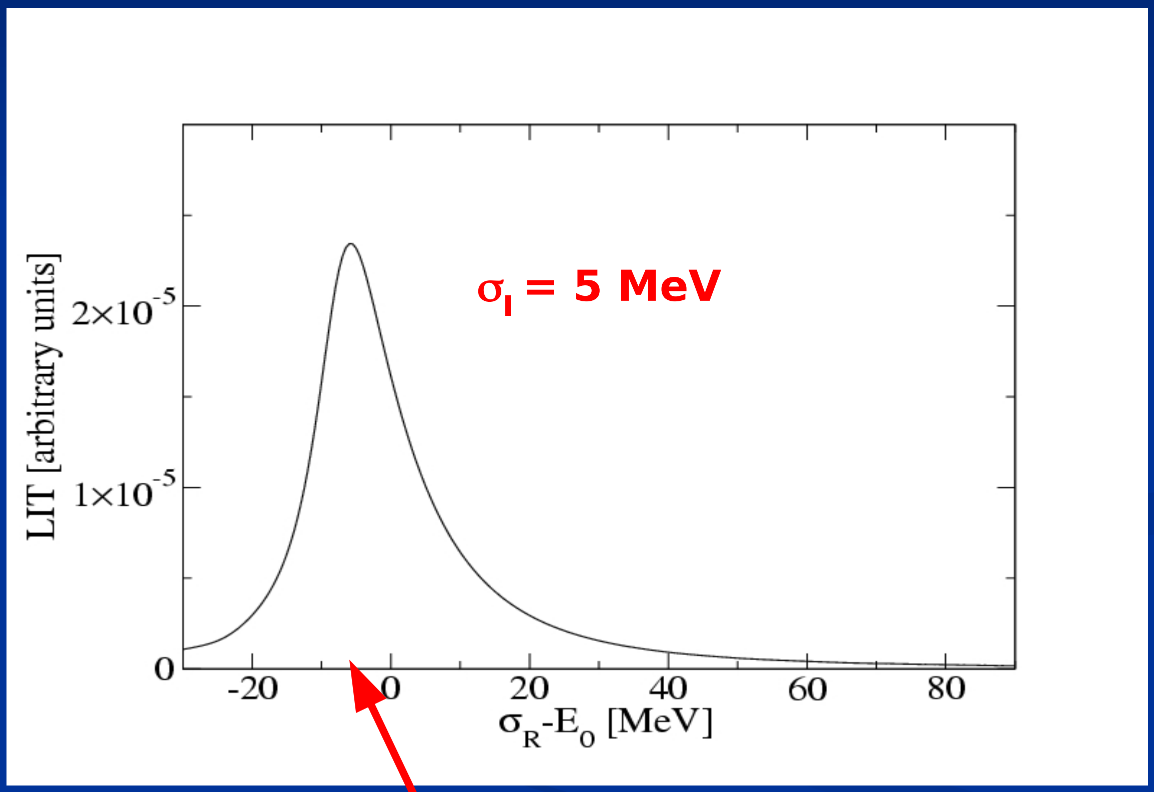

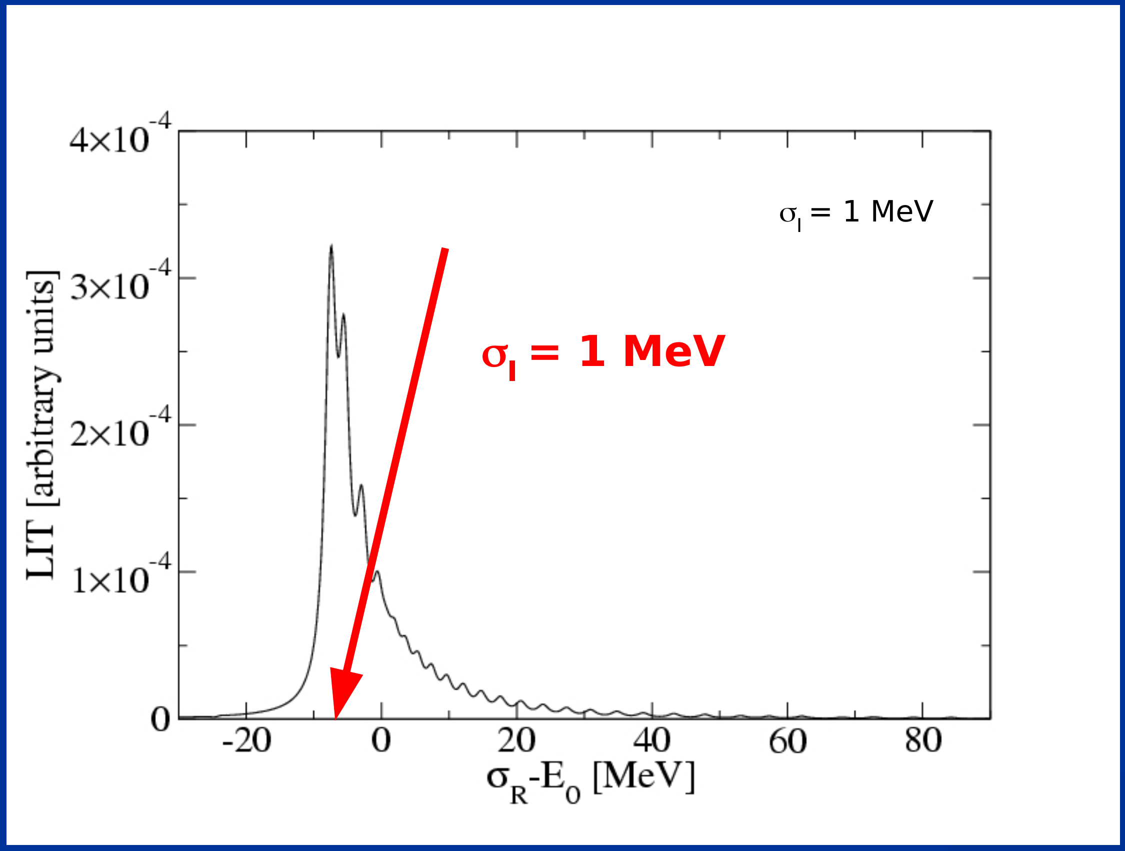

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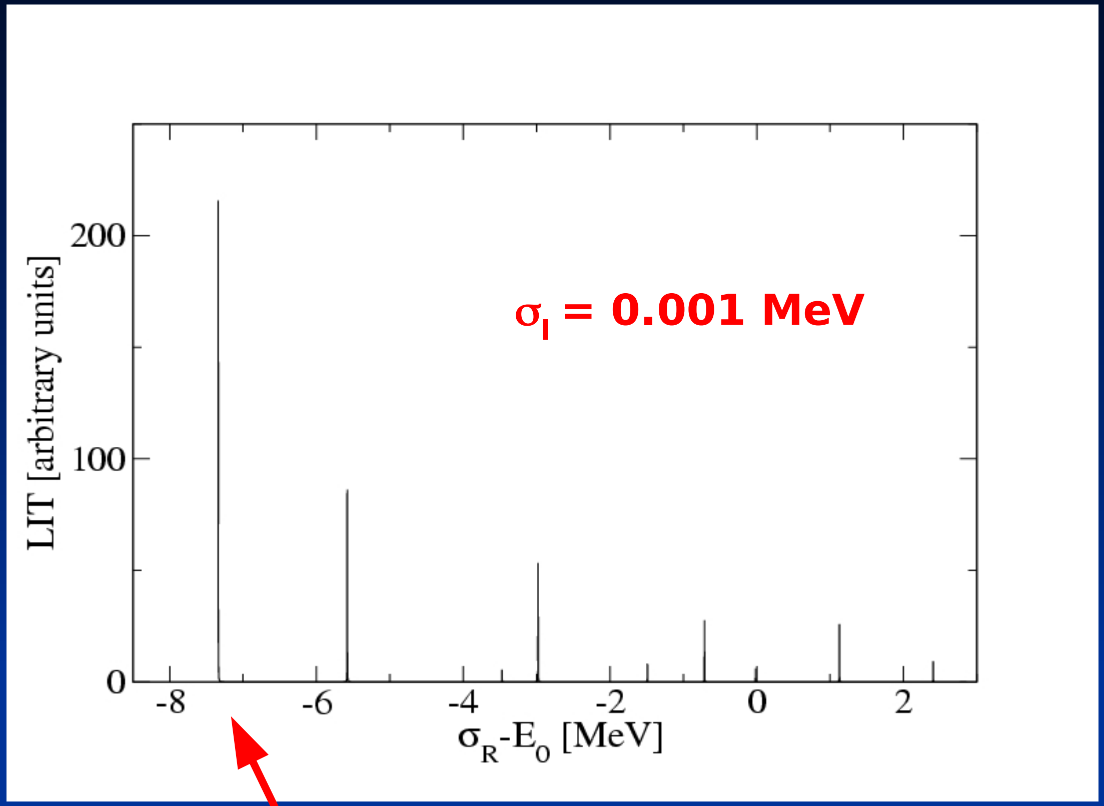

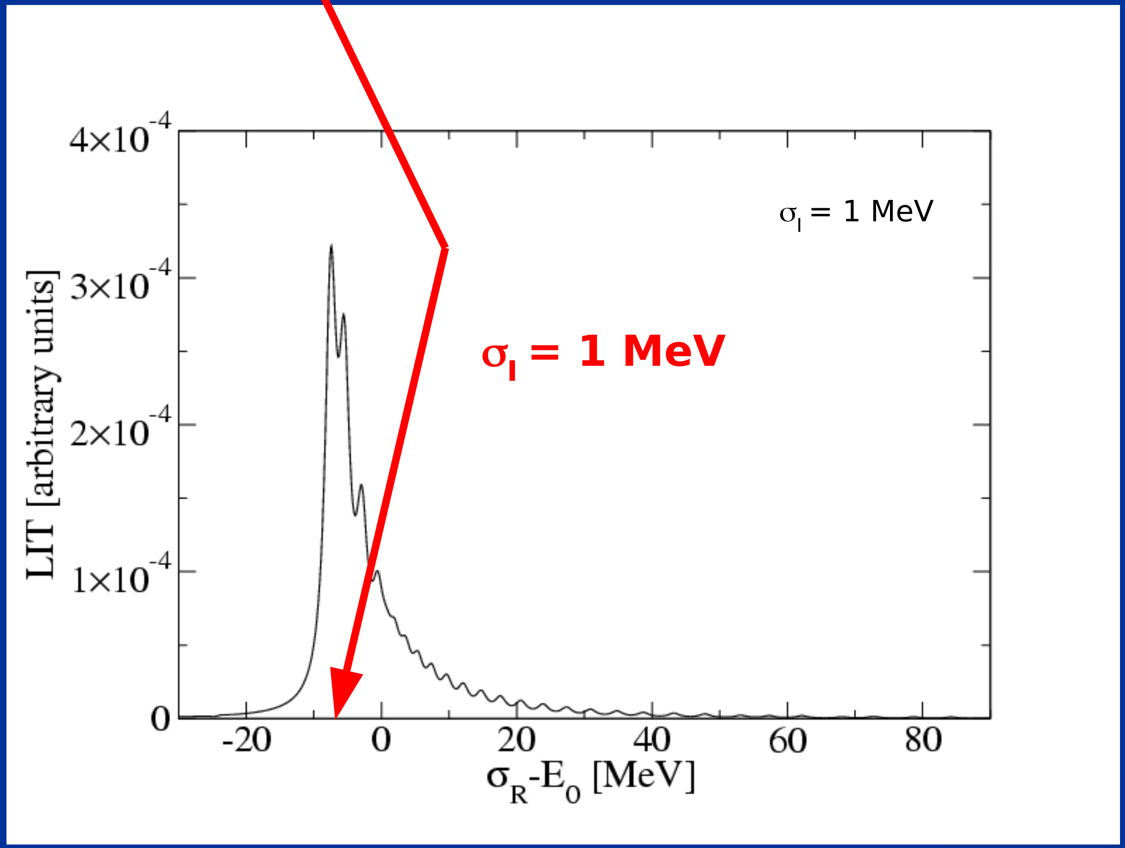

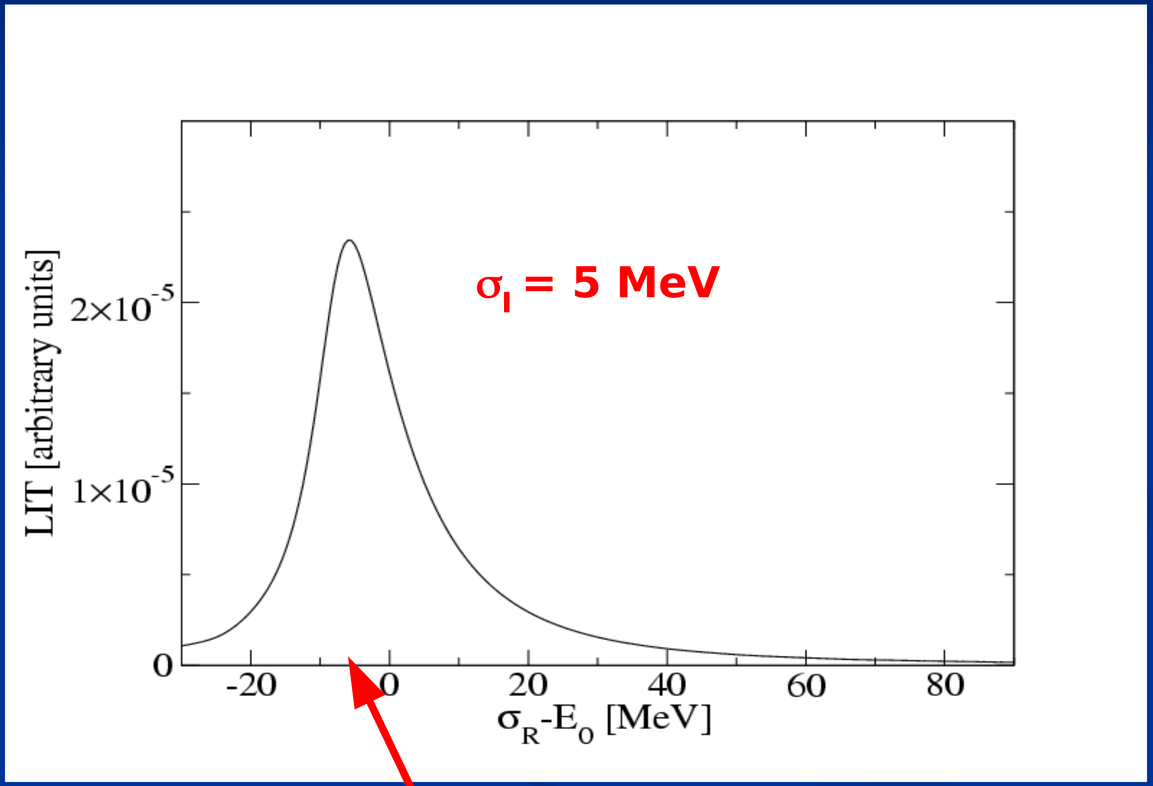

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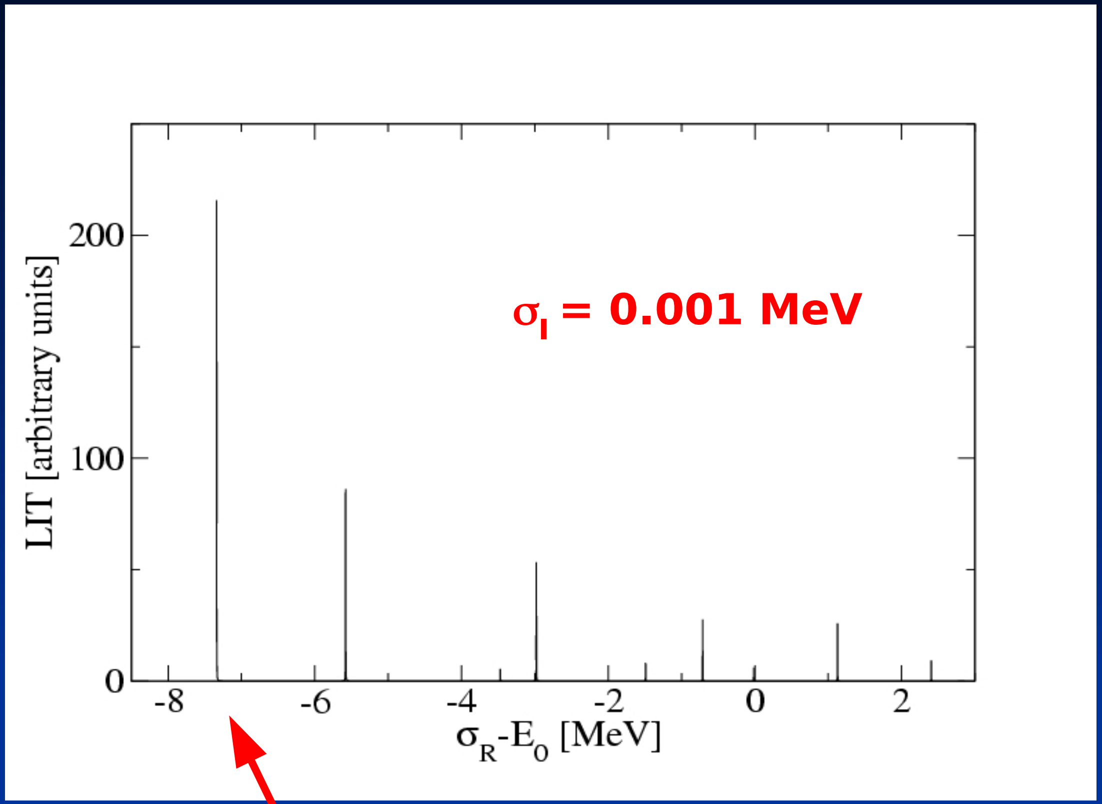
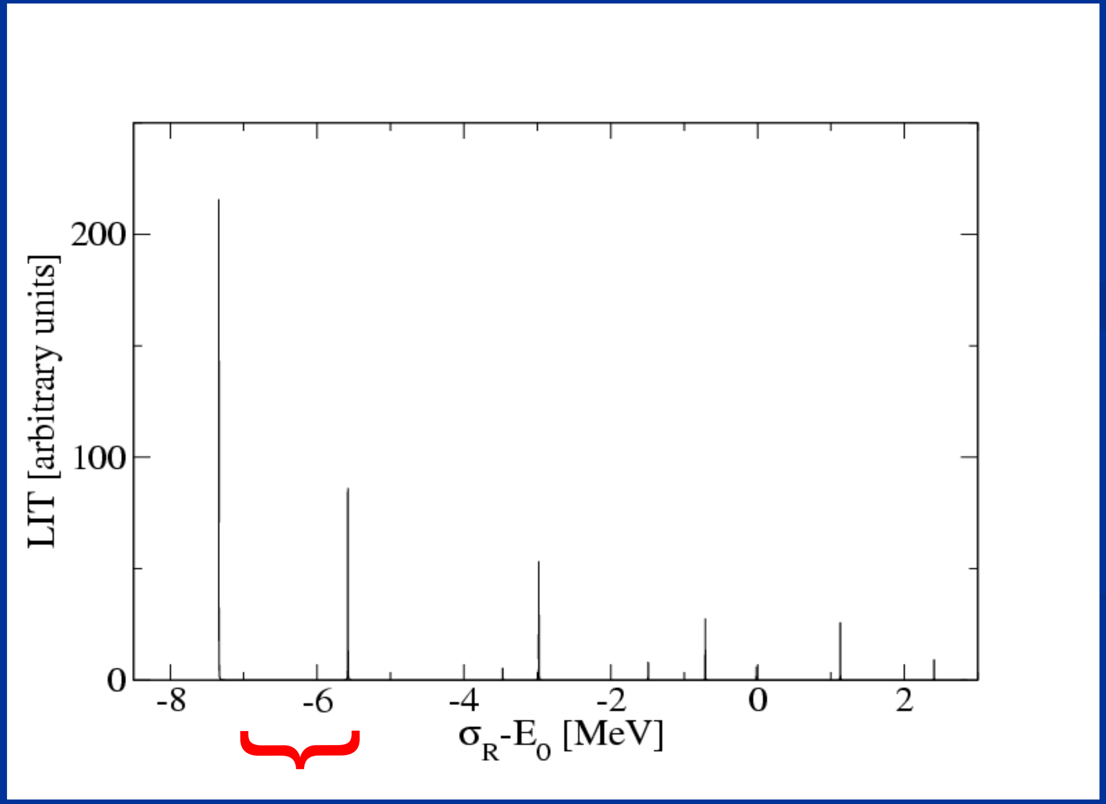


Of 200,000 “states” only very few are close to threshold

The precision of the calculation **did not** allow to resolve the shape of the resonance, therefore the width **could not** be determined.
[*now possible!* *See W.Leidemann, Phys. Rev. C 91, 054001 (2015)*]



too few states!

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However, the strength of the resonance (transition f.f.) **could be determined**!

Of course **not** by taking the strength to the state $|\xi_\nu\rangle$, but by arranging **the inversion** in a suitable way:

LIT - Inversion

Standard LIT inversion method

1) Take the following ansatz for the response function $S_M(q, \omega)$

$$S_M(q, \omega) = \sum_{m=1}^{M} c_m \chi_m(q, \omega, \alpha_i)$$

with given set of functions χ_m, and unknown coefficients c_m

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2) Calculate: $\boldsymbol{\lambda_m\ (q\ ,\ \omega_0\ ,\ \Gamma\ ) = \int d\omega\ \ \chi_m(q\ , \alpha_i)\ L\ (\ \omega, \omega_0\ ,\ \Gamma\ \ )}$

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2) Calculate: $\lambda_m(\mathbf{q}, \omega_0, \Gamma) = \int d\omega \; \chi_m(\mathbf{q}, \alpha_i) \, \mathbf{L}(\omega, \omega_0, \Gamma)$

3) Construct $\Lambda_L(\mathbf{q}, \omega_0, \Gamma) = \sum_{m=1} \mathbf{c_m} \, \lambda_m(\mathbf{q}, \omega_0, \Gamma)$

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3) Construct $\Lambda_L(\,q\,,\omega_0\,,\Gamma\,)=\sum_{m=1} c_m\,\lambda_m(q\,,\omega_0\,,\Gamma\,)$

4) Determine c_m and α_i by best fit on $\Lambda_L(\,q\,,\omega_0\,,\Gamma\,)$

Inversion in this case of a resonance

1) Subtract a Lorentzian centered in E_R with parameter f_R

$$\Lambda'_L(\mathbf{q}, \omega_0, \Gamma, f_R) \equiv \Lambda_L(\mathbf{q}, \omega_0, \Gamma, f_R) - f_R / [(\omega_0 - E_R)^2 + \Gamma^2]$$

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$$\chi_R(\omega) = f_R / [(E_R - \omega)^2 + \Gamma^2 / 4]$$

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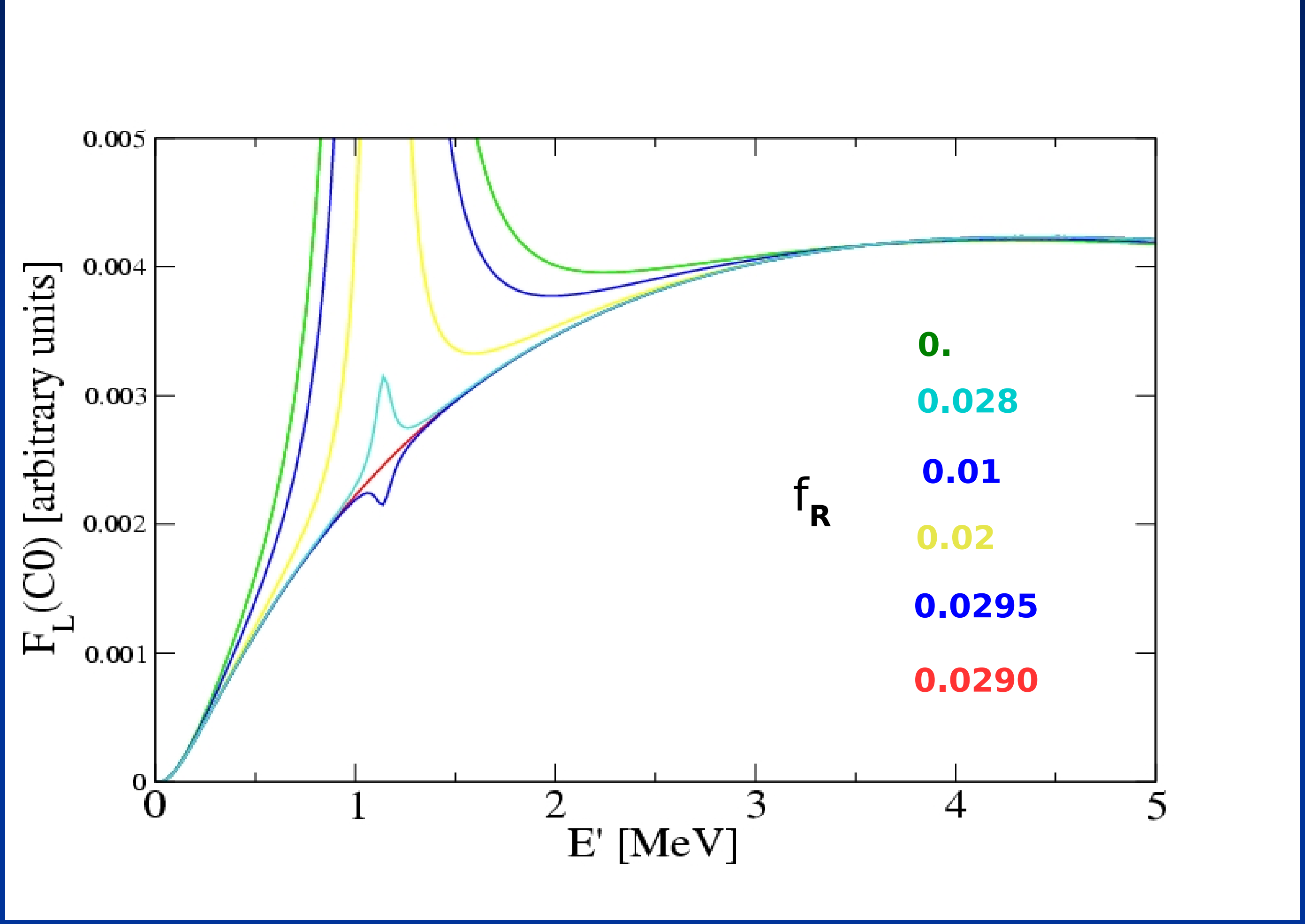
$$\chi_R(\omega) = f_R / [(E_R - \omega)^2 + \Gamma^2 / 4]$$

3) Reduce the strength f_R up to the point that the inversion does not show any resonant structure at the resonance energy E_R

Inversion results with different f_R values

[AV18+UIX, q=300 MeV/c, Γ= 5 MeV)



ENERGIES