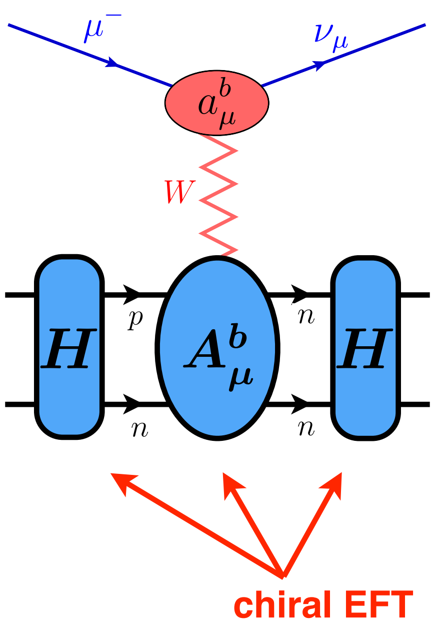
Evgeny Epelbaum, RUB

Two-body currents in electroweak processes

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
FROM QUARKS AND GLUONS TO HADRONS AND NUCLEI



Chiral EFT yields CONSISTENT nuclear forces and exchange currents.

Bundesministerium für Bildung und Forschung
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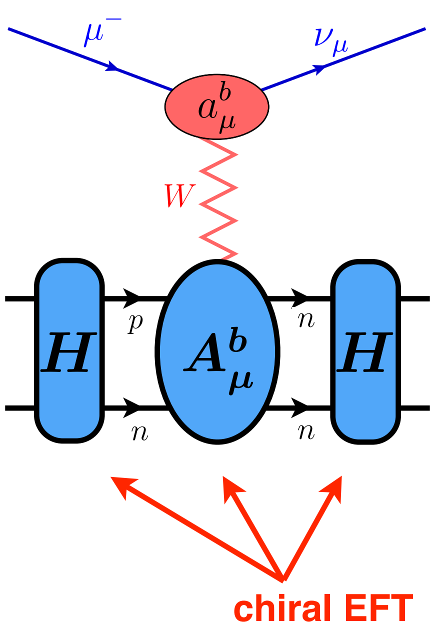

Chiral EFT yields CONSISTENT nuclear forces and exchange currents.

But what does this actually mean??

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From QCD to nuclei

QCD → **lattice**

symmetries (especially the chiral symmetry);
lost of information (LECs)

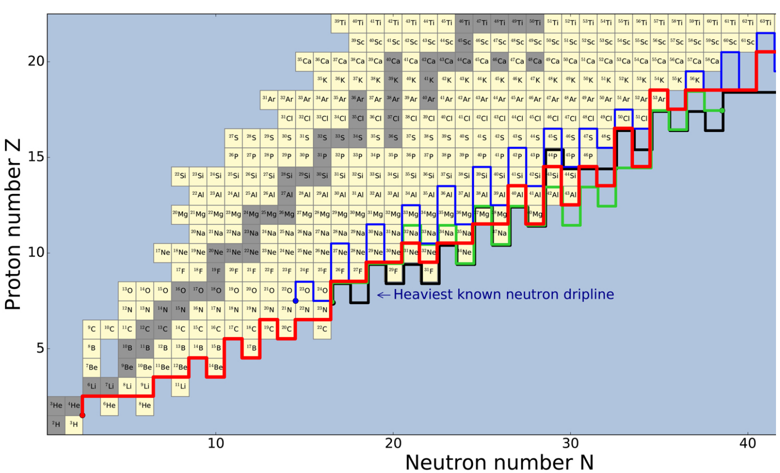
effective chiral Lagrangian $\mathcal{L}_{\text{eff}}(\pi, N)$

integrate out $|\vec{p}| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}| \sim M_\pi$):
Chiral Perturbation Theory

nuclear forces and currents

ab initio many-body methods:
lattice, FY, NCSM,…

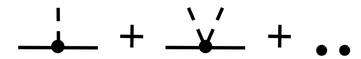
nuclear structure and dynamics



Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

- **Canonical transformation & quantization:** $\mathcal{L}_{\pi \mathrm{N}} \longrightarrow \mathcal{H}_{\pi \mathrm{N}} =$



nucleonic states $|N\rangle, \; |NN\rangle, \; \ldots$

EOM:
$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

← can not solve (infinite-dimensional eq.)

projectors

states with mesons $|N\pi\rangle, \; |N\pi\pi\rangle, \; \ldots$

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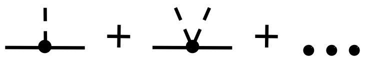
- **Canonical transformation & quantization:** $\mathcal{L}_{\pi \mathrm{N}} \longrightarrow \mathcal{H}_{\pi \mathrm{N}} =$ 

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- **Decouple pions via a suitable UT:** $\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$

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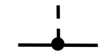
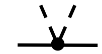
- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$  +  + ...

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(Minimal) ansatz: $$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger (1 + A A^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + A A^\dagger)^{-1/2} \end{pmatrix}, \quad A = \lambda A \eta$$

Okubo '54

Require: $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0$ ⟶ $$\boxed{\lambda \left(H - [A, \ H] - A H A \right) \eta = 0}$$

The decoupling equation is solved perturbatively (chiral expansion)

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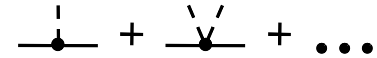
- **Canonical transformation & quantization:** $\mathcal{L}_{\pi\mathrm{N}} \longrightarrow \mathcal{H}_{\pi\mathrm{N}} =$ 

EOM: $$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$ ← **can not solve (infinite-dimensional eq.)**

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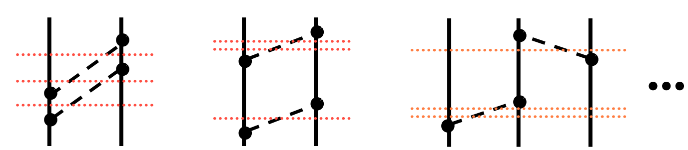
E.g., for the 2-pion exchange $\propto g_A^4$ one finds:



$$V^{(2)} = \eta \left[- H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} + \frac{1}{2} H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} + \frac{1}{2} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} \right] \eta$$

Method of Unitary Transformation

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Contrary to S-matrix, renormalizability of nuclear potentials is not guaranteed

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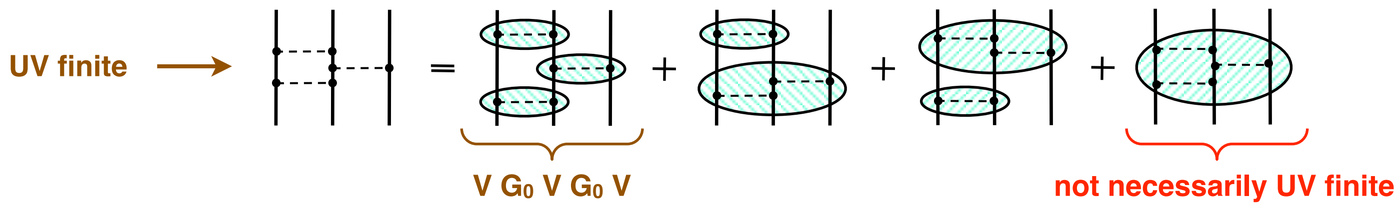

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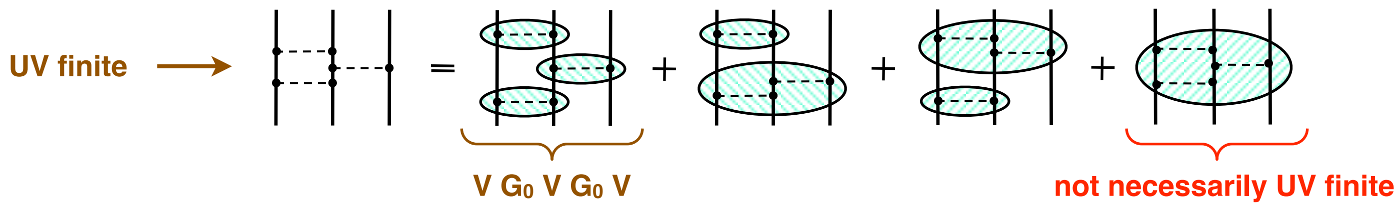
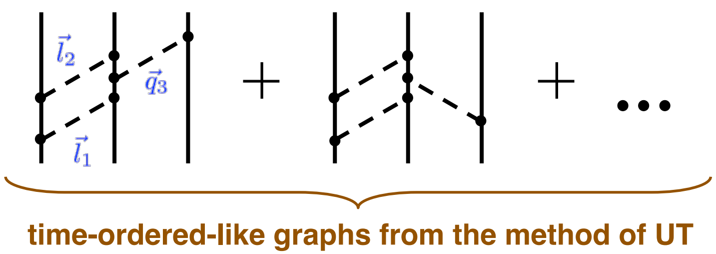


Indeed, explicit calculations of e.g. the 3NF $\propto g_A^6$ yield:



$$V = \ldots = \int d^3l_1\, d^3l_2\, \delta(\vec{l}_1 - \vec{l}_2 - \vec{q}_1) \left[\ldots \right]$$

$$\times \left[2\frac{\omega_1^2 + \omega_2^2}{\omega_1^4\, \omega_2^4\, \omega_3^2} + \frac{8}{\omega_1^2\, \omega_2^2\, \omega_3^4} - \frac{\omega_1 + \omega_2}{\omega_1^3\, \omega_2^3\, \omega_3^3} - \frac{2}{\omega_1^4\, \omega_2^2\, \omega_3\, (\omega_1 + \omega_3)} - \frac{2}{\omega_1^2\, \omega_2^4\, \omega_3\, (\omega_2 + \omega_3)} \right]$$

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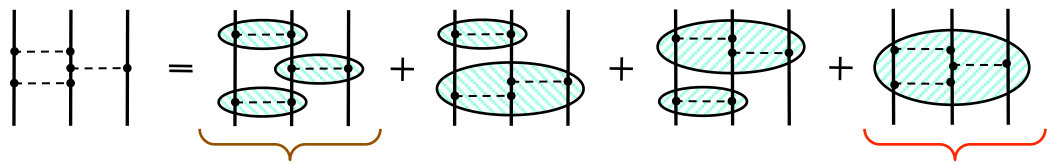
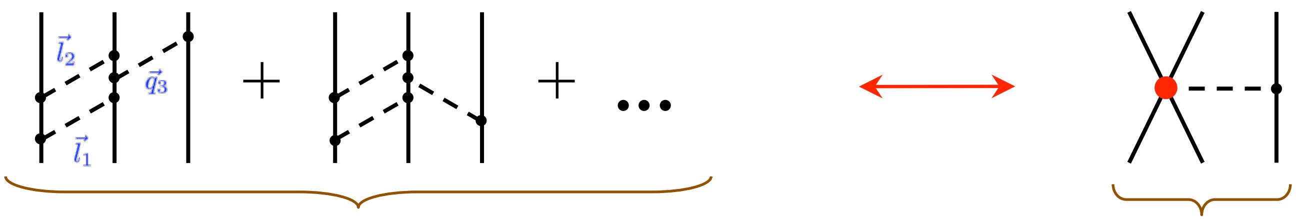
UV finite →



V G₀ V G₀ V

not necessarily UV finite

Indeed, explicit calculations of e.g. the 3NF $\propto g_A^6$ yield:



time-ordered-like graphs from the method of UT

$\propto 1/\omega_3^2$

$$V = \ldots = \int d^3 l_1\, d^3 l_2\, \delta(\vec{l}_1 - \vec{l}_2 - \vec{q}_1) \left[\ldots \right]$$

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→ cannot renormalize the potential !

Method of Unitary Transformation

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Solution [EE '06]

Nuclear potentials are not uniquely defined. Starting from N^3LO, can construct additional UTs in Fock space beyond the (minimal) Okubo UT.

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Solution [EE '06]

Nuclear potentials are not uniquely defined. Starting from N³LO, can construct additional UTs in Fock space beyond the (minimal) Okubo UT.

The UTs relevant for the N³LO contributions $\propto g_A^6$ are $U = e^{\alpha_1 S_1 + \alpha_2 S_2}$, with the generators given by:

$$S_1 = \eta \Big[H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^3} H_I^{(1)} - \text{h.c.} \Big] \eta$$

$$S_2 = \eta \Big[H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} - \text{h.c.} \Big] \eta$$

They induce additional contributions in the Hamiltonian starting from N³LO

$$\delta V^{(4)} = [(H_{\text{kin}} + V^{(0)}), \, S] = -\alpha_1 \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^3} H_I^{(1)} + \ldots .$$

Demanding renormalizability constrains α_1, α_2 and leads to unique static results.

So far, it was always possible to obtain **finite** nuclear potentials and currents.

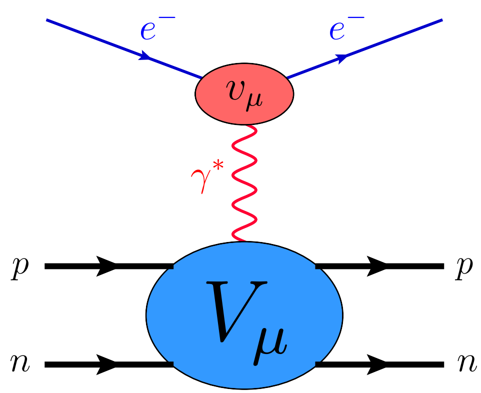
Current operators

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\,r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\,l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

- Decouple π's to get (nonlocal) nuclear $H_{\rm eff}[a, v, s, p]$ (MUT) & get currents via

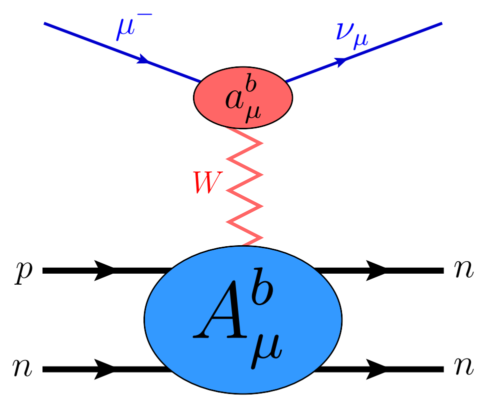
$$V^a_\mu(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta v^\mu_a(\vec x, t)}, \quad A^a_\mu(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta a^\mu_a(\vec x, t)}$$ calculated at $a = v = p = 0,\ s = m_q$.



Park, Min, Rho '95

Pastore et al. (TOPT) '08 — '11: not renormalized…

Kölling, EE, Krebs, Meißner (MUT) '09,'12;
Krebs et al., in preparation: complete (1 loop) & renormalized



Park, Min, Rho '93

Baroni et al. (TOPT) '16: incomplete…

Krebs, EE, Meißner (MUT) '17: complete (1 loop) & renormalized, also derived pseudoscalar currents

— about 250 topologies

— 2-loop/1-loop/tree for 1N/2N/3N operators

Also derived scalar currents, Krebs et al., in preparation

Continuity equations

Krebs, EE, Meißner, Annals Phys. 378 (17) 317

Unexpected result: the continuity equation $\vec{k} \cdot \vec{j} \neq [H_{\rm str}, \rho]$! Why?

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Naive (MUT): $\underbrace{H_{\rm eff}[h]}_{a,\,v,\,s,\,p} = \eta\, U^\dagger_{\rm str}\, H_{\pi N}[h]\, U_{\rm str}\, \eta$ ← cannot renormalize V^a_μ, A^a_μ …

determined in the strong sector ($a = v = p = 0, \; s = m_q$)

Solution: employ a more general class of UT's, namely

$$H_{\rm eff}[h, \dot h] = U^\dagger_\eta[h]\, \eta\, U^\dagger_{\rm str}\, H_{\pi N}[h]\, U_{\rm str}\, \eta\, \underbrace{U_\eta[h]} + i \left(\frac{\partial}{\partial t} U^\dagger_\eta[h] \right) U_\eta[h]$$

subject to the constraint $U_\eta[0, 0, m_q, 0] = \eta$

← *induce* k_0*-dependence in the currents (off-shell effect…)*

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Continuity equations = manifestations of the chiral symmetry, $h(x) \xrightarrow{SU(2)_L \times SU(2)_R} h'(x)$: $H_{\rm eff}[h, \dot h]$ and $H_{\rm eff}[h', \dot h']$ should be unitary equivalent, i.e. there exists such $U(t)$ that

$$H_{\rm eff}[h', \dot h'] = U^\dagger(t) H_{\rm eff}[h, \dot h] U(t) + i \left(\frac{\partial}{\partial t} U^\dagger_\eta(t)\right) U_\eta(t)$$

This implies the relations for currents $V^i_\mu(k) := \frac{\delta H_{\rm eff}}{\delta v^\mu_i(k)}$, $A^i_\mu(k) := \frac{\delta H_{\rm eff}}{\delta a^\mu_i(k)}$, $P^i(k) := \frac{\delta H_{\rm eff}}{\delta p_i(k)}$:

$$\vec{k} \cdot \vec{A}^i(\vec{k}, 0) = \left[H_{\rm str},\, A^i_0(\vec{k}, 0) - \frac{\partial}{\partial k_0}\left(\vec{k} \cdot \vec{A}^i(k) + [H_{\rm str},\, A^i_0(k)] + i m_q P^i(k)\right)\right] + i m_q P^i(\vec{k}, 0)$$

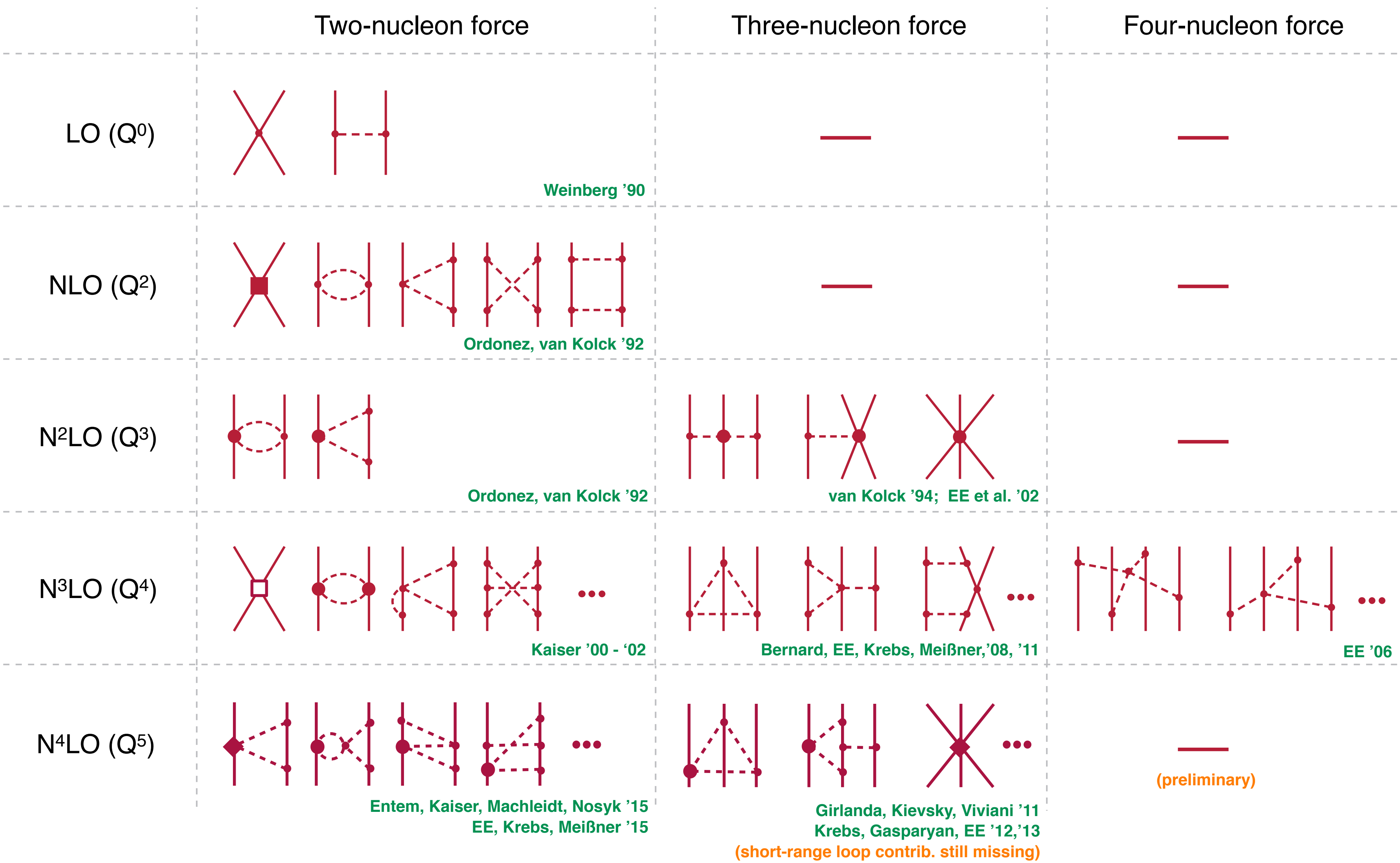
$$\vec{k} \cdot \vec{V}^i(\vec{k}, 0) = \left[H_{\rm str},\, V^i_0(\vec{k}, 0) - \frac{\partial}{\partial k_0}\left(\vec{k} \cdot \vec{V}^i(k) + [H_{\rm str},\, V^i_0(k)]\right)\right]$$

Nuclear forces and currents

— state of the art —

[HB ChPT formulation with π and N as the only explicit DOF]

Chiral expansion of the nuclear forces [W-counting]



— A similar program is being pursued in chiral EFT with explicit Δ(1232) Kaiser et al.; Krebs, Gasparyan, EE, Meißner

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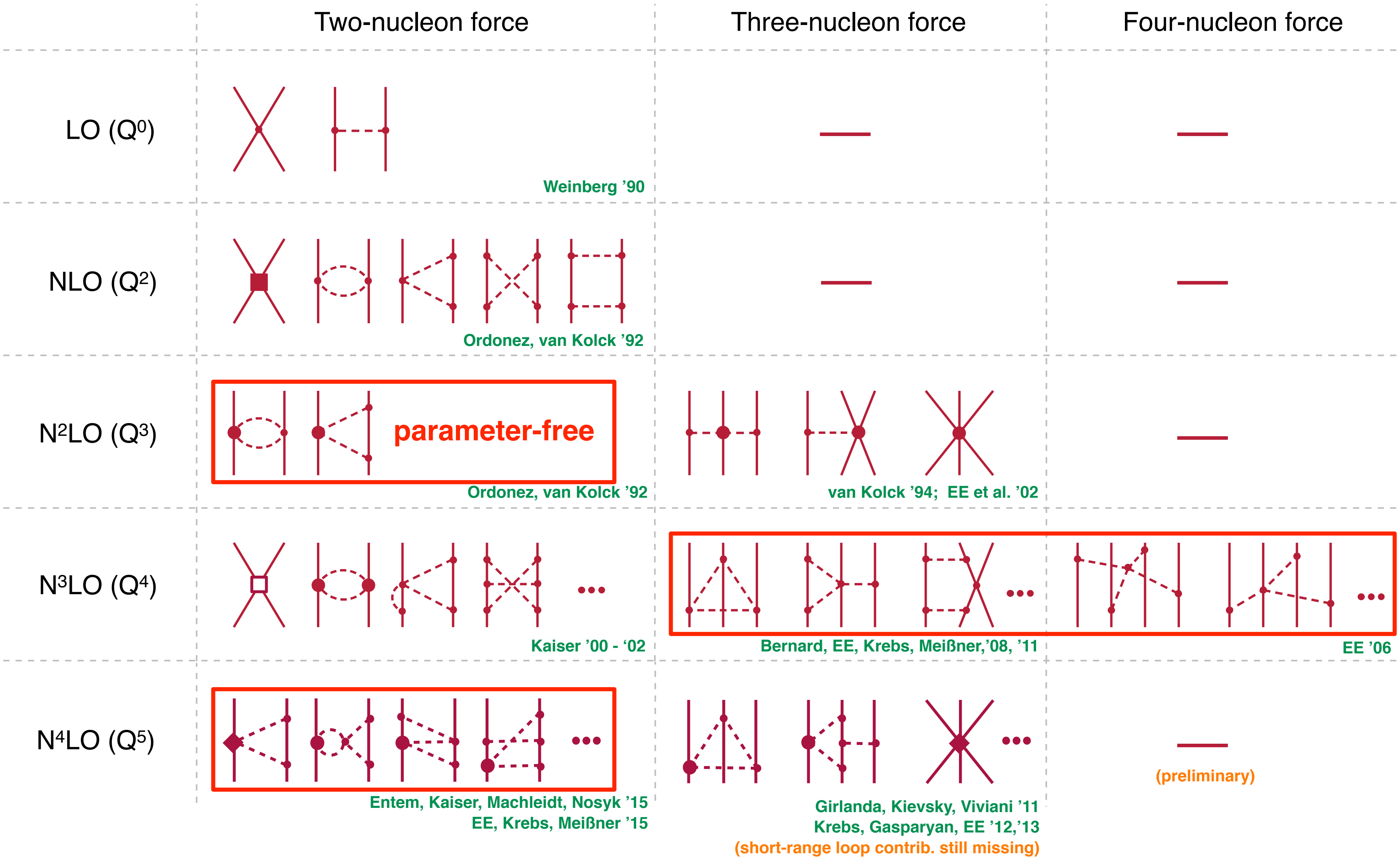

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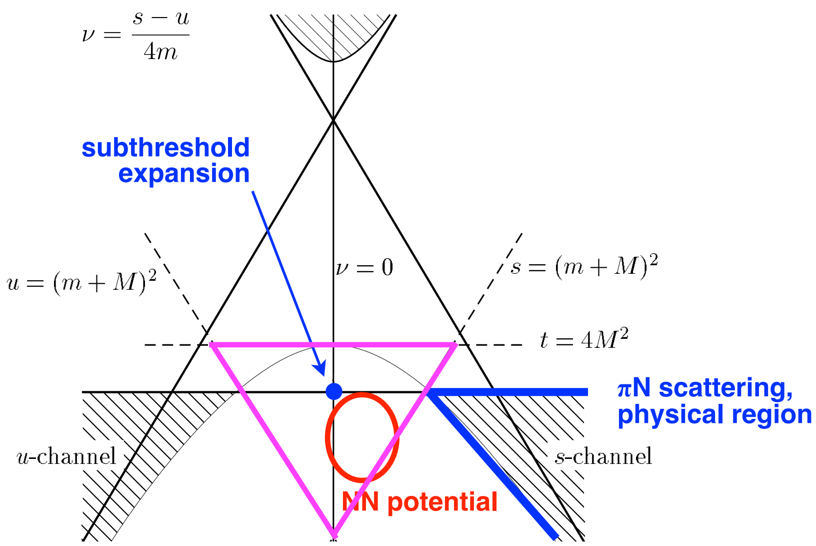
Determination of πN LECs



Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

$$\bar{X} = \sum_{m,n} x_{mn} \, \nu^{2m+k} t^n, \qquad X = \{A^\pm, B^\pm\}$$

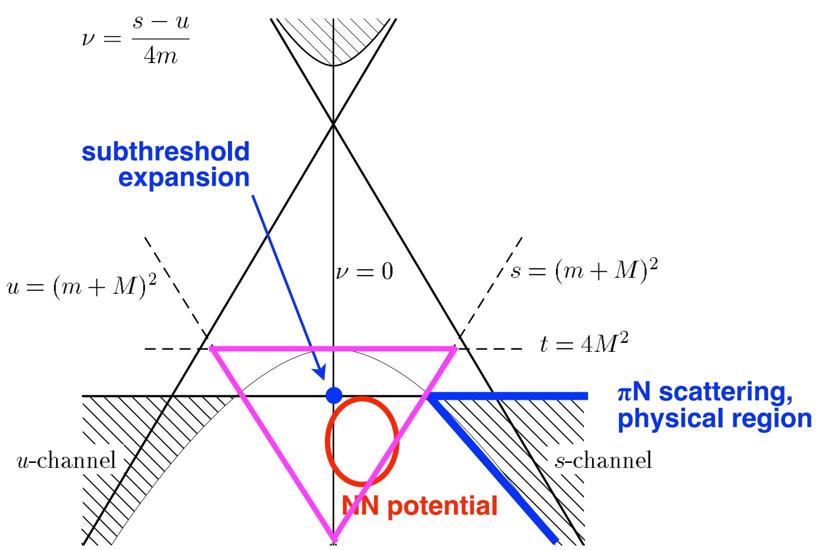
- Closer to the kinematics relevant for nuclear forces…

Relevant LECs (in GeV^{-n}) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$	
$[Q^4]_{\mathrm{HB,NN}}$, GW PWA	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−0.58	Krebs, Gasparyan, EE, PRC85 (12) 054006
$[Q^4]_{\mathrm{HB,NN}}$, KH PWA	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−0.37	(Krebs, Gasparyan, EE, PRC85 (12) 054006)
$[Q^4]_{\mathrm{HB,NN}}$, Roy-Steiner	−1.10	3.57	−5.54	4.17	6.18	−8.91	0.86	−12.18	1.18	−0.18	Hoferichter et al., PRL 115 (15) 092301
$[Q^4]_{\mathrm{covariant}}$, data	−0.82	3.56	−4.59	3.44	5.43	−4.58	−0.40	−9.94	−0.63	−0.90	Siemens et al., PRC94 (16) 014620

— Some LECs show sizable correlations (especially c_1 and c_3)…

Determination of πN LECs



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$[Q^4]_{\rm HB,\,NN}$, GW PWA	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−0.58	Krebs, Gasparyan, EE, PRC85 (12) 054006
$[Q^4]_{\rm HB,\,NN}$, KH PWA	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−0.37	
$[Q^4]_{\rm HB,\,NN}$, Roy-Steiner	−1.10	3.57	−5.54	4.17	6.18	−8.91	0.86	−12.18	1.18	−0.18	Hoferichter et al., PRL 115 (15) 092301
$[Q^4]_{\rm covariant}$, data	−0.82	3.56	−4.59	3.44	5.43	−4.58	−0.40	−9.94	−0.63	−0.90	Siemens et al., PRC94 (16) 014620

— Some LECs show sizable correlations (especially c_1 and c_3)…

— EKM N^{4}LO [EE, Krebs, Meißner, PRL 115 (2015) 122301]: Q^4 fit to KH PWA

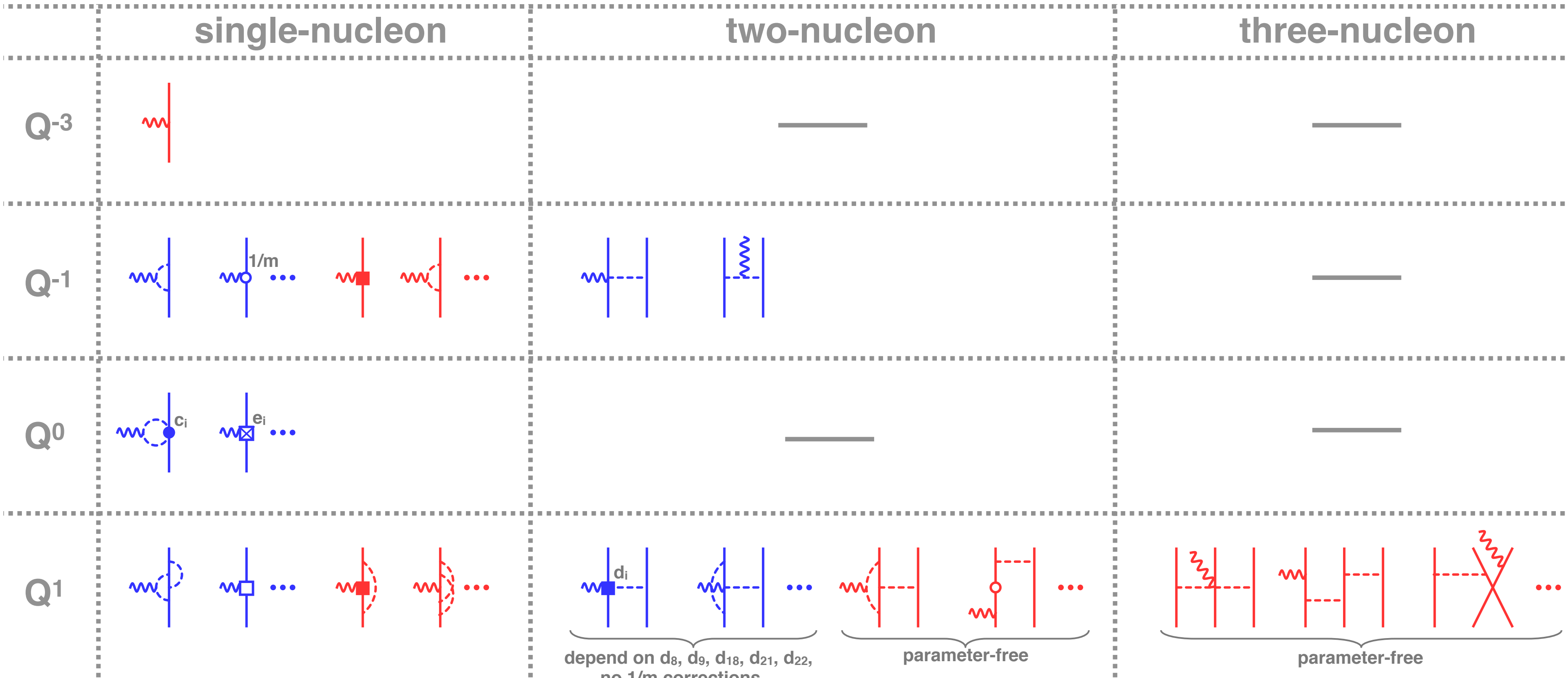
— RKE N^{4}LO [Reinert, Krebs, EE, EPJA 54 (2018) 88]: Q^4 fit to RS and Q^4 fit to KH PWA

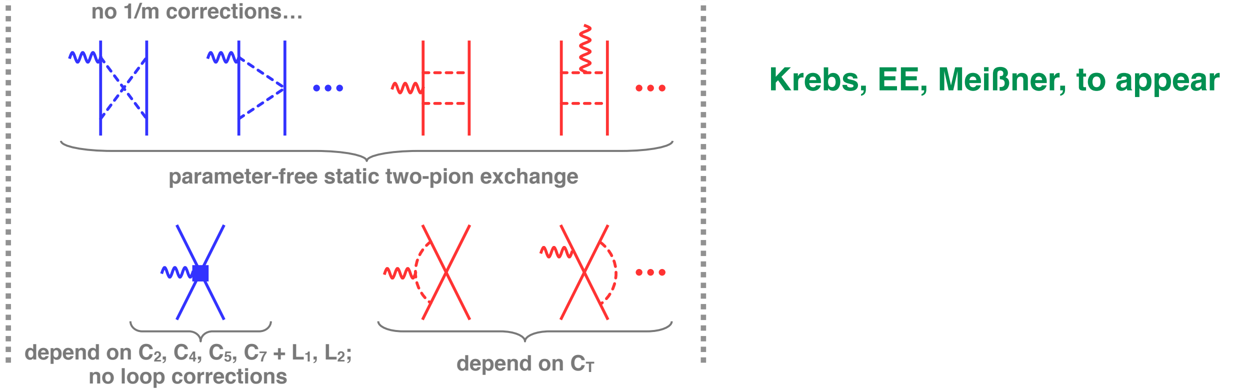
With the LECs taken from πN, the long-range NN force is fixed in a parameter-free way

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502;
PRC 86 (12) 047001

Chiral expansion of the electromagnetic **current** and **charge** operators



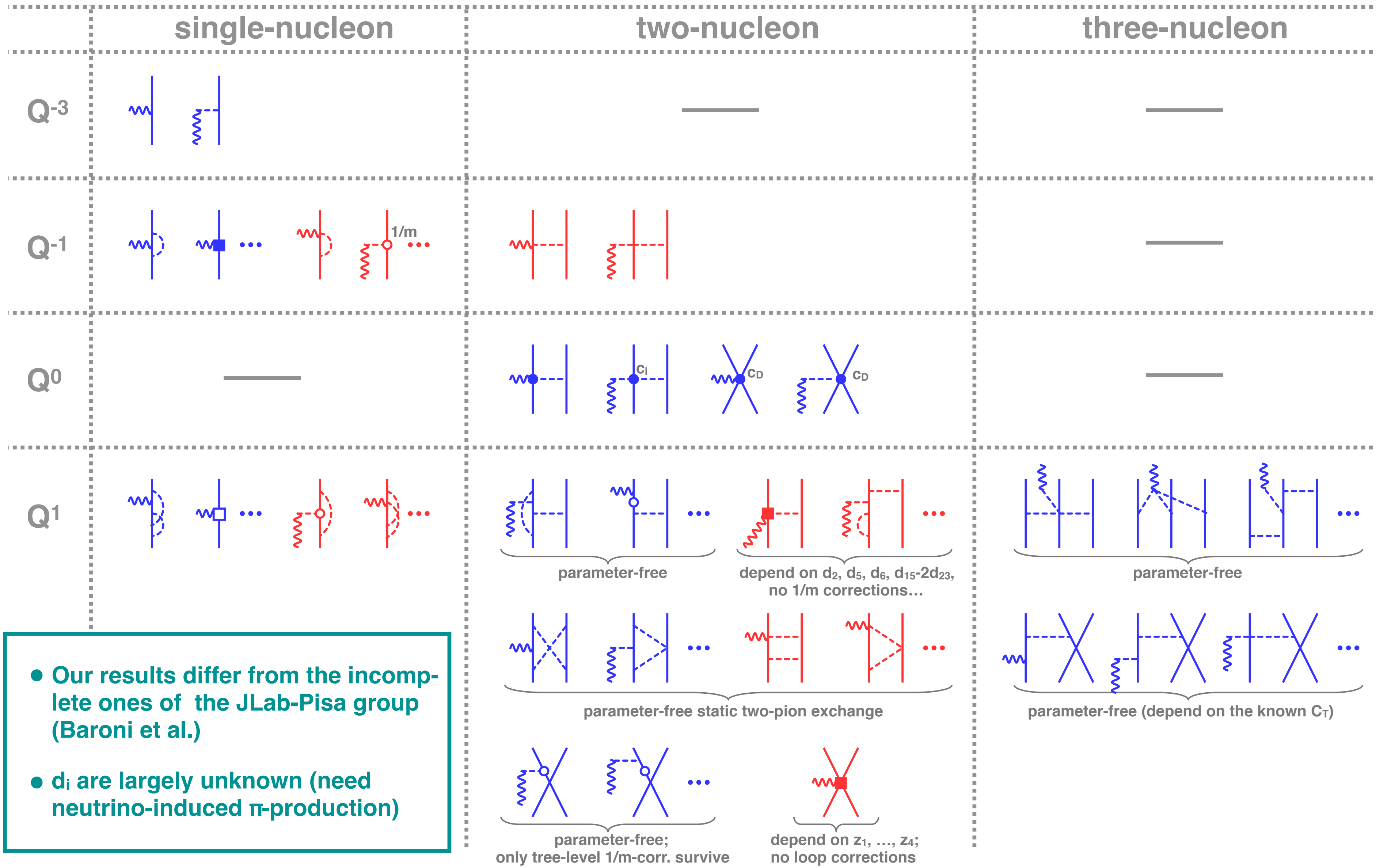


- Exchange currents do not depend on k_0.
- Our results differ from the ones of the JLab-Pisa group (Pastore et al., 08-11)
- d_i can be fixed from GTD, axial radius and π-photoproduction

Axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators



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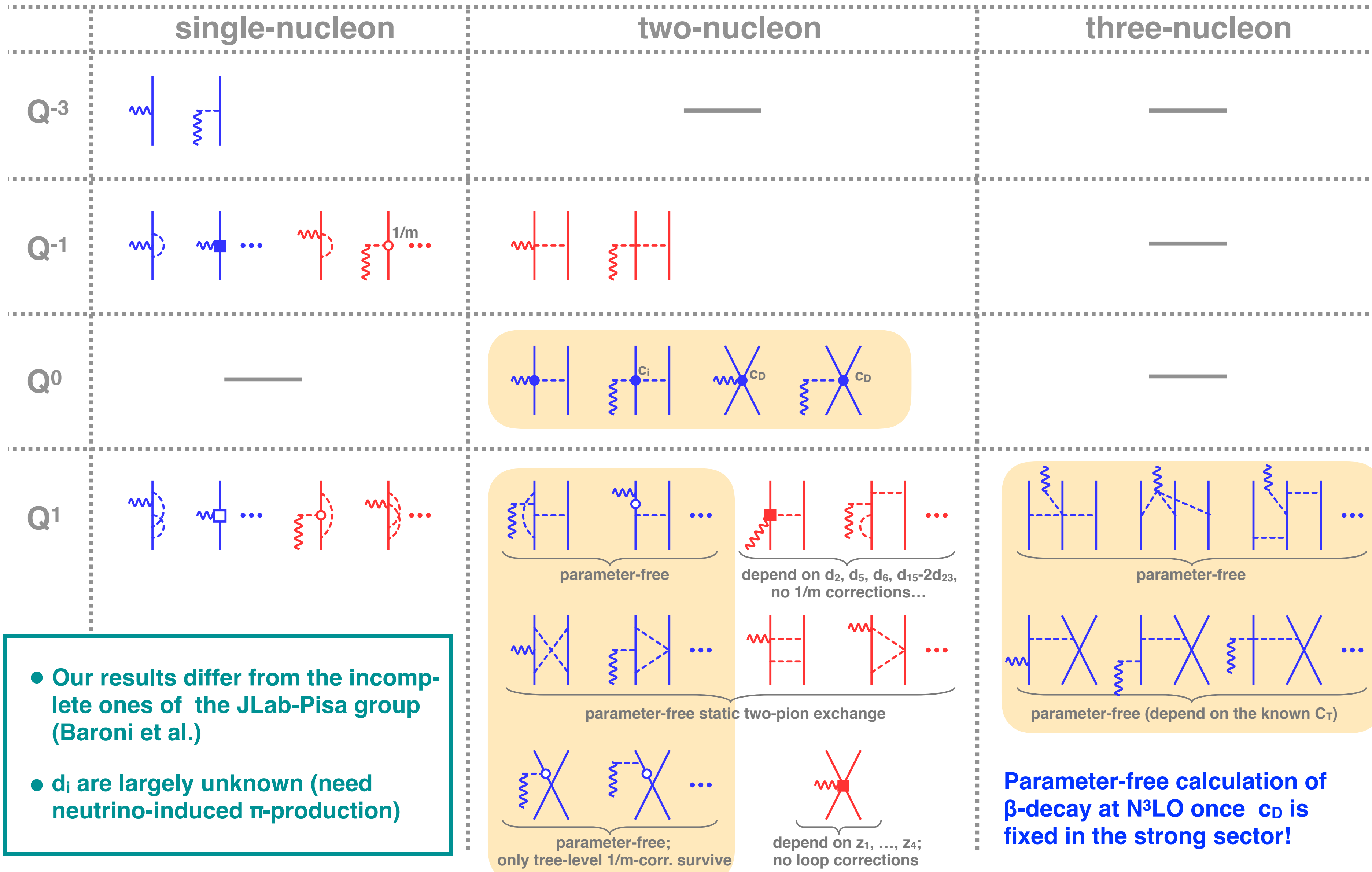

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Parameter-free calculation of β-decay at N³LO once c_D is fixed in the strong sector!

Intermediate summary

- Derivation of nuclear forces complete through N^4LO (a few N^4LO 3NF contributions still missing)

- Derivation of EM, axial and pseudoscalar currents complete through N^3LO

- Loop contributions are calculated using dimensional regularization; off-shell behavior (unitary ambiguity) is chosen to ensure renormalizability.

- **Nuclear forces and currents** (derived by our group) correspond to the same choice of UT and **are** thus **off-shell consistent** with each other.

  This means: $V + V G_0 V + V G_0 V G_0 V + \ldots$, evaluated using DR, reproduce the corresponding contributions to the S-matrix in terms of Feynman diagrams.

Selected applications

✅ **Accurate & precise NN potentials**

— new generation of semilocal r- and p-space NN potentials up to N^4LO^+

— currently the best description of the 2013 Granada data

☑️ **Consistent 3NFs**

— worked out to N^3LO (and even beyond), numerical PWD has been developed

— regularization nontrivial starting from N^3LO, 3NF@N^2LO ready to use

☑️ **Consistent currents**

— worked out to N^3LO, numerical PWD has been developed (at the 2N level…)

— consistent regularization to be done, axial currents@N^2LO ready to use

Regularization in the 2N sector

The cutoff Λ has to be kept finite, $\Lambda \sim \Lambda_b$. In practice, low values of Λ are preferred:

— many-body methods require soft interactions,

— spurious deeply-bound states for $\Lambda > \Lambda^{crit}$ make calculations for $A > 3$ unfeasible…

⟶ it is crucial to employ a regulator that minimizes finite-Λ artifacts!

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Nonlocal: $$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4 + p^4}{\Lambda^4}}}{\vec q^{\,2} + M_\pi^2} \longrightarrow \frac{1}{\vec q^{\,2} + M_\pi^2} \underbrace{\left(1 - \frac{p'^4 + p^4}{\Lambda^4} + \mathcal{O}(\Lambda^{-8})\right)}_{\text{affect long-range interactions...}}$$

EE, Glöckle, Meißner '04;
Entem, Machleidt '03;
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Local (implemented in coordinate space)

$$V_\pi(\vec r) \longrightarrow V_\pi(\vec r)\left[1-\exp(-r^2/R^2)\right]^n$$ used in **EE, Krebs, Meißner (EKM) '15**

— still an ad hoc procedure

— (technically) difficult to apply to 3NF and exchange currents

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[inspired by Thomas Rijken]

Reinert, Krebs, EE '18;

⟶ does not affect long-range physics **at any order** in $1/\Lambda^2$-expansion

— Application to 2π exchange does not require re-calculating the corresponding diagrams:

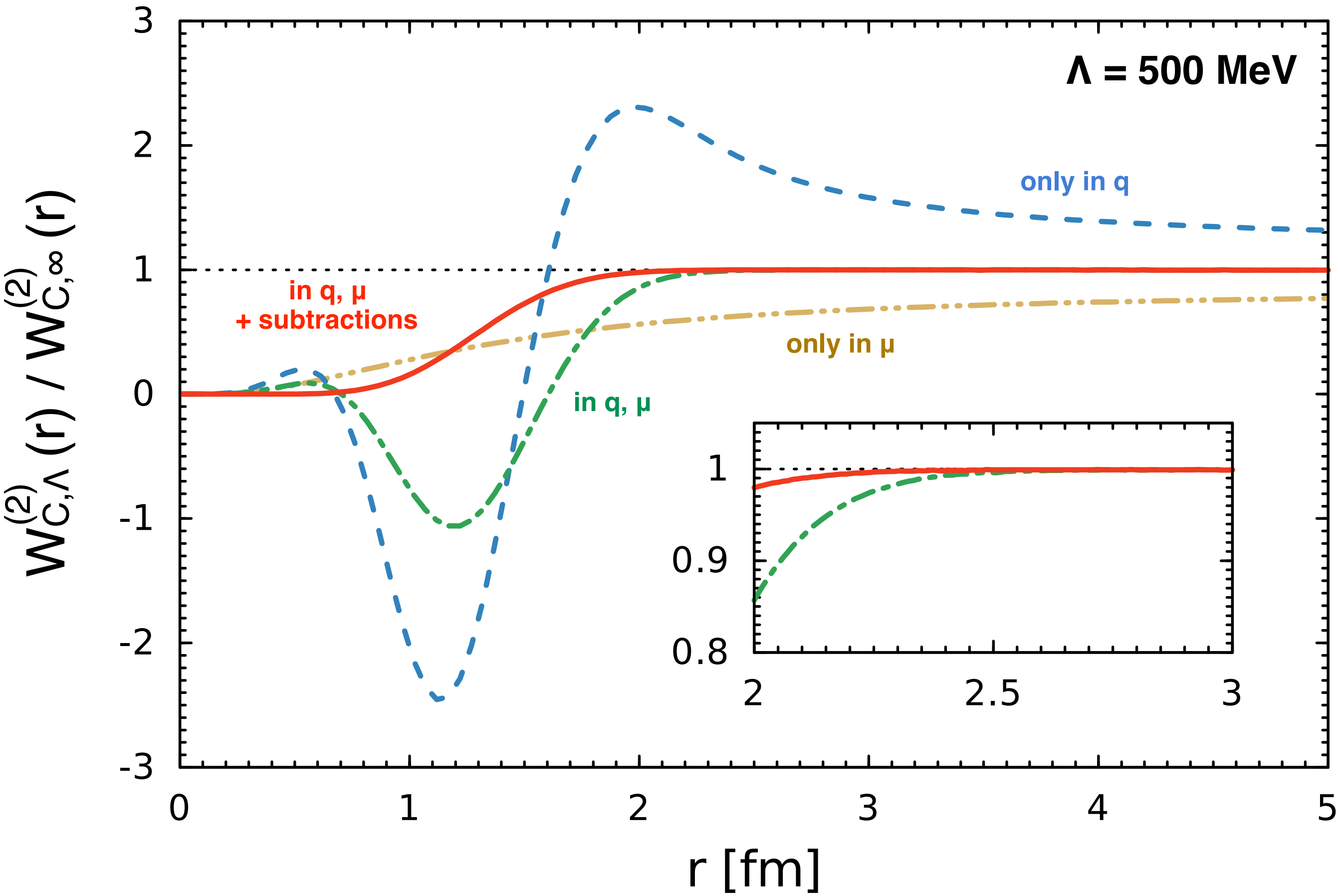
$$V(q)=\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2} + \ldots \xrightarrow{\text{reg.}} V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}}\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}\, e^{-\frac{\mu^2}{2\Lambda^2}} + \underbrace{\ldots}_{\text{polynomial in } q^2,\, M_\pi}$$

— Convention: choose polynomial terms such that $\Delta^n V_{\Lambda,\,\rm long}(\vec r)\Big|_{r=0} = 0$

Regularization in the 2N sector

Regularized 2π-exchange potential: $W_{C,\Lambda}(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi^2}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} \, e^{-\frac{\mu^2}{2\Lambda^2}}$

Various regularization approaches



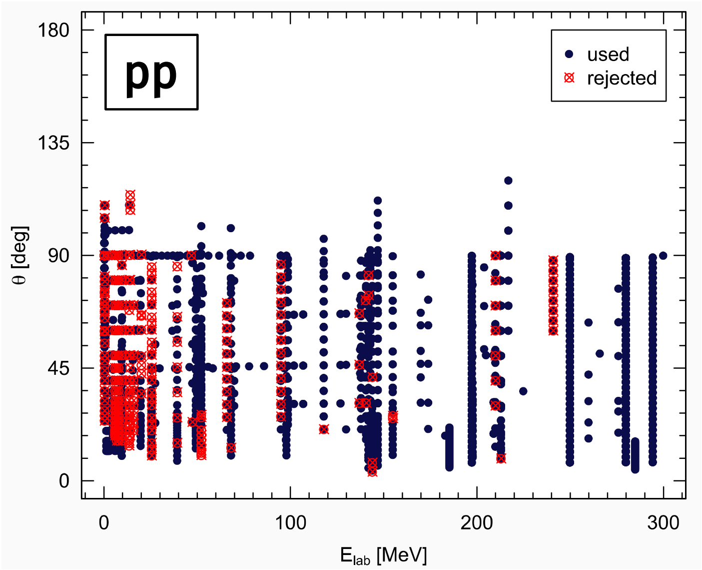
Does it matter in practice?

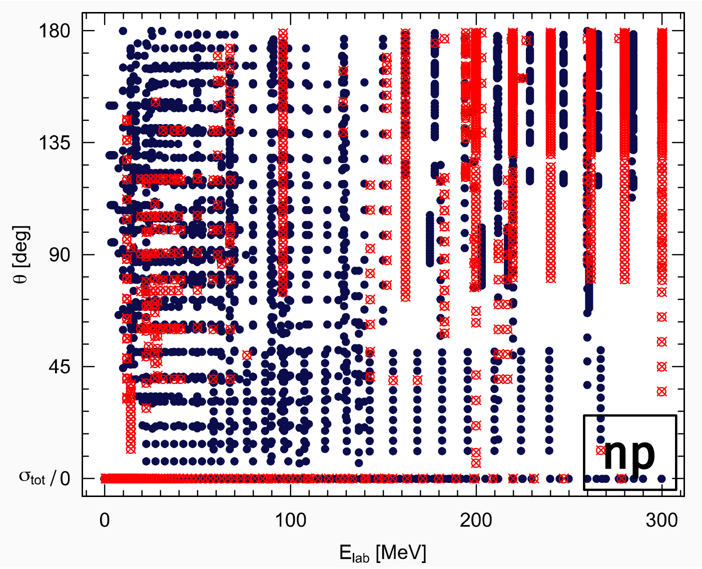
Partial wave analysis of NN data

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

- To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm. Use a simple (nonlocal) Gaussian cutoff for contact terms.
- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been collected.
- However, certain data are mutually incompatible within errors and have to be rejected. 2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:

  2158 proton-proton + 2697 neutron-proton data below $E_{lab} = 300$ MeV





- Careful error analysis: statistical, truncation, πN LECs, maximal energy in the fits

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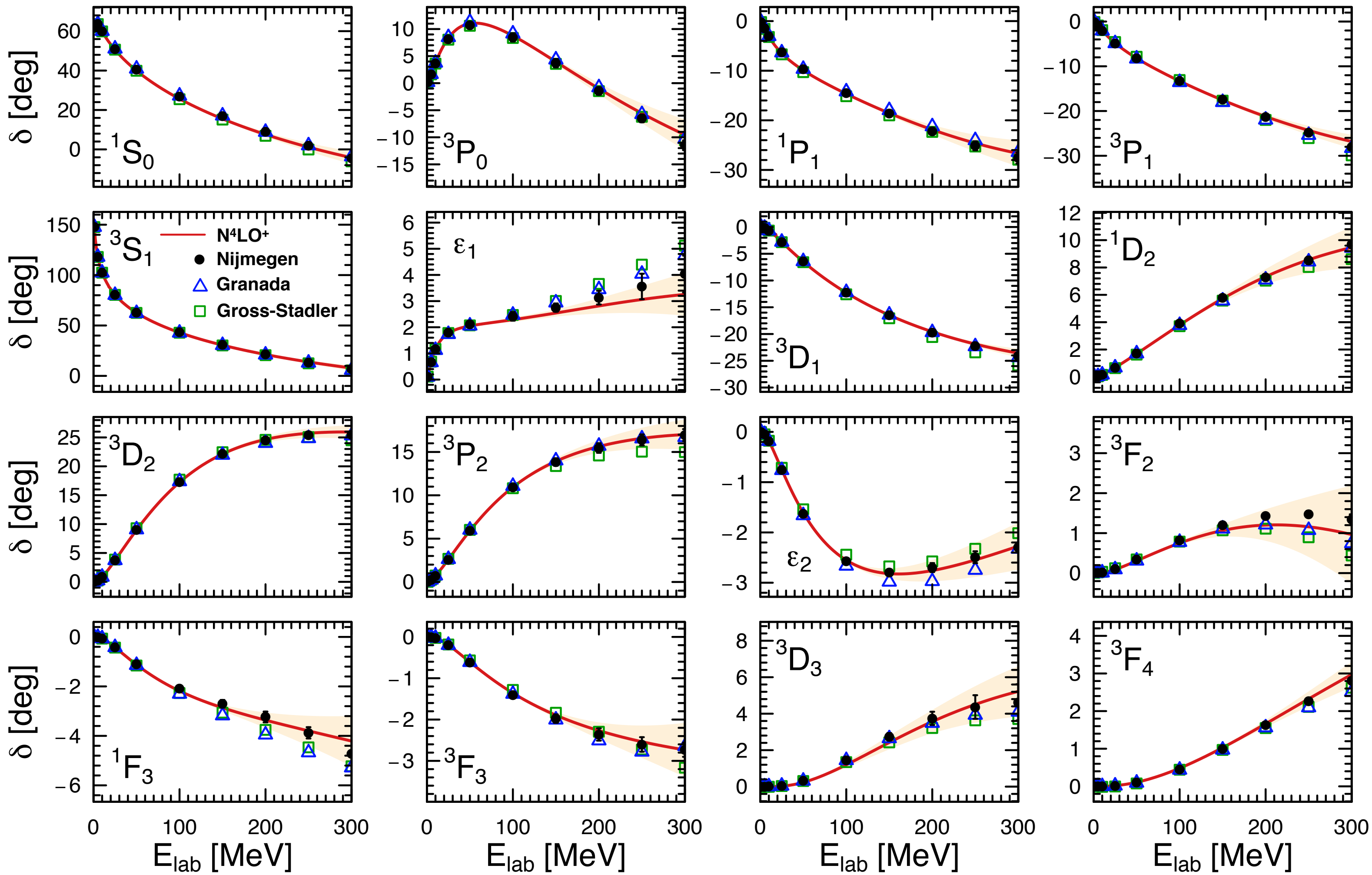
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



— For the first time, chiral EFT potentials qualify for being regarded as PWA

— Clear evidence of the parameter-free chiral 2π exchange

Partial wave analysis of NN data

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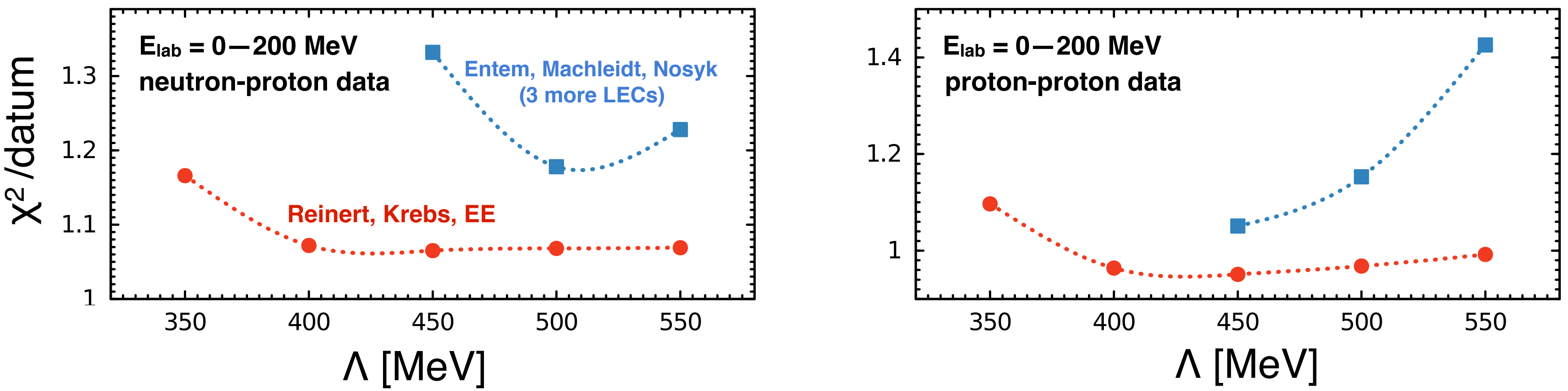
χ^2/datum for the description of the Granada-2013 database: χEFT vs. phenomenology

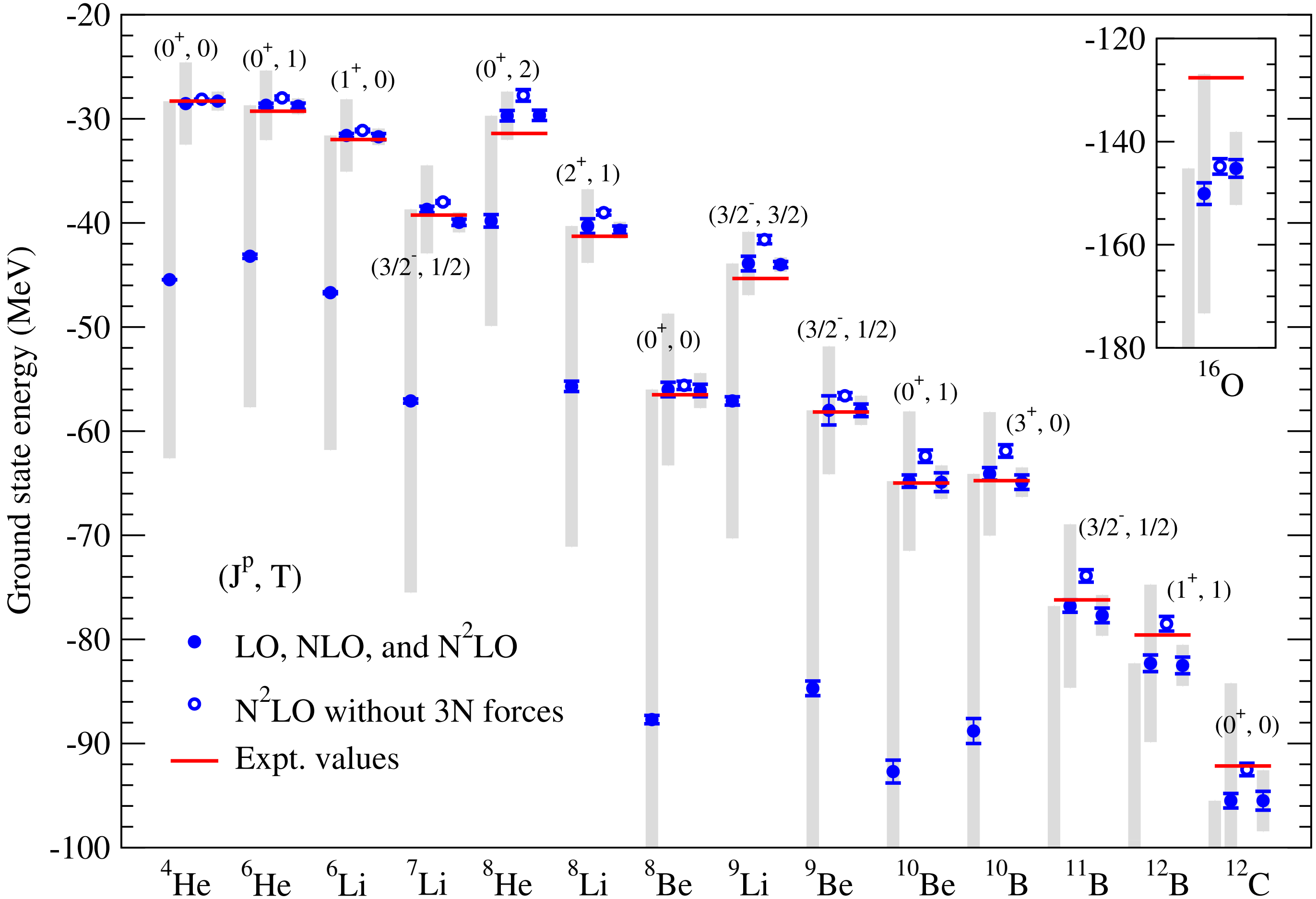
$E_{\rm lab}$ bin	CD Bonn$_{(43)}$	Nijm I$_{(41)}$	Nijm II$_{(47)}$	Reid93$_{(50)}$	N^{4}LO$^+$$_{(27+1)}$, this work
neutron-proton scattering data					
0 – 100	1.08	1.06	1.07	1.08	1.07
0 – 200	1.08	1.07	1.07	1.09	1.07
0 – 300	1.09	1.09	1.10	1.11	1.06
proton-proton scattering data					
0 – 100	0.88	0.87	0.87	0.85	0.86
0 – 200	0.98	0.99	1.00	0.99	0.95
0 – 300	1.01	1.05	1.06	1.04	1.00

N^{4}LO$^+$: semilocal (Reinert, Krebs, EE) vs. nonlocal (Entem, Machleidt, Nosyk)



Light nuclei up to N²LO

EE et al. (LENPIC), arXiv:1807.02848



(c_D and c_E are fixed from ^{3}H BE and Nd scattering)

[based on the EKM potential, R = 1.0 fm]

LENPIC: Low Energy Nuclear Physics International Collaboration

Radii of medium-mass nuclei: A smoking gun?

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$NNLO_{sat}$: r_D = 1.978 fm (+0.13%)

Ekström et al., PRC91 (2015) 051301

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Ekström et al., PRC97 (2018) 024332

However, NN data seem to prefer smaller r_D:

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r_D, ^{2}H (fm)	**1.965 . . . 1.968**	**1.965**	**1.967**	**1.968**	**1.969**	**1.966**	**1.975**

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	r_D, ^2H (fm)	r_p, ^3H (fm)	r_p, ^4He (fm)
AV18 + UIX	**1.967** (−0.4%)	**1.584** (−1%)	**1.44** (−2%)
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- Work in progress: calculations of the EM FFs of A = 2…16 nuclei including consistent MECs and 3NF beyond N^{2}LO **[Vadim Baru, Arseniy Filin, Hermann Krebs, Daniel Möller + LENPIC, in progress]**

The challenge: **consistent** regularization!

Regularization and the chiral symmetry

$$V(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu \, d\mu \frac{\rho(\mu)}{q^2+\mu^2} + \ldots \quad \longrightarrow \quad V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu \, d\mu \frac{\rho(\mu)}{q^2+\mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} + \ldots$$

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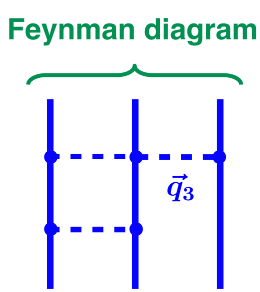
What can go wrong if one uses cutoff regularization of, say, 3NF calculated in dim. reg.?



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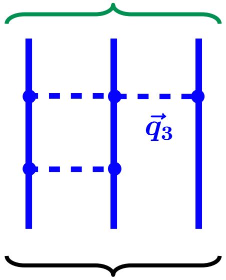
Feynman diagram



(finite in DR)

Linearly divergent: $\propto \Lambda \frac{q_3^a\, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$

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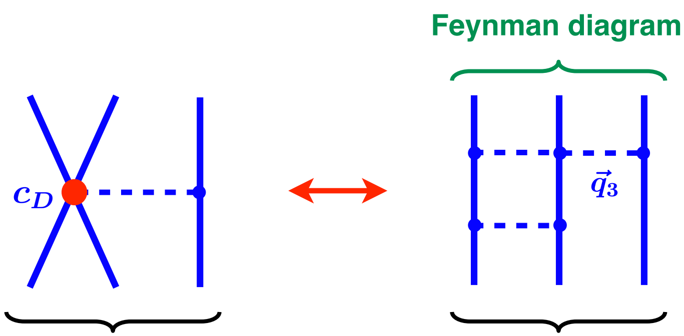

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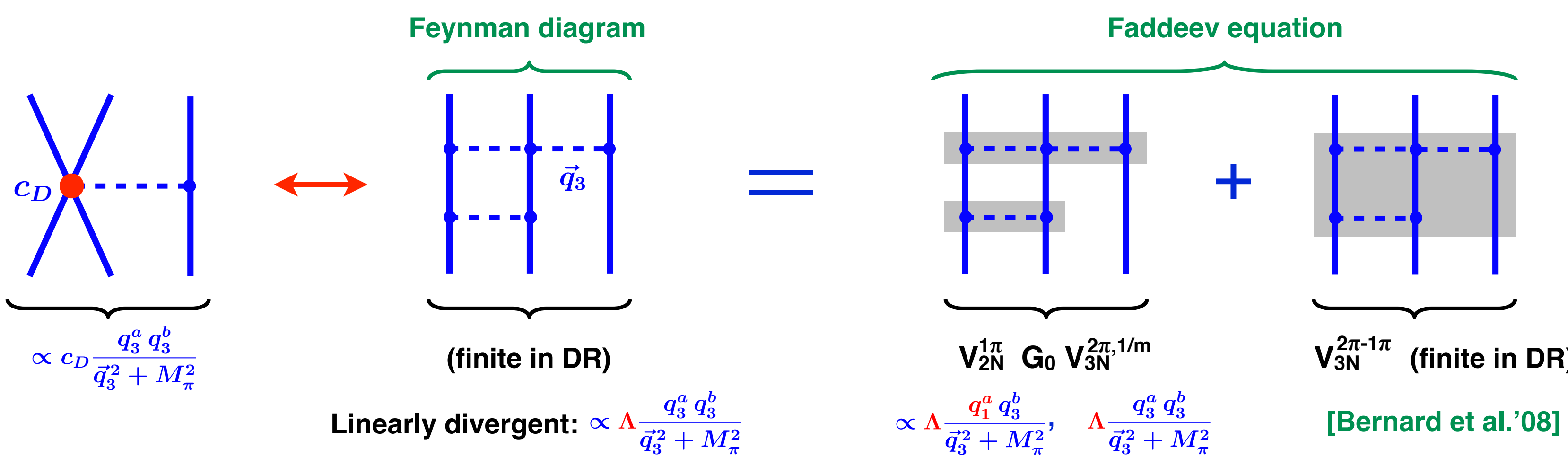

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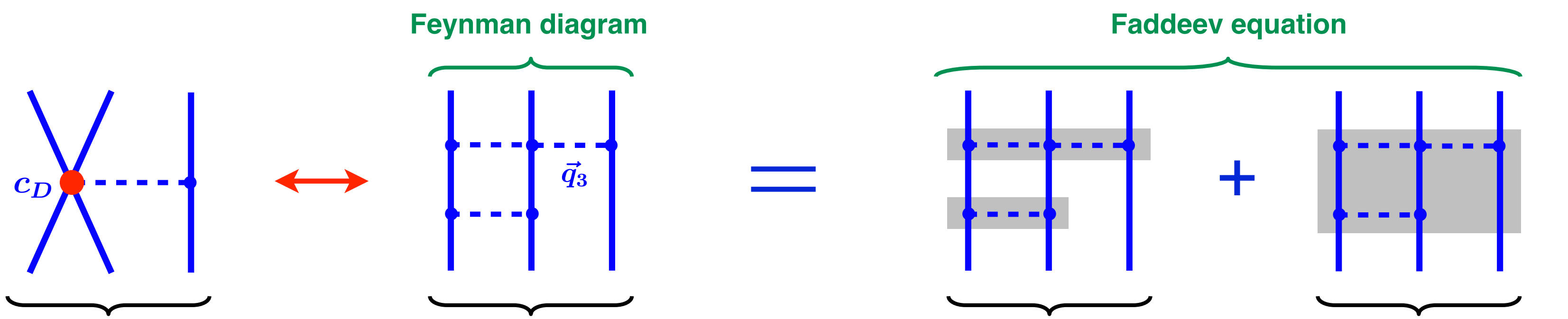

$\propto c_D \frac{q_3^a\, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$ ⟷ (finite in DR) = $V_{2N}^{1\pi}\ G_0\ V_{3N}^{2\pi,1/m}$ + $V_{3N}^{2\pi\text{-}1\pi}$ (finite in DR)

Linearly divergent: $\propto \Lambda \frac{q_3^a\, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$ $\quad$ $\propto \Lambda \frac{q_1^a\, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$, $\Lambda \frac{q_3^a\, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$ $\quad$ [Bernard et al.'08]

Renormalization of the iteration requires χ-symmetry breaking counter terms! This divergence cancels out if $V_{3N}^{2\pi\text{-}1\pi}$ is calculated using (consistent) cutoff regularization.

Regularization and the chiral symmetry

The same problem affects loop contributions to the exchange charge/current operators.

Is it enough to recalculate all loop contributions to the 3NF/exchange currents by modifying the pion propagators via $(\vec{q}^{\,2} + M_\pi^2)^{-1} \longrightarrow \exp[-(\vec{q}^{\,2} + M_\pi^2)/\Lambda^2]\,(\vec{q}^{\,2} + M_\pi^2)^{-1}$?

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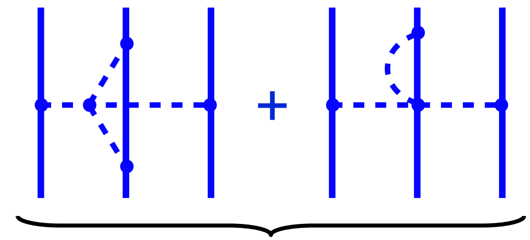
Is it enough to recalculate all loop contributions to the 3NF/exchange currents by modifying the pion propagators via $(\vec{q}^{\,2} + M_\pi^2)^{-1} \longrightarrow \exp[-(\vec{q}^{\,2} + M_\pi^2)/\Lambda^2]\,(\vec{q}^{\,2} + M_\pi^2)^{-1}$?

Not quite… Have to ensure that regularization maintains the chiral symmetry.

$$U(\vec{\pi}) = 1 + \frac{i}{F_\pi}\vec{\tau}\cdot\vec{\pi} - \frac{1}{2F_\pi^2}\vec{\pi}^{\,2} - \frac{i\alpha}{F_\pi^3}(\vec{\tau}\cdot\vec{\pi})^3 - \frac{8\alpha - 1}{8F_\pi^4}\vec{\pi}^{\,4} + \ldots$$



is independent on α in DR, but not of one uses (naive) cutoff regularization

All observables should be α-independent.

Charge form factor of the deuteron

Baru, EE, Filin, Krebs, Möller, in progress

- 1N charge operator expressed in terms of Sachs FFs G_M, G_E
- 2N charge (up to N^3LO): consistently regularized 1/m corrections to the 1π-exchange

(preliminary)

Summary

Chiral EFT can provide **CONSISTENT nuclear forces & currents**

It's a long way to go…

But we are almost there!

Exciting time for precision low-energy few-N physics with chiral EFT (also at MESA!)