# Neutrino reactions with two- and three-nucleon systems

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# Outline

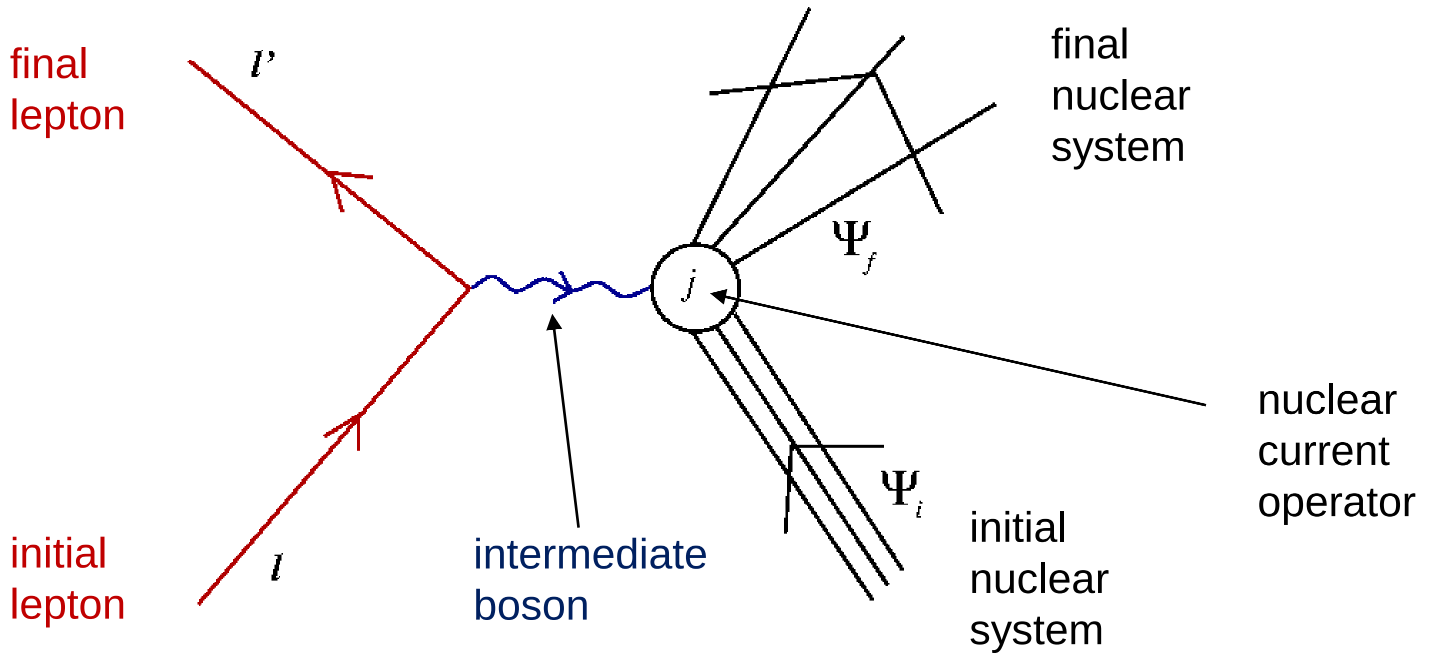
- Elements of our formalism
- Results for selected reactions with (anti)neutrinos
- Conclusions and outlook

# Introduction

A very efficient momentum space framework to deal with nucleon-nucleon scattering, nucleon-deuteron scattering and sevveral electroweak processes with two- and three-nucleon systems has been constructed and tested:
Phys. Rept. 274, 107 (1996); Phys. Rept. 415, 89 (2005);
Eur. Phys. J. A24, 31 (2005)

Limitations: nonrelativistic character and lack of the Coulomb force in the 3N continuum

# Formalism




$$\left|T_{fi}\right|^2 \propto L_{\alpha\beta}\, N^{\alpha}\left(N^{\beta}\right)^{*} \qquad N^{\alpha} \equiv \left\langle \Psi_f \left| j^{\alpha} \right| \Psi_i \right\rangle$$

# Formalism *(cont.)*

$L_{\alpha\beta}$ known analytically !

$$N^{\alpha} = \left\langle \Psi_{f\, m_f} \middle| j^{\alpha} \middle| \Psi_{i\, m_i} \right\rangle$$

from *ab initio* calculations in momentum space

Dynamical ingredients (1): 2N and 3N Hamiltonians

$$H_{2N} = H_0^{2N} + V_{12}$$

$$H_{3N} = H_0^{3N} + V_{23} + V_{13} + V_{12} + V_{123} \equiv H_0^{3N} + V_1 + V_2 + V_3 + V_4$$

$$\equiv H_0^{3N} + V_1 + V_2 + V_3 + \underbrace{V_4^{(1)} + V_4^{(2)} + V_4^{(3)}}_{V_4}$$

used to generate nuclear bound and scattering states contain 2N and 3N potentials

# Formalism *(cont.)*

Dynamical ingredients (2): nuclear EM and weak single-nucleon, 2N and 3N current operators

$$j_{2N} = j_1 + j_2 + j_{12}$$

$$j_{3N} = j_1 + j_2 + j_3 + j_{12} + j_{23} + j_{13} + j_{123}$$

$$\equiv j_1 + j_{23} + j_2 + j_{13} + j_3 + j_{12} + \underbrace{j_{123}^{(1)} + j_{123}^{(2)} + j_{123}^{(3)}}_{j_{123}}$$

$$\equiv \underbrace{j_1 + j_{23} + j_{123}^{(1)}}_{j(1)} + \underbrace{j_2 + j_{13} + j_{123}^{(2)}}_{j(2)} + \underbrace{j_3 + j_{12} + j_{123}^{(3)}}_{j(3)}$$

describe interactions of the electroweak probe with nuclear system

# Formalism *(reactions with $^2$H)*

$$H_{2N}\left|\psi_d\right\rangle = E_d\left|\psi_d\right\rangle$$ deuteron state with $E_d < 0$

$$N^\alpha \equiv \left\langle \psi'_d \right| j^\alpha \left|\psi_d\right\rangle$$ elastic channel

$$N^\alpha \equiv \left\langle \psi^{(-)} \right| j^\alpha_{2N} \left|\psi_d\right\rangle = {}_a\left\langle \vec{p}_o \right| \left(1 + t_{12} G_0^{2N}\right) j^\alpha_{2N} \left|\psi_d\right\rangle$$ break-up channel

$$H_{2N}\left|\psi^{(-)}\right\rangle = E\left|\psi^{(-)}\right\rangle, \qquad E = \frac{p_0^2}{m} > 0$$

$$t_{12} = V_{12} + t_{12}\, G_0^{2N}\left(E + i\varepsilon\right) V_{12}$$ Lippmann-Schwinger equation

$$G_0^{2N}(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E + i\varepsilon - H_0^{2N}}$$ free 2N propagator

# Formalism *(reactions with $^2$H)*

$$H_{2N}\left|\psi^{(-)}\right\rangle = E\left|\psi^{(-)}\right\rangle, \qquad E=\frac{p^2}{m}>0$$

break-up channel

Scattering radial wave function can be calculated directly in ccordinate space or generated from the solution of the Lippmann-Schwinger equation $t_{12}(E+i\varepsilon)$ obtained in momentum space as

$$\psi^{j}_{l'sls}(r)=\delta_{l'l}\, j_l(pr)+i^{l-l'}m\int_0^\infty \frac{dk\,k^2\, j_{l'}(kr)}{p^2-k^2+i\varepsilon}\left\langle k(l's)j\right|t_{12}\left(E+i\varepsilon\right)\left|p(ls)j\right\rangle$$

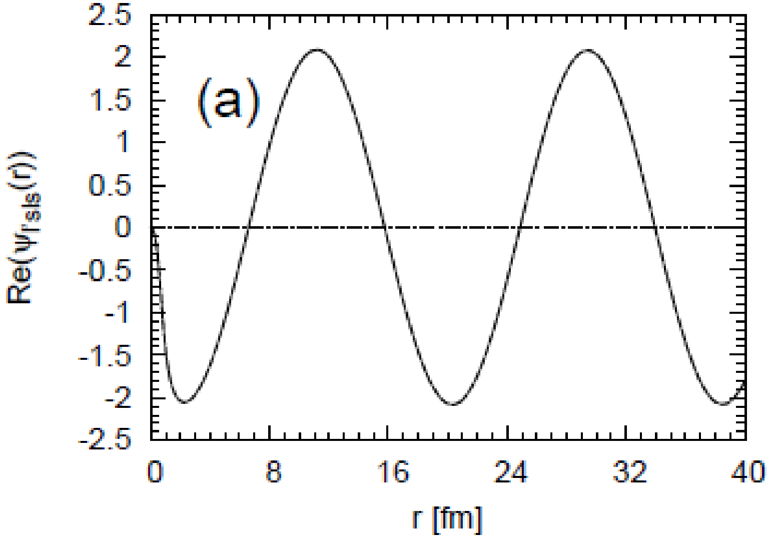
Sharply cut off Coulomb potential with the parameter $R_C$ in the 2N partial wave basis

$$\left\langle p'(l's')j'm_{j'}\right|V_{RC}\left|p(ls)jm_j\right\rangle=\delta_{l'l}\,\delta_{s's}\,\delta_{j'j}\,\delta_{m_{j'}m_j}\,8\alpha\int_0^{R_C} dr\,r\,j_l(p'r)\,j_l(pr)$$

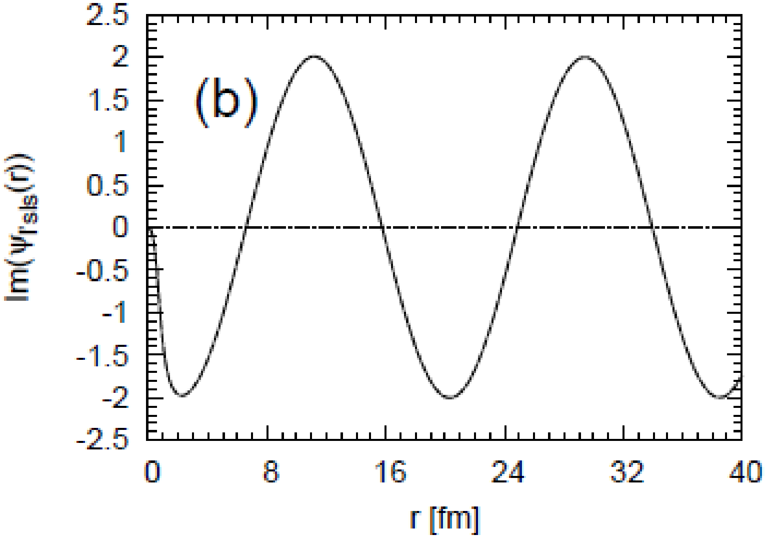
# Formalism *(reactions with $^2H$)*

Cross-checking
two-proton radial scattering wave function
E= 5 MeV and $R_C$= 40 fm
$^1S_0$ channel

real part




imaginary part

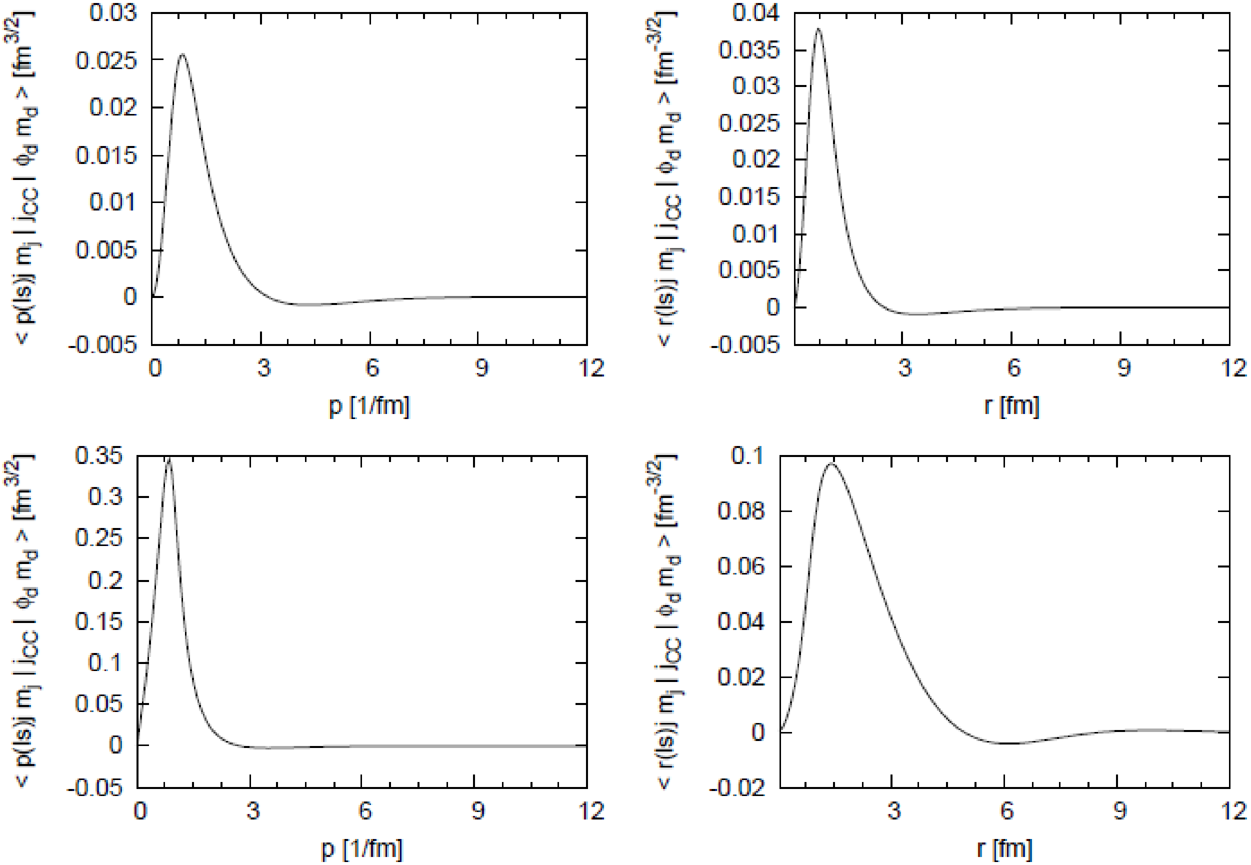



2N scattering states pose no problem (nonrelativistically)

# Formalism *(reactions with $^2H$)*

Examples of 2N current matrix elements Q= 350 MeV

momentum space r-space




$$(m_d = 0,\ j^0_{CC},\ {}^1S_0)$$

$$(m_d = \text{-}1,\ j_{CC\,+1},\ {}^3P_2),$$

# Formalism *(reactions with $^3He$ and $^3H$)*

$$H_{3N} \left| \Psi \right\rangle = E_b \left| \Psi \right\rangle$$

3N bound state with $E_b < 0$ generated by the Faddeev equation

$$N^{\lambda} = \left\langle \Psi' \right| j^{\lambda}_{3N} \left| \Psi \right\rangle$$

elastic or quasielastic channel with initial and final bound states

$$N^{\lambda} = \left\langle \Psi^{(-)}_{f} \right| j^{\lambda}_{3N} \left| \Psi_{i} \right\rangle$$

two-body or three-body break-up channel with final scattering statek

$$\left| \Psi^{(-)}_{f} \right\rangle = \lim_{\varepsilon \to 0^{+}} \frac{-i\varepsilon}{E - i\varepsilon - H_{3N}} \left| \phi_{f} \right\rangle \longleftarrow$$

formal definition including the channel state

# Formalism *(reactions with $^3$He and $^3$H)*

Operators in 3N space:

(1) 3N force decomposed as $$V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}$$

$V_4^{(i)}$ is symmetric under the exchange of nucleons j and k, i≠j≠k≠i

(2) free 3N propagator $$G_0^{3N}(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E + i\varepsilon - H_0^{3N}}$$

(3) 2N off-shell t-matrix generated via LSE: $$t_1 = V_1 + V_1 G_0^{3N} t_1$$

(4) permutation operator: $$P = P_{12} P_{23} + P_{13} P_{23}$$

# Formalism *(reactions with $^3He$ and $^3H$)*

Auxiliary equation for $\left| U^{\lambda} \right\rangle \equiv \left| U(j^{\lambda}, E_{c.m.}, Q) \right\rangle$

3N internal energy

magnitude of the three momentum transfer

$$\left| U^{\lambda} \right\rangle = \left\{ t_1 G_0^{3N} + \frac{1}{2}(1+P) V_4^{(1)} G_0^{3N} \left(1 + t_1 G_0^{3N}\right) \right\} (1+P)\, j^{\lambda}(1) \left| \Psi_i \right\rangle$$

$$+ \left\{ t_1 G_0^{3N} P + \frac{1}{2}(1+P) V_4^{(1)} G_0^{3N} \left(1 + t_1 G_0^{3N}\right) P \right\} \left| U^{\lambda} \right\rangle$$

# Formalism *(reactions with $^3$He and $^3$H)*

Quadratures

$$N_{Nd}^{\lambda} = \langle \phi_{Nd} | (1+P)\, j^{\lambda}(1) | \Psi_i \rangle + \langle \phi_{Nd} | P | U^{\lambda} \rangle$$

$$N_{3N}^{\lambda} = \langle \phi_{3N} | (1+P)\, j^{\lambda}(1) | \Psi_i \rangle + \langle \phi_{3N} | t_1\, G_0^{3N} (1+P)\, j^{\lambda}(1) | \Psi_i \rangle$$
$$+ \langle \phi_{3N} | P | U^{\lambda} \rangle + \langle \phi_{3N} | t_1\, G_0^{3N}\, P | U^{\lambda} \rangle$$

to obtain nuclear matrix elements for arbitrary exclusive kinematics !
Semi-exclusive and inclusive observables are calculated by suitable integrations over the phase space domains.

# Formalism *(reactions with $^3He$ and $^3H$)*

A supplementary scheme for inclusive reactions !

Using the completeness relation for final nuclear states with *E>0*, response functions $R^{inc}$ can be calculated without explicit integrations over the full two-body and three-body phase space !

$$R_{AB}^{inc} \equiv \sum_{m_i, m_f} \int df \, \delta(E - E_f) \left\langle \Psi_f^{(-)} \middle| A \middle| \Psi_i \right\rangle \left\langle \Psi_f^{(-)} \middle| B \middle| \Psi_i \right\rangle^*$$

$$= \sum_{m_i} \left\langle \Psi_i \middle| B^+ \delta(E - H_{3N}) A \middle| \Psi_i \right\rangle$$

$A, B$ are the $j^0$, $j_z$ , $j_{\pm 1}$ components of the current operator

# Formalism *(reactions with $^3$He and $^3$H)*

$$R_{AB}^{inc} = \frac{1}{2\pi i} \sum_{m_i} \left( \left\langle \Psi_i \left| A^+ \right| \Psi_B \right\rangle^* - \left\langle \Psi_i \left| B^+ \right| \Psi_A \right\rangle \right)$$

$$= \frac{3}{2\pi i} \sum_{m_i} \left( \left\langle \Psi_i \left| A_1^+ G_0^{3N} (1+P) \right| U_B \right\rangle^* - \left\langle \Psi_i \left| B_1^+ G_0^{3N} (1+P) \right| U_A \right\rangle \right),$$

where

$$\left| \Psi_C \right\rangle = G_0^{3N} (1+P) \left| U_C \right\rangle$$

and

$$\left| U_C \right\rangle = \left(1 + t_1 G_0^{3N}\right) j_C(1) \left| \Psi_i \right\rangle + \left\{ t_1 G_0^{3N} P + \left(1 + t_1 G_0^{3N}\right) V_4^{(1)} G_0^{3N} (1+P) \right\} \left| U_C \right\rangle$$

# Formalism *(reactions with $^3$He and $^3$H)*

The two methods for inclusive response functions provide an excellent test of numerics.

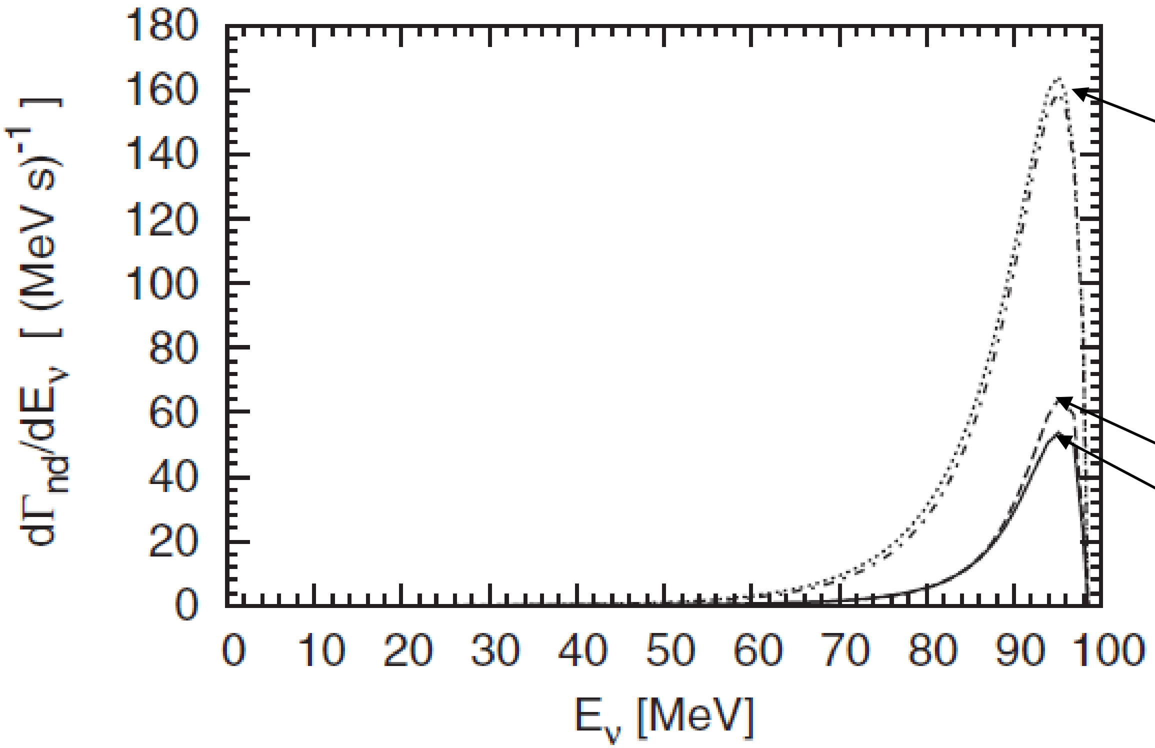
They have been applied to electron scattering, photodisintegration reactions, muon capture, pion absorption, radiative pion capture, CC and NC neutrino induced reactions.

The case without 3N force is particularly interesting, since

$$\left| U_A \right\rangle = j_A(1) \left| \Psi_i \right\rangle + \left| U^A \right\rangle$$

# Muon capture: $\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + n + d$

JG *et al.*, Phys. Rev. C **90**, 024001 (2014)

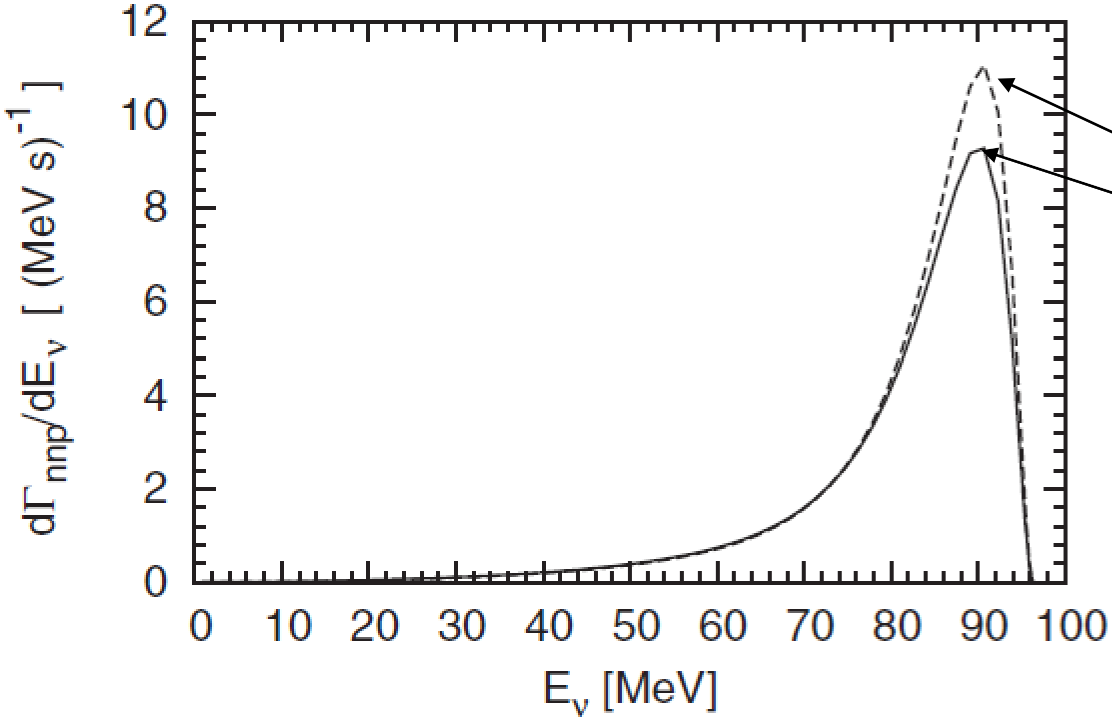



Plane wave predictions with AV18

Full predictions:
with AV18
with AV18 + Urbana IX

# Muon capture: $\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + n + n + p$

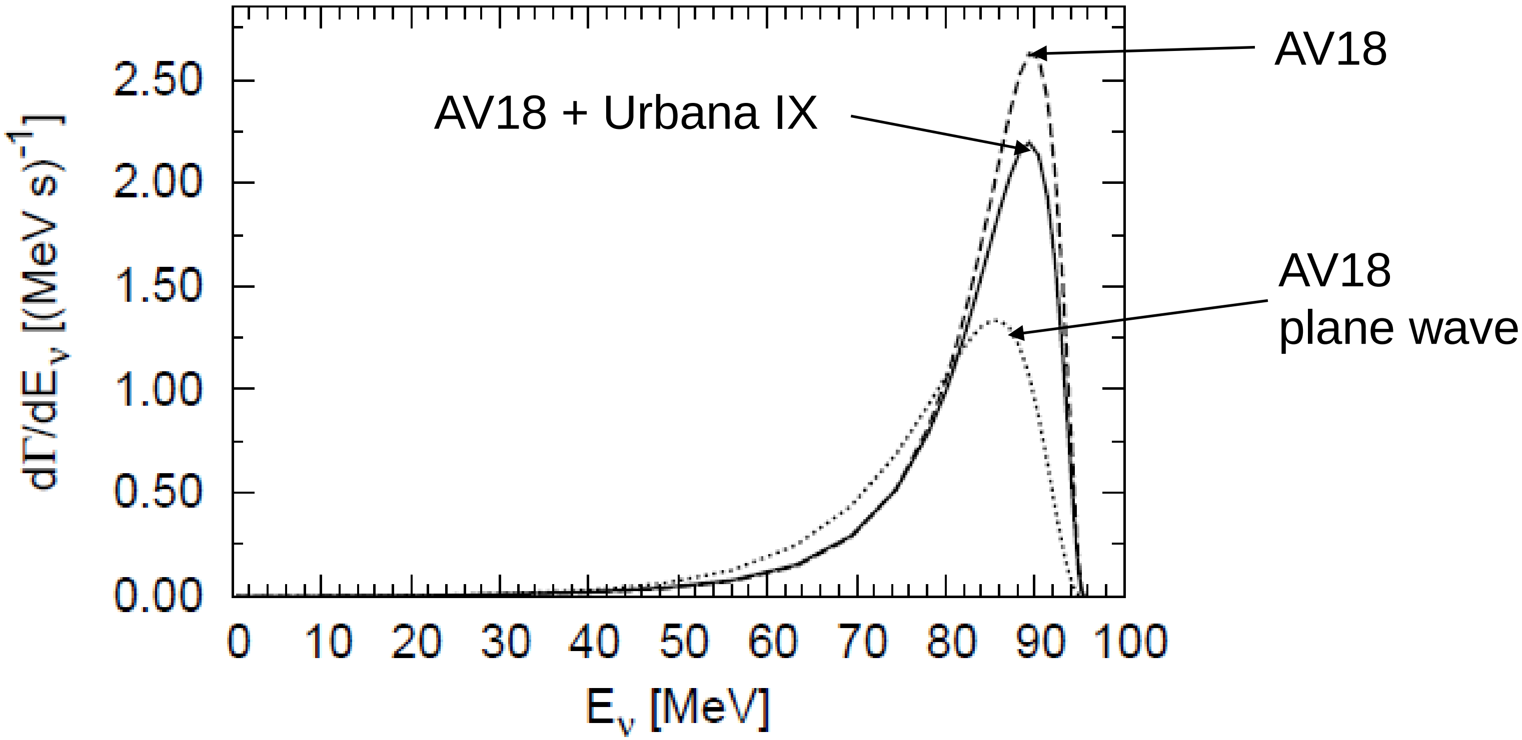
JG *et al.*, Phys. Rev. C **90**, 024001 (2014)




Full predictions:
AV18
AV18 + Urbana IX

# Muon capture: $\mu^- + {}^3H \rightarrow \nu_\mu + n + n + n$

JG *et al.*, Phys. Rev. C **94**, 034002 (2016)



# Reactions with (anti)neutrinos

We follow papers by:

- S. Nakamura *et al*., Phys. Rev. C 63, 034617 (2001); Erratum Phys. Rev. C 73, 049904 (2006)
- Doron Gazit and Nir Barnea, Phys. Rev. C 70, 048801 (2004)
- E. O'Connor *et al.,* Phys. Rev. C 75, 055803 (2007)
- Doron Gazit and Nir Barnea, Phys. Rev. Lett. 98, 192501 (2007)
- G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)
- A. Baroni and R. Schiavilla, C 96, 014002 (2017)

# Reactions with (anti)neutrinos

Simple recipe If you start with $e + d \to e + p + n$ , replace for the charged current (CC) driven reactions

$$\bar{u}(k',s')\gamma_\alpha u(k,s)\frac{e^2}{q^2} N^\alpha_{EM}$$

$$\bar{u}(k',s')\gamma_\alpha(1-\gamma_5)\, u(k,s)\frac{G_F\cos\Theta_C}{\sqrt{2}} N^\alpha_{CC,+1}$$

$$\nu_l + d \to l^- + p + p$$

$$\bar{v}(k,s)\gamma_\alpha(1-\gamma_5)\, v(k',s')\frac{G_F\cos\Theta_C}{\sqrt{2}} N^\alpha_{CC,-1}$$

$$\bar{\nu}_l + d \to l^+ + n + n$$

# Reactions with (anti)neutrinos *(cont.)*

For reactions with the neutral current (NC), construct the corresponding nuclear current operator and replace

$$\bar{u}(k',s')\gamma_\alpha\, u(k,s)\frac{e^2}{q^2} N^\alpha_{EM}$$

$$\bar{u}(k',s')\gamma_\alpha(1-\gamma_5)\, u(k,s)\frac{G_F}{\sqrt{2}} N^\alpha_{NC}$$

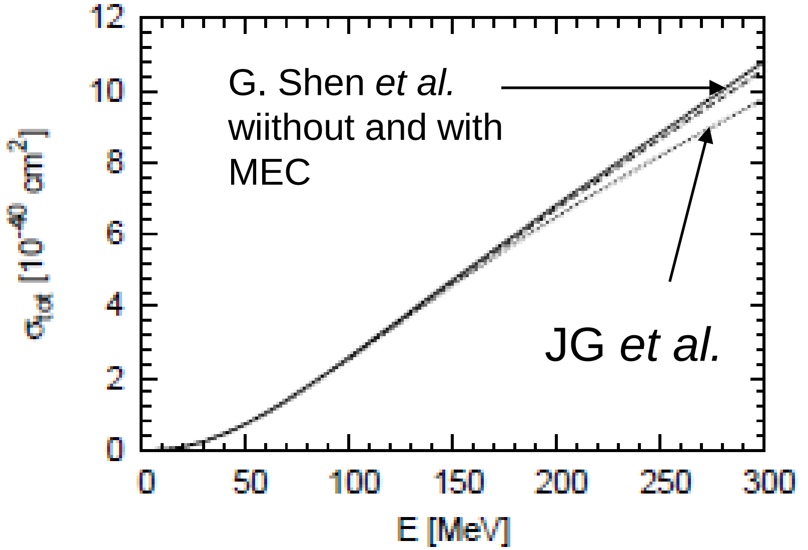
$$\nu_l + d \rightarrow \nu_l + p + n$$

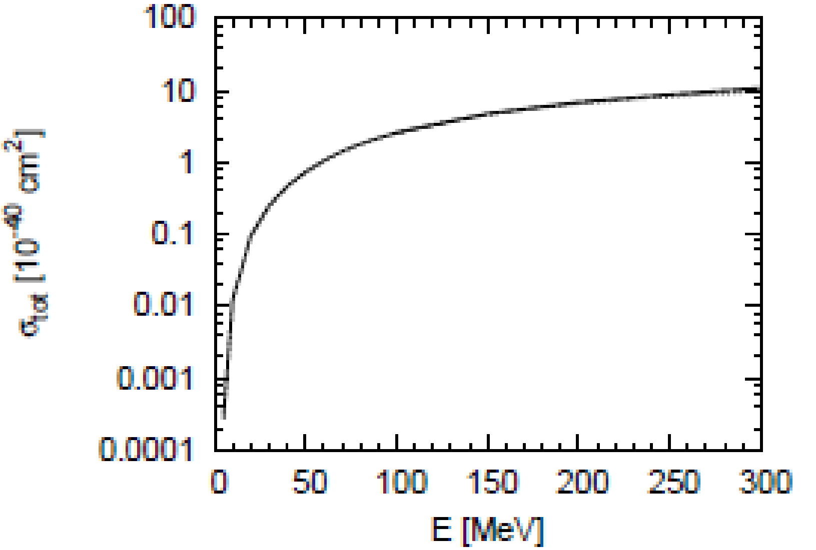
$$\bar{v}(k,s)\gamma_\alpha(1-\gamma_5)\, v(k',s')\frac{G_F}{\sqrt{2}} N^\alpha_{NC}$$

$$\overline{\nu_l} + d \rightarrow \overline{\nu_l} + p + n$$

# Reactions with (anti)neutrinos *(cont.)*

$$\bar{\nu}_e + {}^2H \to e^+ + n + n$$



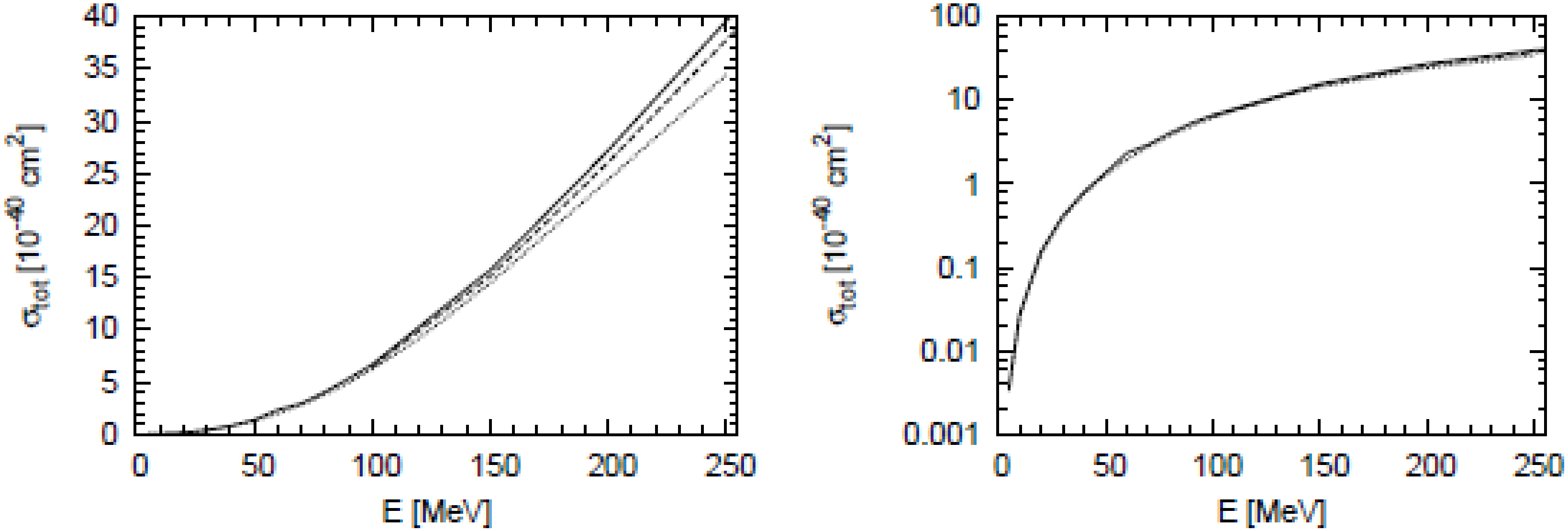

Total CC cross section as a function of the antineutrino energy

Results based on AV18 with the single nucleon current

Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)

# Reactions with (anti)neutrinos *(cont.)*

$$\nu_e + {}^2H \rightarrow e^- + p + p$$



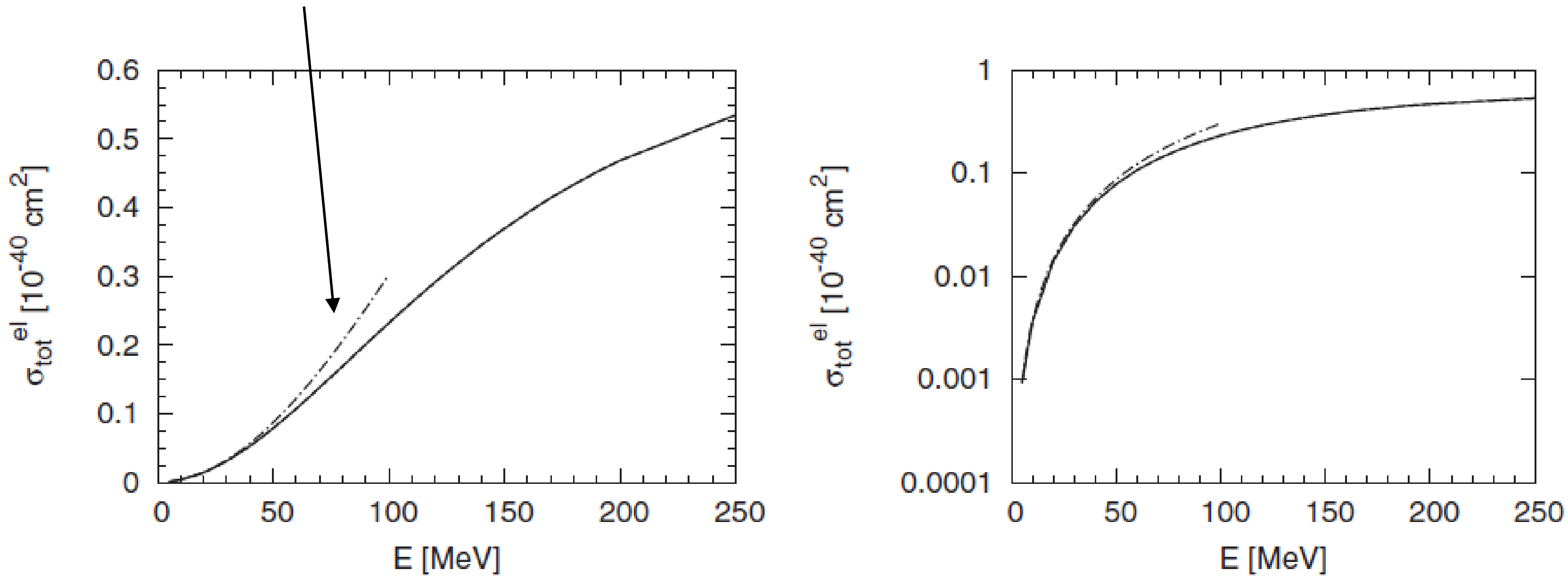

Total CC cross section as a function of the neutrino energy

Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)

# Reactions with (anti)neutrinos *(cont.)*

$$\nu_l + {}^2H \rightarrow \nu_l + {}^2H \quad \text{and} \quad \overline{\nu}_l + {}^2H \rightarrow \overline{\nu}_l + {}^2H$$

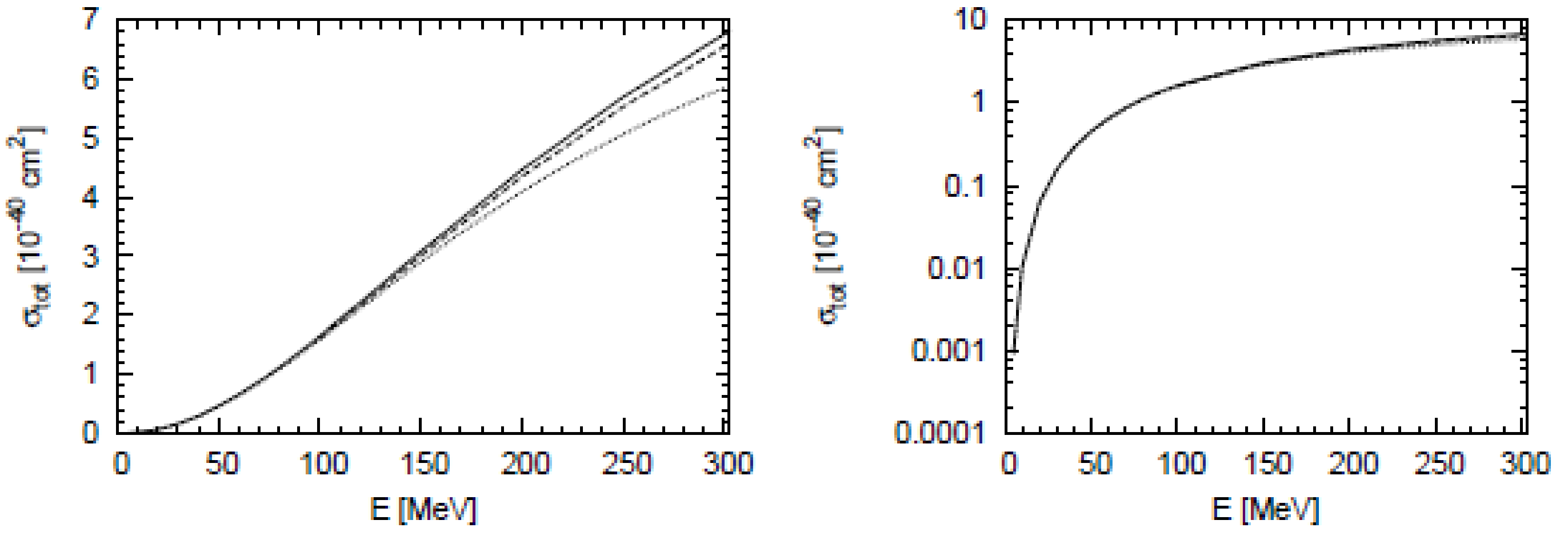
LO prediction from
M. Butler, J.-W. Chen,
Nucl. Phys. A**675**, 575 (2000)




Total NC (flavor independent !) cross section as a function of the neutrino energy

# Reactions with (anti)neutrinos *(cont.)*

$$\bar{\nu}_e + {}^2H \rightarrow \bar{\nu}_e + p + n$$

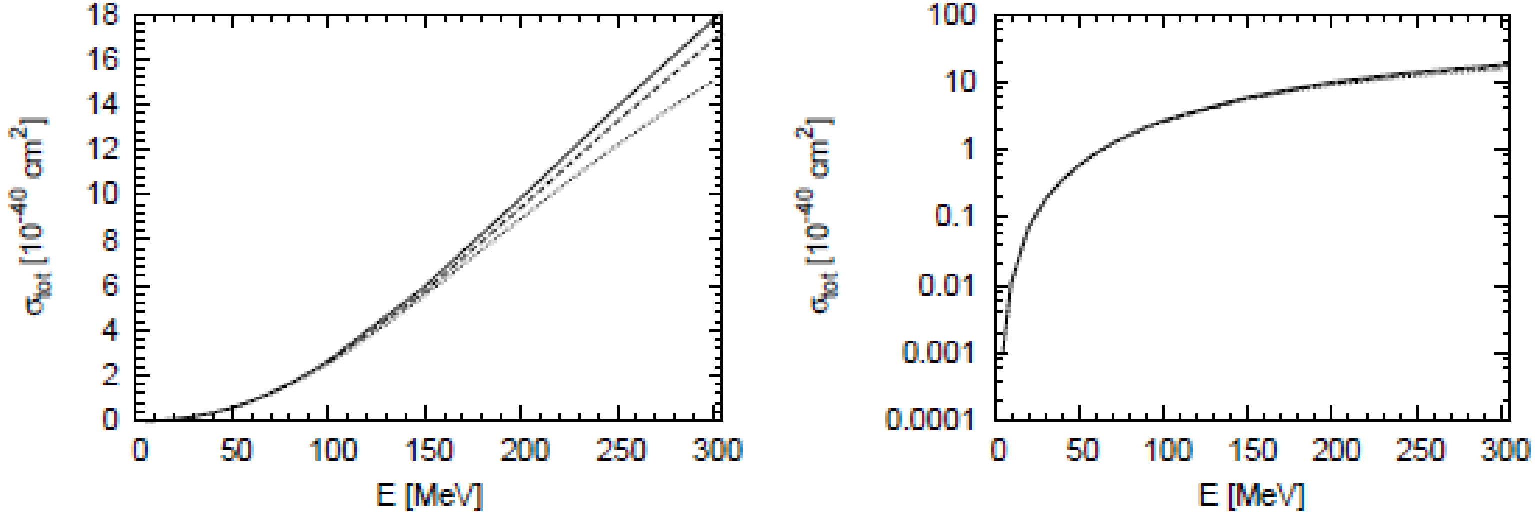



Total NC cross section as a function of the antineutrino energy

Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)

# Reactions with (anti)neutrinos *(cont.)*

$$\nu_e + {}^2H \rightarrow \nu_e + p + n$$

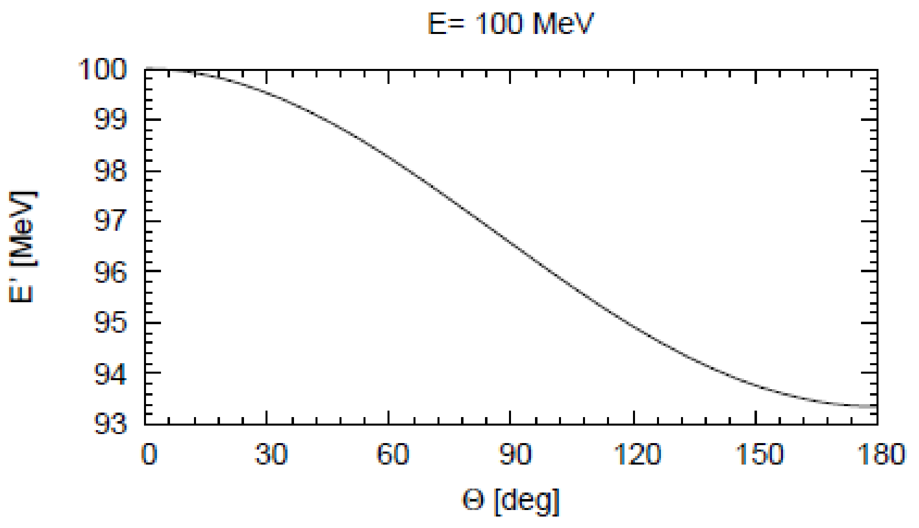



Total NC cross section as a function of the neutrino energy

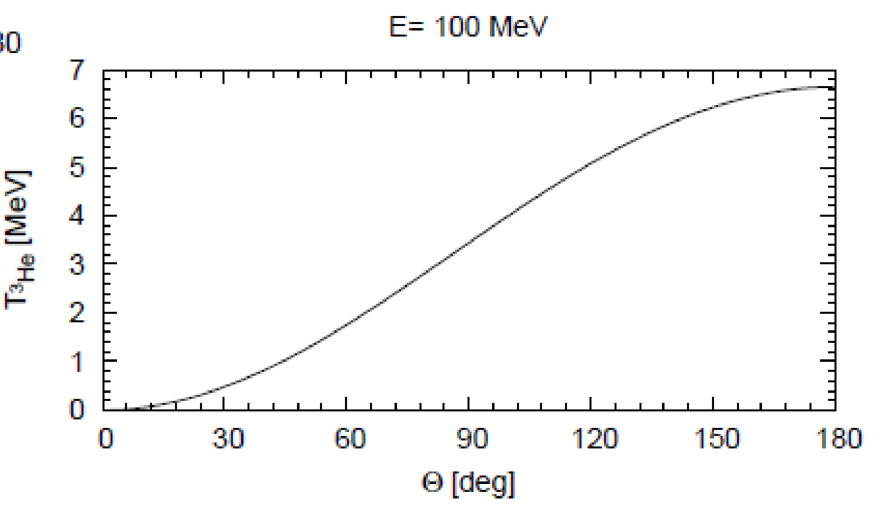
Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)

# Reactions with (anti)neutrinos *(cont.)*

Elastic (anti)neutrino scattering on $^3$He and $^3$H

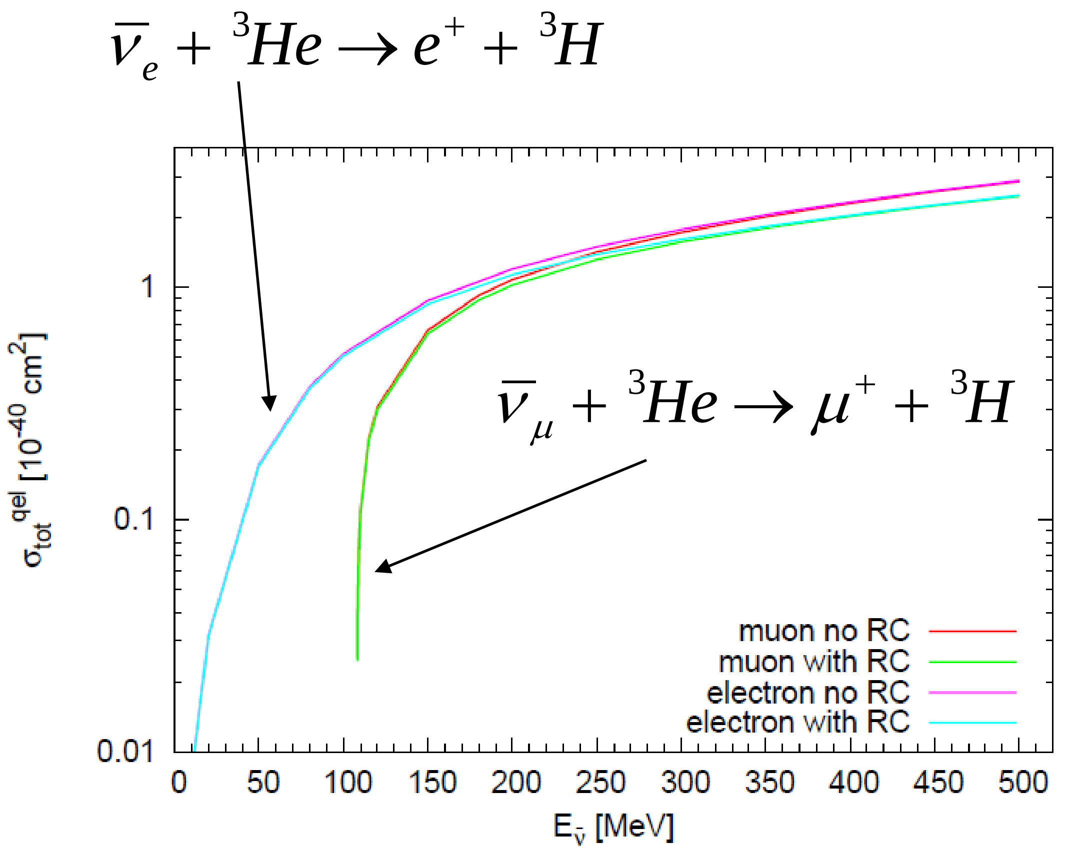



neutrino takes all the energy …



# Reactions with (anti)neutrinos *(cont.)*

Quasielastic antineutrino scattering on $^3$He



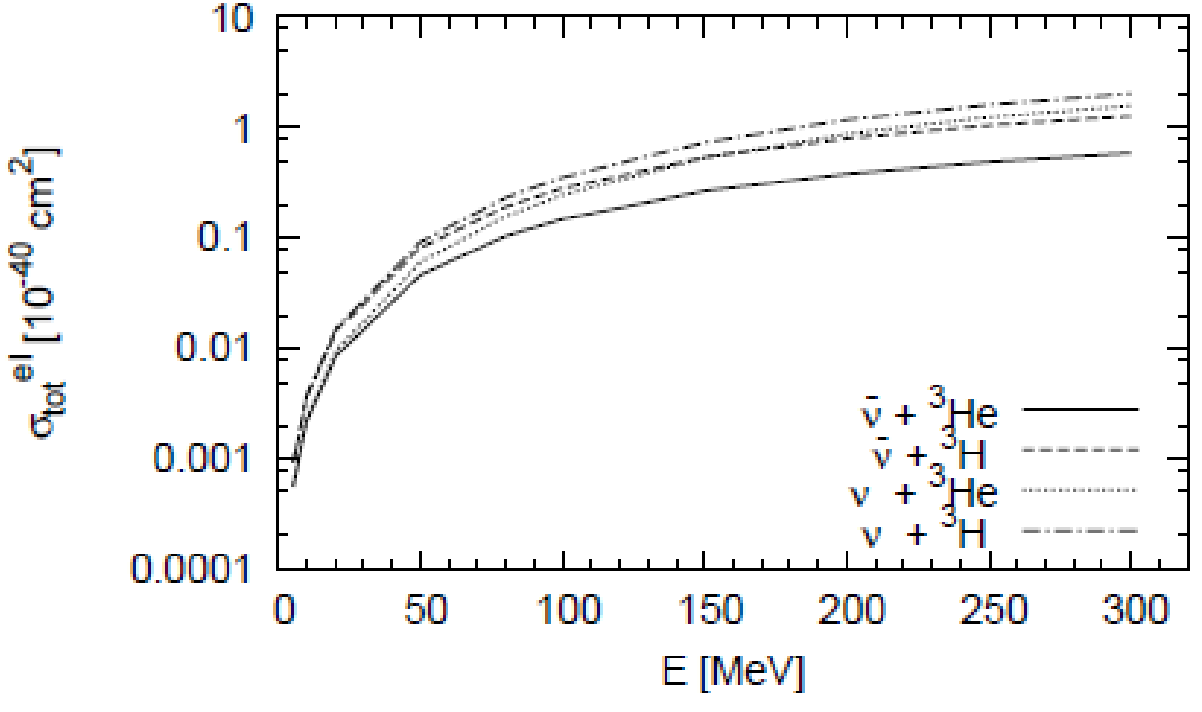

results based on AV18 with the single nucleon current with and without relativistic corrections (RC)

# Reactions with (anti)neutrinos *(cont.)*

Elastic (anti)neutrino scattering on $^3$He and $^3$H

$$\bar{\nu}_e + {}^3He \rightarrow \bar{\nu}_e + {}^3He \quad etc.$$

$$\sigma_{tot}^{el} = 2\pi \int d\theta \sin\theta \frac{d^2\sigma}{d\Omega}$$




results based on AV18 with the single nucleon current including relativistic corrections

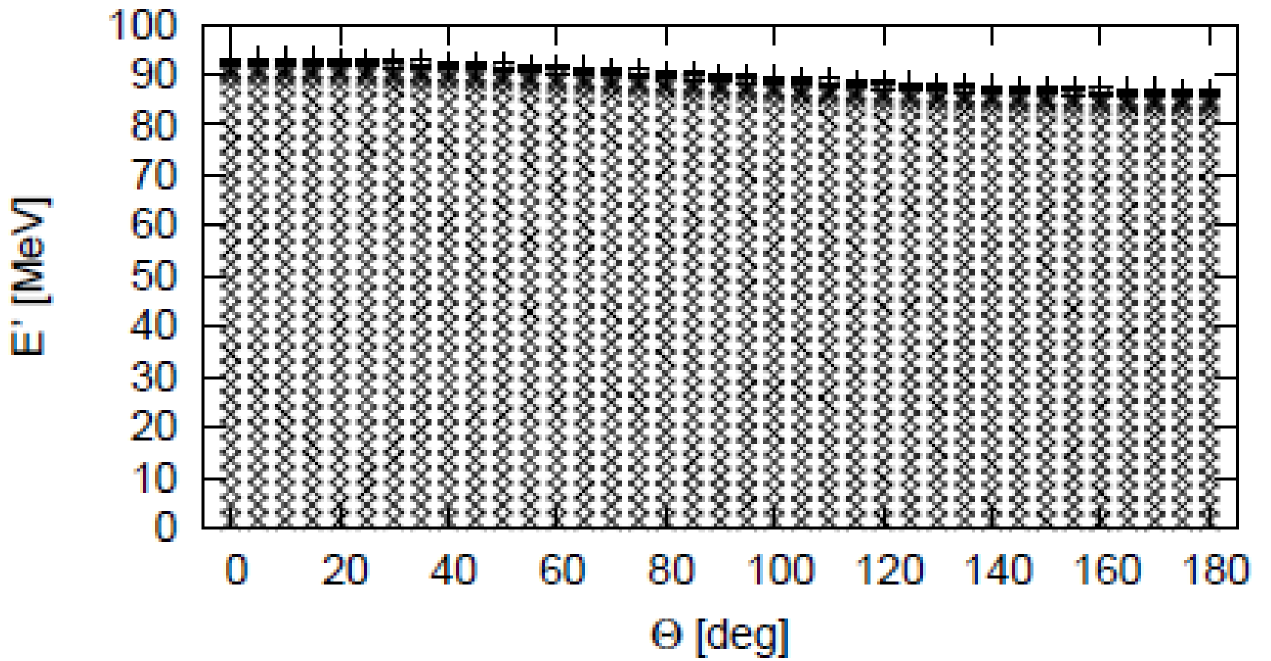
# Reactions with (anti)neutrinos *(cont.)*

Inclusive inelastic (anti)neutrino NC scattering on $^3H$ at E=100 MeV

$$\bar{\nu}_e + {}^3H \rightarrow \bar{\nu}_e + n + d$$

$$\bar{\nu}_e + {}^3H \rightarrow \bar{\nu}_e + n + n + p$$

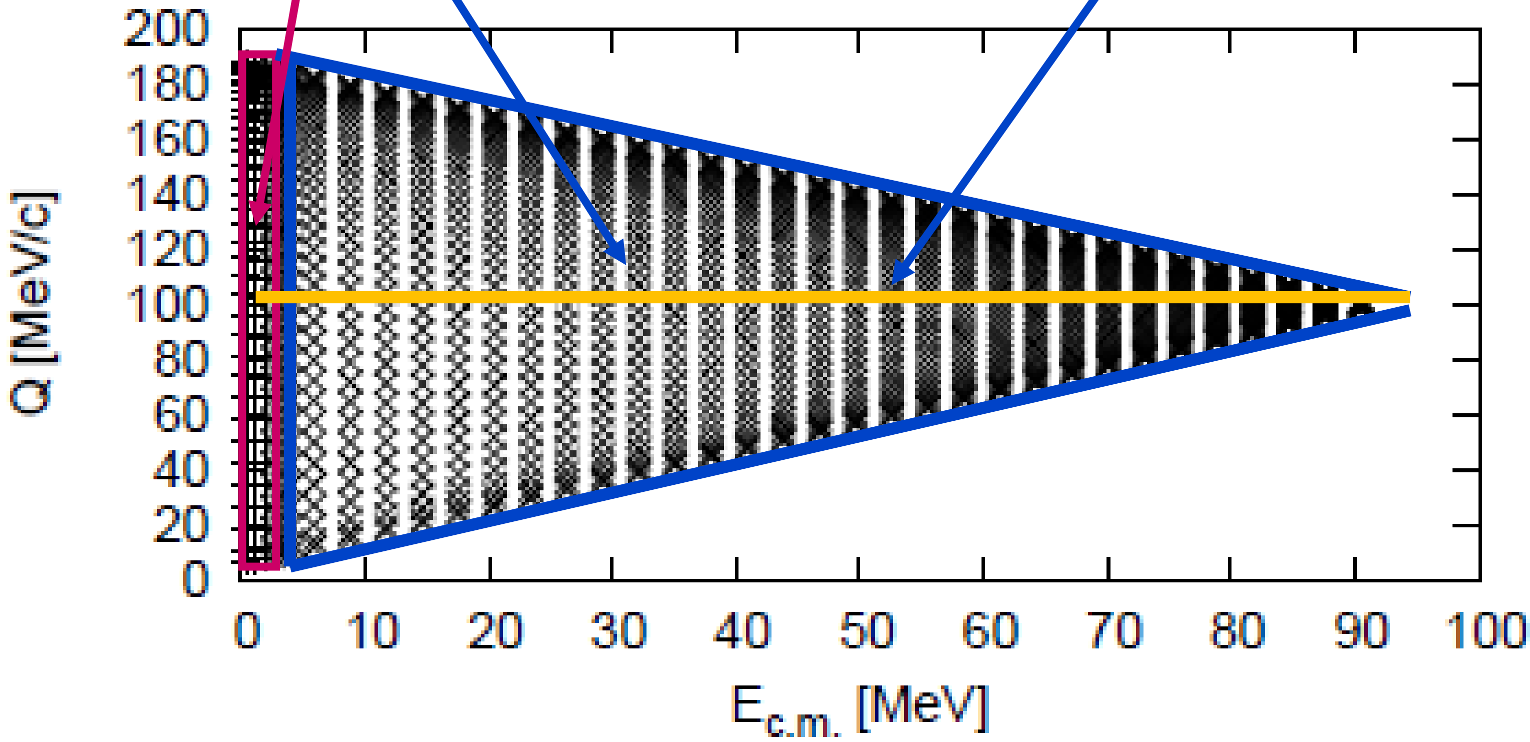
$$\sigma = 2\pi \int d\theta \sin\theta \int dE' \frac{d^3\sigma}{d\Omega\, dE'}$$



# Reactions with (anti)neutrinos *(cont.)*

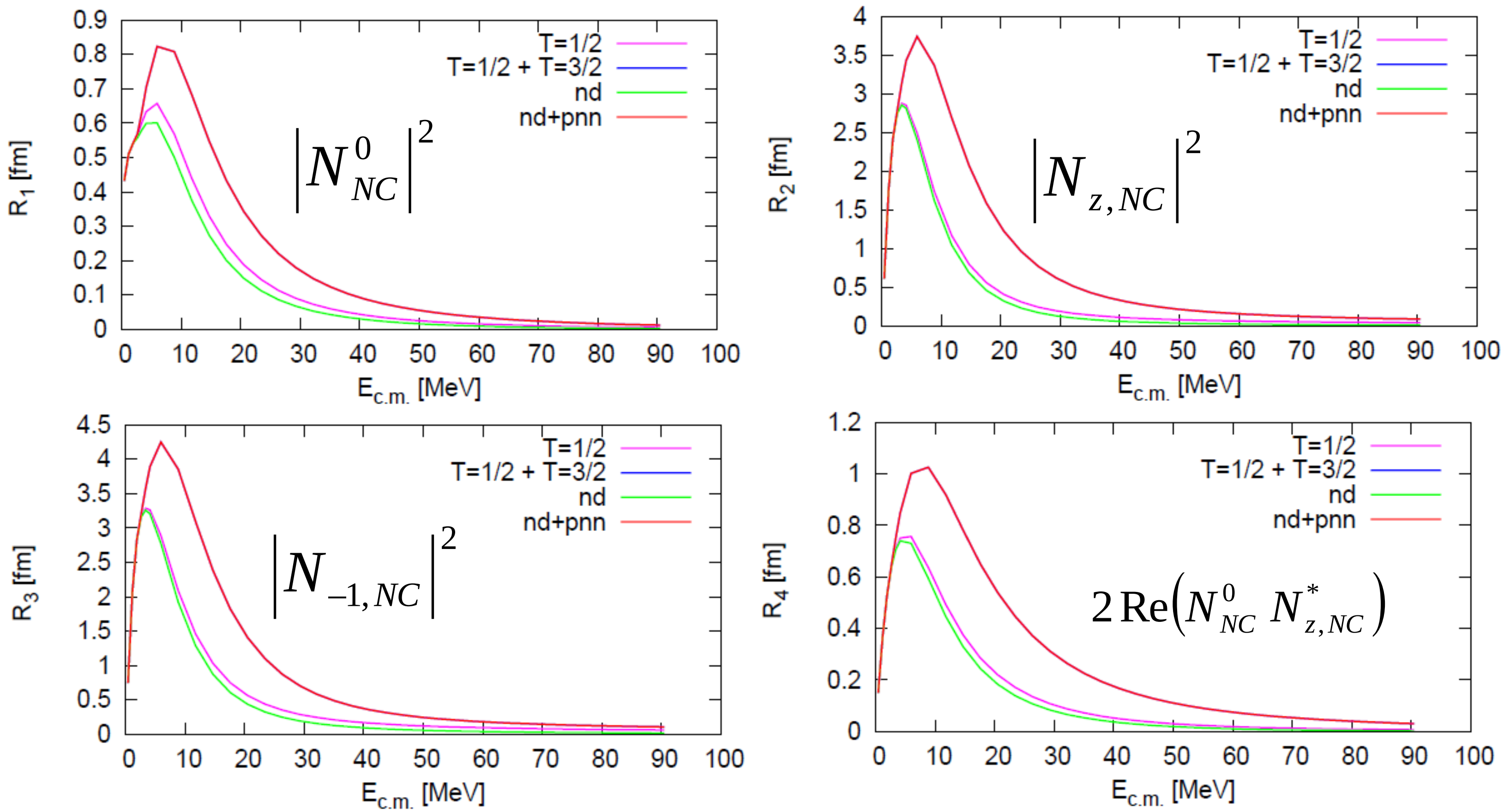
Inclusive inelastic (anti)neutrino NC scattering on $^3$H at E=100 MeV

$$\bar{\nu}_e + {}^3H \rightarrow \bar{\nu}_e + n + d \qquad \bar{\nu}_e + {}^3H \rightarrow \bar{\nu}_e + n + n + p$$



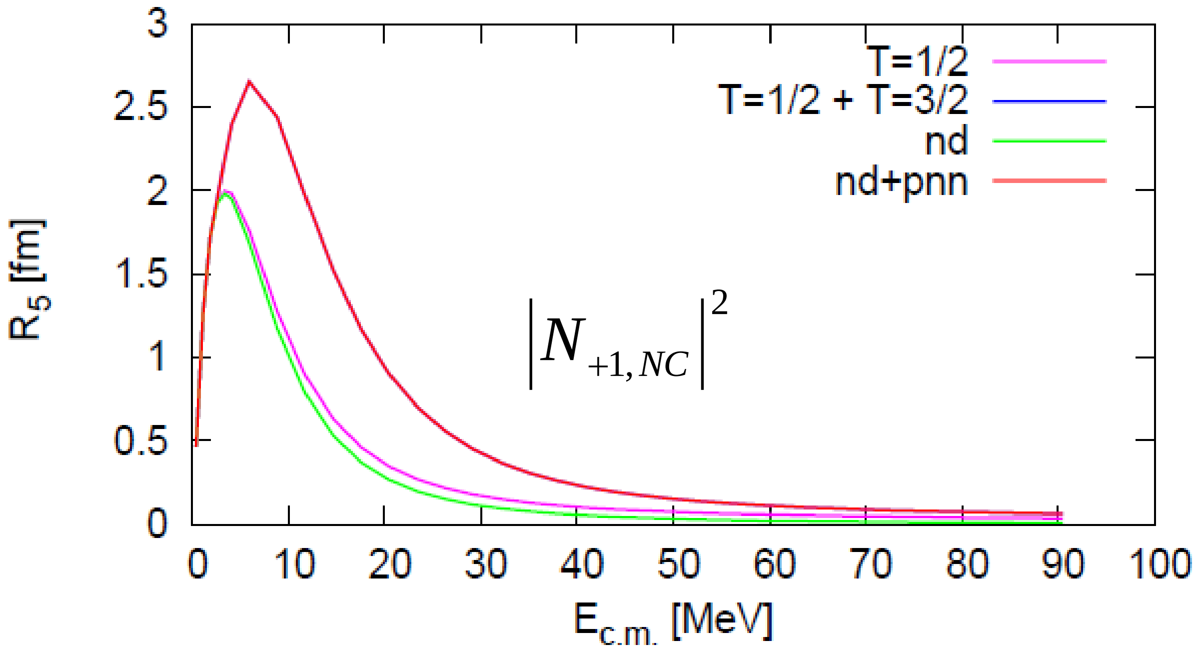
# Reactions with (anti)neutrinos *(cont.)*

Inelastic (anti)neutrino scattering off $^{3}H$ at E=100 MeV
Inclusive weak NC response functions for *Q*= 100 MeV



# Reactions with (anti)neutrinos *(cont.)*

Note: response functions depend only on $E_{c.m.}$ and $Q$ !

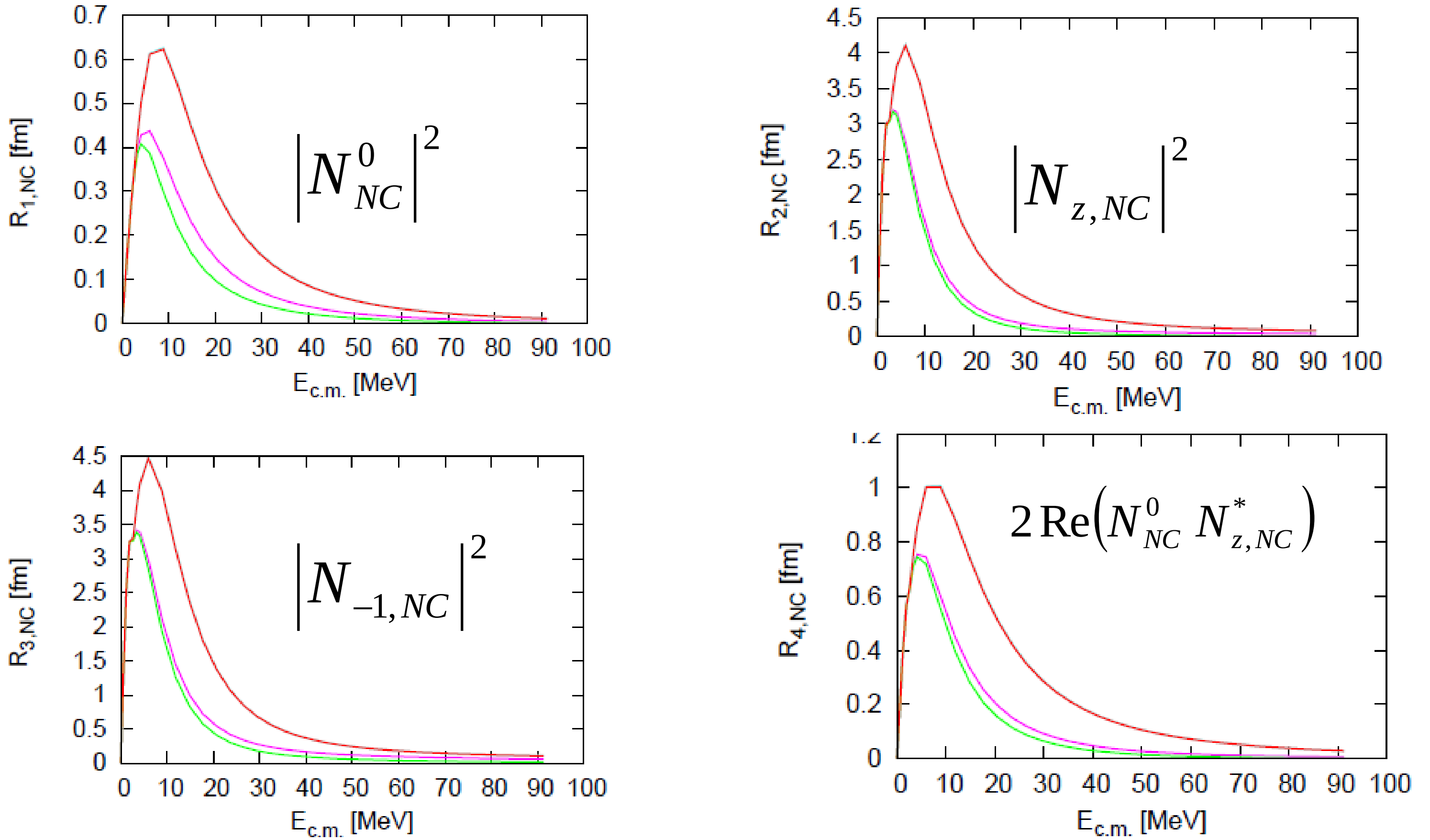



$^3H$

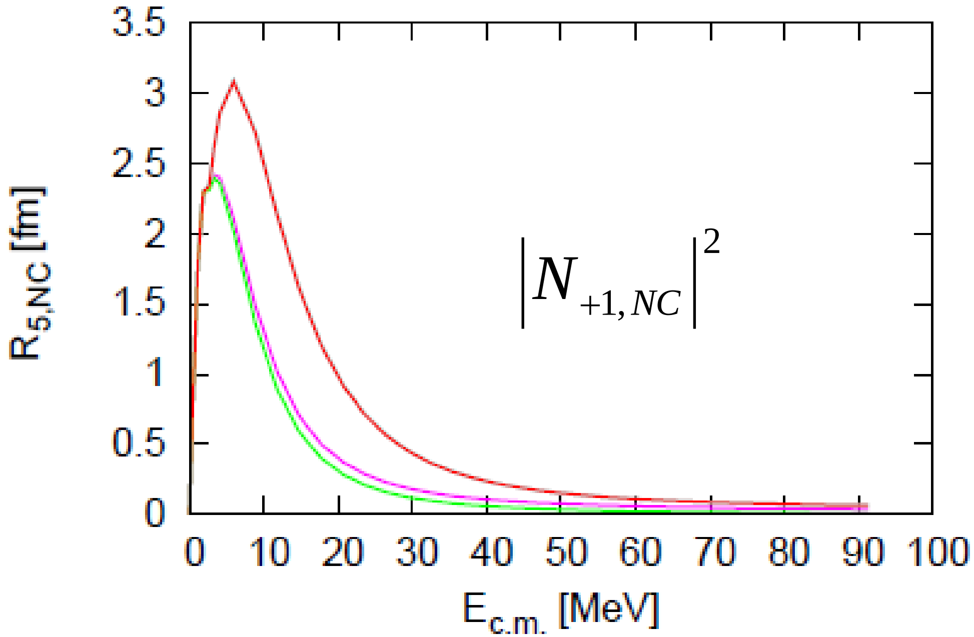
$$\frac{d^3\sigma}{d\Omega\, dE'} = \sum_{i=1}^{5} v_i(E,\theta,E')\, R_i(E_{c.m.},Q)$$

# Reactions with (anti)neutrinos *(cont.)*

Inelastic (anti)neutrino scattering off $^3$He at E=100 MeV
Inclusive weak NC response functions for $Q$= 100 MeV



# Reactions with (anti)neutrinos *(cont.)*

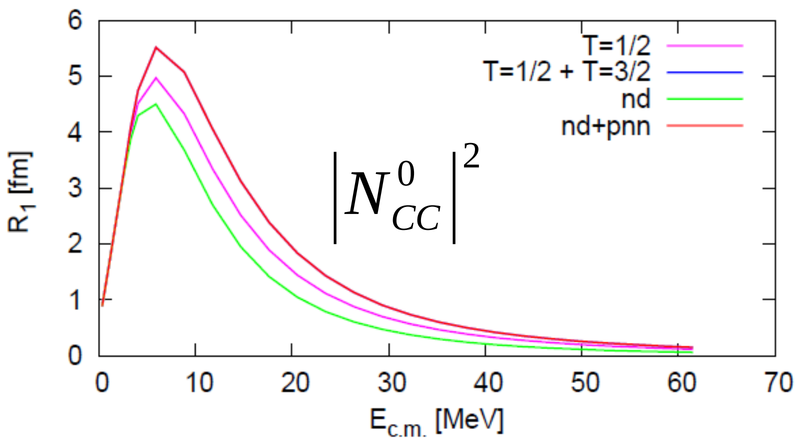


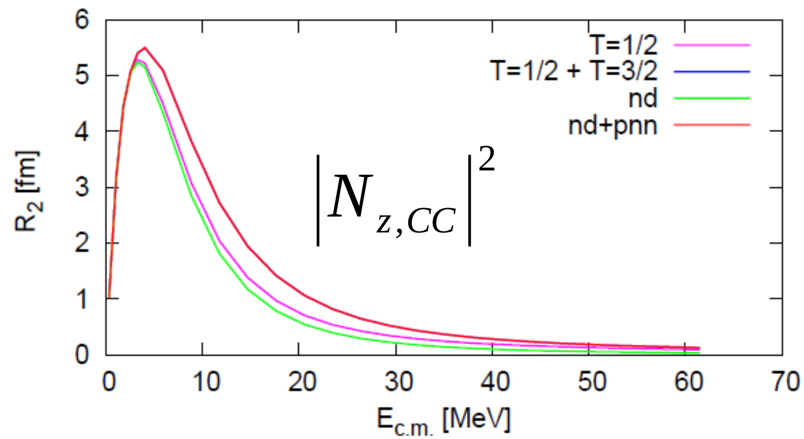

$^3$He

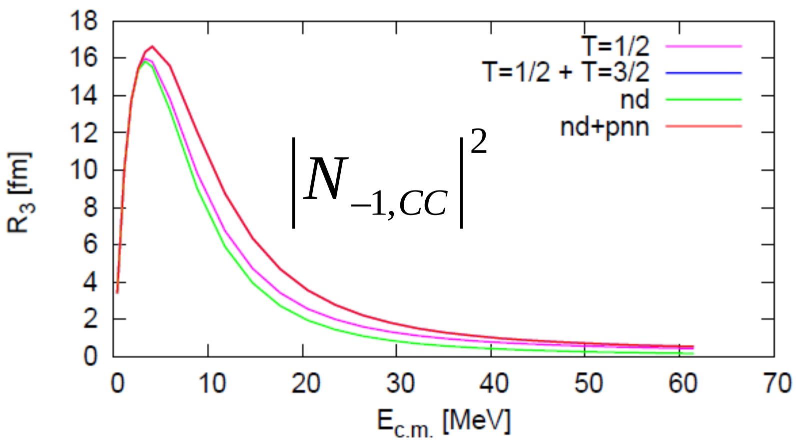
$$\frac{d^3\sigma}{d\Omega\, dE'} = \sum_{i=1}^{5} v_i(E,\theta,E')\, R_i(E_{c.m.},Q)$$

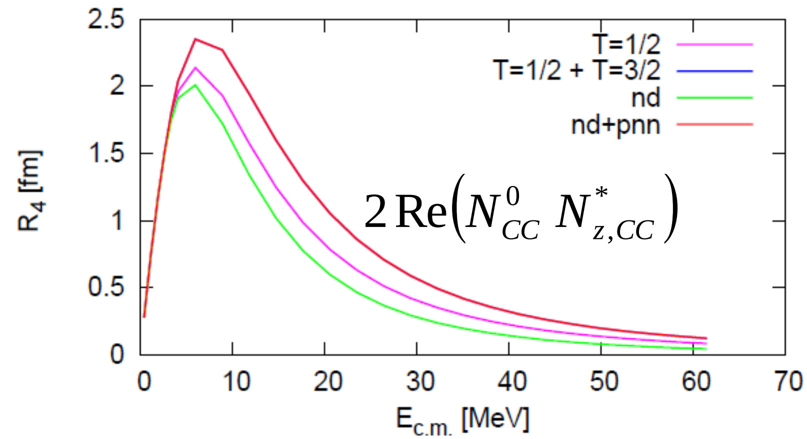
# Reactions with (anti)neutrinos *(cont.)*

Inelastic antineutrino CC scattering off $^3$He at E=100 MeV
Inclusive weak CC response functions for $Q$= 100 MeV



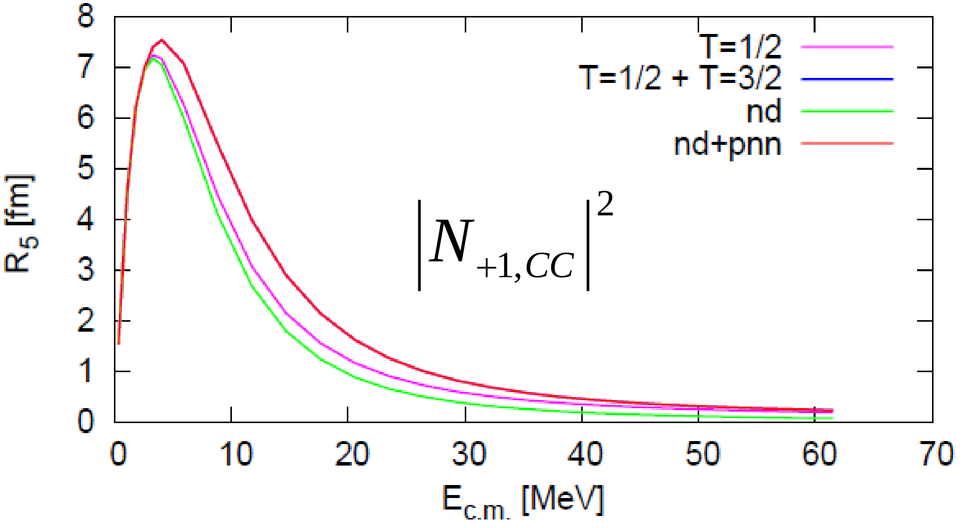



# Reactions with (anti)neutrinos *(cont.)*

Inelastic antineutrino scattering on $^3$He at E=100 MeV
Inclusive weak CC response functions for $Q$=100 MeV




Response functions depend only on $E_{c.m.}$ and $Q$ !

# Conclusions and outlook

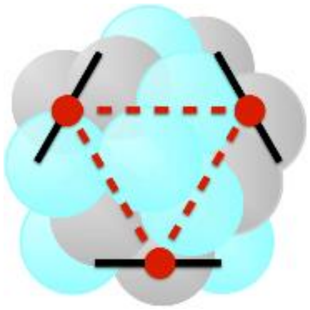
- Momentum space framework to deal with various electroweak processes in 2N and 3N systems has been established
- Several important numerical tests passed successfully
- Results of calculations utilizing the single nucleon current agree with the corresponding results obtained previously by other groups
- Robust methods to deal with PWD of a single- and two-nucleon current operator are available
- Calculations without PWD, especially for 2N system, are also possible
- Systematic calculations of inclusive response functions (and cross sections) are planned
- Unsolved problems: relativity, distortion of $e^{\pm}$ and $\mu^{\pm}$ energy spectra, Coulomb interactions of three protons
- Real progress requires consistent 2N and 3N forces as well as vector and axial parts of the nuclear weak current operator

# Conclusions and outlook *(cont.)*

It's a long way to go (EE) but we hope that consistent forces and currents will be applied within LENPIC „to understand nuclear structure and reactions with chiral forces"

http://www.lenpic.org

Low Energy Nuclear Physics
International Collaboration



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Thank you !