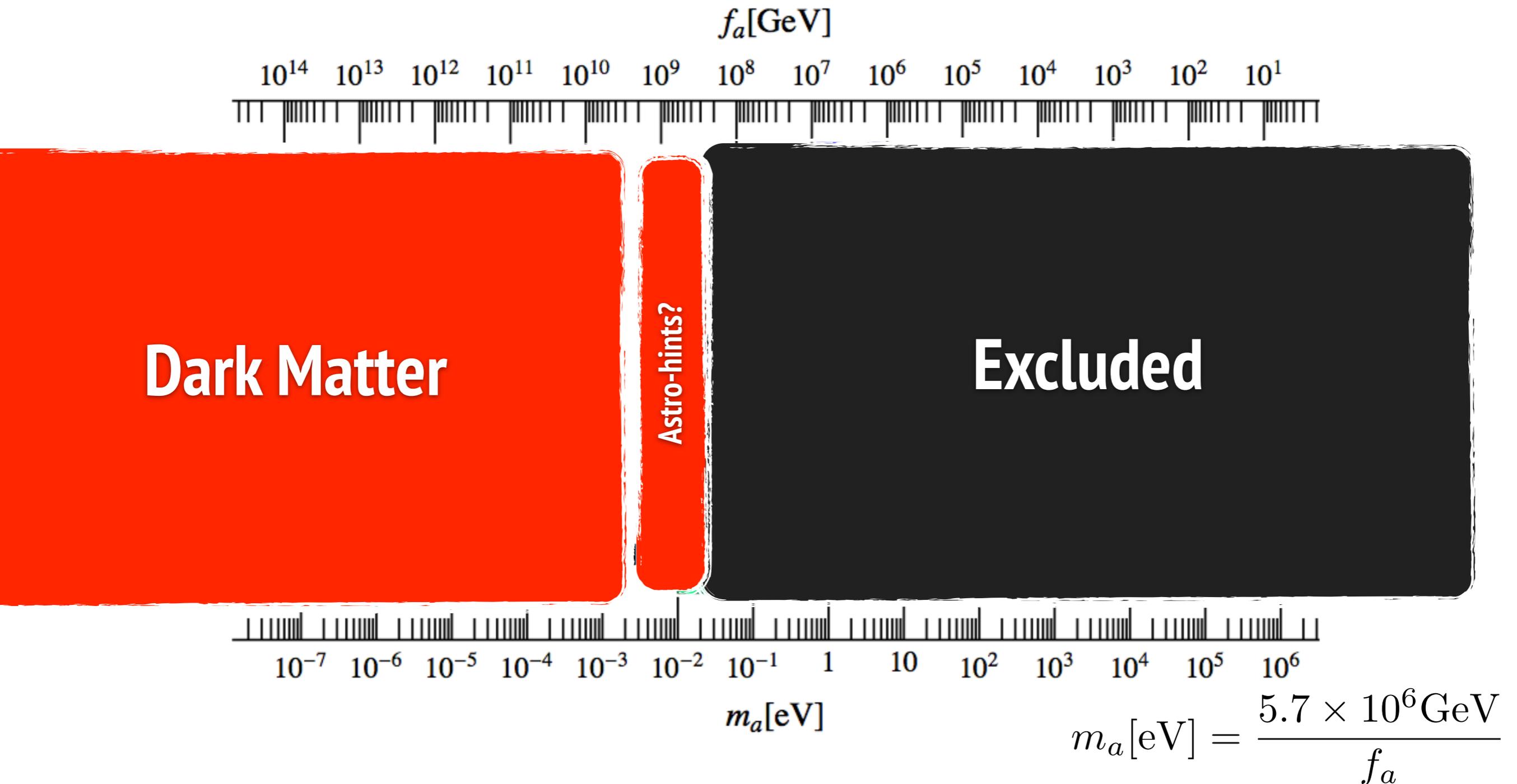


Axion dark matters



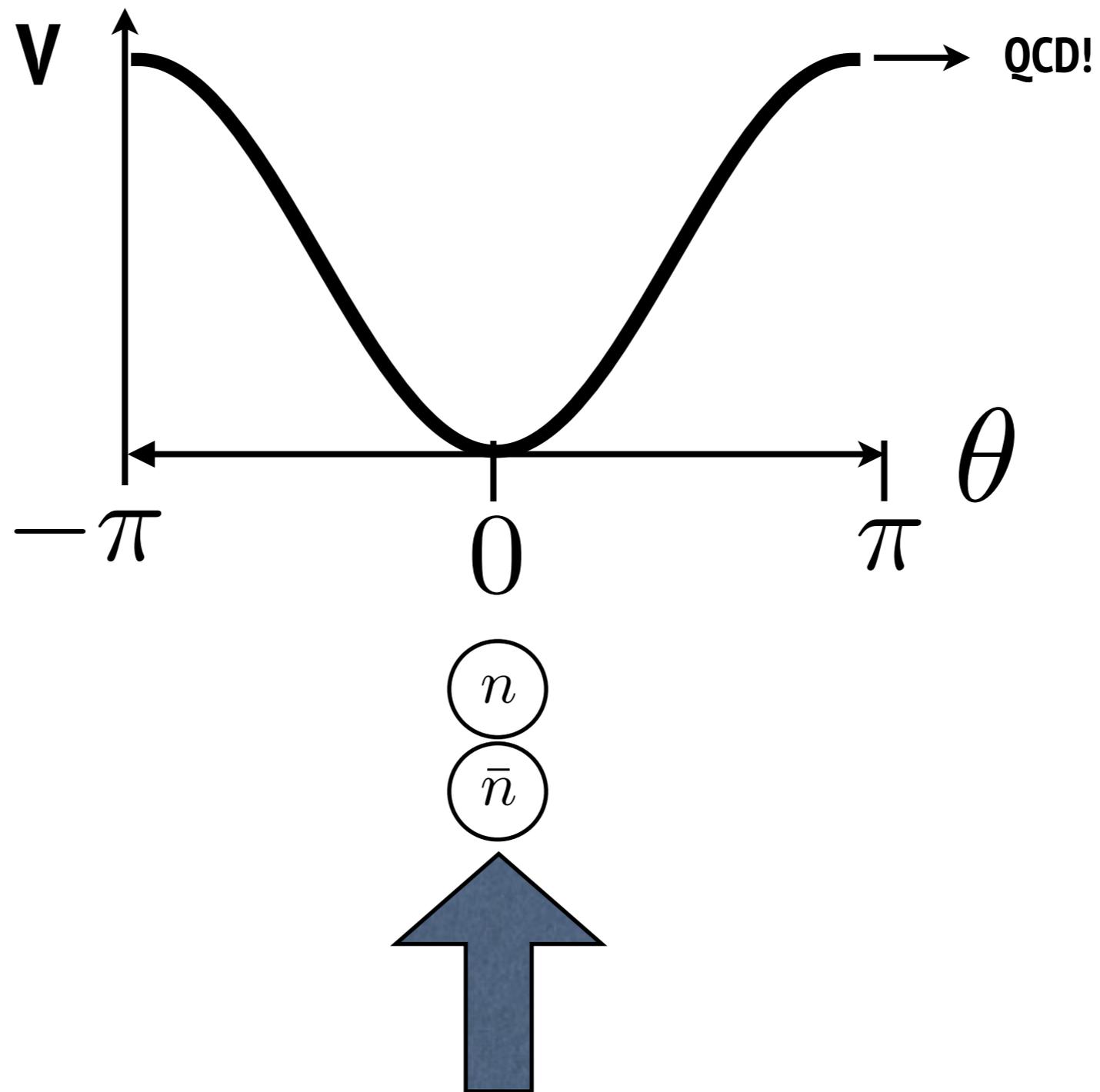
Landscape, what do we know?



If axions exist, they are very light and VERY weakly interacting!

Axions

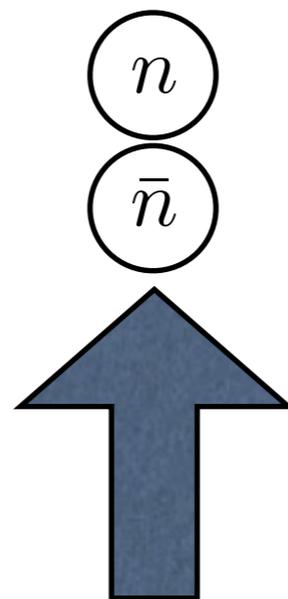
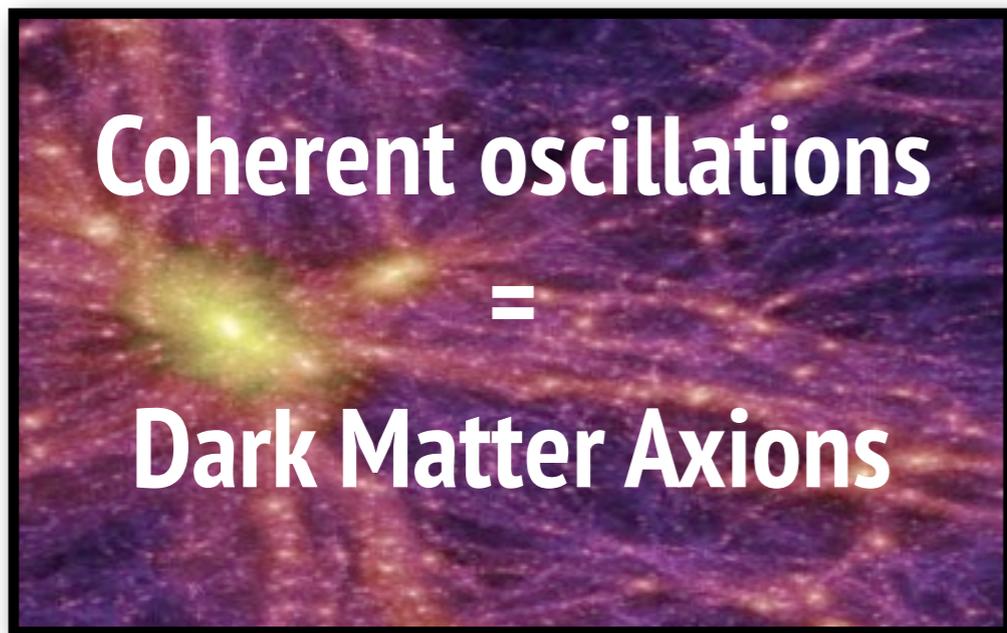
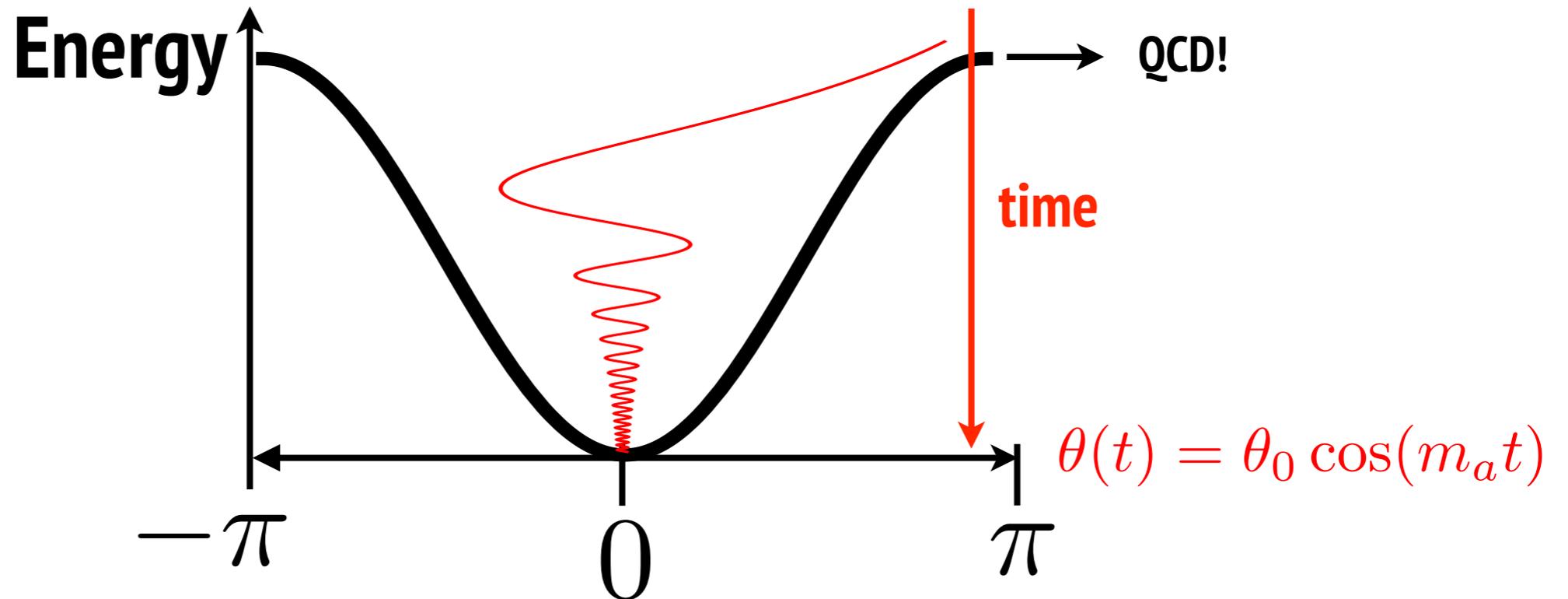
- theta is dynamical $\theta(t, \mathbf{x})$



Measured today $|\theta| < 10^{-10}$ (strong CP problem)

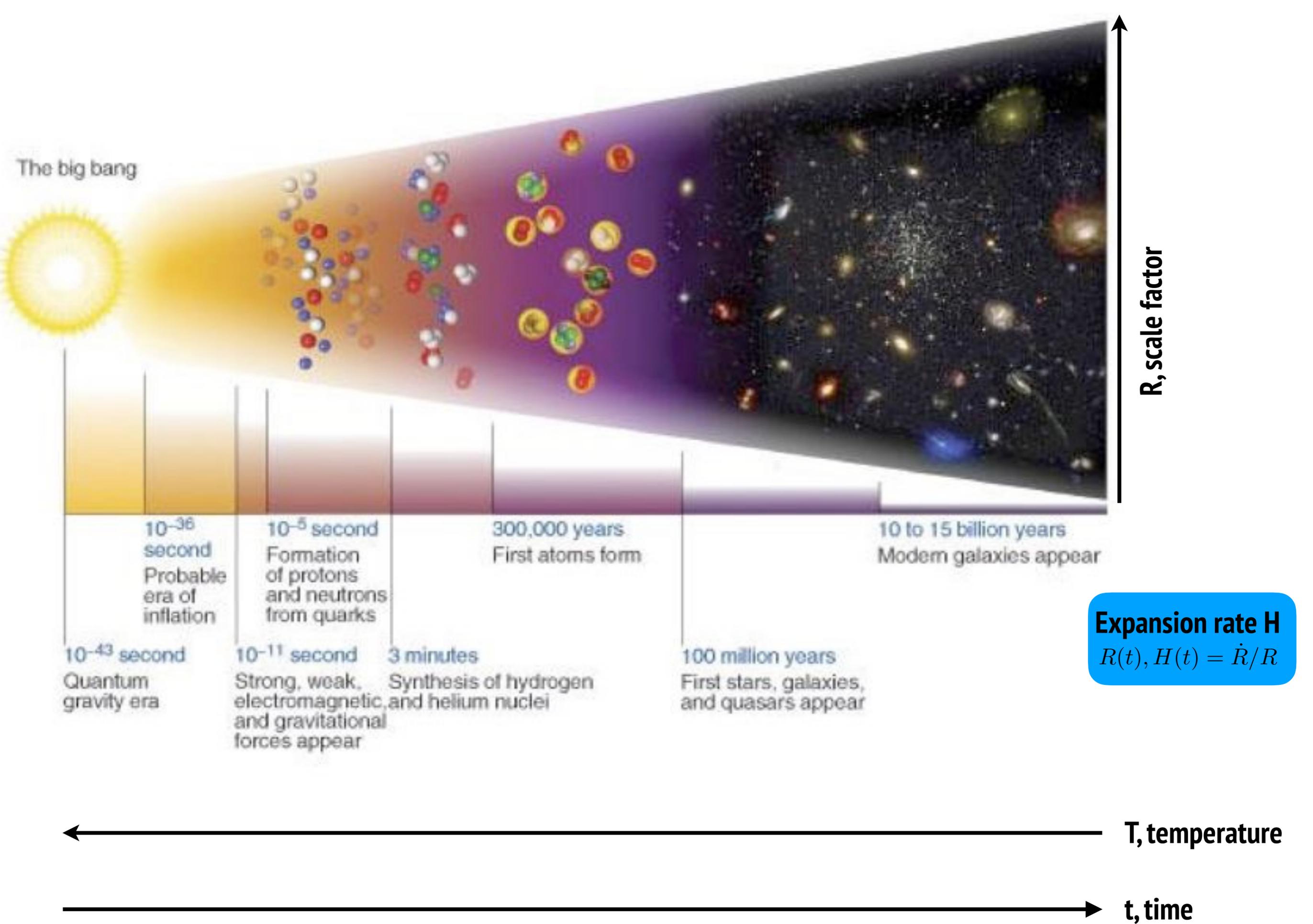
Axions imply dark matter

- theta is dynamical $\theta(t, \mathbf{x})$

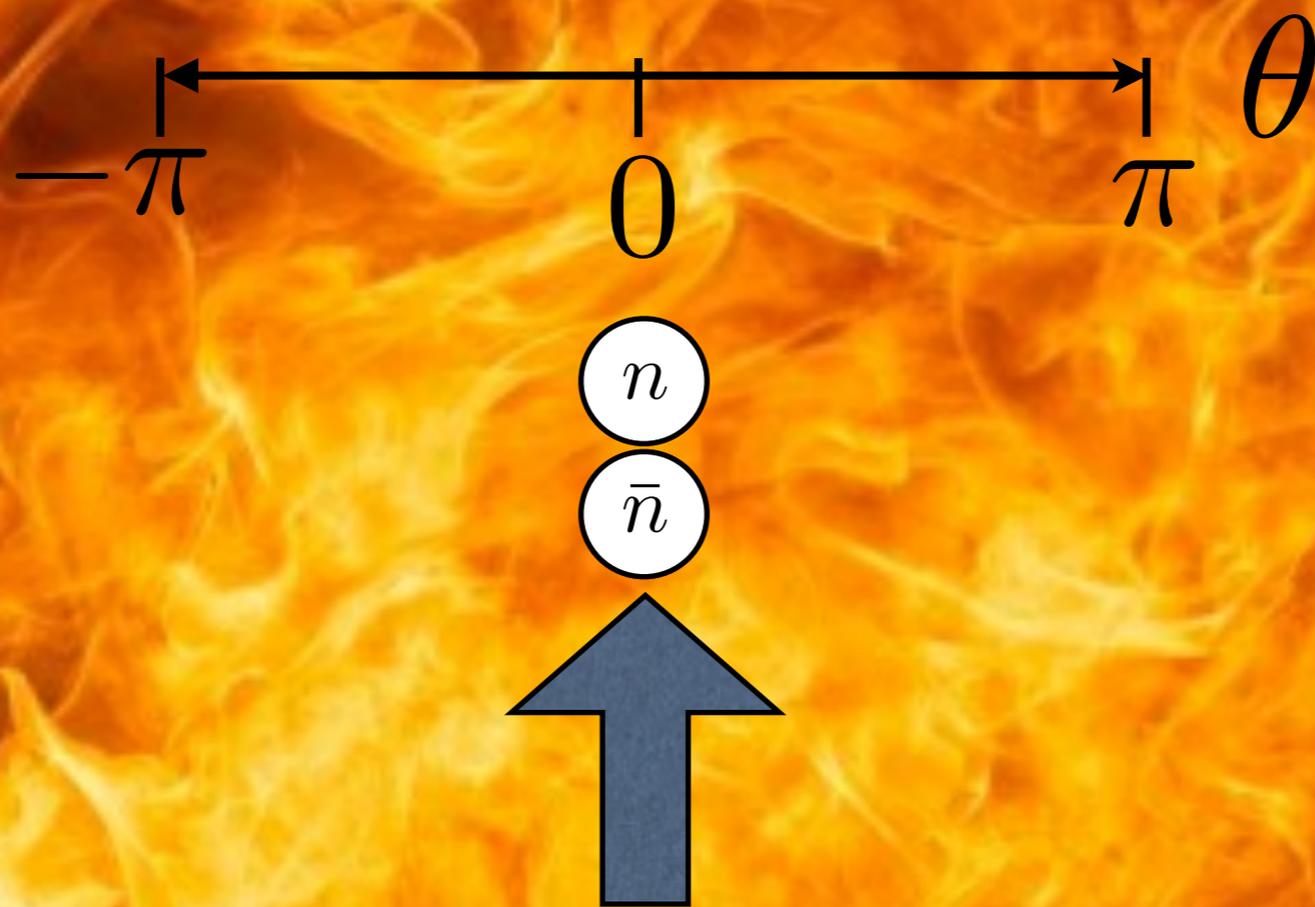


Initial conditions ?
= model-dependency

CP conserving



high Temperature ($T \gg \Lambda_{\text{QCD}}$)

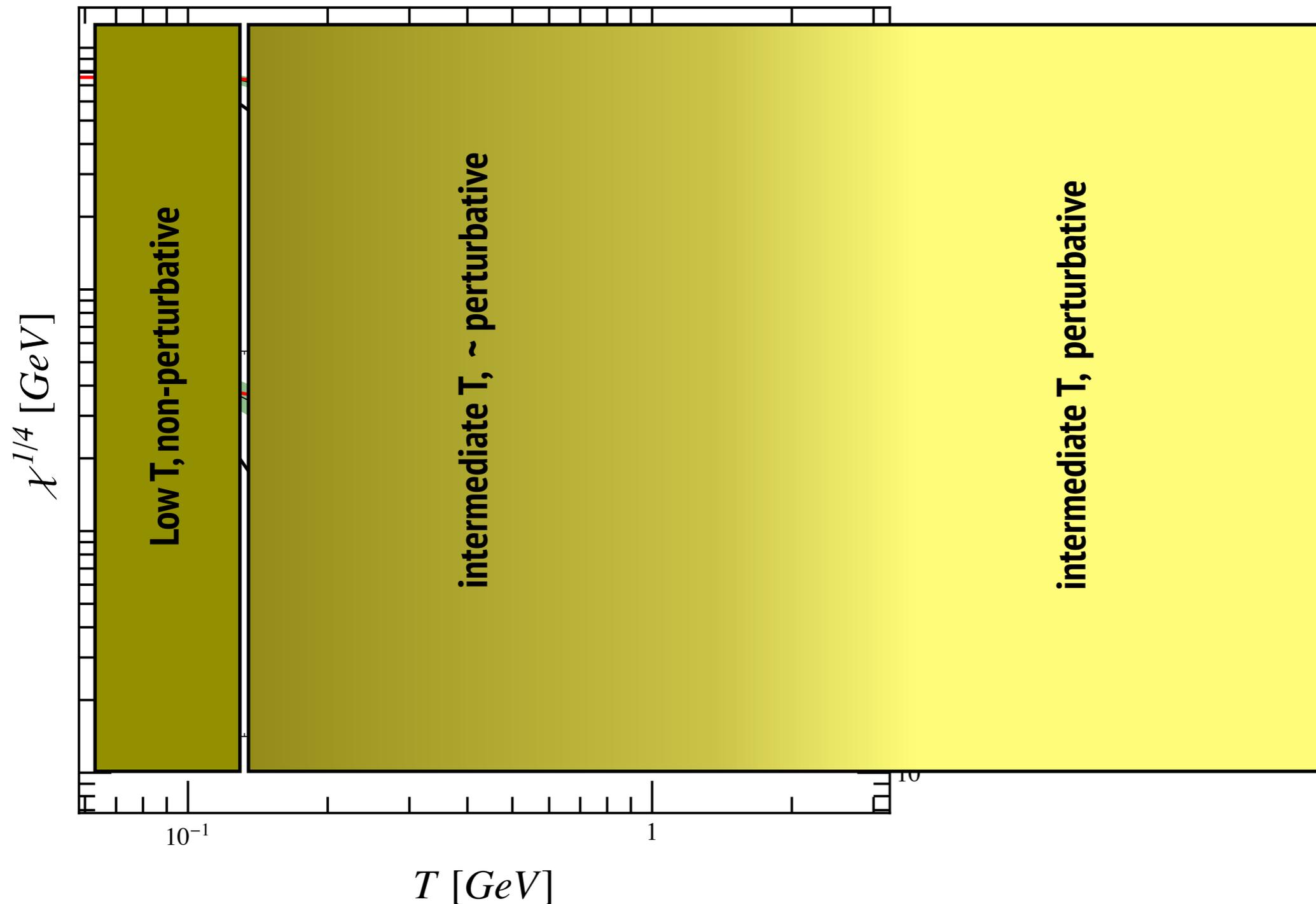


Measured today $|\theta| < 10^{-10}$ (strong CP problem)

Effective mass, lattice calculations

Axion mass/potential depend on non-perturbative QCD parameters $\langle \bar{q}q \rangle$, Λ^4

$T > T_c \sim 160$ MeV, running QCD coupling decreases, QCD becomes perturbative
quark condensate quickly disappears, Lambda also decreases



Effective mass, lattice calculations

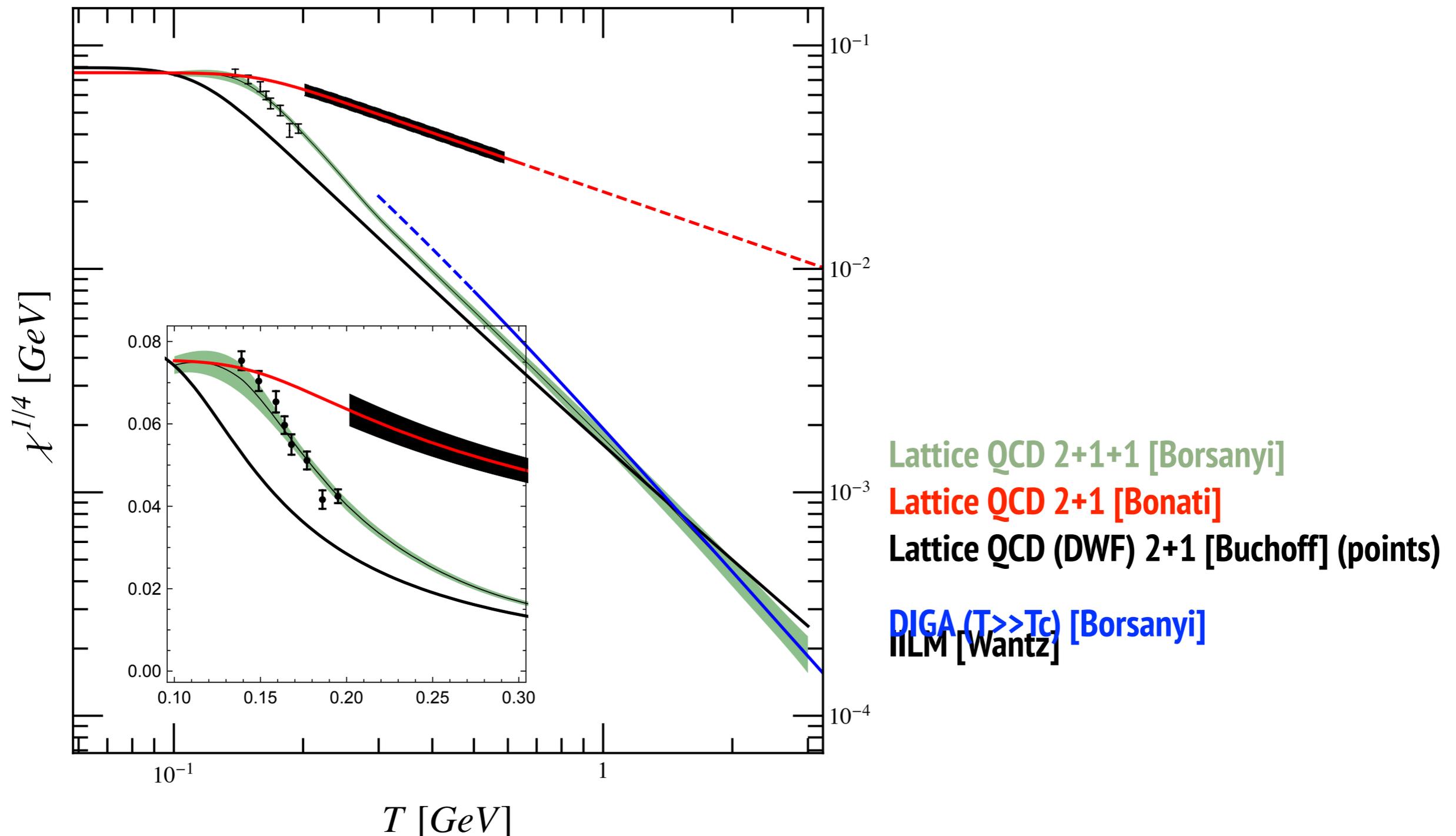
Lattice QCD: we can compute axion mass

$$m_a^2 f_a^2 = \chi(T)$$

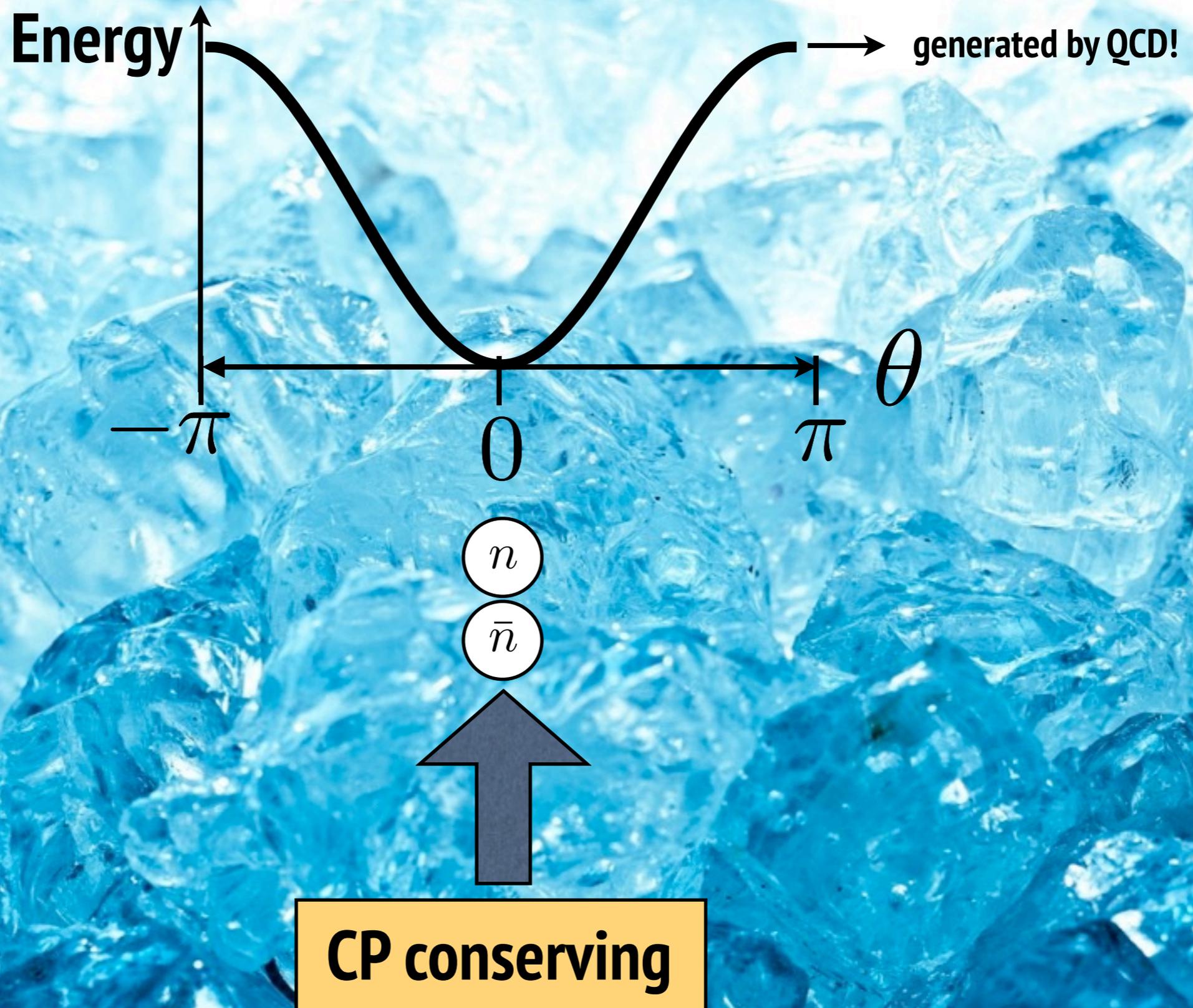
At ultra-high T (no mesons)

we can analytically compute potential (DIGA)

$$V(\theta) = -\chi(T) \cos \theta$$



at low temperatures ...

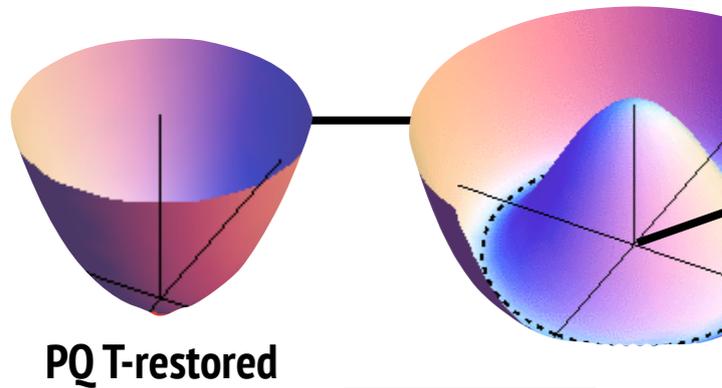


2 “typical” scenarios for initial conditions

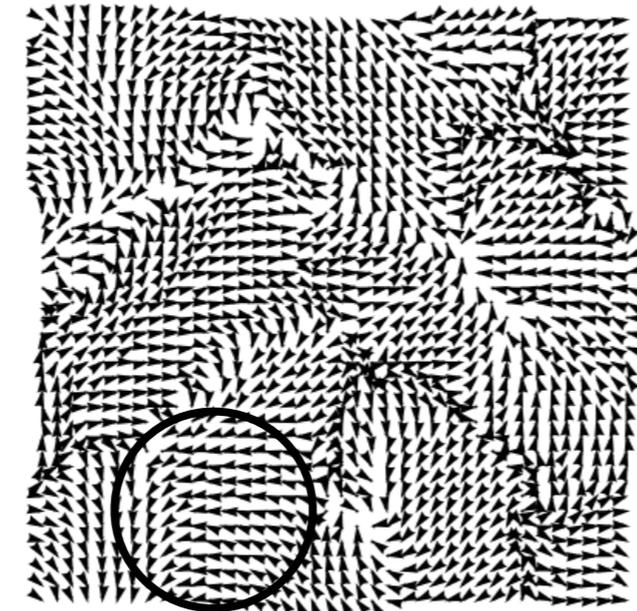
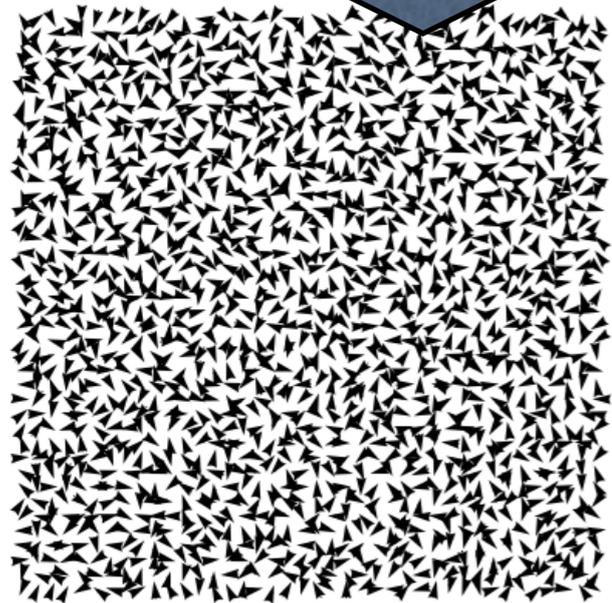
- Initial conditions are cosmological and axion-model dependent
- Two typical (and extreme) scenarios are:
 - random initial conditions in patches of our Universe
 - One common initial condition for the whole Universe

1st typical scenario: random initial conditions in our Universe

- Example KSVZ



SSB

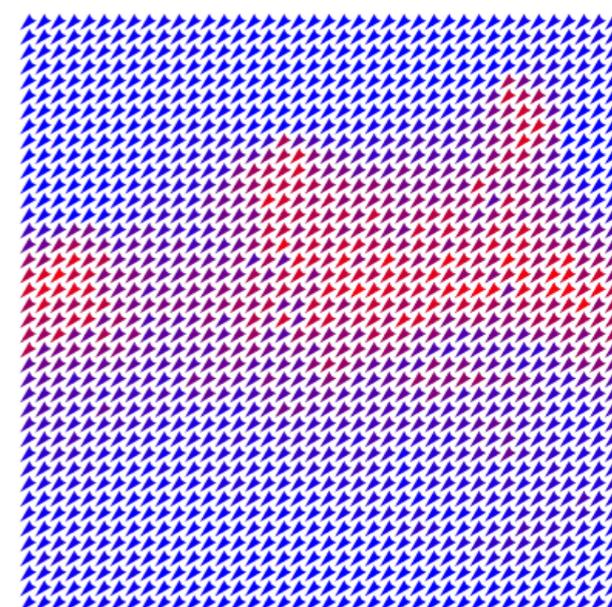
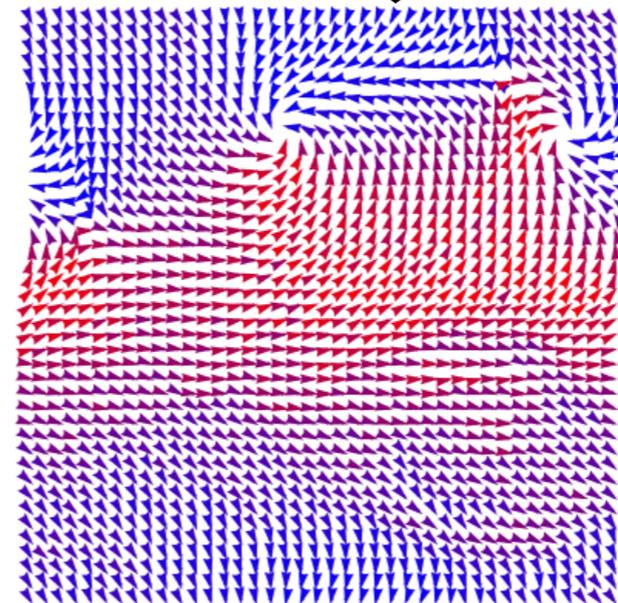


$2t$

smoothing

SCENARIO-I
realignment+CS+DWs
 $O(1)$ inhomogeneous DM

QCD
D. Walls

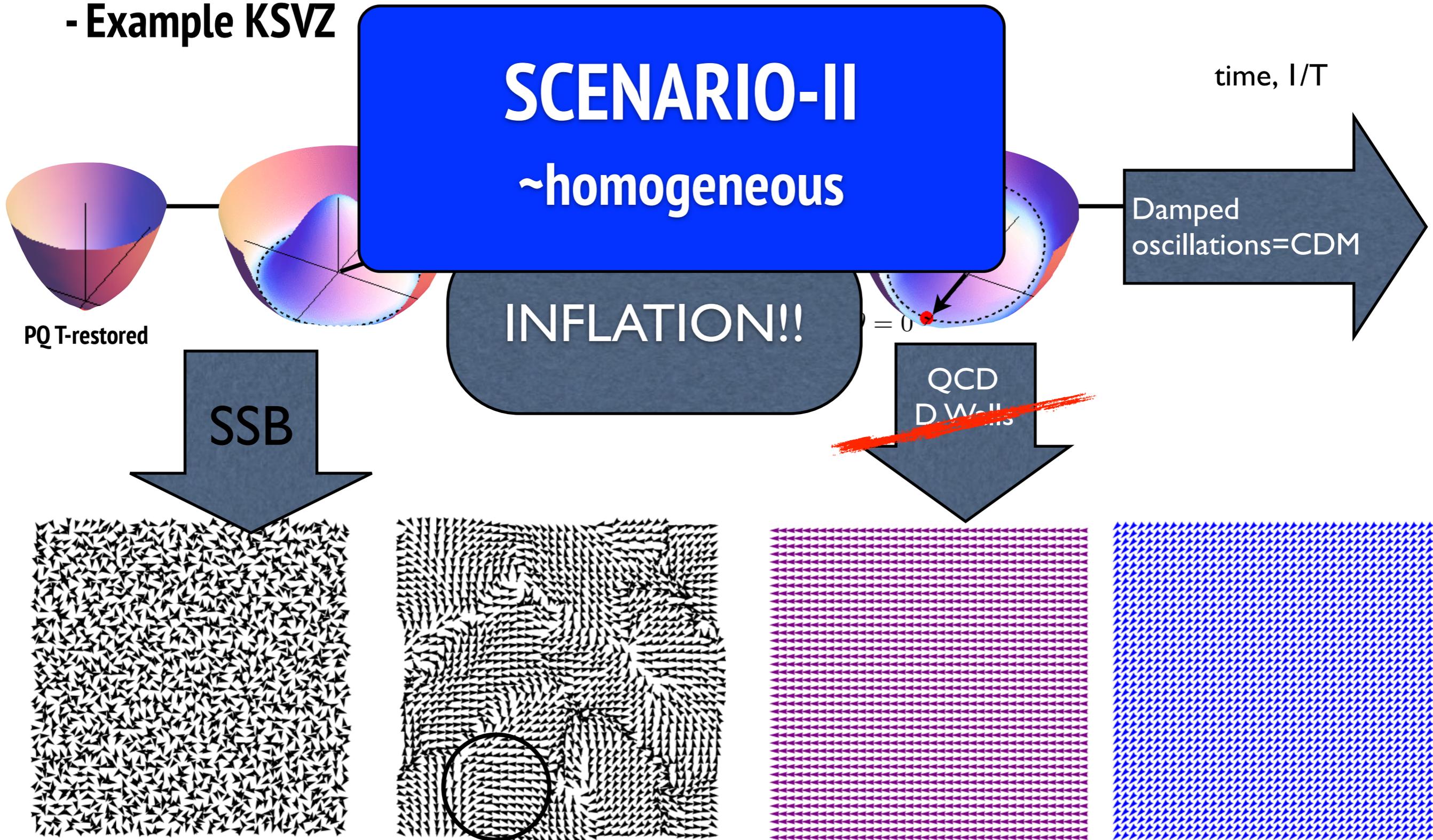


time, $1/T$

Damped
oscillations=CDM

2nd typical scenario: 1 initial condition for our whole Universe

- Example KSVZ



SCENARIO-II

~homogeneous

INFLATION!!

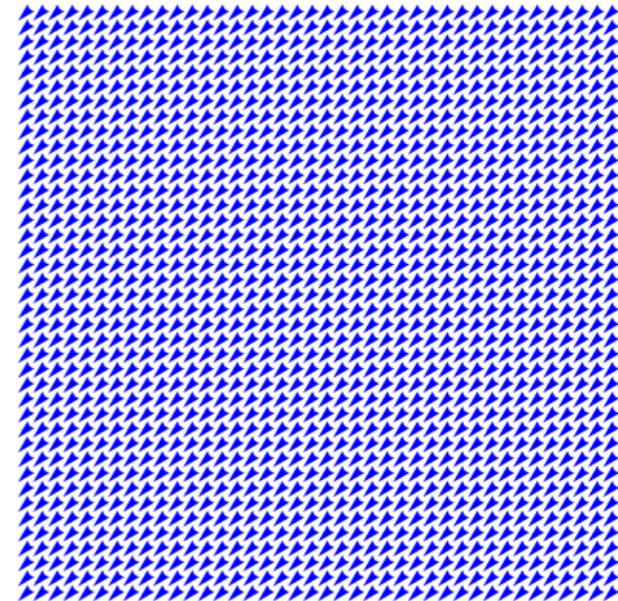
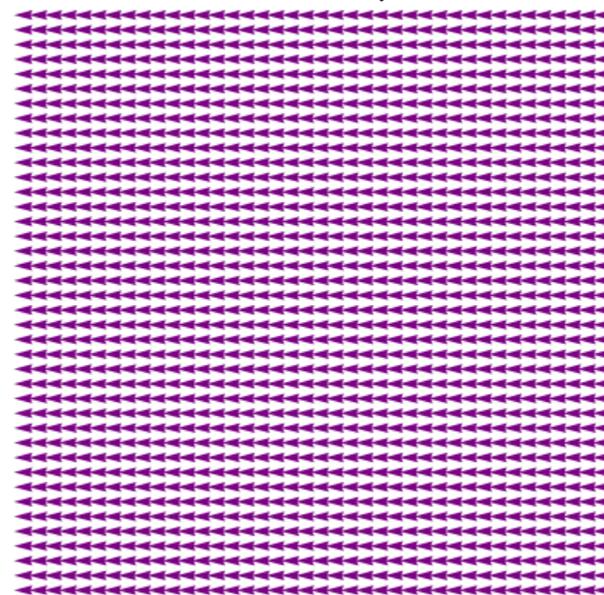
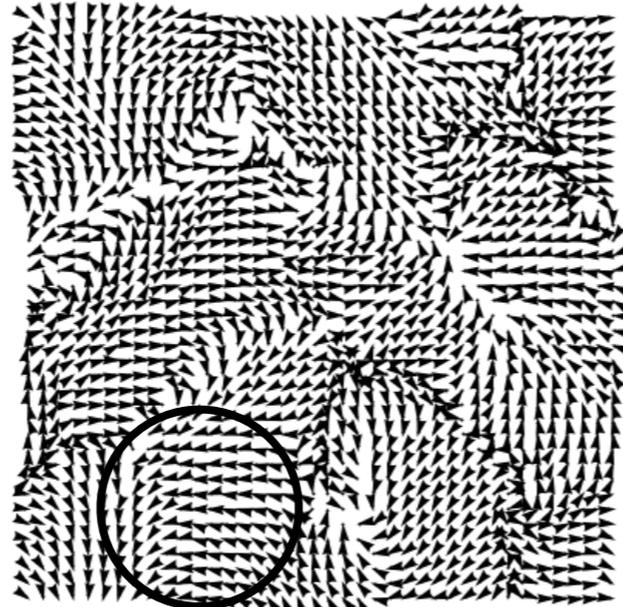
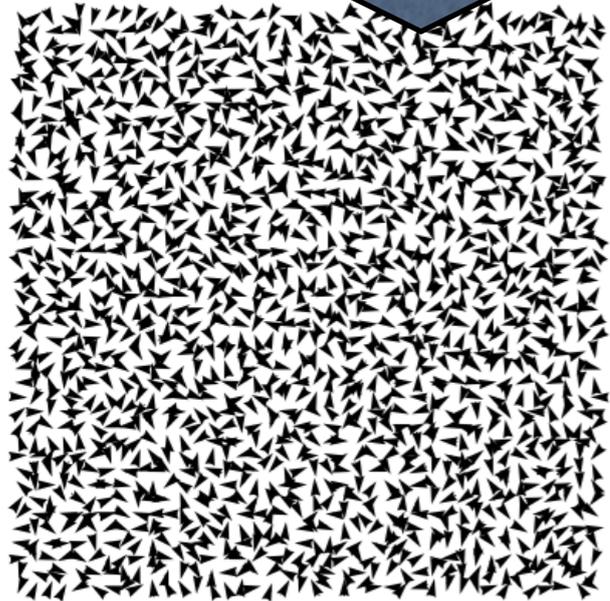
~~QCD
DWs~~

Damped
oscillations=CDM

time, $1/T$

PQ T-restored

SSB

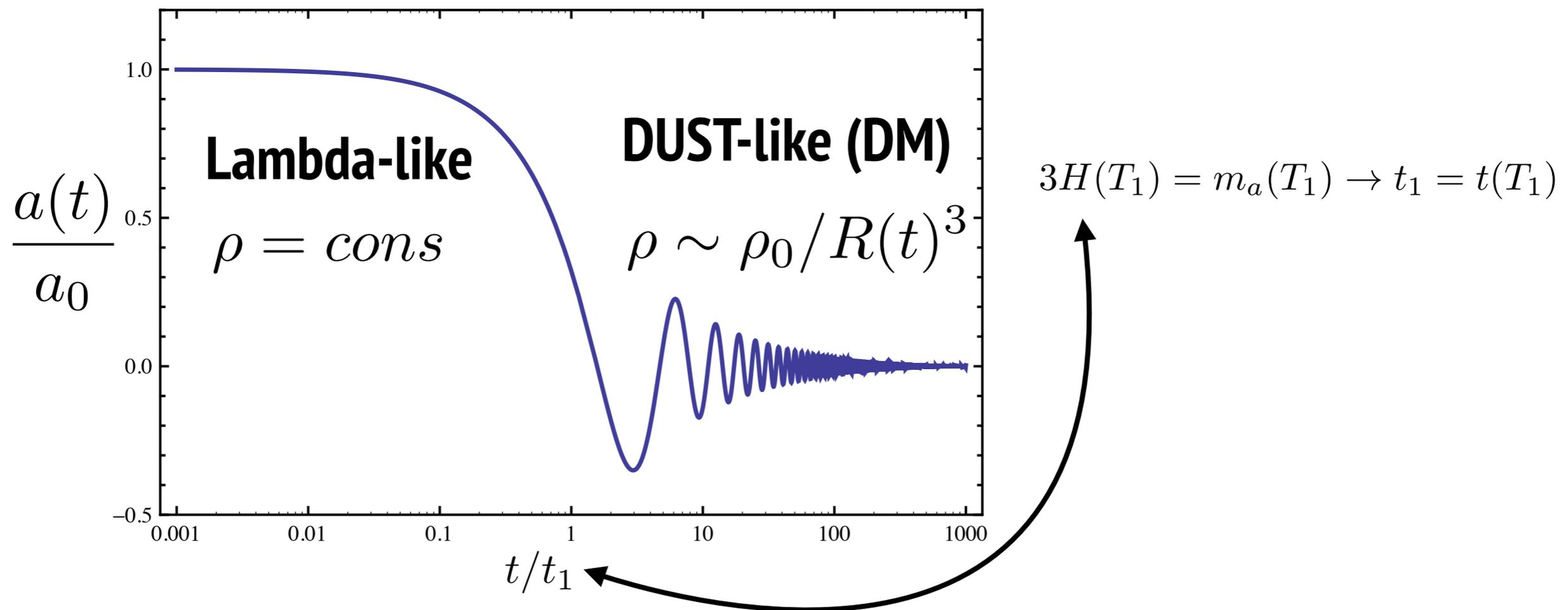


Axion Dark Matter Evolution (scenario 2)

Non-thermal, decoupled = Initial conditions + simple evolution

E.O.M. $\ddot{a} + 3H\dot{a} + m_a^2 a \simeq 0$ (SIMPLIFIED)

Energy density (harmonic osc) $\rho = \frac{1}{2}(\dot{a})^2 + \frac{1}{2}(\nabla a)^2 + V(\theta) \sim \frac{1}{2}(\dot{a})^2 + \frac{1}{2}m_a^2 a^2$

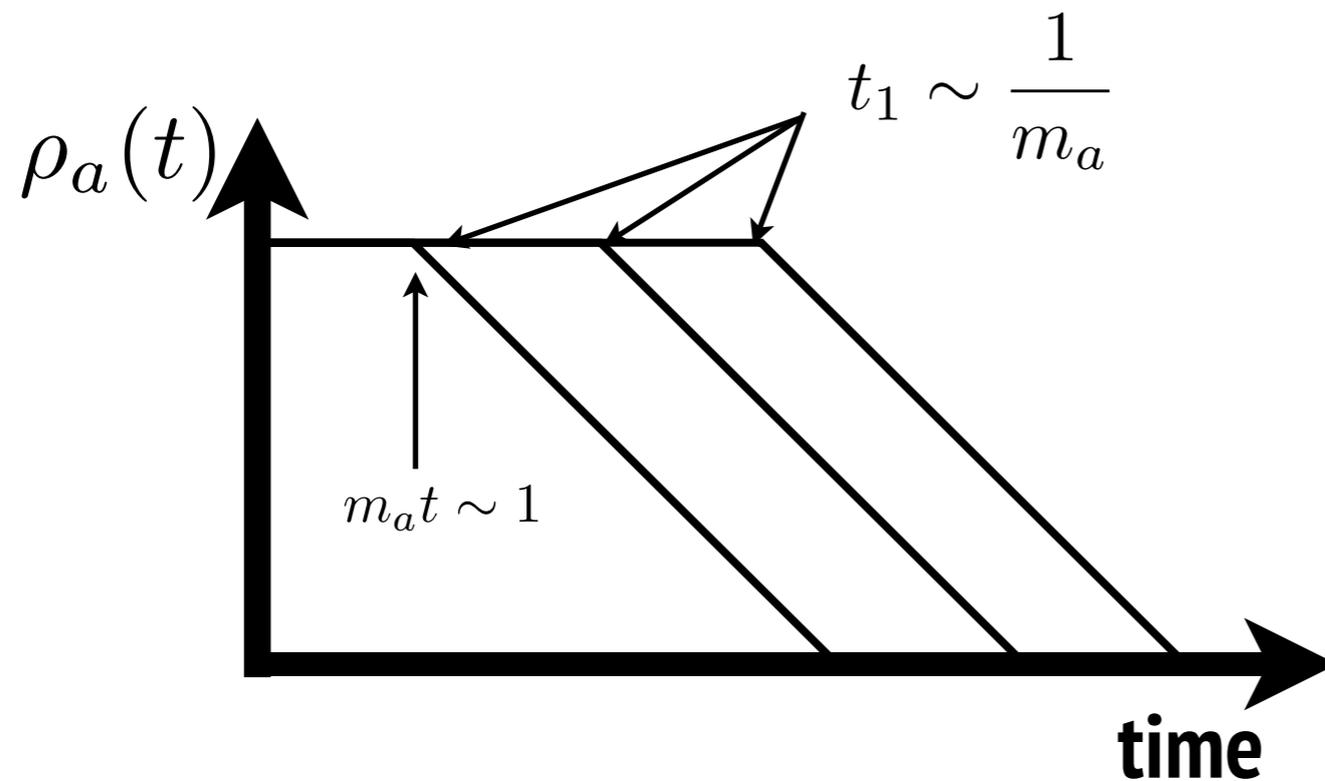


Note: No spatial dependence -> no momentum, no velocity (up to H)= ultracold dark matter

Axion dark matter

- The amount of axion DM produced depends on m_a (eq. fa)

$$H = \frac{1}{2t} \rightarrow t_1 \sim \frac{1}{m_a(T(t_1))} \propto f_a$$



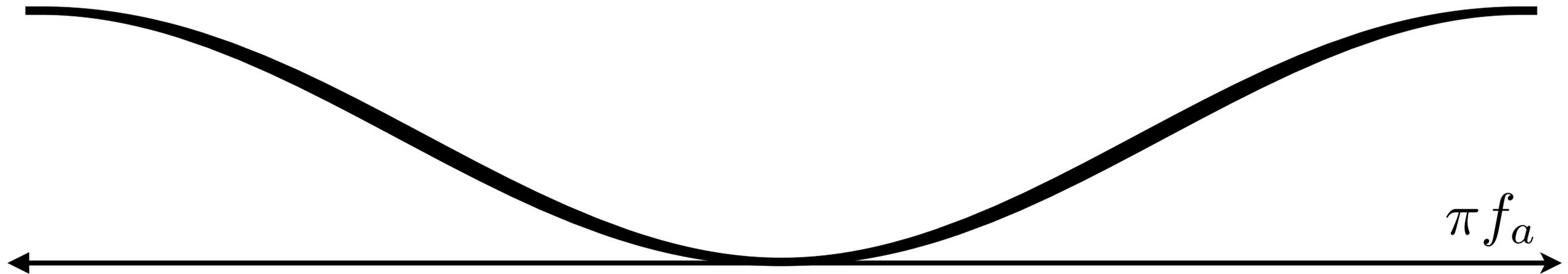
- Axion starts to oscillate later, \sim initial energy \rightarrow more DM

Axion dark matter

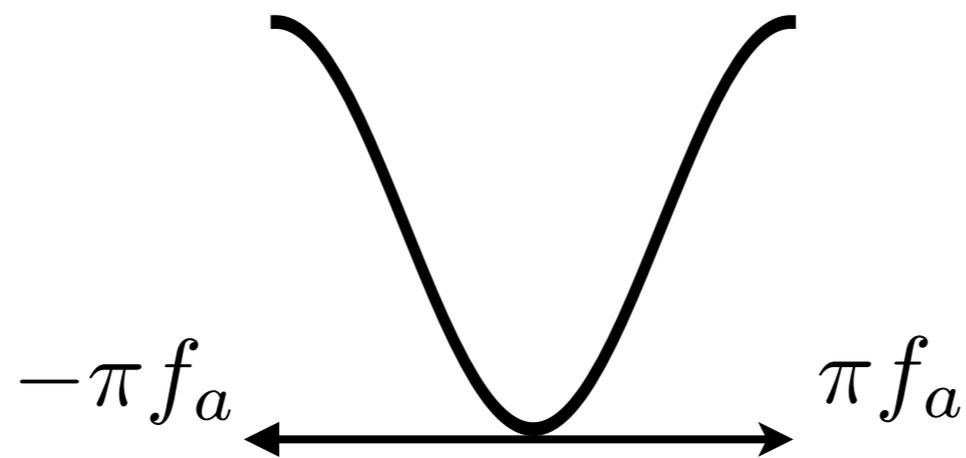
- The amount of axion DM produced depends on f_a

$$H = \frac{1}{2t} \rightarrow t_1 \sim \frac{1}{m_a(T(t_1))} \propto f_a$$

- large f_a , small curvature, oscillations start later \rightarrow more DM



- small f_a , large curvature, oscillations start earlier \rightarrow less DM



Accounting for the axion mass rapid increase

At late times ... $m_a \gg H$ (m increases, H decreases ... e.g. in radiation epoch $H \simeq 1/2t$)

$$\ddot{a} + 3H\dot{a} + m_a^2(t)a \simeq 0$$

can be solved perturbatively in the adiabatic limit $H/m_a \ll 1$ (weak damping)

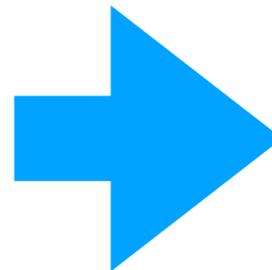
$$\ddot{a} + m_a^2 \simeq 0 \rightarrow a = Ae^{i \int^t m_a dt'} + O(\dot{m}_a/m_a \sim H/m_a) + h.c.$$

$$a = A(t)e^{i \int^t m_a dt'} + h.c.$$

$O(m^2)$ $-m_a^2 a + \dots + m_a^2 a = 0$

$O(m)$ $2im_a \dot{A} + i\dot{m}_a A + 3\frac{\dot{R}}{R} im_a A + \dots = 0$

$O(1)$...



$$\frac{d}{dt} (m_a A^2 R^3) = \frac{d}{dt} \left(\frac{\rho_a}{m_a} R^3 \right) = 0$$

adiabatic invariant!

number of axions in a "comoving volume" $V_c = V * R^3$

Prob: Relic abundance calculation

DM density ($m_a \gg 3H$)

$$\rho_a \sim \frac{\rho_1}{m_a(t_1)} m_a(t) \left(\frac{R_1}{R(t)} \right)^3$$

initial number density, mass, dilution due to Universe expansion

$$\rho_1 \sim \frac{1}{2} m_a^2(t_1) f_a^2 \theta_1^2$$

$$H^2 = \frac{8\pi}{M_p^2} \frac{\pi^2}{30} g_* T^4$$

$$m_a^2 = \chi(T) / f_a^2$$

$$3H(T_1) = m_a(T_1) \rightarrow t_1 = t(T_1) \rightarrow T_1$$

Entropy conservation from T1 until now

$$g_S(T) (RT)^3 = \text{cons}$$

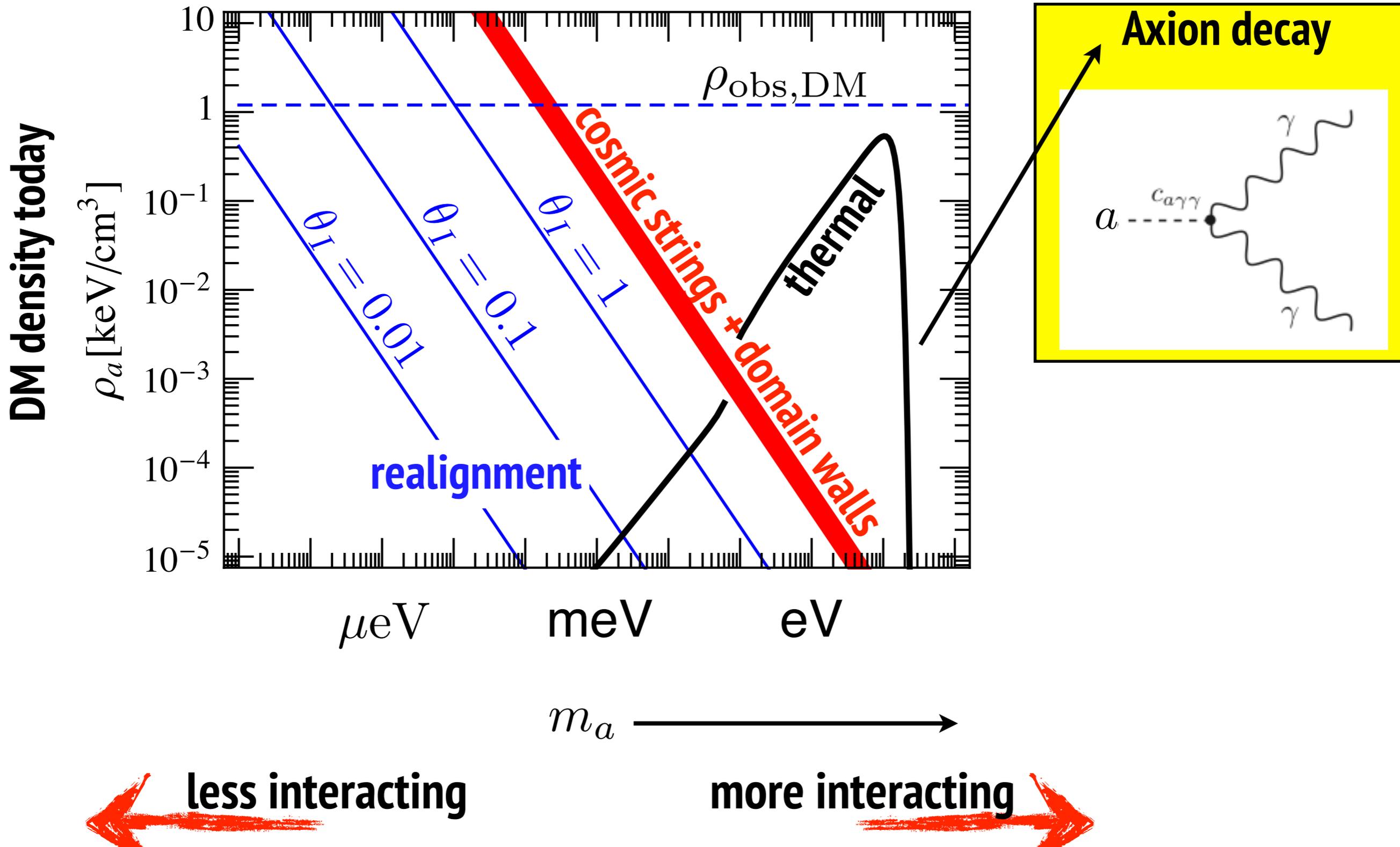
$$g_S(T_1) (R_1 T_1)^3 = g_S(T_0) (R_0 T_0)^3$$

Compute $\rho_a(\text{today}), \Omega_{\text{DM}} h^2 = \frac{\rho_a(\text{today})}{\rho_c}$

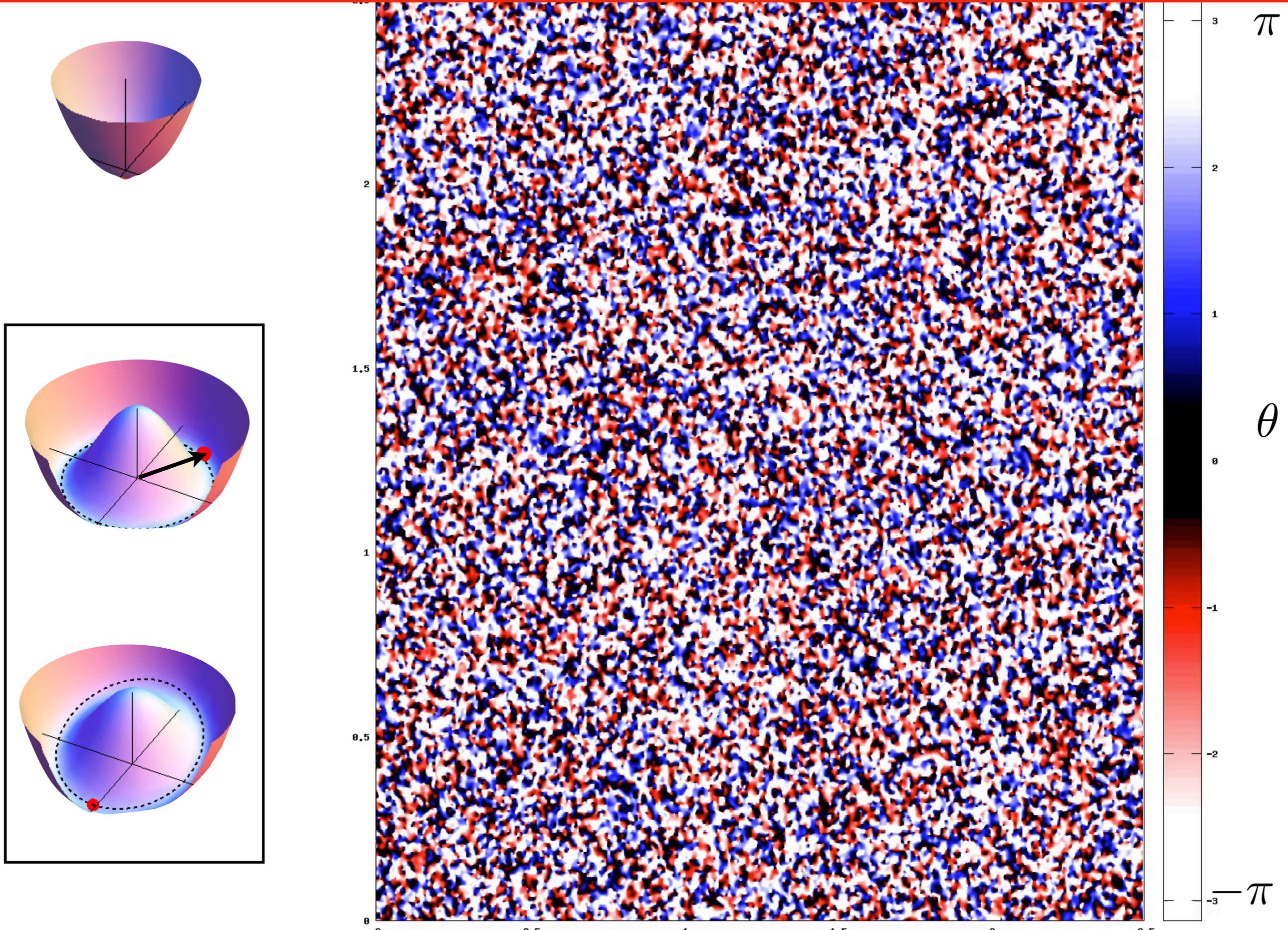
As a function of θ_1, f_a **and** θ_1, m_a

$$M_p = 1.22 \times 10^{19} \text{ GeV}$$

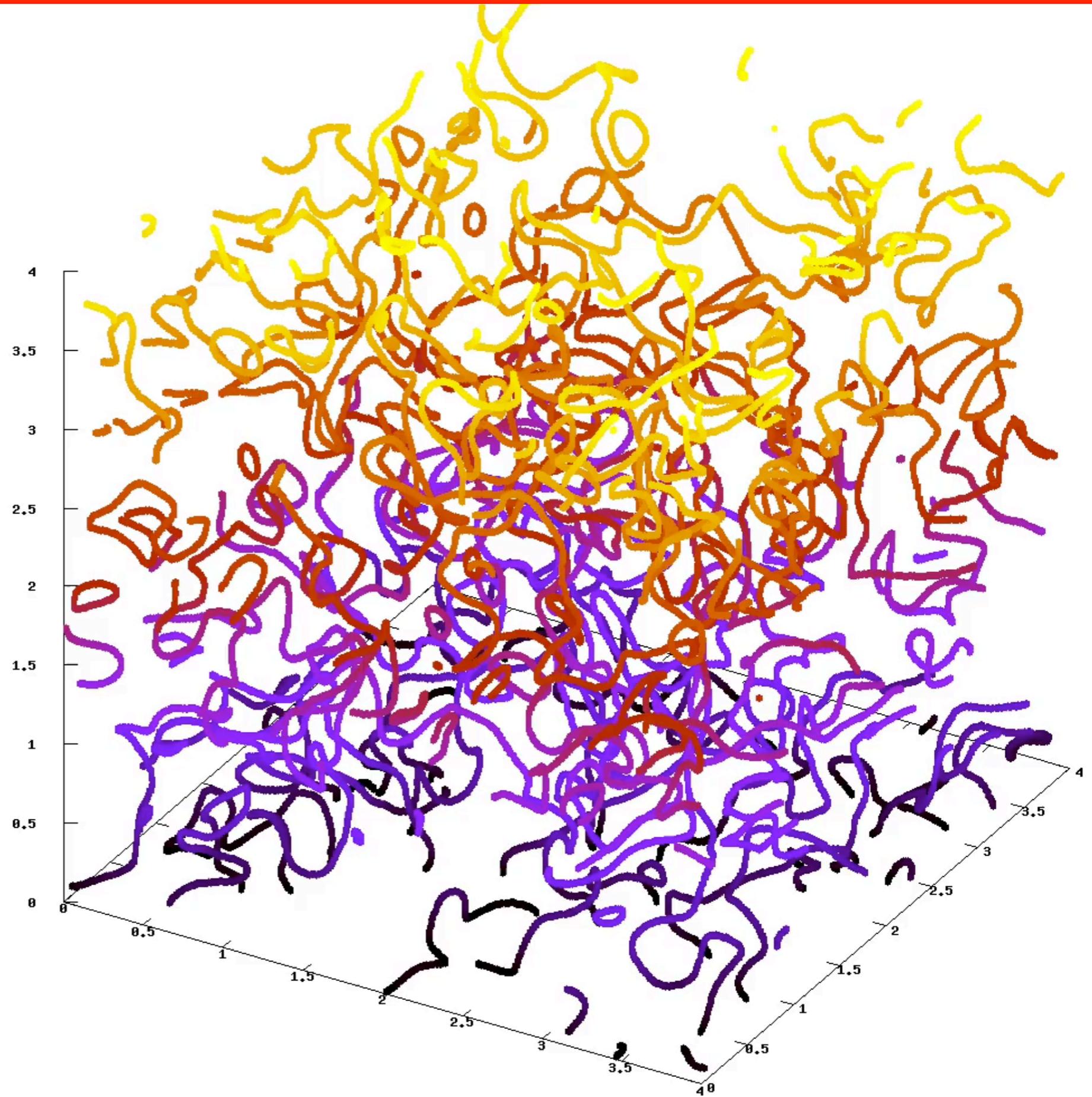
Axion DM, how much



SCENARIO I (N=1): axion evolution around t1

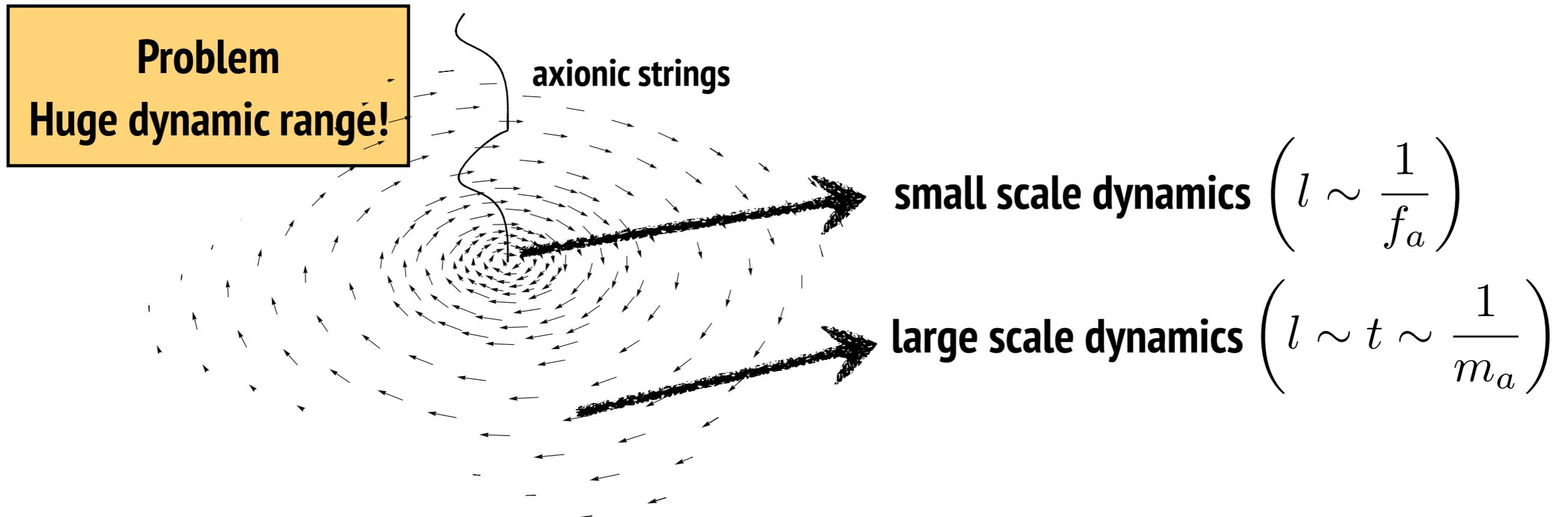


Strings



Prob: Relic abundance calculation

DM density can be obtained from numerical simulations



We usually split it into two pieces:

- averaged misalignment

$$\langle \rho_a \rangle_{\theta_1} = \int \frac{d\theta_1}{2\pi} \rho_a(\theta_1)$$

- axions radiated from topological defects

(extrapolated from simulations) [work in progress]

Length scales

- Time scale

$$3H(T_1) = m_a(T_1) \quad t_1 \sim \frac{1}{2H_1}$$

- Horizon size (shorter wavelengths deca)

$$L_1 = 2t_1 \sim \frac{1}{H_1}$$

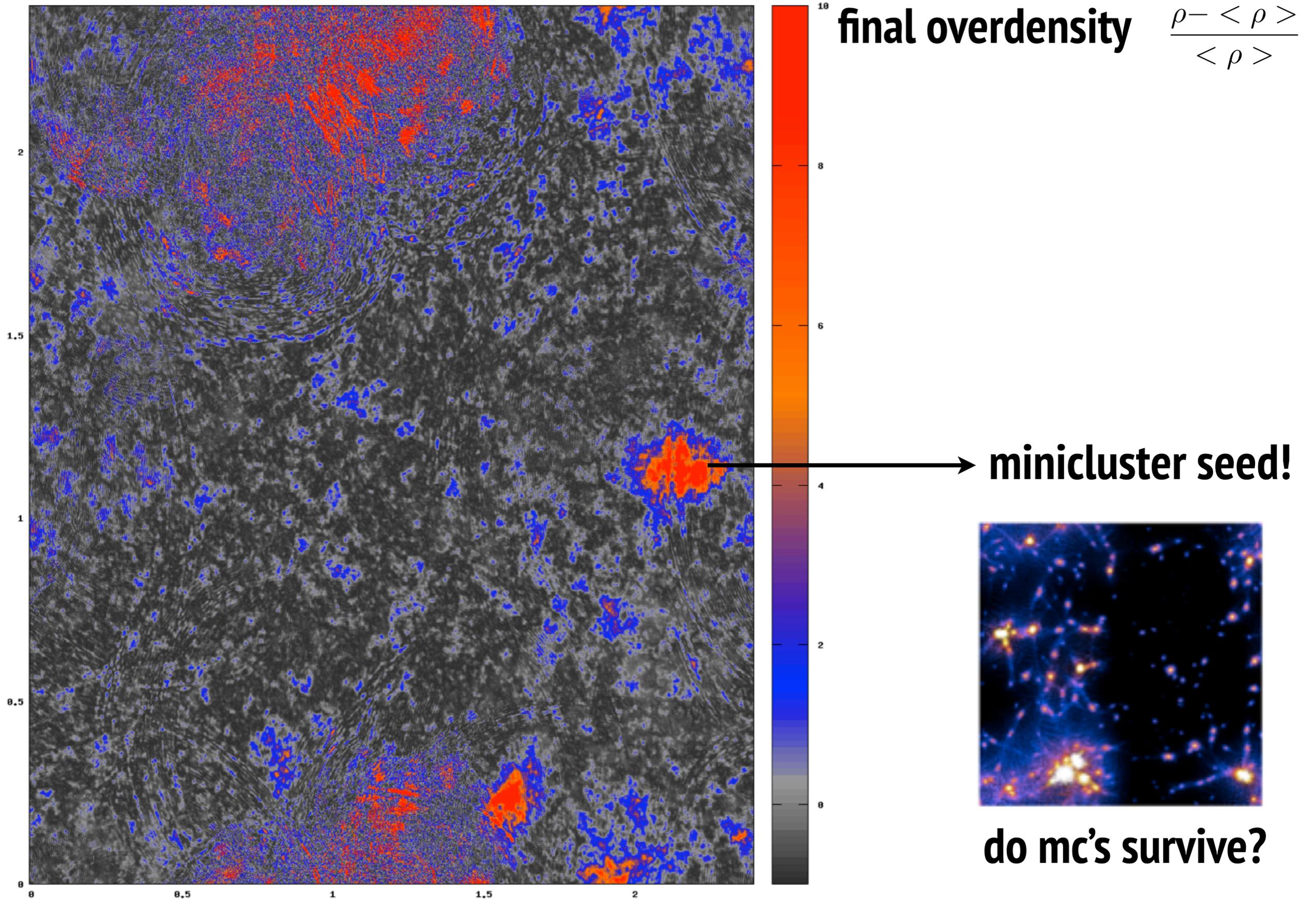
- Full Axion DM in this model $f_a \sim 10^{11} \text{ GeV}$

$$T_1 \sim 1.5 \text{ GeV} \left(\frac{10^{11} \text{ GeV}}{f_a} \right)^{0.16}$$

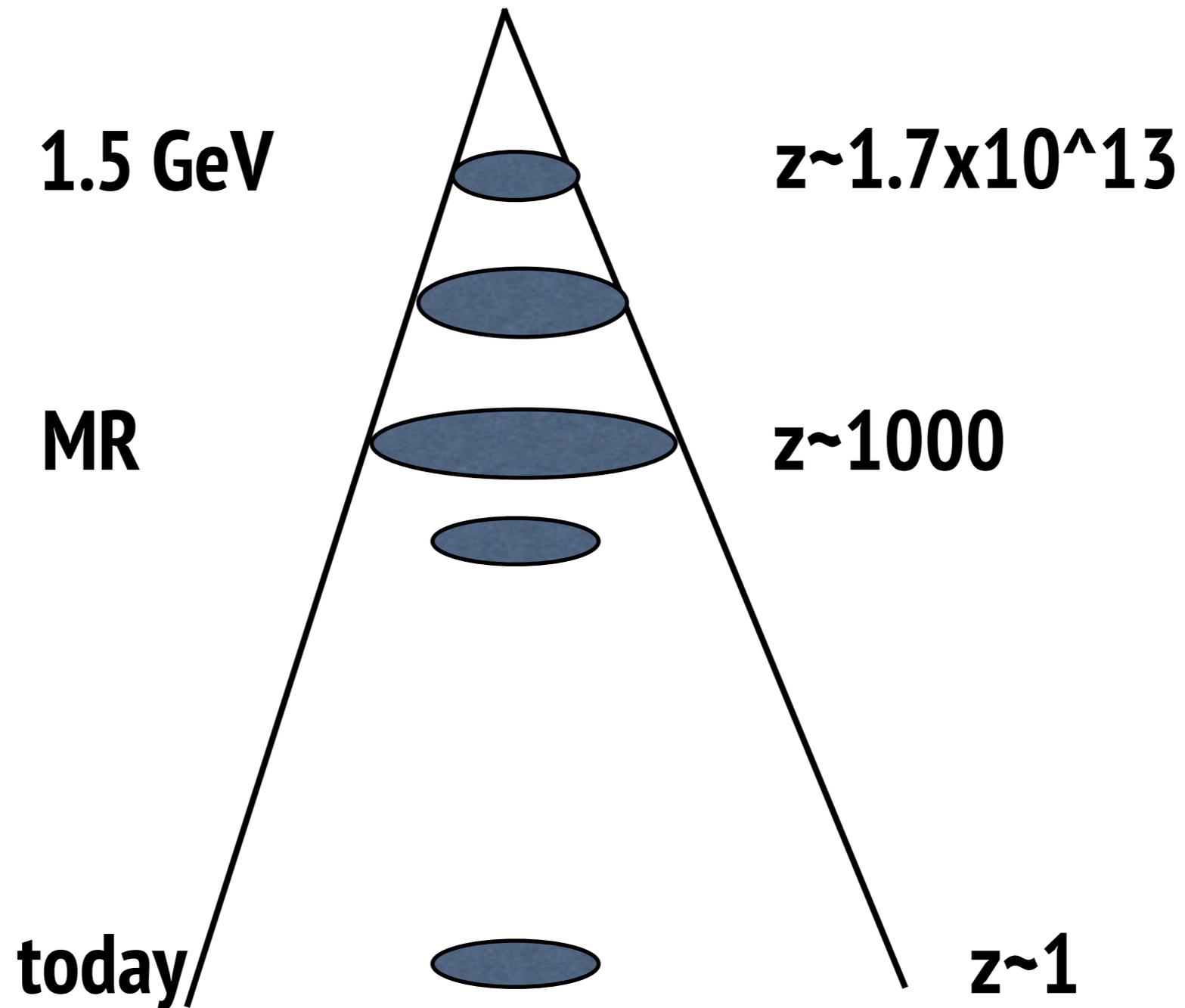
- Horizon scale at t1

$$L \sim 100 \text{ AU} \quad (\text{comoving})$$

Dark matter density, inhomogeneous at comoving mpc scales



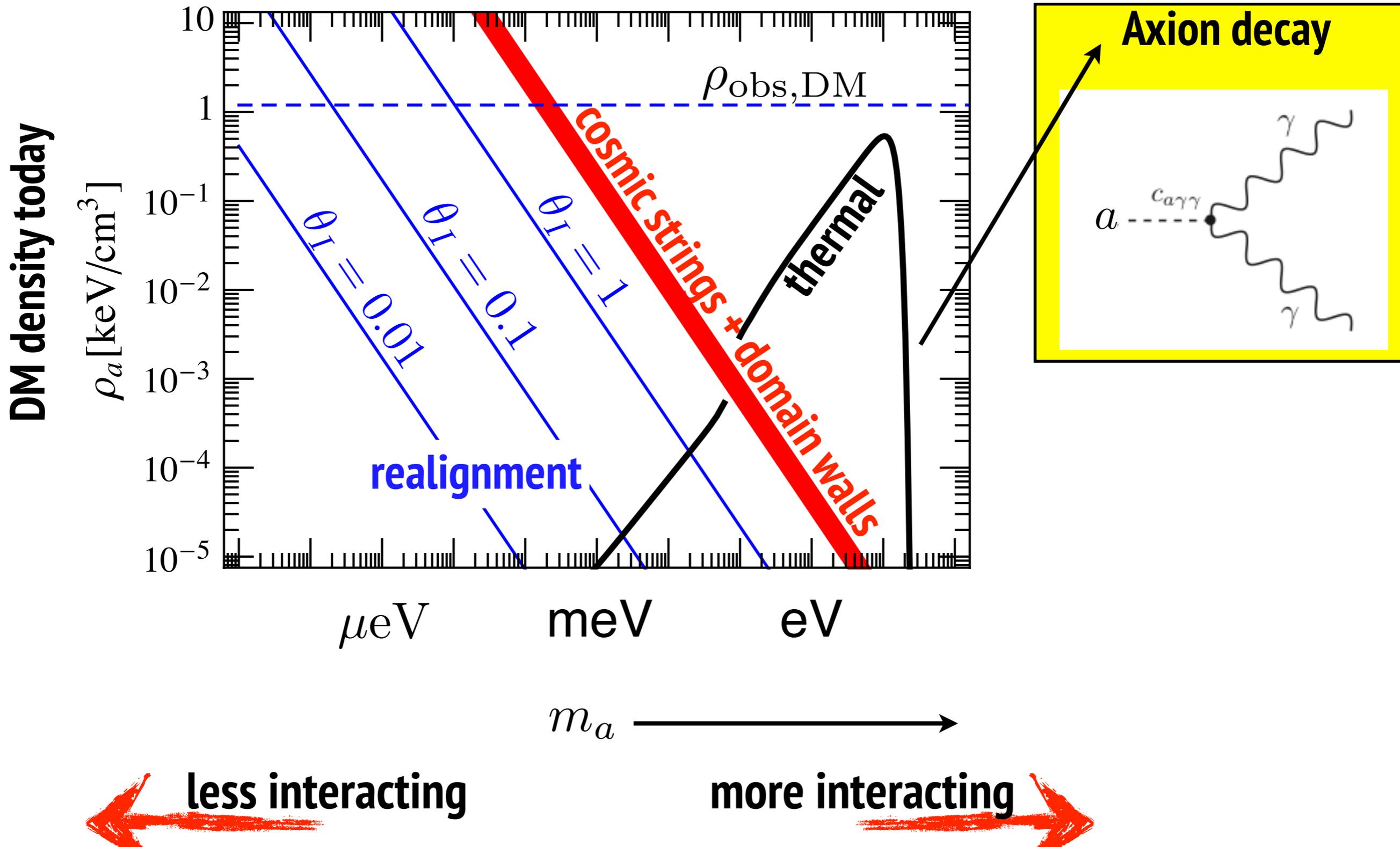
Minicluster size



They expand with the Universe until \sim Matter-radiation equality ($z \sim 1000$)

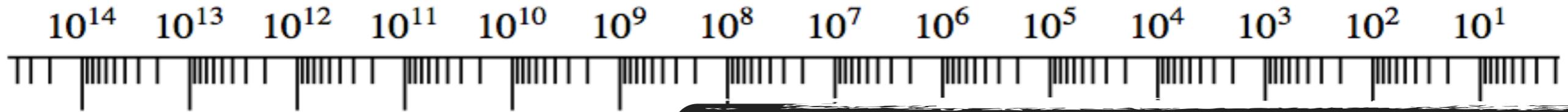
$L \sim \mathcal{O}(0.1) \text{A.U.}$

Axion DM, how much

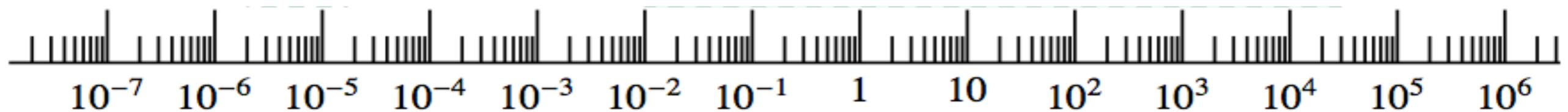
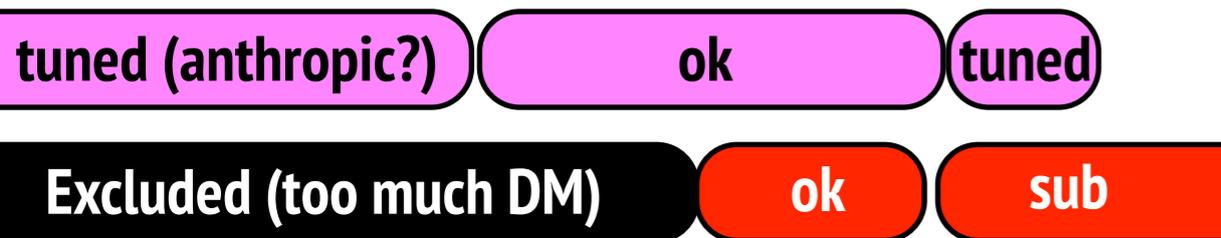


Axion dark matter

f_a [GeV]



- Axion DM scenarios



m_a [eV]

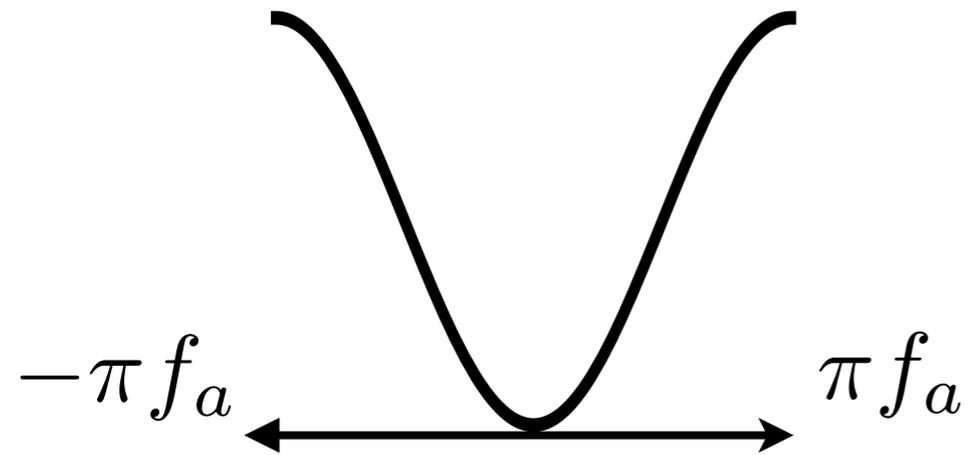
Initial conditions set by :

Inflation smooth

$$\Omega_{\text{aDM}} h^2 \simeq \theta_I^2 \left(\frac{80 \mu\text{eV}}{m_a} \right)^{1.19}$$

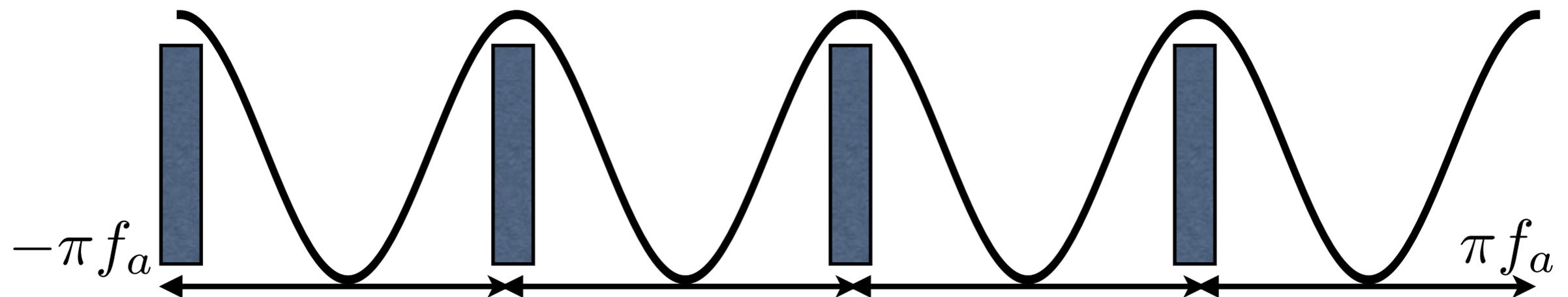
**Phase transition (N=1)
strings+unstable DW's**

SCENARIO I, N=1

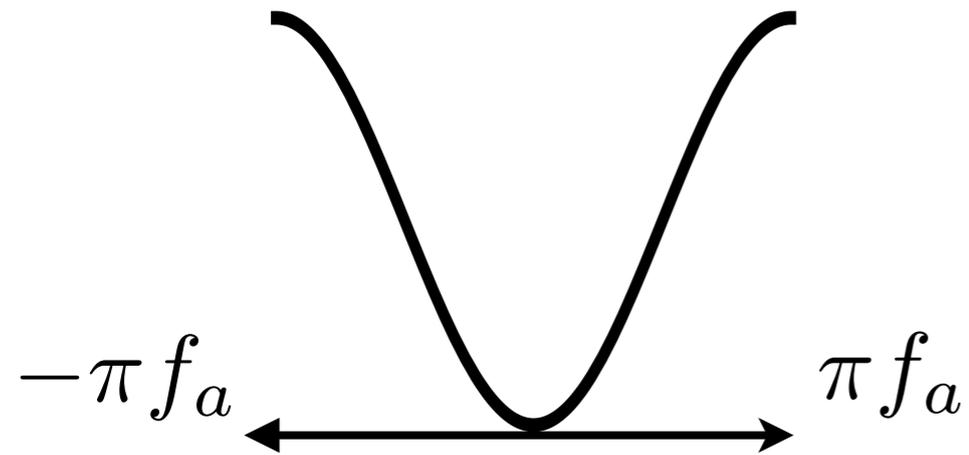


$$\frac{a}{f_a} = N\theta$$

SCENARIO I, N>1, Domain Walls stable -> cosmological disaster

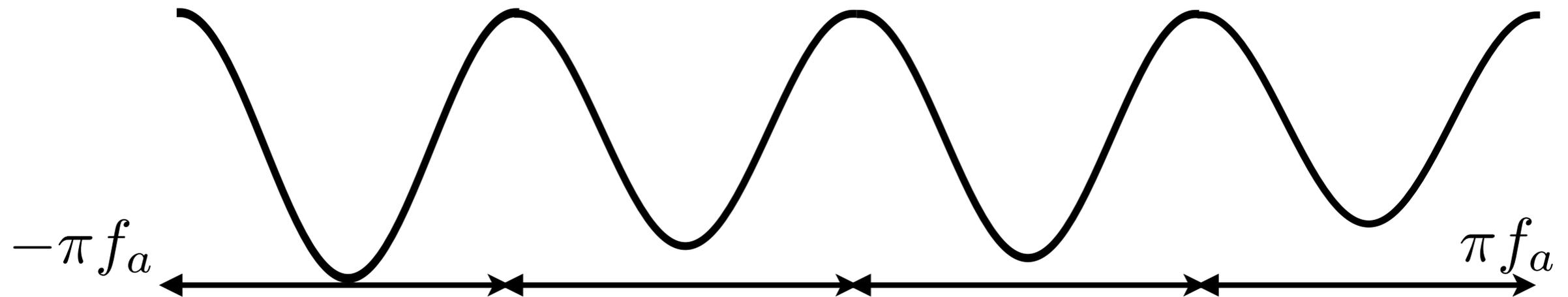


SCENARIO I, N=1



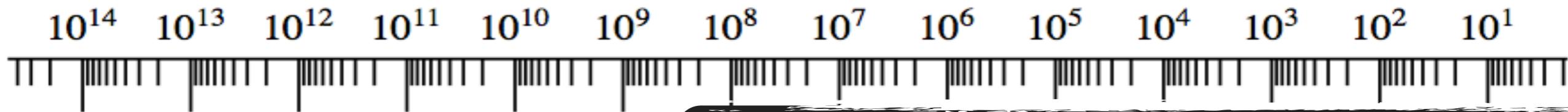
$$\frac{a}{f_a} = N\theta$$

SCENARIO I, N>1, break slightly degeneracy (but tuning...)

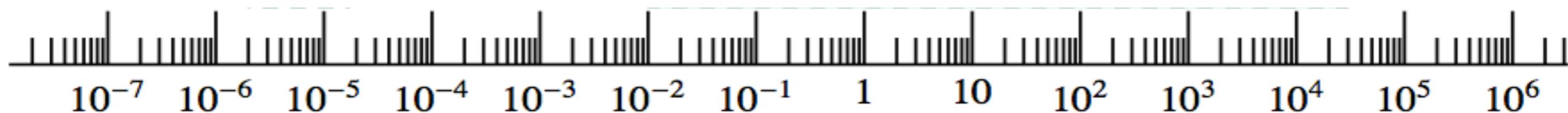
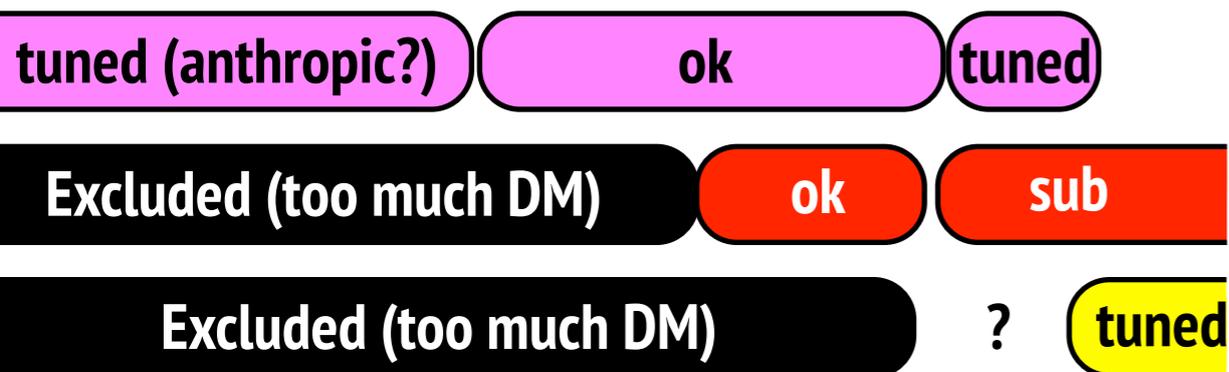


Axion dark matter

$f_a[\text{GeV}]$



- Axion DM scenarios



$m_a[\text{eV}]$

Initial conditions set by :

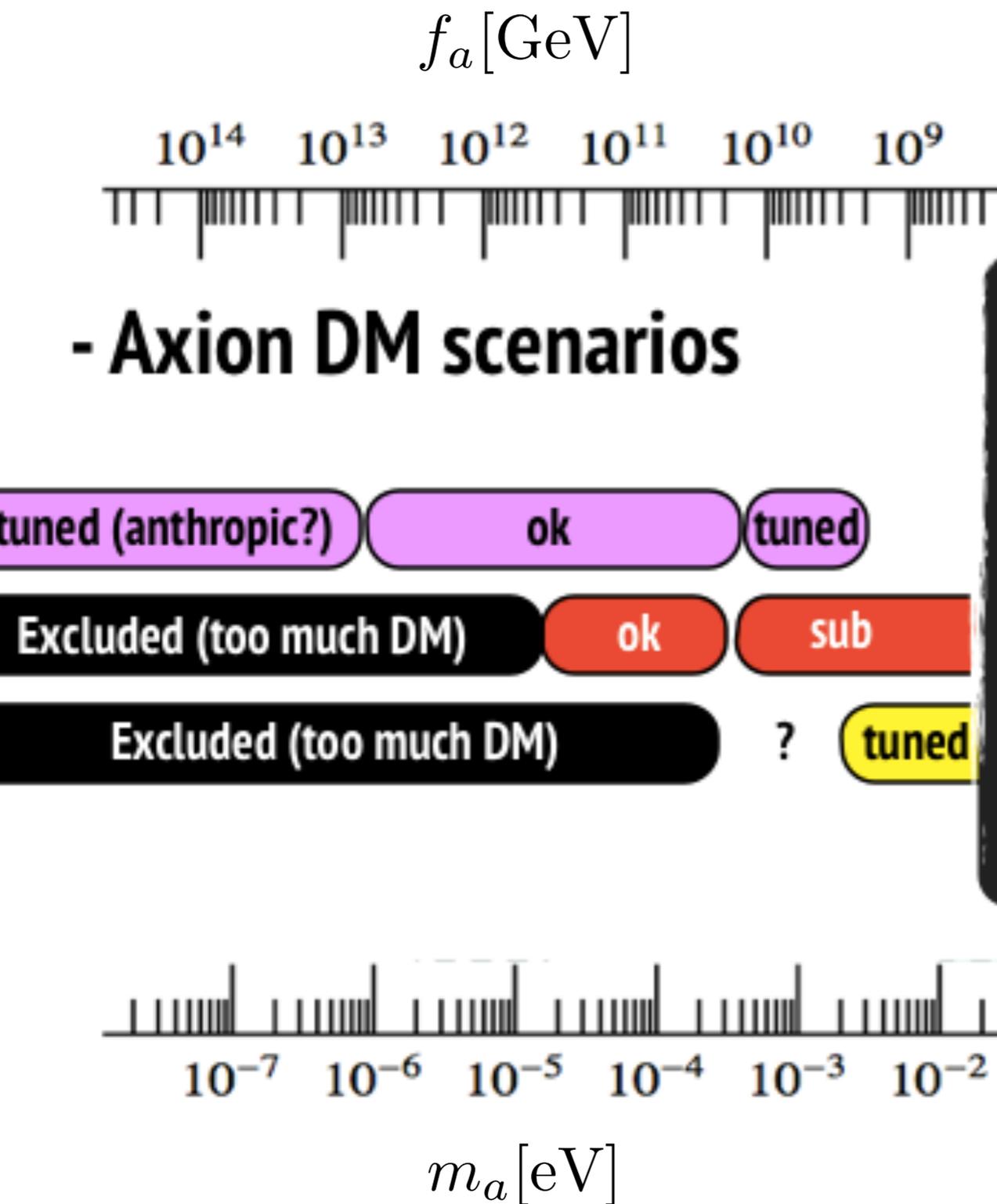
Inflation smooth

$$\Omega_{\text{aDM}} h^2 \simeq \theta_I^2 \left(\frac{80 \mu\text{eV}}{m_a} \right)^{1.19}$$

Phase transition (N=1)
strings+unstable DW's

Phase transition (N>1)
strings+long-lived DWs

Conclusions



- Some axion DM is inevitable
- DM ab. depends on i.c. and f_a
- Some scenarios “excluded”

- standard thermal history
- extrapolations
- isocurvature constraints

- $f_a \sim 10^{11}$ GeV favoured (?)
- larger f_a possible (anthropic)
- much do to!