

INTRODUCTION TO CHIRAL EFFECTIVE FIELD THEORY

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Outline

- Effective Field Theories
- QCD at Low Energies
 - ▶ QCD and chiral symmetry
 - ▶ Chiral EFT
 - ▶ Nuclear bound states and reactions
 - ▶ Example: nuclear EDMs
 - ▶ Renormalization issues
 - ▶ Outlook & summary

Reference:

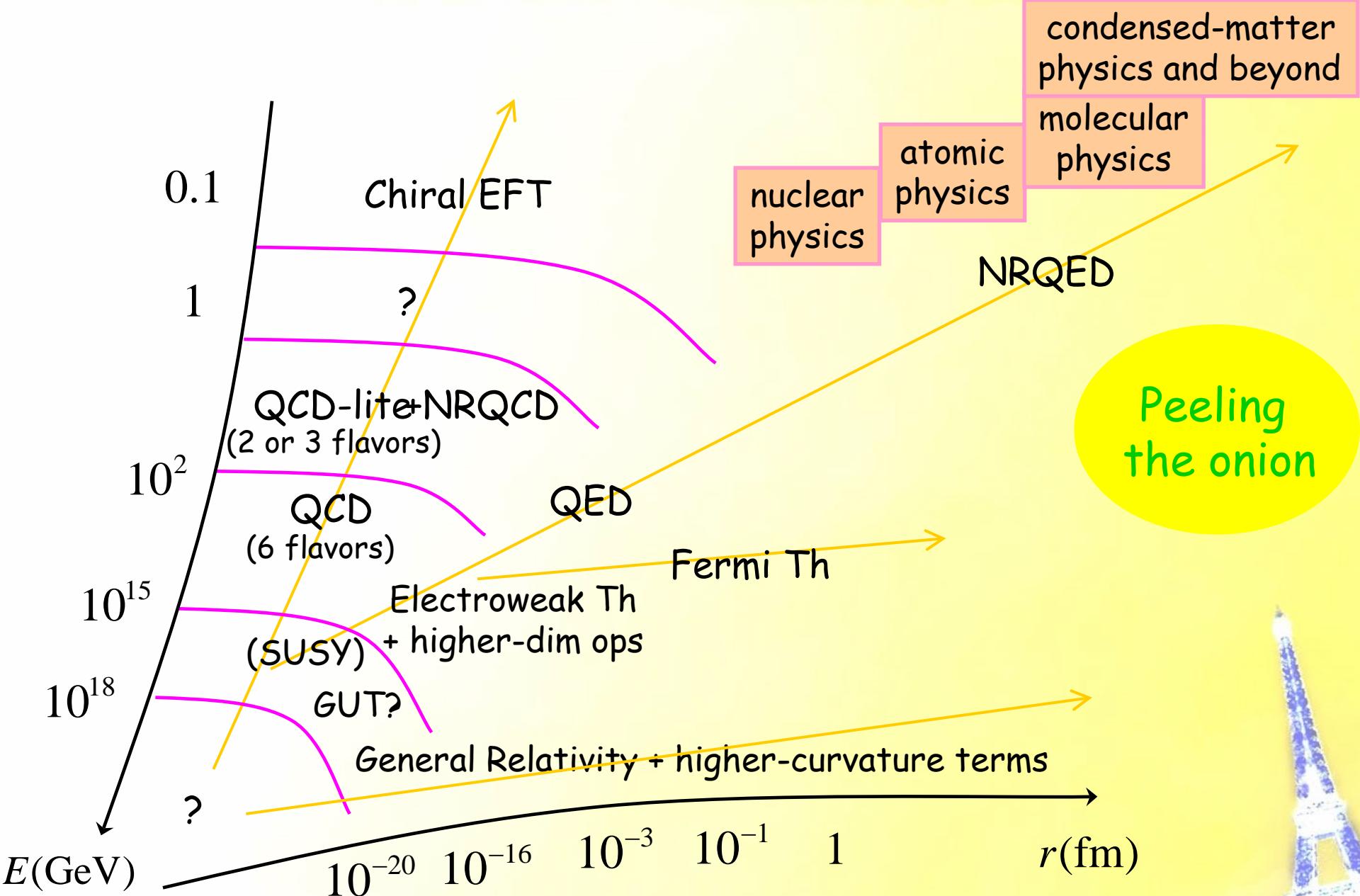
U. van Kolck,

Effective field theories of loosely bound nuclei,

in *The Euroschool on Exotic Beams, Vol. IV*

C. Scheidenberger and M. Pfützer (eds.), Springer, Berlin Heidelberg (2014)

Lect. Notes Phys. **879** (2014) 123



EFT at a few GeV = underlying theory for nuclear physics

d.o.f.s leptons: $l_f = \begin{pmatrix} l^+ \\ \nu \end{pmatrix}_f$ quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ photon: A_μ gluons: G_μ^a

symmetries: SO(3,1) global, U_e(1) gauge, SU_c(3) gauge

$$\begin{aligned} \mathcal{L}_{\text{und}} = & \sum_{f=1}^3 \bar{l}_f \left(i\cancel{D} + eQ_l \cancel{A} - m_f \right) l_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \bar{q} \left(i\cancel{D} + eQ_q \cancel{A} + g_s \cancel{G} \right) q - \frac{1}{2} \text{Tr} \left[G_{\mu\nu} G^{\mu\nu} \right] \\ & - \frac{1}{2} (m_u + m_d) \bar{q} q - \frac{1}{2} (m_u - m_d) \bar{q} \tau_3 q \\ & + \frac{m_u m_d}{m_u + m_d} \bar{\theta} \bar{q} i\gamma_5 q + \dots \end{aligned}$$

higher-dimension interactions:
suppressed by larger masses

unnaturally small T violation
(strong CP problem)

$$Q_l = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} Q_q = & \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \\ = & \frac{1+3\tau_3}{6} \end{aligned}$$

e.g.

$$G_F \propto 1/M_{W,Z}^2$$

$$\bar{\theta} \lesssim 10^{-9}$$

Focus on strong-interacting sector: four parameters

1) $m_u = m_d = 0, e = 0, \bar{\theta} = 0$

"chiral limit"

single, dimensionless parameter

$$\int d^4x \mathcal{L}_{\text{QCD}} = \int d^4x \left\{ \bar{q} (i\partial + g_s G) q - \frac{1}{2} \text{Tr } G^{\mu\nu} G_{\mu\nu} \right\}$$

invariant under scale transformations

but in

$$Z = \int DG \int D\bar{q} \int Dq \exp \left(i \int d^4x \mathcal{L}_{\text{QCD}} \right)$$

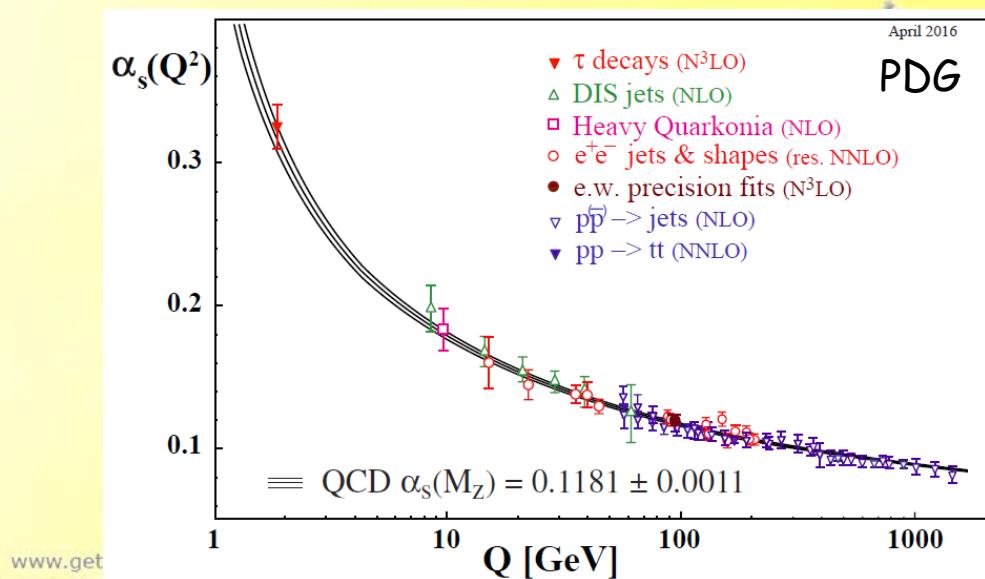
$$\begin{cases} x \rightarrow \lambda^{-1}x \\ q \rightarrow \lambda^{3/2}q \\ G \rightarrow \lambda G \end{cases}$$

scale invariance
"anomalously broken"
by dimensionful regulator

⇒ coupling runs

$$\alpha_s(Q \sim 1 \text{ GeV}) \sim 1$$

("dimensional transmutation")



Non-perturbative physics at $Q \sim 1$ GeV

Assumption 1: confinement

only colorless states ("hadrons") are asymptotic

Observation: (almost) all hadron masses $\gtrsim 1$ GeV

Assumption 2: naturalness

masses are determined by characteristic scale

$$\rightarrow M_{\text{QCD}} \sim 1 \text{ GeV}$$

Observation: pion mass $m_\pi \simeq 140$ MeV $\ll M_{\text{QCD}}$

breakdown of naturalness? NO!

"spontaneous breaking" of chiral symmetry

Why is the pion special?

$$\mathcal{L}_{QCD} = \bar{q}_L (i\partial + g_s G) q_L + \bar{q}_R (i\partial + g_s G) q_R - \frac{1}{2} \text{Tr} G^{\mu\nu} G_{\mu\nu}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{array}{c} \xrightarrow{\hspace{1cm}} \\[-1ex] \xleftarrow{\hspace{1cm}} \end{array} \frac{1-\gamma_5}{2} q \quad \begin{array}{c} \xrightarrow{\hspace{1cm}} \\[-1ex] \xleftarrow{\hspace{1cm}} \end{array} \frac{1+\gamma_5}{2} q$$

invariant under

$$q_{L(R)} \rightarrow \underbrace{\exp(i\alpha_{L(R)} \cdot \tau)}_{\equiv L(R)} q_{L(R)}$$

chiral symmetry

$$\text{SU}(2)_L \times \text{SU}(2)_R \sim \text{SO}(4)$$

broken by vacuum down to

$$q \rightarrow \exp(i\alpha_V \cdot \tau) q$$

isospin

$$\text{SU}(2)_V \sim \text{SO}(3)$$

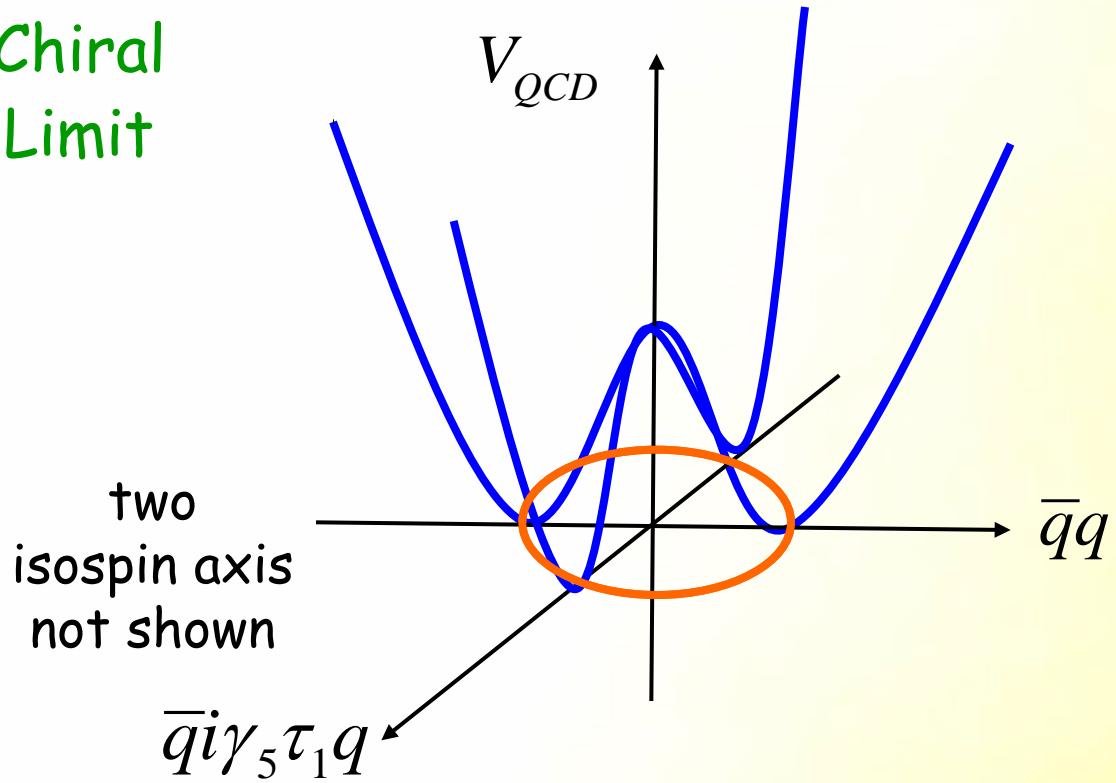
$$m_\sigma \gg m_\pi$$

$$m_{N_-} \gg m_{N_+}$$

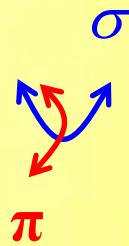
$$\alpha_{R/L} = \alpha_V \pm \alpha_A$$

(axial transformations broken)

Chiral Limit



chiral circle



pion decay constant (in chiral limit)

$$\mathcal{L}_{\text{EFT}} = \text{piece invariant under } \pi \rightarrow \pi + \varepsilon \quad [\text{function of } \overbrace{\partial_\mu \pi \text{ on chiral circle}}{\left(1 - \frac{\pi^2}{4f_\pi^2} + \dots\right) \partial_\mu \pi}]$$

2) $\textcolor{red}{m}_u \neq 0 \neq \textcolor{red}{m}_d$, $e = 0$, $\bar{\theta} = 0$

$$\mathcal{L}_{\text{QCD}} = \bar{q} \left(i\partial + g_s G \right) q - \frac{1}{2} \text{Tr} G^{\mu\nu} G_{\mu\nu}$$

v.K. '93

$$+ \frac{1}{2} (\textcolor{red}{m}_u + \textcolor{red}{m}_d) \underbrace{\bar{q}q}_{\text{4}^{\text{th}} \text{ component of SO(4) vector}} + \frac{1}{2} (\textcolor{red}{m}_u - \textcolor{red}{m}_d) \underbrace{\bar{q}\tau_3 q}_{\text{3}^{\text{rd}} \text{ component of SO(4) vector}} + \dots$$

4th component of SO(4) vector

$$S = (\bar{q}i\gamma_5 \tau q, \bar{q}q)$$

3rd component of SO(4) vector

$$P = (\bar{q}\tau q, \bar{q}i\gamma_5 q)$$

break

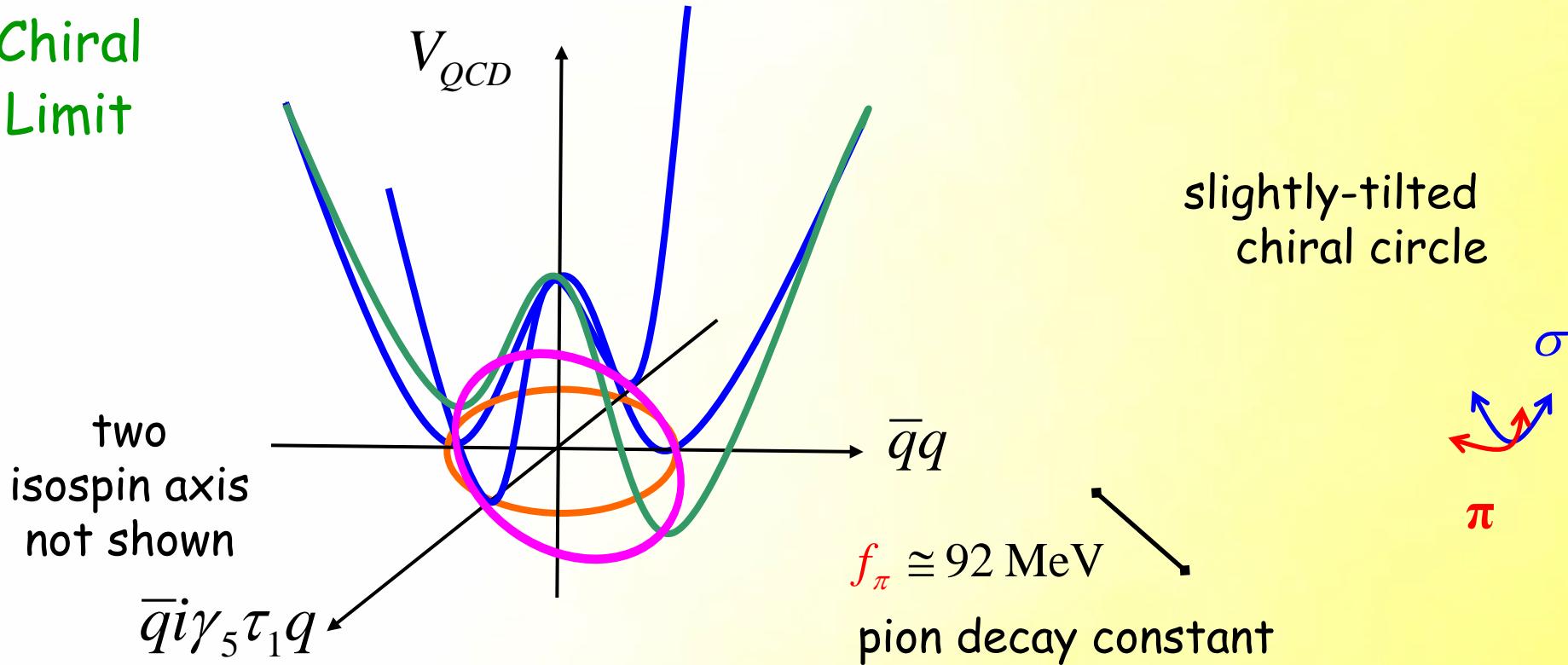
$$\text{SO}(4) \rightarrow \text{SO}(3)$$

(explicit chiral-symmetry breaking)

$$\rightarrow \text{U}(1)$$

(isospin violation)

Chiral Limit



$\mathcal{L}_{\text{EFT}} = \text{piece invariant under } \pi \rightarrow \pi + \varepsilon \text{ [function of } \partial_\mu \pi] \propto Q$

+ piece in $\bar{q}q$ direction [function of π explicitly] $\propto (m_u + m_d)$

+ isospin breaking $\propto (m_u - m_d)$

CHIRAL SYMMETRY \rightarrow WEAK PION INTERACTIONS

$$3) \ e \neq 0, \bar{\theta} = 0$$

Two types of interactions:

➤ "soft" photons - explicit d.o.f. in the EFT

$$D_\mu = \partial_\mu - ieQ_q A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

➤ "hard" photons - "integrated out" of EFT

v.K. '93

$$\mathcal{L}_{\text{und}} = \dots - e^2 \overline{q} Q_q \gamma_\mu q D^{\mu\nu} (\partial^2) \overline{q} Q_q \gamma_\nu q + \dots$$


34 comp of
antisymmetric tensor

$$F_\mu = \begin{pmatrix} \epsilon_{ijk} \bar{q} i \gamma_\mu \gamma_5 \tau_k q & \bar{q} i \gamma_\mu \tau_j q \\ -\bar{q} i \gamma_\mu \tau_i q & 0 \end{pmatrix}$$

breaks SO(4) (and SO(3) in particular) \rightarrow U(1)



$\mathcal{L}_{\text{EFT}} =$ soft photons $\propto e$
+ further isospin breaking $\propto \alpha / 4\pi$

4) $\bar{\theta} \neq 0$

$$\mathcal{L}_{\text{QCD}} = \dots + \frac{m_u m_d}{m_u + m_d} \bar{\theta} \underbrace{\bar{q} i \gamma_5 q}_{\text{ }} + \dots$$

4th component of SO(4) vector $P = (\bar{q} \tau q, \bar{q} i \gamma_5 q)$

T violation linked to isospin violation: in EFT, combination is

$$-\frac{1}{2} (m_u - m_d) P_3 + \frac{m_u m_d}{m_u + m_d} \bar{\theta} P_4$$

Hockings,
Mereghetti + v.K., '10

5) continue with higher-order operators,

e.g. T-violating quark EDM and color-EDM

P-violating four-quark operators

De Vries, Mereghetti,
Timmermans + v.K., '12

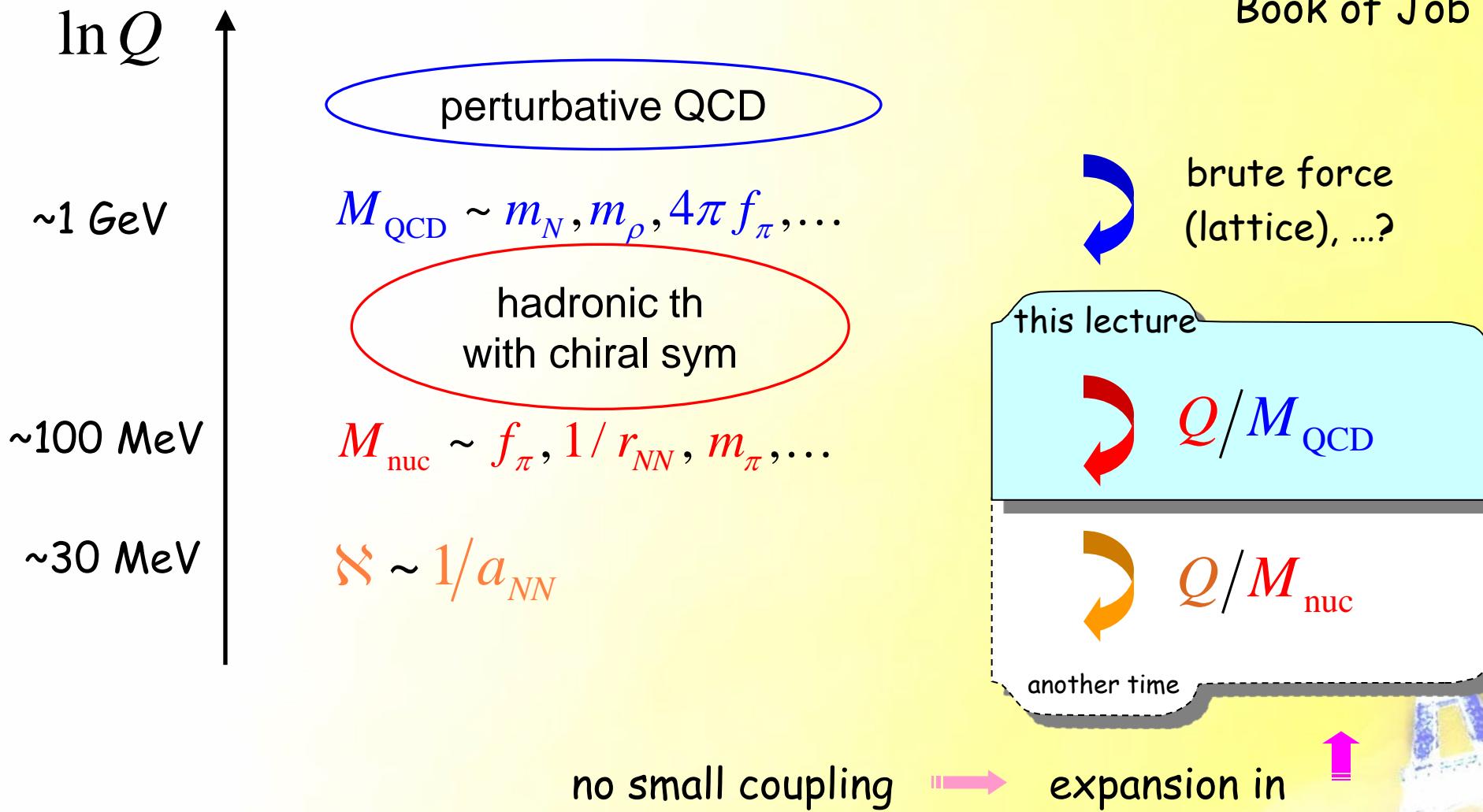
...

Kaplan + Savage '96

Zhu, Maekawa, Holstein, Musolf + v.K. '02

Nuclear physics scales

"His scales are His pride",
Book of Job



Chiral EFT

$$Q \sim m_\pi \ll M_{\text{QCD}}$$

- d.o.f.s: nucleons, pions, deltas, Roper? ($m_\Delta - m_N \sim 2m_\pi$, $m_{N'} - m_N \sim 3m_\pi$)

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ -i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix} \quad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix} \quad N' = \begin{pmatrix} p' \\ n' \end{pmatrix}$$

- symmetries: Lorentz, ~~P, T, chiral~~

Weinberg '68

Non-linear realization of chiral symmetry

Callan, Coleman, Wess + Zumino '69

chiral invariants

(chiral)
covariant
derivatives

pion	$\mathbf{D}_\mu \equiv \left(\frac{\partial_\mu \pi}{f_\pi} \right) \left(1 - \frac{\pi^2}{4f_\pi^2} + \dots \right)$
fermions	$\mathcal{D}_\mu \equiv \left(\partial_\mu - \frac{i}{2} \boldsymbol{\tau} \cdot \mathbf{E}_\mu \right)$

+ S_4 's, P_3 's, F_{34} 's

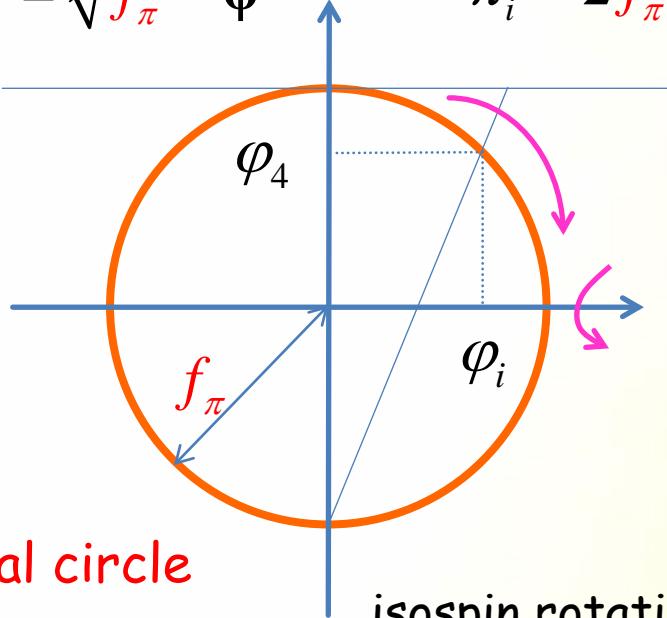
$$m_\pi^2 = \mathcal{O}\left((m_u + m_d)M_{\text{QCD}}\right)$$

$$\Rightarrow m_u + m_d = \mathcal{O}\left(\frac{m_\pi^2}{M_{\text{QCD}}}\right)$$

$$\mathbf{E}_\mu \equiv \frac{\pi}{f_\pi} \times \mathbf{D}_\mu$$

Open parentheses on parametrizations

$$\varphi_4 = \sqrt{f_\pi^2 - \Phi^2}$$



$$\begin{cases} \Phi \rightarrow \Phi + i\mathbf{a}_V \times \Phi \\ \varphi_4 \rightarrow \varphi_4 \end{cases}$$

$$\Rightarrow \pi \rightarrow \pi + i\mathbf{a}_V \times \pi$$

$$\Rightarrow D_\mu \rightarrow D_\mu + i\mathbf{a}_V \times D_\mu$$

rotates

$$\pi_i = 2f_\pi \frac{\varphi_i}{f_\pi + \varphi_4}$$

stereographic projection

Weinberg '68

axial rotation

$$\begin{cases} \Phi \rightarrow \Phi - \mathbf{a}_A \cdot \varphi_4 \\ \varphi_4 \rightarrow \varphi_4 + \mathbf{a}_A \cdot \Phi \end{cases}$$

$$\Rightarrow \pi \rightarrow \left(1 - \frac{\mathbf{a}_A \cdot \pi}{2f_\pi} \right) \pi - f_\pi \left(1 - \frac{\pi^2}{4f_\pi^2} \right) \mathbf{a}_A$$

nonlinear!

$$\Rightarrow D_\mu \rightarrow D_\mu - \left(\mathbf{a}_A \times \frac{\pi}{2f_\pi} \right) \times D_\mu$$

$$\text{rotates with } \mathbf{a}_V \rightarrow i \left(\mathbf{a}_A \times \frac{\pi}{2f_\pi} \right)$$

analogous for \mathcal{D}_μ

isospin symmetry of covariant objects ensures chiral invariance

$$\tilde{\Sigma} \equiv \varphi_4 + i\boldsymbol{\tau} \cdot \boldsymbol{\varphi} = f_\pi \exp\left(i \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{f_\pi}\right) \equiv f_\pi \Sigma$$

"sigma-model
parametrization"

"exponential
parametrization"

Choice of fields does
not affect observables:
all parametrizations
are equivalent

e.g. $\Sigma^\dagger \partial_\mu \Sigma = 2i\boldsymbol{\tau} \cdot \boldsymbol{D}_\mu$

$$\Sigma \rightarrow \exp\left(\frac{i}{2} \boldsymbol{\alpha}_L \cdot \boldsymbol{\tau}\right) \Sigma \exp\left(\frac{i}{2} \boldsymbol{\alpha}_R \cdot \boldsymbol{\tau}\right)$$

invariants constructed with Tr

e.g. $\text{Tr}[(\partial^\mu \Sigma^\dagger) \partial_\mu \Sigma] = 8 \boldsymbol{D}^\mu \cdot \boldsymbol{D}_\mu$

less intuitive but
easier to generalize to
 $SU(3)_L \times SU(3)_R$

chiral-breaking terms --- an example: common quark mass

- stereographic projection 4th component of SO(4) vector

$$S = (\bar{q}i\gamma_5 \tau q, \bar{q}q) \quad \Rightarrow \quad S = \left(\left(1 + \frac{\pi^2}{4f_\pi^2} \right)^{-1} \frac{\pi}{f_\pi}, 1 - \underbrace{\left(1 + \frac{\pi^2}{4f_\pi^2} \right)^{-1} \frac{\pi^2}{2f_\pi^2}}_{= 1 - \frac{\pi^2}{2f_\pi^2} + \frac{\pi^4}{8f_\pi^4} + \dots} \right)$$

pion mass term

- exponential parametrization

$$\frac{m_u + m_d}{2} \bar{q}q \rightarrow \frac{1}{2} [\bar{q}_L \bar{M}(x) q_R + \bar{q}_R \bar{M}^\dagger(x) q_L]$$

invariant if $\bar{M}(x) \rightarrow L \bar{M}(x) R^\dagger$

"spurion field"

$$\Rightarrow \frac{1}{2} \text{Tr} [\Sigma^\dagger \bar{M} + \bar{M}^\dagger \Sigma] = \frac{m_u + m_d}{2} \left(\frac{\pi^2}{f_\pi^2} + \dots \right)$$

pion mass term

chiral symmetry
broken explicitly
just as in QCD

Close parentheses

Schematically,

$$\mathcal{L}_{\text{EFT}} = \sum_{\{n,p,f\}} c_{\{n,p,f\}} \left(\frac{\mathbf{D}, \mathcal{D}, m_R - m_N}{M_{\text{QCD}}} \right)^n \left(\frac{m_\pi^2}{M_{\text{QCD}}^2} \frac{\pi^2}{f_\pi^2} \right)^{p/2} \left(\frac{\psi^+ \psi}{f_\pi^2 M_{\text{QCD}}} \right)^{f/2} f_\pi^2 M_{\text{QCD}}^2$$

{ calculated from QCD: lattice, ...
fitted to data

$$= \mathcal{O}(1) \\ = \mathcal{O}\left(\epsilon, \frac{\alpha}{4\pi}\right)$$

isospin conserving

isospin breaking

NDA: naïve
dimensional
analysis

$$= \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)}$$

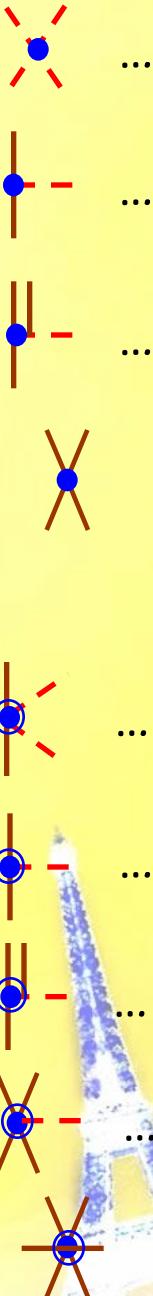
$$\Delta \equiv n + p + \frac{f}{2} - 2 \equiv d + \frac{f}{2} - 2 \geq 0$$

"chiral index"

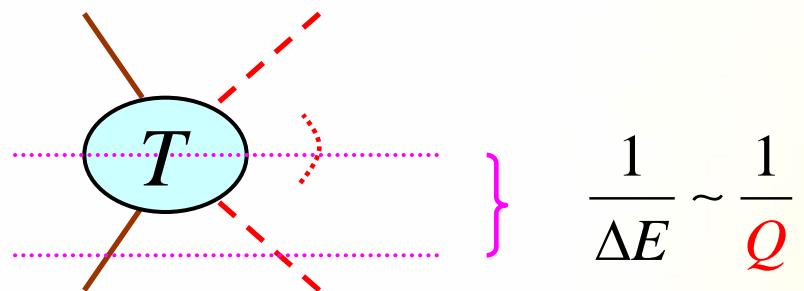
chiral symmetry

$$\begin{aligned}
\mathcal{L}^{(0)} &= \frac{1}{2} (\partial_\mu \boldsymbol{\pi})^2 \left(1 - \frac{\boldsymbol{\pi}^2}{2 f_\pi^2} + \dots \right) - \frac{1}{2} \mathbf{m}_\pi^2 \boldsymbol{\pi}^2 \left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \dots \right) \\
&\quad + N^+ \left[i \partial_0 - \frac{1}{4 f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) + \dots \right] N + \frac{g_A}{2 f_\pi} N^+ \boldsymbol{\tau} \vec{\sigma} N \cdot (\vec{\nabla} \boldsymbol{\pi}) \left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \dots \right) \\
&\quad + \Delta^+ \left[i \partial_0 - (\mathbf{m}_\Delta - \mathbf{m}_N) + \dots \right] \Delta + \dots + \frac{h_A}{2 f_\pi} \left(N^+ \mathbf{T} \vec{S} \Delta + \text{H.c.} \right) \cdot (\vec{\nabla} \boldsymbol{\pi}) (1 + \dots) \\
&\quad - C_S \left(N^+ N \right)^2 - C_T \left(N^+ \vec{\sigma} N \right)^2 \\
\mathcal{L}^{(1)} &= N^+ \left[\frac{1}{2 \mathbf{m}_N} \left(\vec{\nabla} + \frac{1}{4 f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \vec{\nabla} \boldsymbol{\pi}) + \dots \right)^2 + \frac{1}{2} (\mathbf{m}_p - \mathbf{m}_n) \left(\tau_3 - \frac{1}{2 f_\pi^2} \pi_3 \boldsymbol{\pi} \cdot \boldsymbol{\tau} + \dots \right) \right] N \\
&\quad + \frac{1}{f_\pi^2} N^+ \left[b_2 (\partial_0 \boldsymbol{\pi})^2 - b_3 (\vec{\nabla} \boldsymbol{\pi})^2 - 2 b_1 \mathbf{m}_\pi^2 \boldsymbol{\pi}^2 + i b_4 \epsilon_{ijk} \epsilon_{abc} \sigma_k \tau_c (\partial_i \pi_b) (\partial_j \pi_c) \right] N + \dots \\
&\quad - \frac{g_A}{4 \mathbf{m}_N f_\pi} \left[i N^+ \boldsymbol{\tau} \vec{\sigma} \cdot \vec{\nabla} N + \text{H.c.} \right] \cdot (\partial_0 \boldsymbol{\pi}) (1 + \dots) \\
&\quad - \frac{h_A}{4 \mathbf{m}_N f_\pi} \left[i \Delta^+ \mathbf{T} \vec{S} \cdot \vec{\nabla} N + \text{H.c.} \right] \cdot (\partial_0 \boldsymbol{\pi}) (1 + \dots) \\
&\quad + \frac{d}{f_\pi} N^+ N N^+ \boldsymbol{\tau} \vec{\sigma} N \cdot (\vec{\nabla} \boldsymbol{\pi}) (1 + \dots) \\
&\quad - E \left(N^+ N \right)^3 \\
\mathcal{L}^{(2)} &= \dots
\end{aligned}$$

*Form of pion interactions
determined by
chiral symmetry*



$A=0, 1$: chiral perturbation theory



$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$

$$\sim \sum_{\nu} c_{\nu} \left(\frac{Q}{M_{\text{QCD}}} \right)^{\nu} F_{\nu} \left(\frac{Q}{m_{\pi}} \right)$$

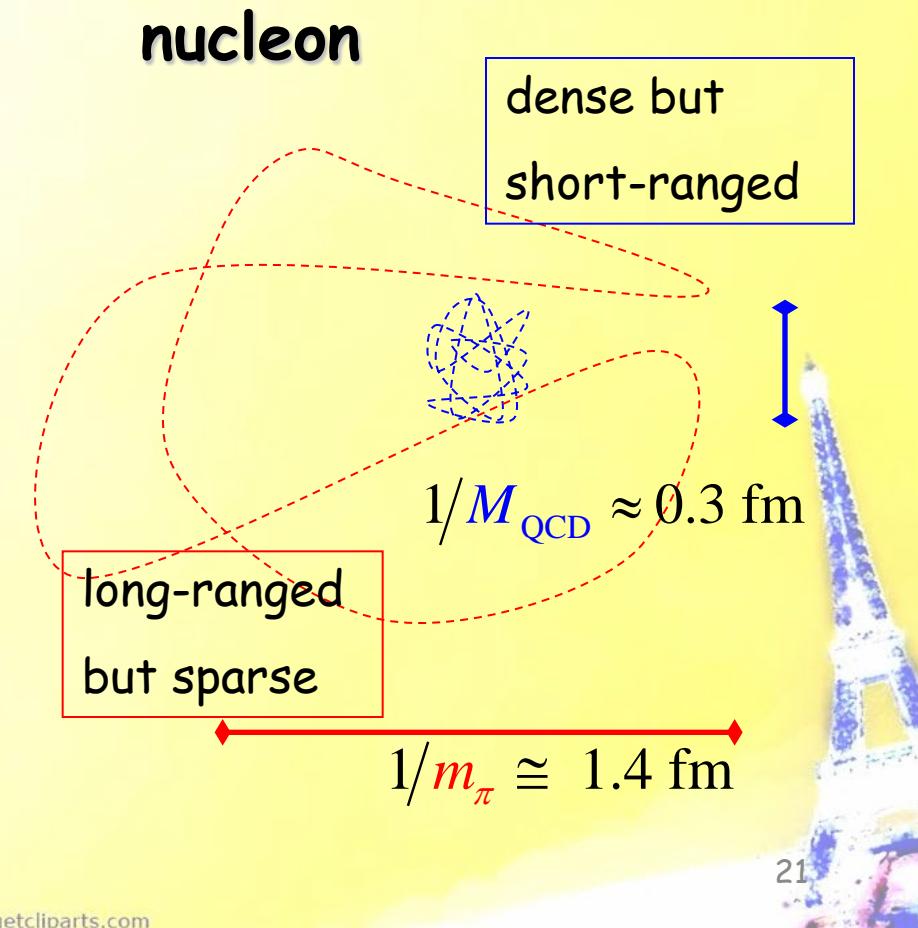
$$\nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\min} = 2 - A$$

loops # vertices of type i

expansion in

$$\frac{Q}{M_{\text{QCD}}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_{\rho}, \dots & \text{multipole} \\ Q/4\pi f_{\pi} & \text{pion loop} \end{cases}$$

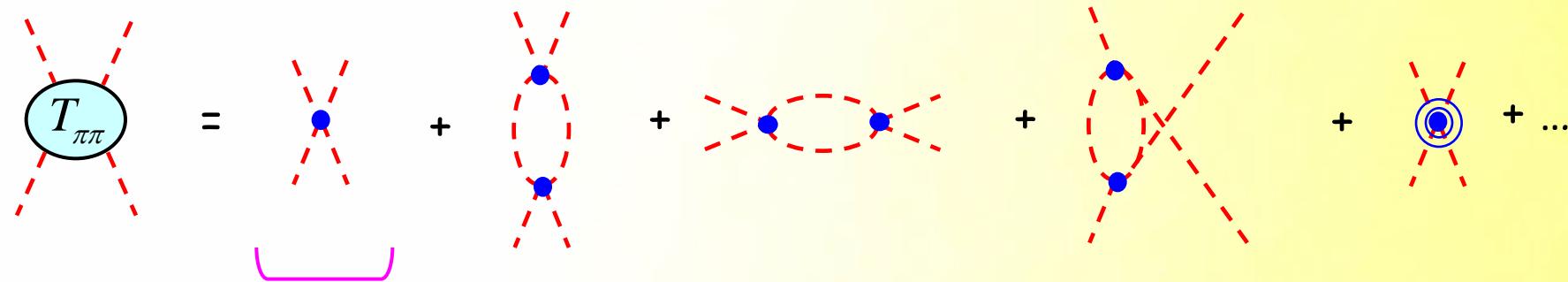
Gasser, Sainio + Švarc '87
Bernard, Kaiser + Meißner '90
Jenkins + Manohar '91
...



Analogous to NRQED...

Weinberg '79

Gasser + Leutwyler '84



current
algebra

Weinberg '66

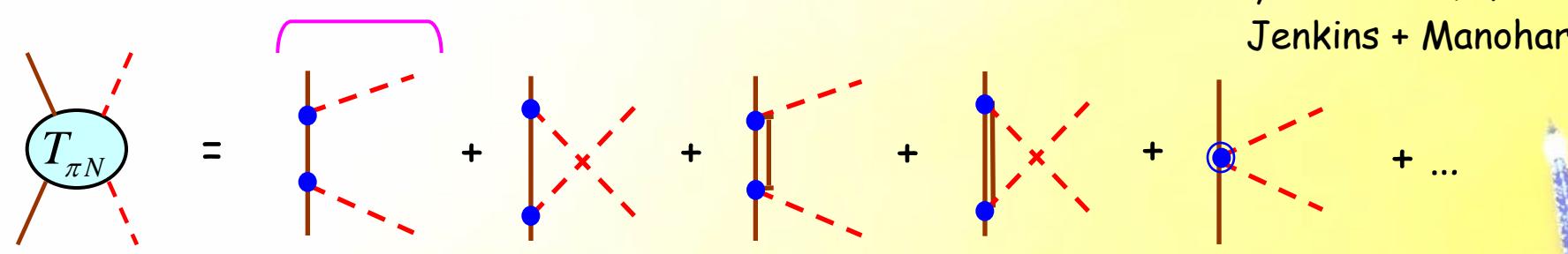
...

Gasser, Sainio + Švarc '87

Bernard, Kaiser + Meißner '90

Jenkins + Manohar '91

...



etc.

N.B. For $|E - (m_\Delta - m_N)| \lesssim \mathcal{O}\left(\frac{Q^3}{M_{\text{QCD}}^2}\right)$: resummation necessary

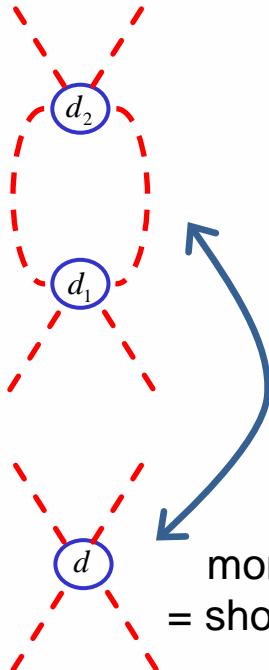
Phillips + Pascalutsa '02

Long + v.K. '08

$\Rightarrow |E - (m_{N'} - m_N)| = \mathcal{O}(Q)$: Roper important

Long + v.K. '09

NDA



momenta ~ cutoff
= short-range physics

$$\sim \frac{Q^4}{(4\pi)^2} \frac{Q^{d_1+d_2}}{Q^4} c_{d_1}(\Lambda) c_{d_2}(\Lambda)$$

$$\sim \frac{\Lambda^{d_1+d_2-d}}{(4\pi)^2} c_{d_1}(\Lambda) c_{d_2}(\Lambda) Q^d + \dots$$

$$\sim c_d(\Lambda)$$

$$d_1 = d_2 = d$$



$$c_d(\Lambda) \sim \frac{(4\pi)^2}{\Lambda^d}$$

naturalness



$$c_d(M_{\text{QCD}}) \sim \frac{(4\pi)^2}{M_{\text{QCD}}^d}$$

arbitrary
diagram

- fermions
- more loops, vertices
- other interactions

naïve dimensional analysis
(NDA)

(perturbative renormalization)

number of fields
in operator



$$c_i = \mathcal{O}\left(\frac{(4\pi)^{N-2}}{M_{\text{QCD}}^{D-4}} c_i^{\text{red}}\right)$$

Georgi + Manohar '86

dimension of
operator

reduced
coupling

$$c_i^{\text{red}} = \mathcal{O}\left(\left(g^{\text{red}}\right)^\#\right)$$

reduced
underlying theory
parameter

insertions

Example

$$\mathcal{L}_{\text{QCD}} = \dots + \frac{1}{2} (\textcolor{red}{m}_u + m_d) \bar{q}q + \dots$$

$$(\textcolor{red}{m}_u + m_d)^{\text{red}} = \frac{\textcolor{blue}{M}_{\text{QCD}}^{3-4}}{(4\pi)^{2-2}} (\textcolor{red}{m}_u + m_d) = \frac{m_u + m_d}{\textcolor{blue}{M}_{\text{QCD}}}$$

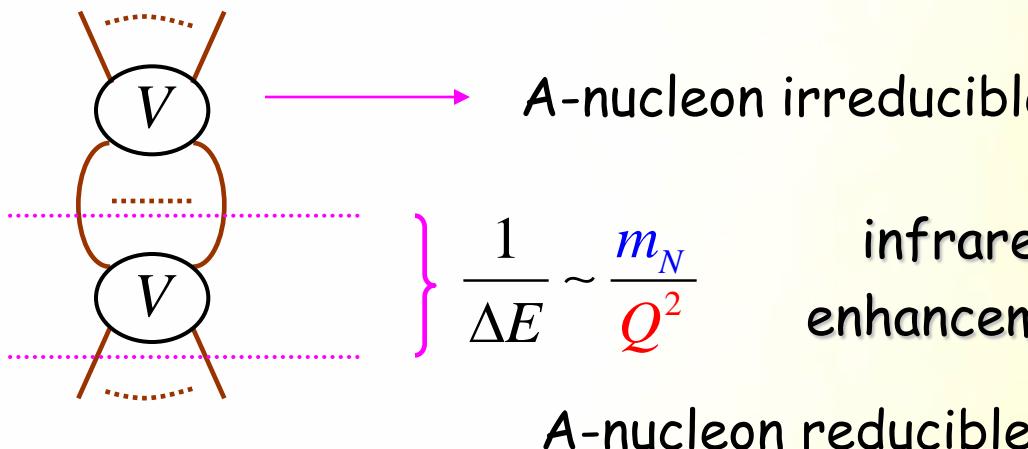
$$\mathcal{L}_{\text{EFT}} = \dots - \frac{1}{2} \textcolor{red}{m}_\pi^2 \boldsymbol{\pi}^2 + \dots$$

$$(\textcolor{red}{m}_\pi^2)^{\text{red}} = \frac{\textcolor{blue}{M}_{\text{QCD}}^{2-4}}{(4\pi)^{2-2}} \textcolor{red}{m}_\pi^2 = \frac{\textcolor{red}{m}_\pi^2}{\textcolor{blue}{M}_{\text{QCD}}^2}$$

$$(\textcolor{red}{m}_\pi^2)^{\text{red}} = \mathcal{O}\left((\textcolor{red}{m}_u + m_d)^{\text{red}}\right) \Rightarrow \textcolor{red}{m}_\pi^2 = \mathcal{O}\left((\textcolor{red}{m}_u + m_d) \textcolor{blue}{M}_{\text{QCD}}\right)$$

$A \geq 2$: resummed chiral perturbation theory

Weinberg '90, '91



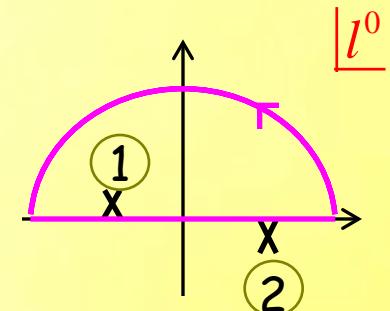
$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$

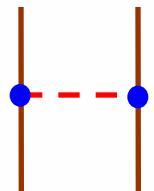
infrared
enhancement!

A-nucleon reducible

e.g.

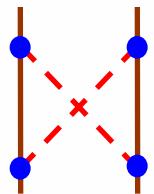
$$\begin{aligned}
 & \simeq i \int \frac{d^4 l}{(2\pi)^4} V \frac{1}{l^0 + k^2/m_N - l^2/m_N - i\varepsilon} \frac{1}{-l^0 + k^2/m_N - l^2/m_N - i\varepsilon} V \\
 & = \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \dots \quad \sim \mathcal{O}\left(\underbrace{\frac{m_N Q}{4\pi}}_{\text{instead of }} V^2\right) \\
 & E = \frac{k^2}{m_N}
 \end{aligned}$$



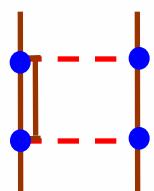


$$\sim i \left(\frac{g_A}{2 f_\pi} \right)^2 \left(\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right) \frac{(S_{12}(\hat{q}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2)}{3} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim \frac{1}{f_\pi^2} \quad \text{tensor force}$$

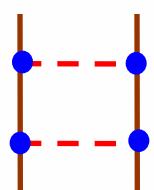
$S_{12}(\hat{q}) = 3 \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$



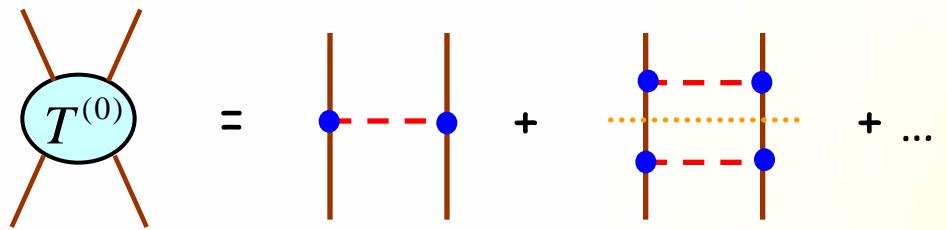
$$\sim \frac{1}{f_\pi^4} \frac{Q^3}{(4\pi)^2} \frac{1}{Q} \frac{1}{Q} \frac{Q^2}{Q^2} \frac{Q^2}{Q} \sim \frac{1}{f_\pi^2} \frac{Q^2}{(4\pi f_\pi)^2} \underbrace{= \mathcal{O}\left(\frac{Q^2}{M_{QCD}^2}\right)}$$



$$\sim \frac{1}{f_\pi^4} \frac{Q^3}{(4\pi)^2} \frac{1}{m_\Delta - m_N} \frac{1}{Q} \frac{Q^2}{Q^2} \frac{Q^2}{Q} \sim \frac{1}{f_\pi^2} \frac{\overbrace{Q^2}}{(4\pi f_\pi)^2} \frac{Q}{\underbrace{m_\Delta - m_N}_{= \mathcal{O}(1)}}$$



$$\sim \frac{1}{f_\pi^4} \frac{Q^3}{4\pi} \frac{m_N}{Q^2} \frac{Q^2}{Q^2} \frac{Q^2}{Q^2} \sim \frac{1}{f_\pi^2} \frac{\cancel{m_N}}{\cancel{4\pi f_\pi}} \frac{Q}{\cancel{f_\pi}} \sim \frac{1}{f_\pi^2} \frac{Q}{M_{NN}} \equiv \frac{1}{M_{NN}} = \mathcal{O}(1) \text{ for } Q \sim M_{NN}$$

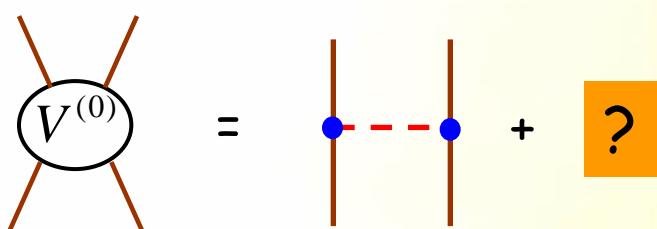
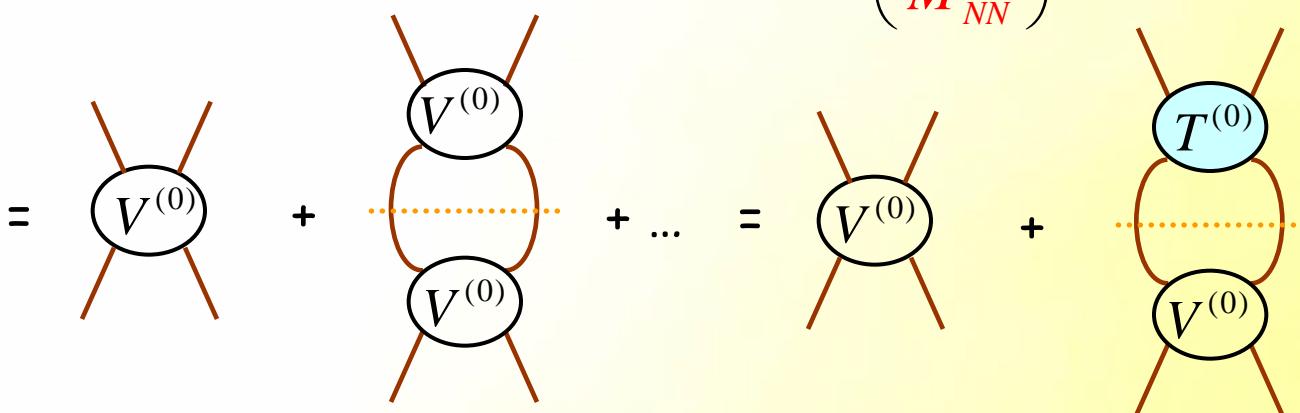


bound state at
 $Q \sim M_{NN}$ $-E \sim \frac{Q^2}{m_N} \sim \frac{M_{NN}^2}{M_{QCD}}$

$$\sim \frac{1}{f_\pi^2} \left\{ 1 + \mathcal{O}\left(\frac{Q}{M_{NN}}\right) + \dots \right\} \sim \frac{1}{f_\pi^2} \frac{1}{1 - \mathcal{O}\left(\frac{Q}{M_{NN}}\right)}$$

$$M_{\text{nuc}} = M_{NN} \sim \frac{4\pi f_\pi}{m_N} f_\pi \approx f_\pi$$

Nuclear scale
arises naturally
from
chiral symmetry



Is 1PE all there is in leading order?
That is, are observables cutoff
independent with 1PE alone?

Issue: relative importance of pion exchange and short-range interactions

$\sim i \left(\frac{g_A}{2 f_\pi} \right)^2 \left(\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right) \frac{(S_{12}(\hat{q}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2)}{3} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim \frac{4\pi}{m_N M_{NN}}$

$$\begin{cases} V(r) = \left(\frac{g_A}{2 f_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(-\delta^{(3)}(\vec{r}) + \frac{m_\pi^2}{4\pi r} e^{-m_\pi r} \right) & S=0 \\ V(r) = \left(\frac{g_A}{2 f_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \frac{1}{3} \left(\delta^{(3)}(r) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r} \right) + \frac{m_\pi^2}{4\pi r} \left(\frac{1}{(m_\pi r)^2} + \frac{1}{m_\pi r} + \frac{1}{3} \right) e^{-m_\pi r} \langle S_{12}(\hat{r}) \rangle \right\} & S=1 \end{cases}$$

much more singular --and complicated!-- than

$\sim \frac{i e^2}{(\vec{p} - \vec{p}')^2 - i\varepsilon} \sim \frac{4\pi\alpha}{Q^2} \rightarrow V(r) = \frac{\alpha}{r}$

$\langle S_{12} \rangle$	$j-1$	j	$j+1$
$j-1$	$-2 \frac{j-1}{2j+1}$	0	$6 \frac{\sqrt{j(j+1)}}{2j+1}$
j	0	2	0
$j+1$	$6 \frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2 \frac{j+2}{2j+1}$

Assume contact interactions are driven by heavier dofs, and scale with M_{QCD}
according to naïve dimensional analysis
(W power counting)

Weinberg '90, '91
Ordóñez + v.K. '92
Ordóñez, Ray + v.K. '96
...

Entem + Machleidt '03
Epelbaum, Glöckle + Meiñner '04
...

$$\sim C_0^{(0)} \frac{(\vec{\sigma}_1 \cdot \vec{\sigma}_2 + 1)}{4} - C_0^{(1)} \frac{(\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3)}{4}$$

$$\equiv \frac{4\pi}{m_N M^{(0)}} \quad \equiv \frac{4\pi}{m_N M^{(1)}}$$

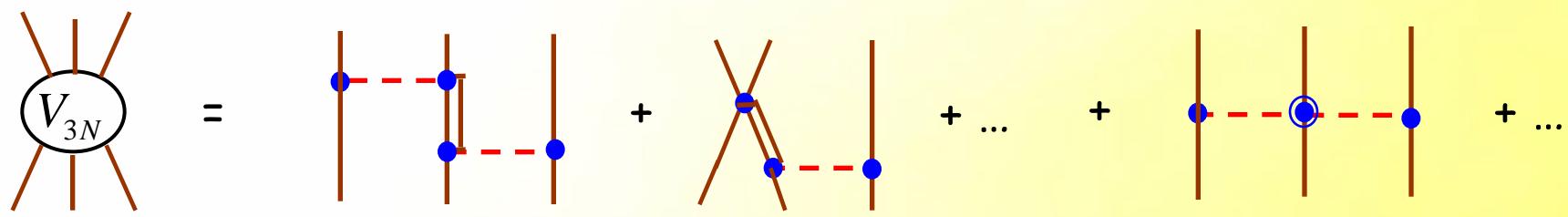
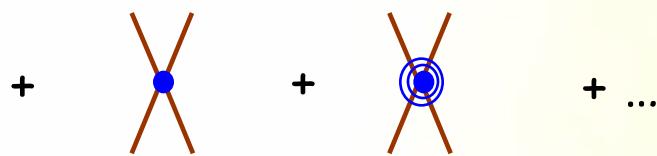
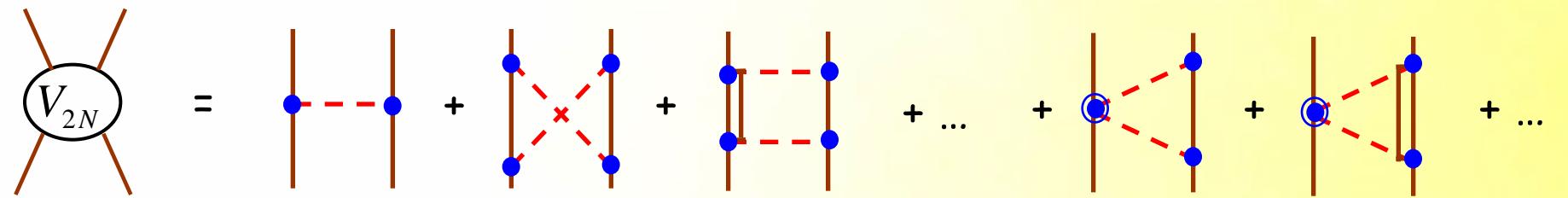
$$\left\{ \begin{array}{ll} V(r) = \frac{4\pi}{m_N M^{(0)}} \delta^{(3)}(\vec{r}) & S = 0 \\ V(r) = \frac{4\pi}{m_N M^{(1)}} \delta^{(3)}(\vec{r}) & S = 1 \end{array} \right.$$

$M^{(i)} \sim M_{NN} \rightarrow C_0^{(i)}$ in LO

$$\sim \frac{4\pi}{m_N M_{NN}} \frac{Q^2}{M_{\text{QCD}}^2} \rightarrow \text{in NNLO}$$

(NLO terms, linear in Q/M_{QCD} , break P, T)

etc.



etc.

↓
more nucleons

higher powers of Q

2-body

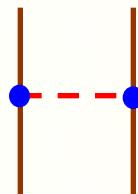
3-body

4-body

...

LO

$$\mathcal{O}\left(\frac{1}{f_\pi^2}\right)$$



in German

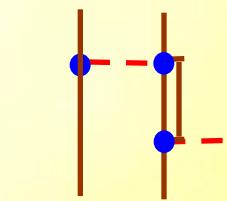
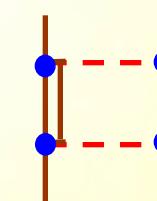
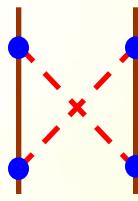
NLO

$$\mathcal{O}\left(\frac{1}{f_\pi^2} \frac{Q}{M_{\text{QCD}}}\right)$$

(parity violating)

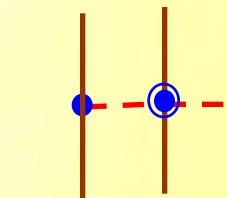
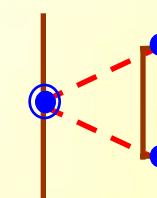
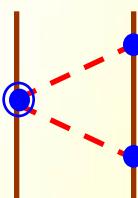
NNLO

$$\mathcal{O}\left(\frac{1}{f_\pi^2} \frac{Q^2}{M_{\text{QCD}}^2}\right)$$



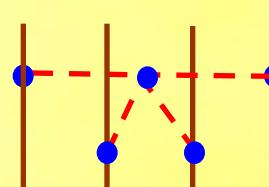
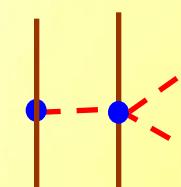
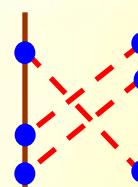
NNNLO

$$\mathcal{O}\left(\frac{1}{f_\pi^2} \frac{Q^3}{M_{\text{QCD}}^3}\right)$$



NNNNLO

$$\mathcal{O}\left(\frac{1}{f_\pi^2} \frac{Q^4}{M_{\text{QCD}}^4}\right)$$



ETC.

Hierarchies

many-body forces

$$V_{2N} \gg V_{3N} \gg V_{4N} \gg \dots$$

A canon emerges!

Weinberg '90, '91

Similar explanation for

isospin-breaking forces

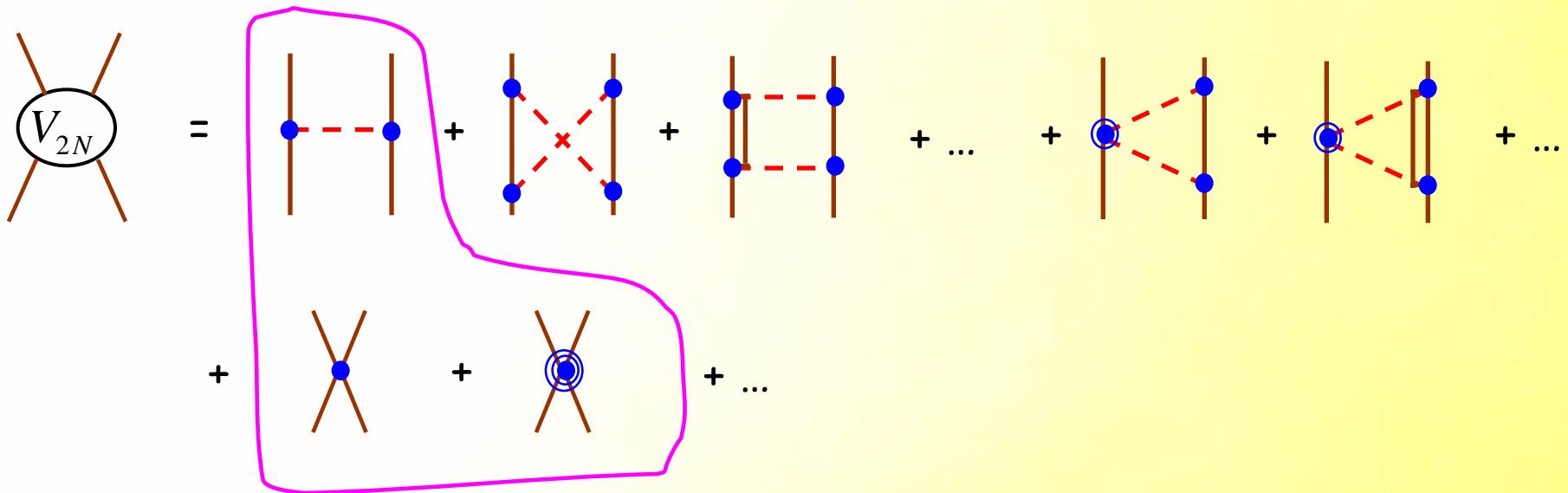
$$\left\{ \begin{array}{l} V_{IS} \gg V_{IV} \gg V_{CSB} \\ J_{1N} \gg J_{2N} \gg J_{3N} \gg \dots \end{array} \right.$$

v.K. '93

Rho '92

external currents

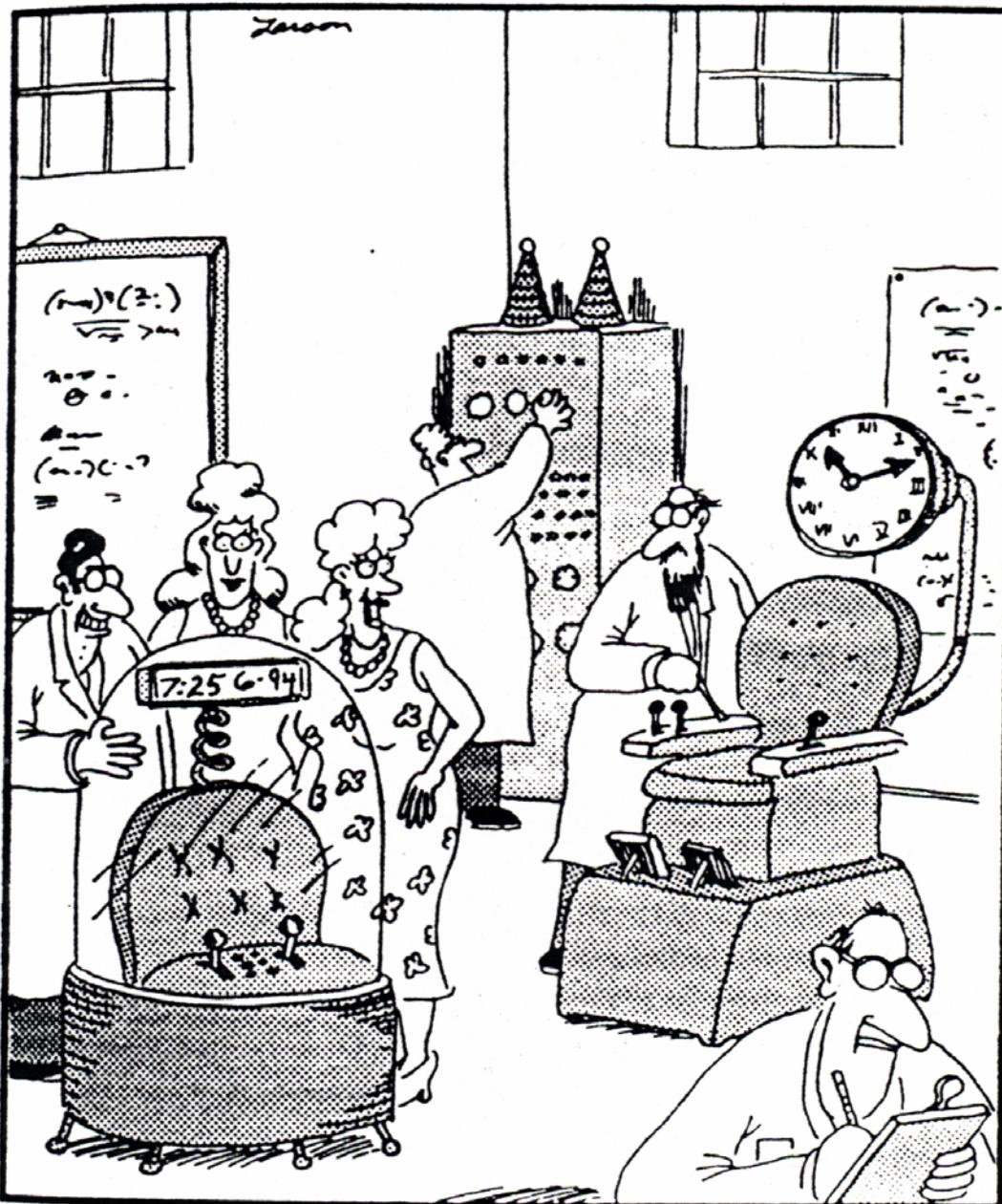
other canons emerge!



similar to phenomenological
potential models,

e.g. AV18 - $(OPE)^2$ + non-local terms

Stoks, Wiringa + Pieper '94

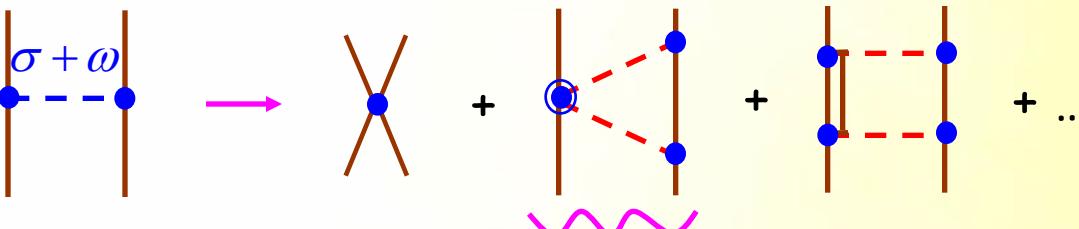


"Oh, Professor DeWitt! Have you seen Professor Weinberg's time machine? ... It's digital!"

But: NOT your usual potential!

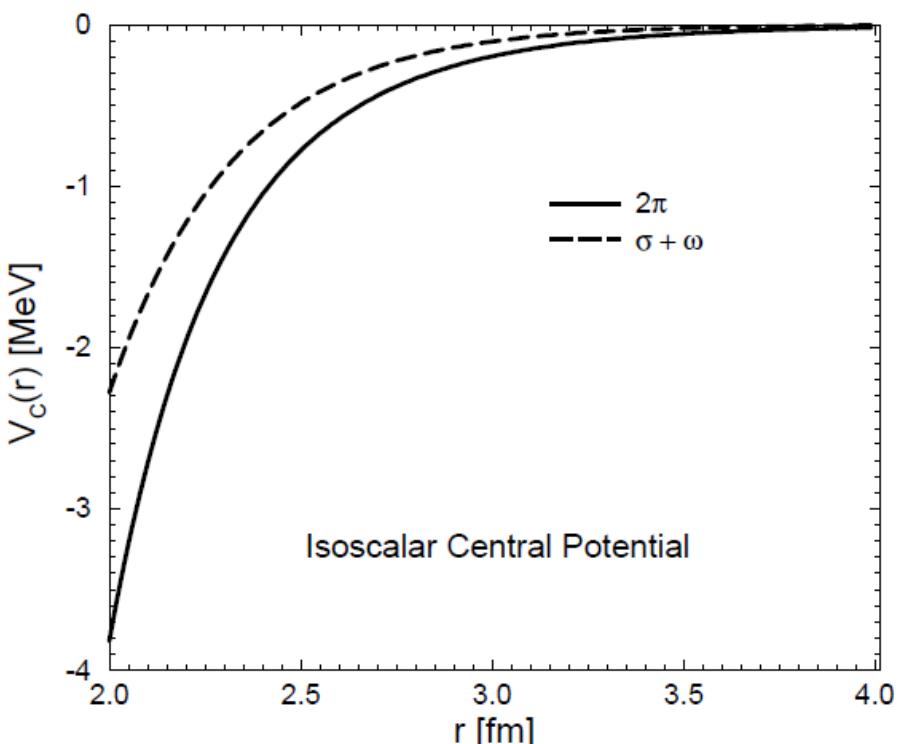
Ordóñez + v.K. '92
(cf. Stony Brook TPE)

e.g.,



$$\text{chiral v.d. Waals force } \sim \frac{1}{r^6} \text{ for } m_\pi^2 \rightarrow 0$$

Kaiser, Brockmann + Weise '97



Similar results in other channels,
e.g. spin-orbit force!

Rentmeester et al. '01, '03

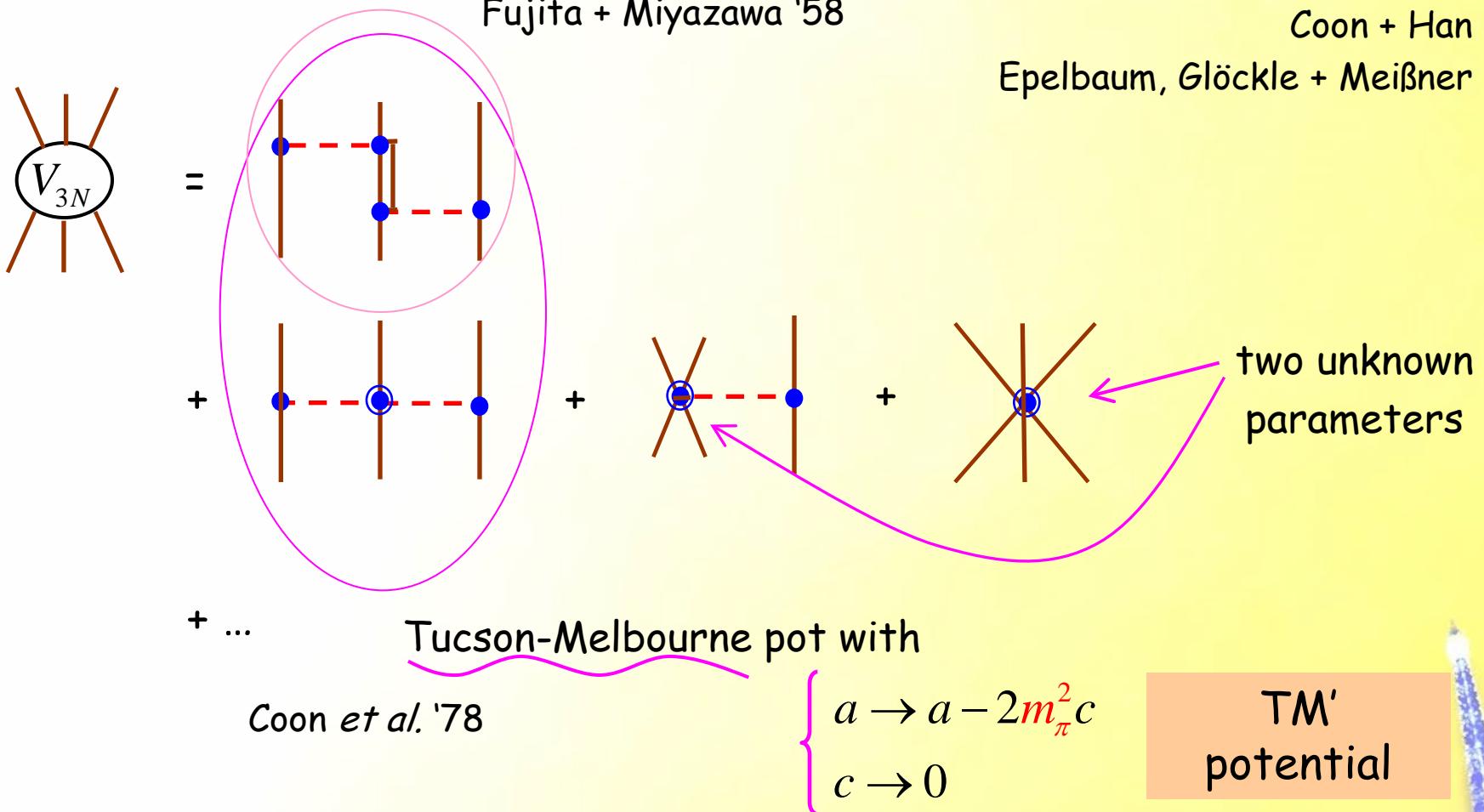
Nijmegen PSA of 1,951 pp data

long-range pot	#bc	χ^2_{\min}
OPE	31	2026.2
OPE + TPE (<i>lo</i>)	28	1984.7
OPE + TPE (<i>nlo</i>)	23	1934.5
Njm78	19	1968.7

parameters found
consistent with πN data!

at least
as good!

models with σ, ω, \dots
might be misleading...



$$(t_{\pi N}(\vec{q}, \vec{q}'))_{\alpha\beta} = \delta_{\alpha\beta} \left[a + b \vec{q} \cdot \vec{q}' + c (\vec{q}^2 + \vec{q}'^2) \right] - d \epsilon_{\alpha\beta\gamma} \tau_{3\gamma} \vec{\sigma} \cdot \vec{q} \times \vec{q}' + \dots$$

Many successes of Weinberg's counting, e.g.,

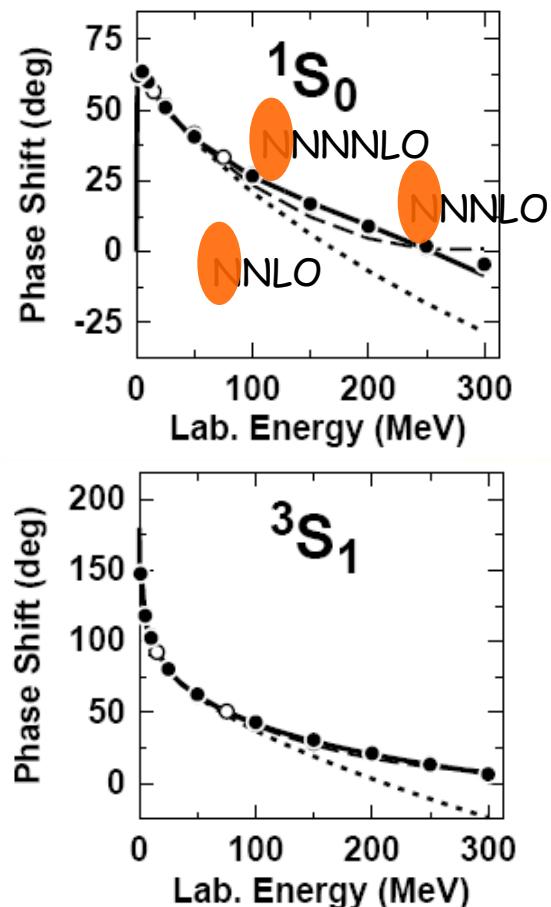
Ordóñez, Ray + v.K. '96

...

- ✓ To NNNNLO (w/o deltas), fit to NN phase shifts comparable to those of "realistic" phenomenological potentials

Epelbaum, Glöckle + Meiñner '02
Entem + Machleidt '03

...



Entem + Machleidt '03

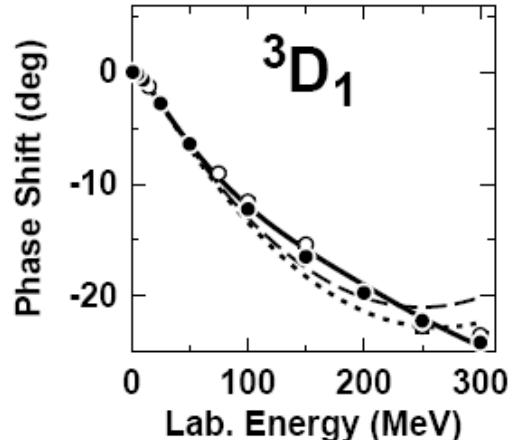
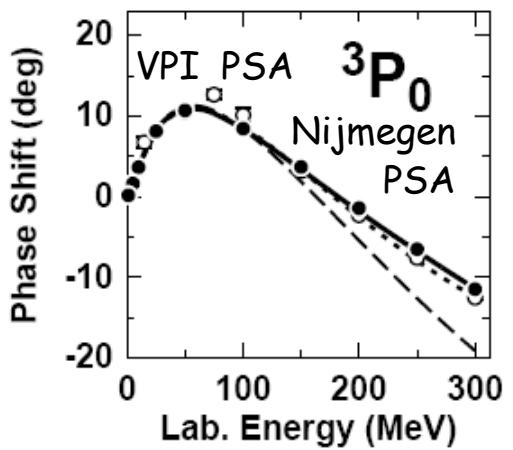


TABLE II. χ^2/datum for the reproduction of the 1999 np database [40] below 290 MeV by various np potentials.

Bin (MeV)	No. of data	N^3LO^a	NNLO ^b	NLO ^b	AV18 ^c
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

TABLE V. Two- and three-nucleon bound-state properties. (Deuteron binding energy B_d ; asymptotic S state A_S ; asymptotic D/S state η ; deuteron radius r_d ; quadrupole moment Q ; D -state probability P_D ; triton binding energy B_t .)

	N^3LO^a	CD-Bonn [10]	AV18 [22]	Empirical ^b
Deuteron				
$B_d(\text{MeV})$	2.224575	2.224575	2.224575	2.224575(9)
$A_S(\text{fm}^{-1/2})$	0.8843	0.8846	0.8850	0.8846(9)
η	0.0256	0.0256	0.0250	0.0256(4)
$r_d(\text{fm})$	1.978 ^c	1.970 ^c	1.971 ^c	1.97535(85)
$Q(\text{fm}^2)$	0.285 ^d	0.280 ^d	0.280 ^d	0.2859(3)
$P_D(\%)$	4.51	4.85	5.76	
Triton				
$B_t(\text{MeV})^e$	7.855	8.00	7.62	8.48

✓ With NNNNLO 2N and NNNLO 3N potentials (w/o deltas),
good description of

- 3N observables and 4N binding energy
- levels of p-shell nuclei

Gueorguiev *et al.* '07

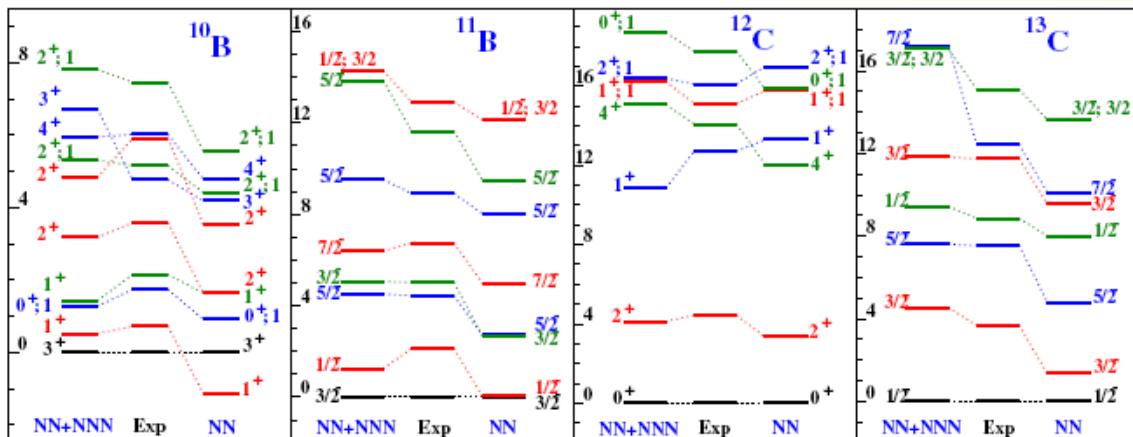


FIG. 4 (color online). States dominated by p -shell configurations for ^{10}B , ^{11}B , ^{12}C , and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin $T = 0$ or $1/2$, the isospin label is explicitly shown only for states with $T = 1$ or $3/2$. The excitation energy scales are in MeV.

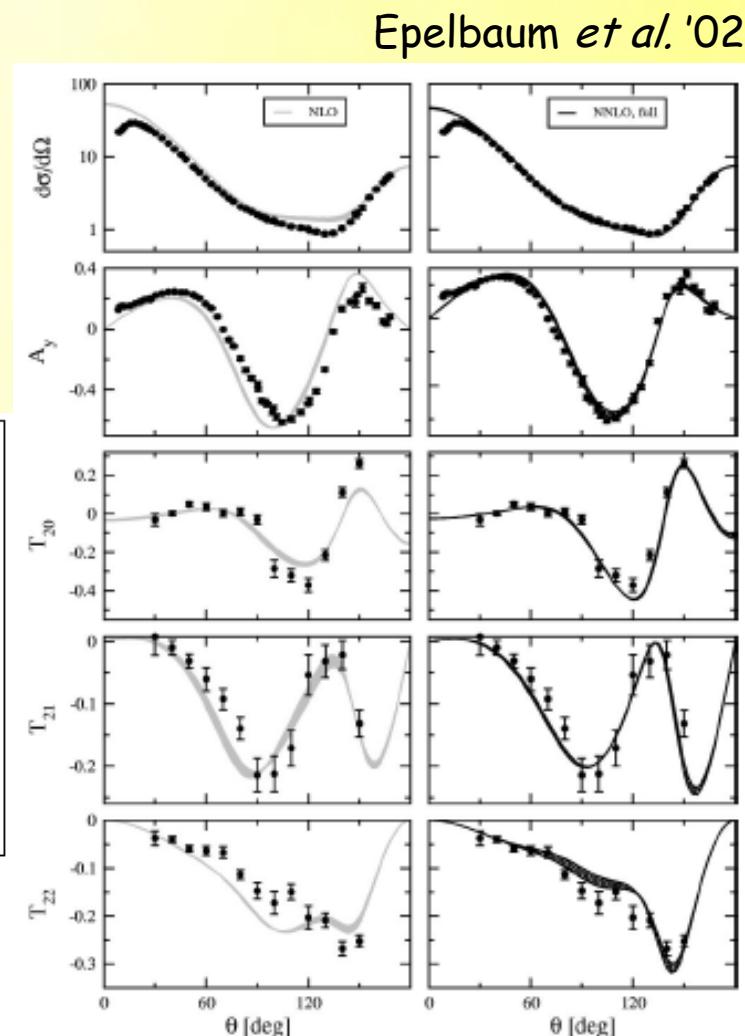


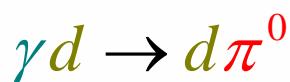
FIG. 6. nd elastic scattering observables at 65 MeV at NLO (left column) and NNLO (right column). The filled circles are pd data [63,69]. The bands correspond to the cutoff variation between 500 and 600 MeV. The unit of the cross section is mb/sr.

Many reactions



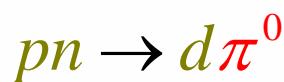
measured: Illinois '94, SAL '00, Lund '03

extracted nucleon polarizabilities: Beane, Malheiro, McGovern,
Phillips + v.K. '04



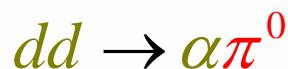
threshold amplitude predicted: Beane, Bernard, Lee, Meißner
+ v.K. '97

confirmed: SAL '98, Mainz '01



CSB asymmetry sign predicted: Miller, Niskanen + v.K. '00

confirmed: TRIUMF '03



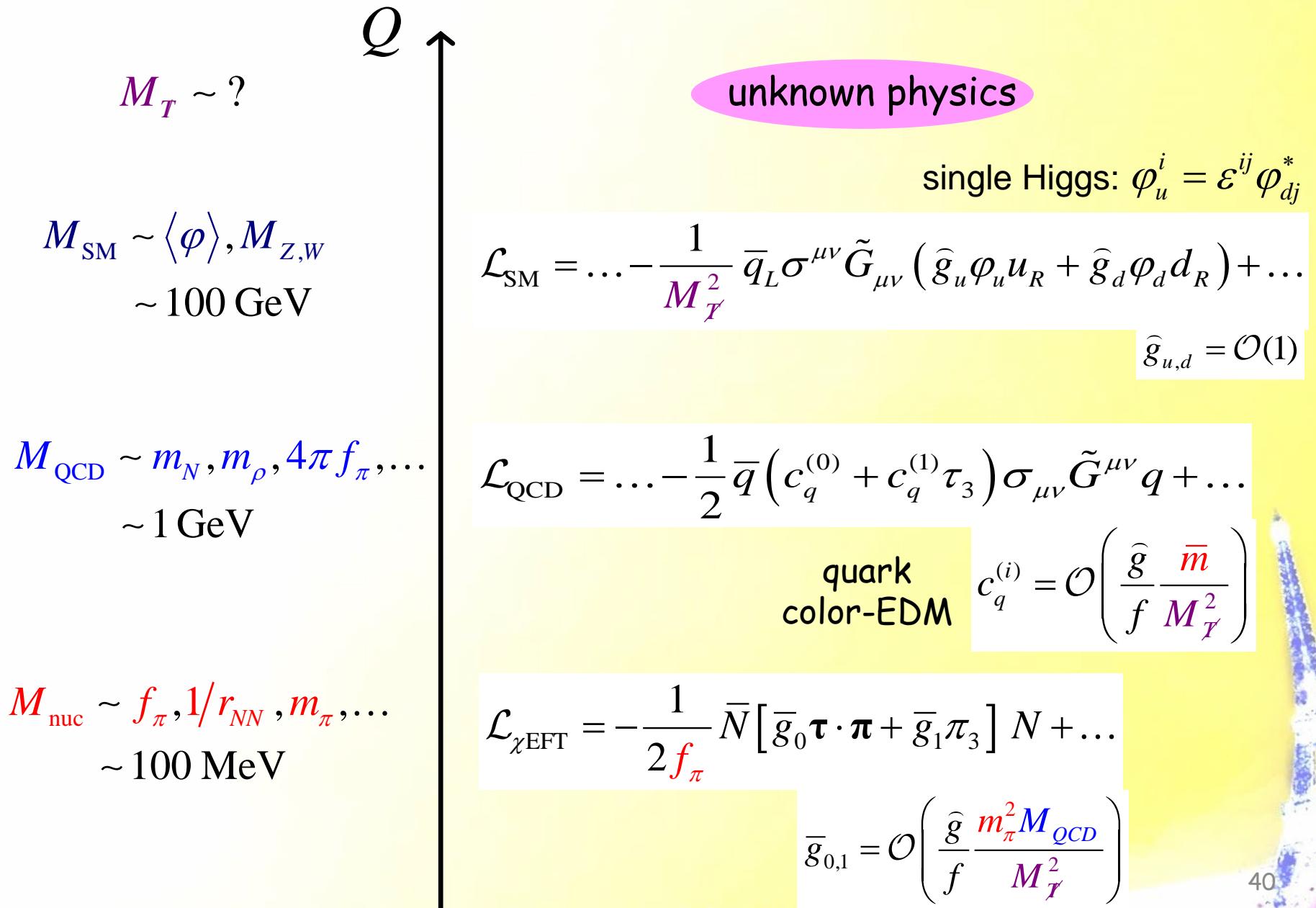
estimated: Gårdestig, Miller + v.K.

discovered: IUCF '03

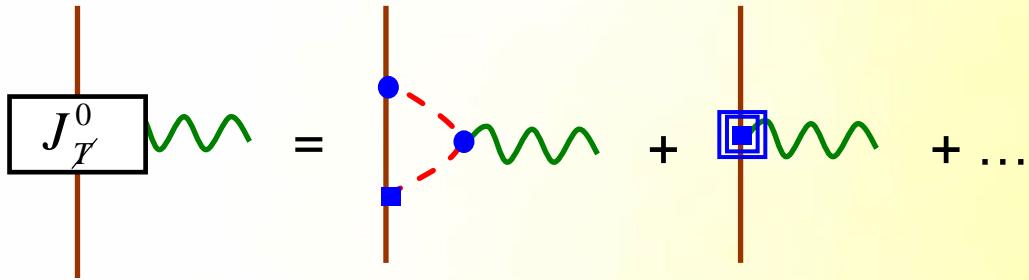


+ many more, including **PARITY**, **TIME-REVERSAL VIOLATION**, etc.

Example: T violation



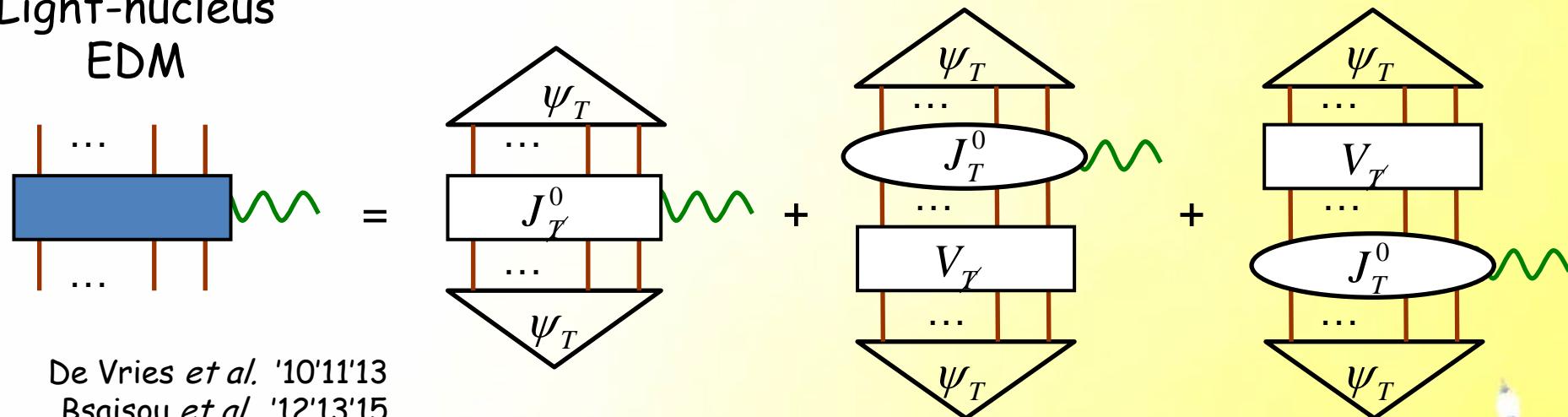
Nucleon
EDM



Crewther et al. '79

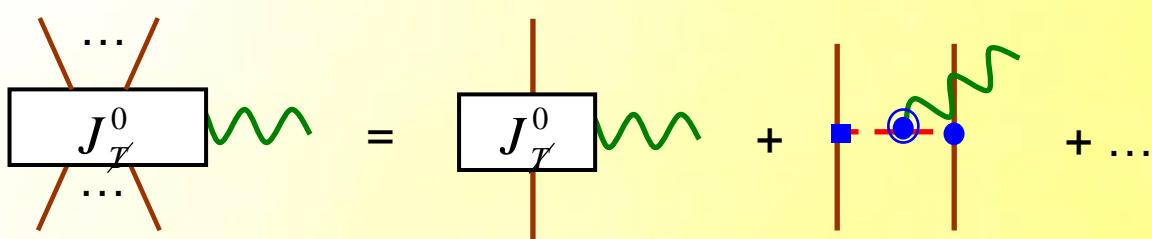
Hockings + v.K. 05
Narison '08
Ottnad et al. '10
De Vries et al. '11

Light-nucleus
EDM

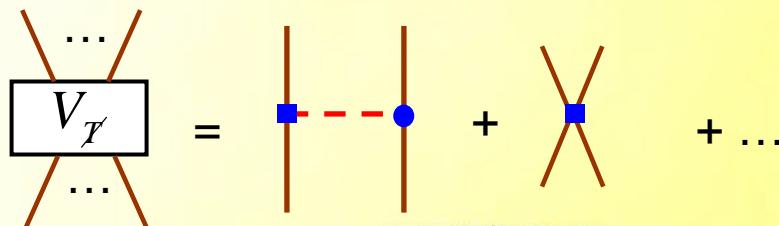


De Vries et al. '10'11'13
Bsaisou et al. '12'13'15
Dekens et al. '14

De Vries et al. '11



Maekawa et al. '11



$$\begin{aligned}
\mathcal{L}_{\text{QCD}} = & \dots + \frac{\bar{m}}{2} (1 - \varepsilon^2) \bar{\theta} \bar{q} i \gamma_5 q - \frac{1}{2} \bar{q} (c_q^{(0)} + c_q^{(1)} \tau_3) \sigma_{\mu\nu} \tilde{G}^{\mu\nu} q - \frac{1}{2} \bar{q} (d_q^{(0)} + d_q^{(1)} \tau_3) \sigma_{\mu\nu} q \tilde{F}^{\mu\nu} \\
& + \frac{c_G}{6} f^{abc} G_{\mu\nu}^a \tilde{G}^{b\nu\rho} G_{\rho}^{c\mu} + \frac{C_1}{4} (\bar{q} q \bar{q} i \gamma_5 q - \bar{q} \tau q \cdot \bar{q} i \gamma_5 \tau q) + \frac{C_8}{4} (\bar{q} \lambda^a q \bar{q} i \gamma_5 \lambda^a q - \bar{q} \tau \lambda^a q \cdot \bar{q} i \gamma_5 \tau \lambda^a q) \\
& + \frac{D_1}{4} \varepsilon_{3ij} \bar{q} \tau_i \gamma^\mu q \bar{q} \tau_j \gamma_\mu \gamma_5 q + \frac{D_8}{4} \varepsilon_{3ij} \bar{q} \tau_i \gamma^\mu \lambda^a q \bar{q} \tau_j \gamma_\mu \gamma_5 \lambda^a q + \dots
\end{aligned}$$

De Vries *et al.* '10'11'13
 Bsaisou *et al.* '12'13'15
 Dekens *et al.* '14

Neutron EDM d_n can be produced by any source, **but**
 different chiral properties of sources → different magnitudes for nuclear EDMs

	θ term	qCEDM	qEDM	gCEDM, PS4QO	LR4QO
¹ H	d_p/d_n	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
² H	d_d/d_n	$\mathcal{O}(1)$	$\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{m_\pi^2}\right)$	$\mathcal{O}(1)$	$\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{m_\pi^2}\right)$
³ He	d_h/d_n	$\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{m_\pi^2}\right)$	$\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{m_\pi^2}\right)$	$\mathcal{O}(1)$	$\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{m_\pi^2}\right)$
³ H	d_t/d_h	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$

+ specific relations

Farley *et al.* '04

storage-ring measurements (CERN?) could teach us about sources!

Chiral EFT has been recognized as
the basis for nuclear physics.

Now it is the favorite input for
the blossoming "*ab initio*" methods
that are revolutionizing
nuclear structure/reaction physics.

BUT

Is Weinberg's power counting consistent?

No!

$$\text{Diagram: Two vertical lines with blue dots connected by a dashed red line.} \sim \left(\frac{g_A}{2f_\pi} \right)^2 \frac{m_\pi^3}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \frac{S_{12}(\hat{r})}{(m_\pi r)^3} + \dots \right\} e^{-m_\pi r}$$

attractive in
some channels

singular
potential

not enough contact interactions
for renormalization-group invariance even at LO

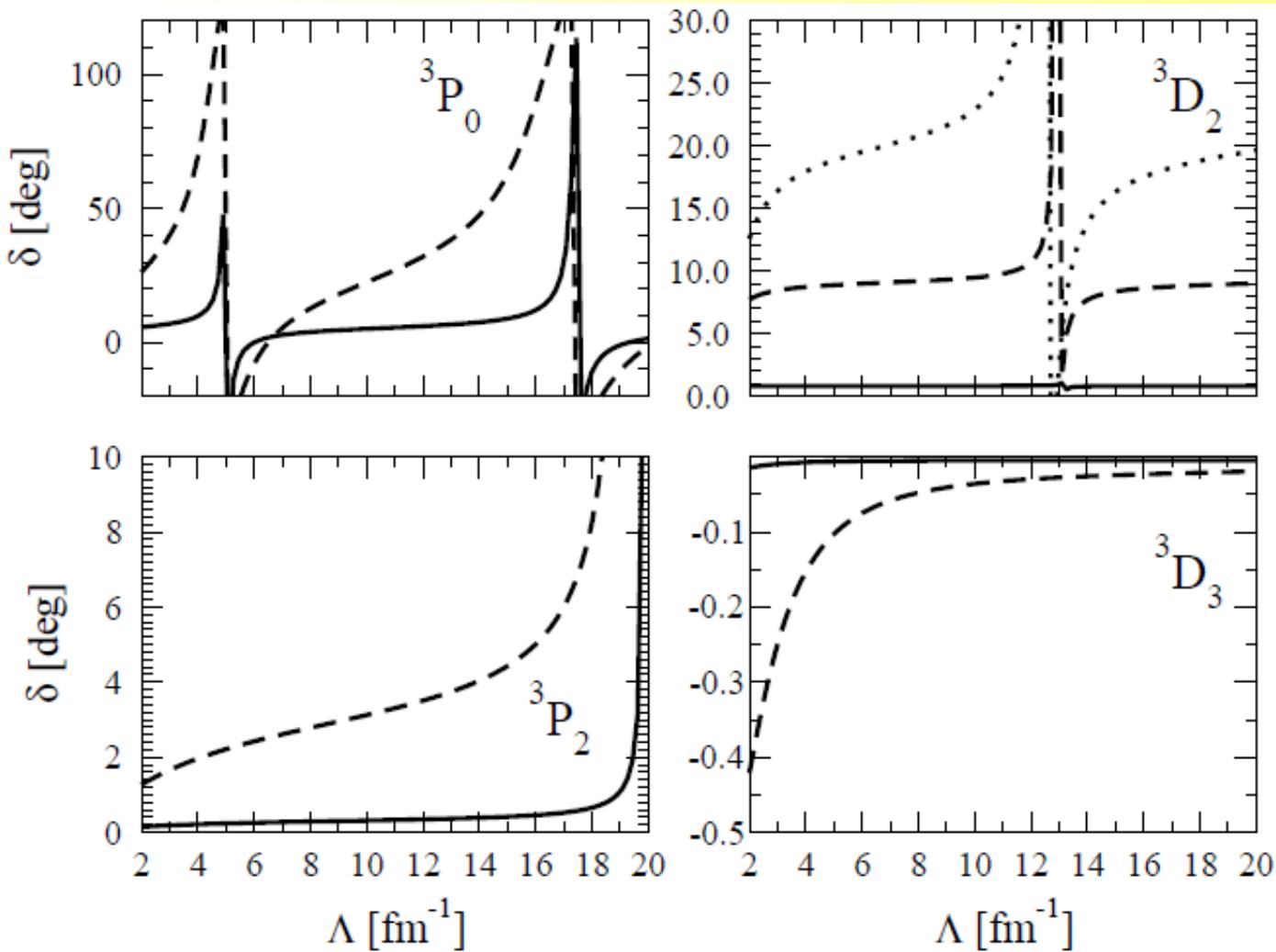
Problems!

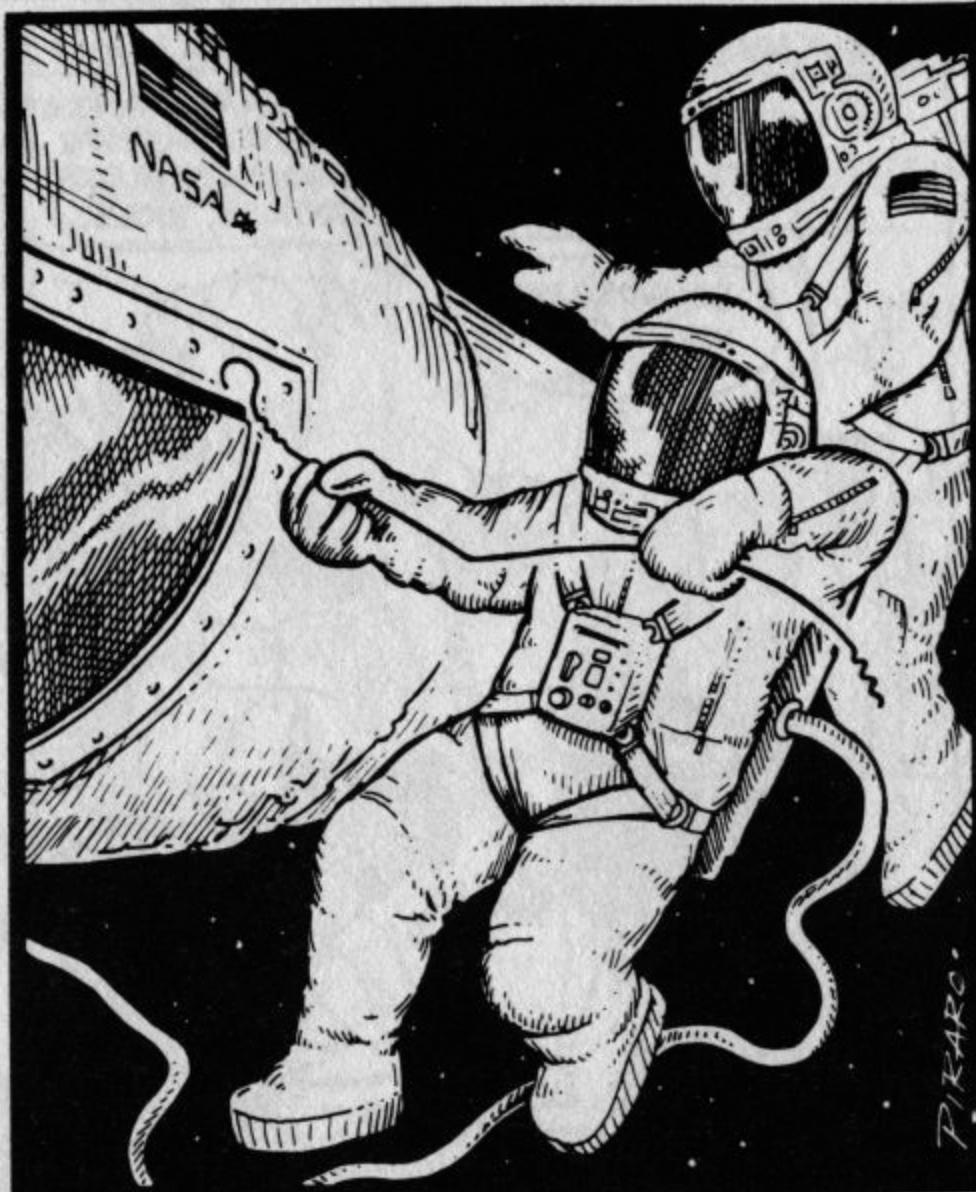
E (MeV)

— 10
 - - - 50
 100

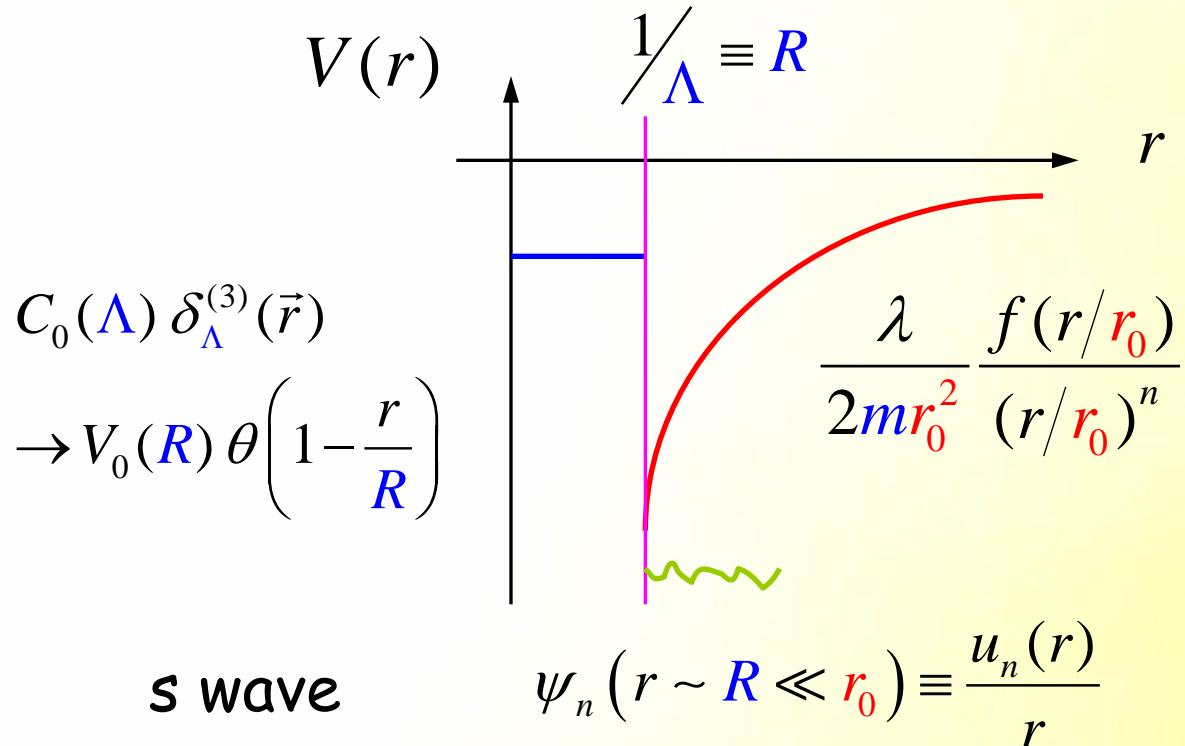
incorrect
 renormalization...

Attractive-tensor channels:





Renormalization of the $-1/r^n$ potential



$$\frac{\lambda}{2mr_0^2} \frac{f(r/r_0)}{(r/r_0)^n}$$

OPE:

$$m = m_N/2$$

$$r_0 = 1/m_\pi$$

$$\lambda = m_\pi/M_{NN}$$

$$\left\{ \begin{array}{l} f(r/r_0) = \exp(-r/r_0) \end{array} \right.$$

so that

$$\frac{R}{T_s} \frac{\partial T_s}{\partial R} (k \sim 1/r_0) = \mathcal{O}\left(\frac{R}{r_0}\right)$$

$n \geq 2$

Beane, Bedaque, Childress, Kryjevski, McGuire + v.K. '02

Two **regular** solutions
that oscillate!

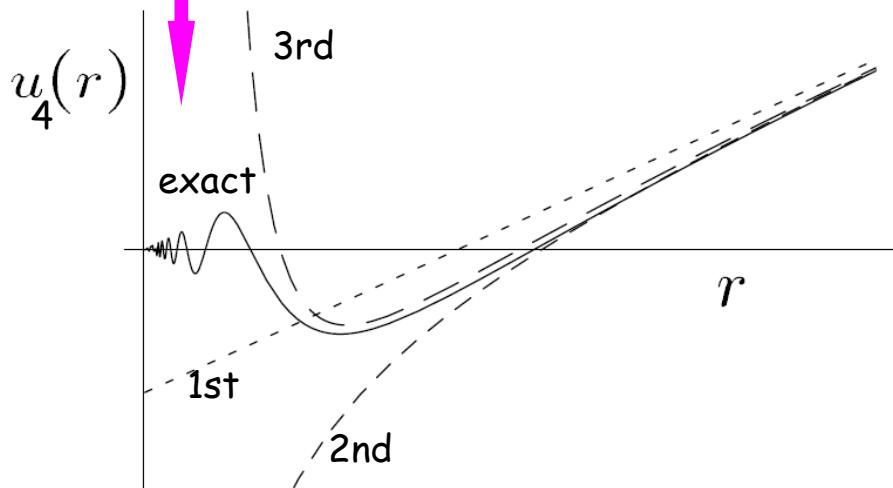
if no counterterm, will depend on cutoff R

➡ model dependence

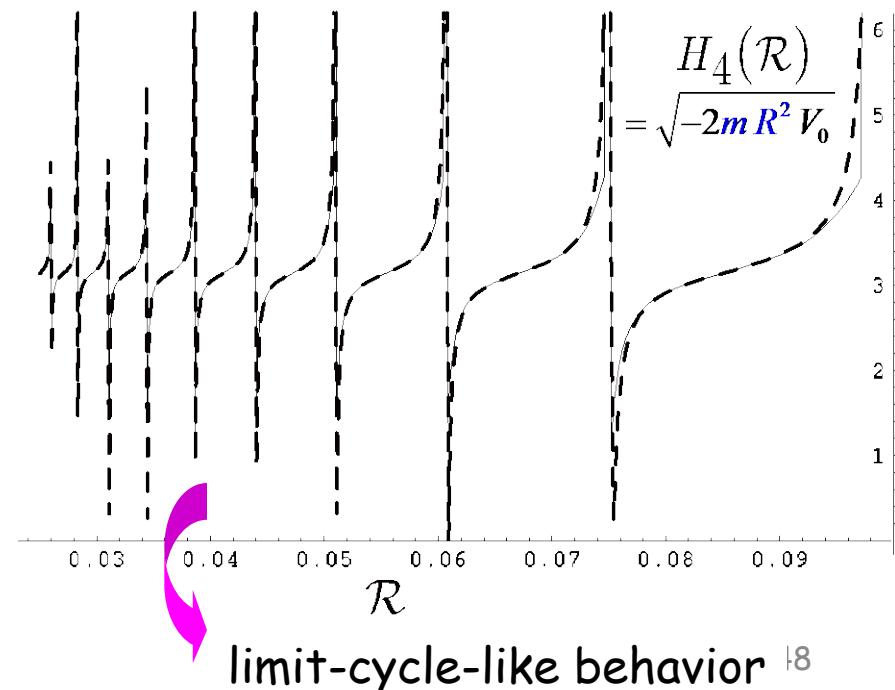
determined by
low-energy data

$$u_n(r \ll r_0) = \left(\frac{\lambda}{(r/r_0)^n} \right)^{-\frac{1}{4}} \cos \left(\frac{\sqrt{\lambda}}{(n/2-1)(r/r_0)^{n/2-1}} + \delta_n \right) + \dots$$

$$F_n(\lambda, r_0, R) = \frac{n}{4} - \sqrt{\lambda} \left(\frac{R}{r_0} \right)^{1-n/2} \tan \left(\frac{\sqrt{\lambda}}{(n/2-1)(R/r_0)^{n/2-1}} + \delta_n \right) + \dots$$

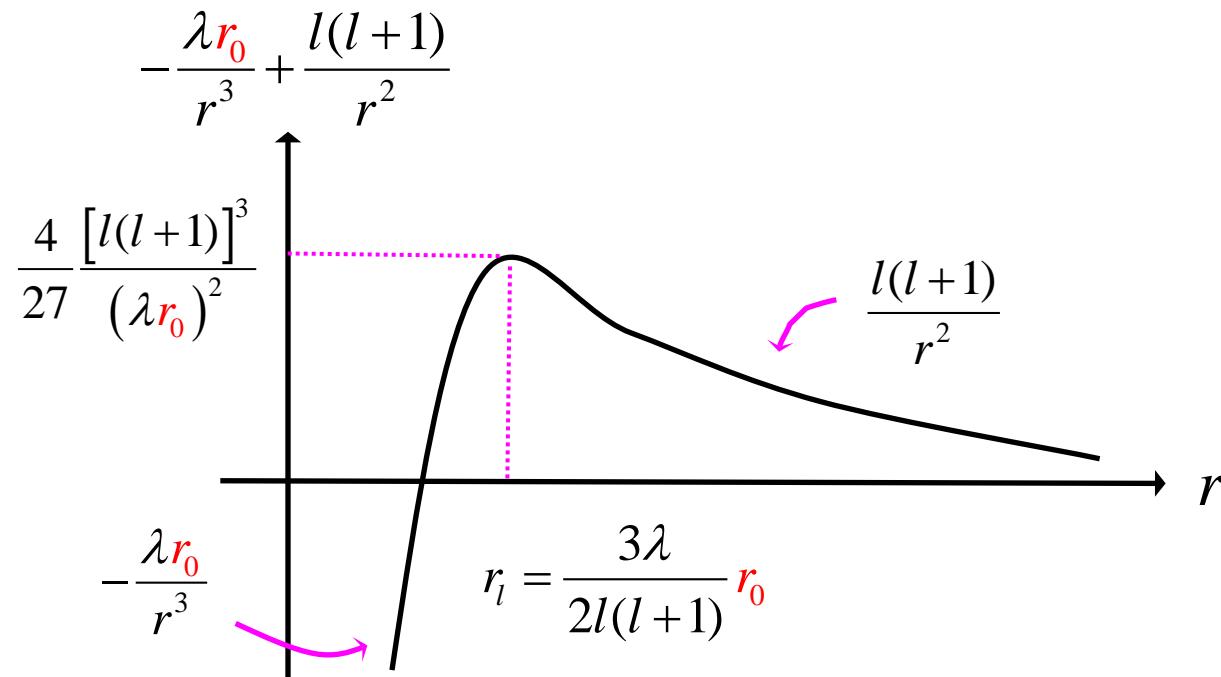


exact vs perturbation th



limit-cycle-like behavior 18

Same is true in all channels where attractive singular potential is iterated



but $r_l \sim \frac{1}{M} \ll r_0$ for $l(l+1) \gg \lambda$



singular potential only needs to be iterated in a few waves,
where counterterms are needed

"Perturbative pions" $\lambda = \frac{m_\pi}{M_{NN}} \ll 1$

Kaplan, Savage + Wise '98

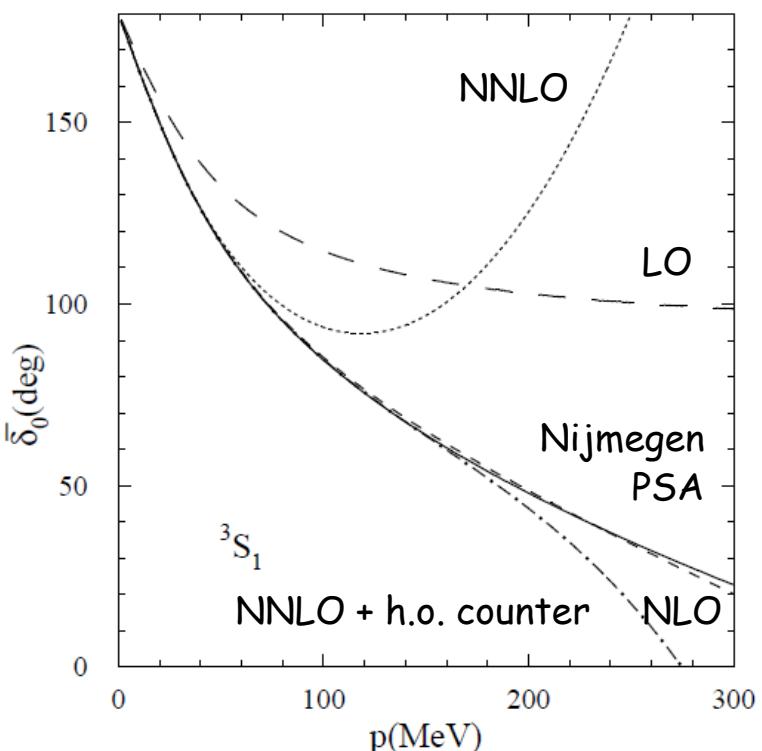
Fleming, Mehen + Stewart '01

→ $M_{NN} \sim f_\pi$ indeed

Partly perturbative pions

$$l(l+1) \gtrsim \frac{3M_{\text{QCD}}}{2M_{NN}} \sim 5 \rightarrow l \gtrsim 2$$

+ subleading orders: in perturbation theory, as in NRQED



Beane, Bedaque, Savage + v.K. '02

Nogga, Timmermans + v.K. '05

Pavón Valderrama + Ruiz-Arriola '06

Birse, '06'07

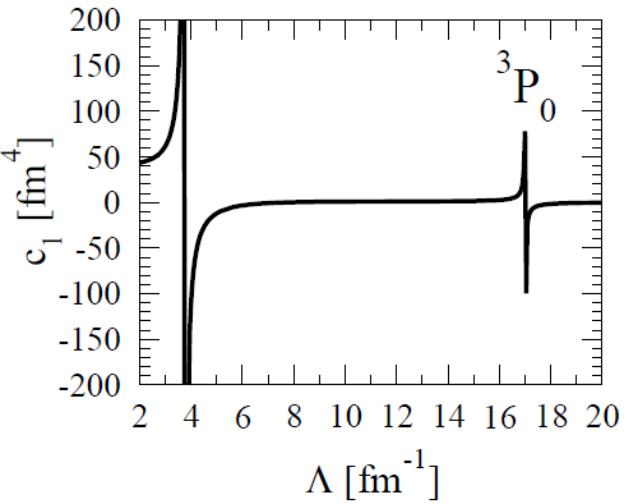
Long + v.K. '07

Pavón Valderrama '10'11

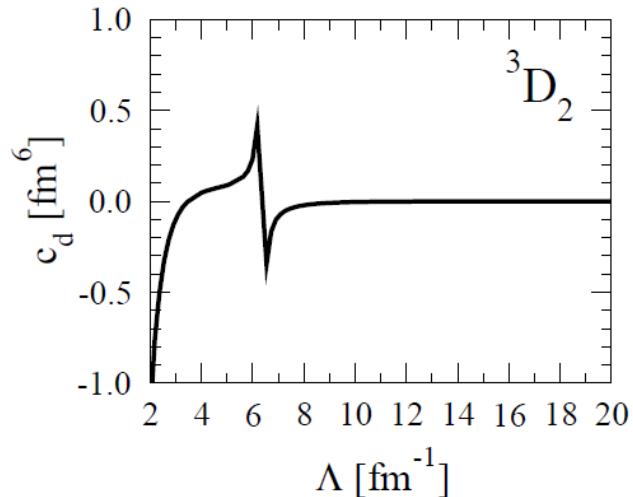
Long + Yang '11'12

...

$$V_{l=1,j=0} = \frac{c_1}{(2\pi)^3} \textcolor{red}{pp'}$$

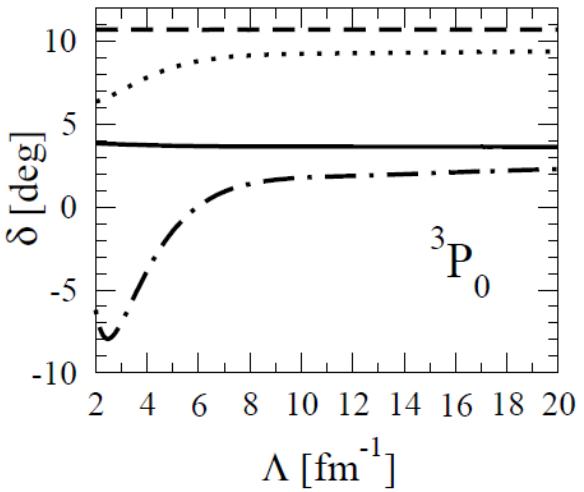


$$V_{l=2,j=2} = \frac{c_d}{(2\pi)^3} \textcolor{red}{p^2 p'^2}$$

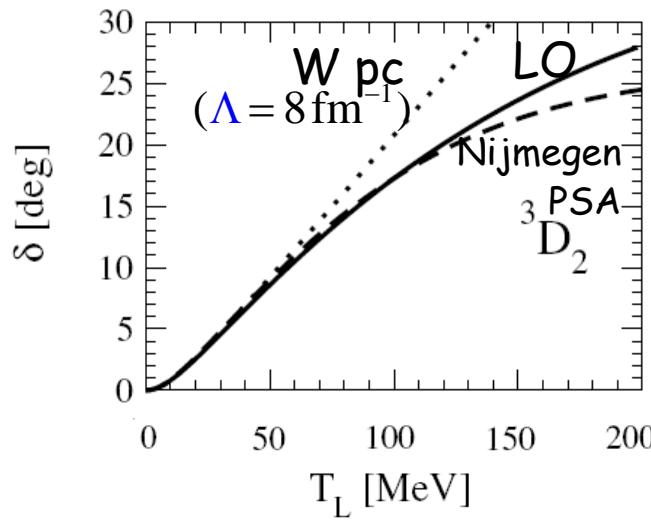
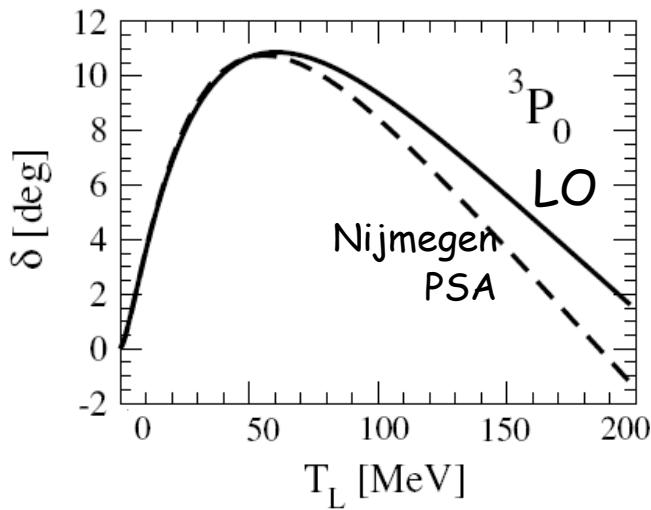
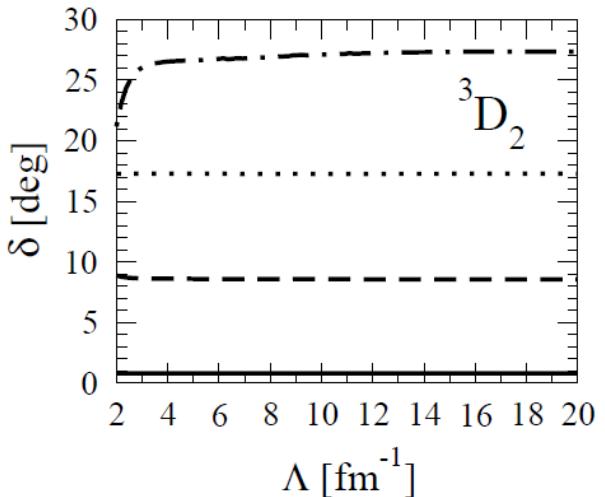


Add
counterterms

Nogga, Timmermans + v.K. '05
(cf. Pavón Valderrama + Ruiz-Arriola, '06)



E (MeV) 10 —
 50 - - -
 100
 190 - . -



certain counterterms that in Weinberg's counting

were assumed suppressed by powers of $\frac{Q}{M_{\text{QCD}}}$

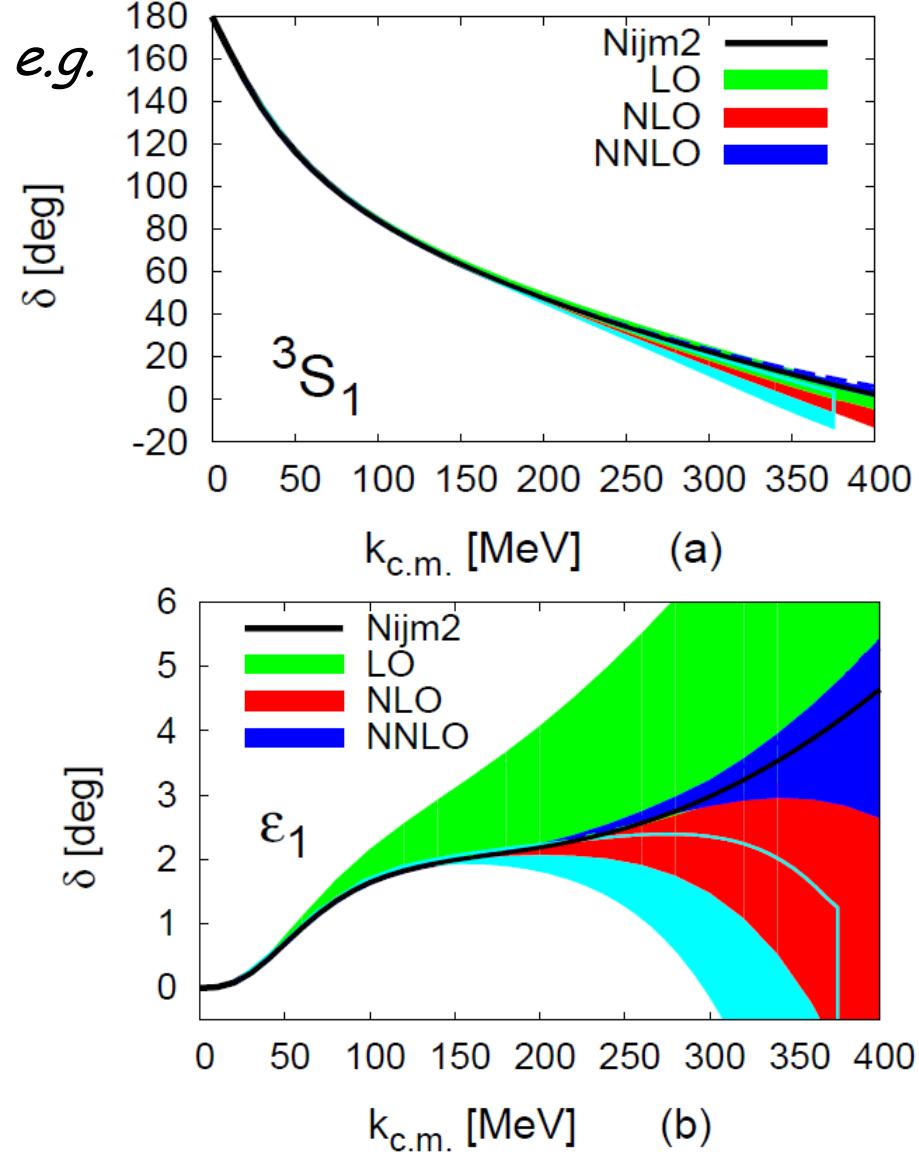
are in fact suppressed by powers of $\frac{Q}{lf_\pi}$



short-range physics more important than assumed by Weinberg's;
most qualitative conclusions unchanged,
but quantitative results need improvement

ACTIVE RESEARCH AREA

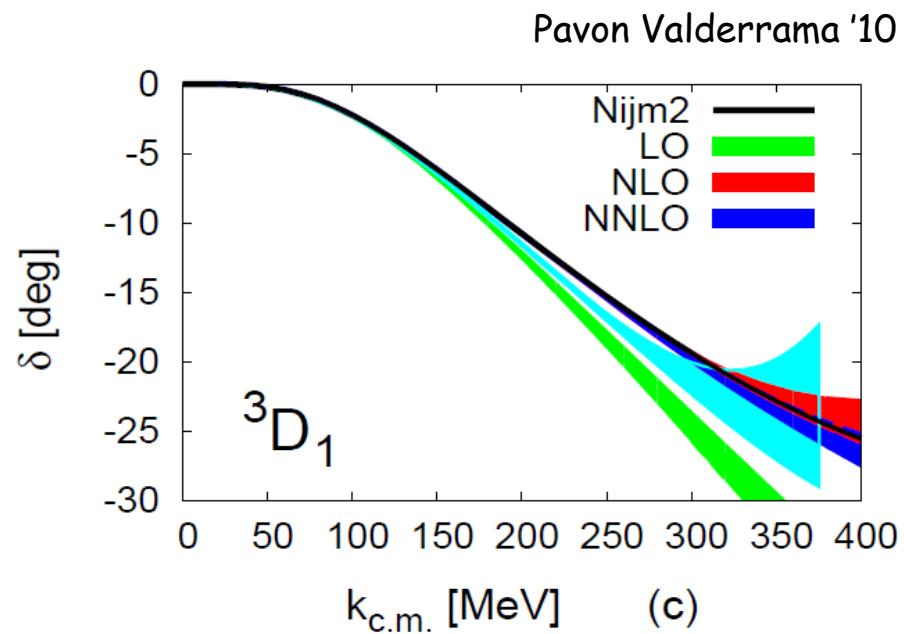
new PC



Fits to data

Pavón Valderrama '10'11
Long + Yang '11'12

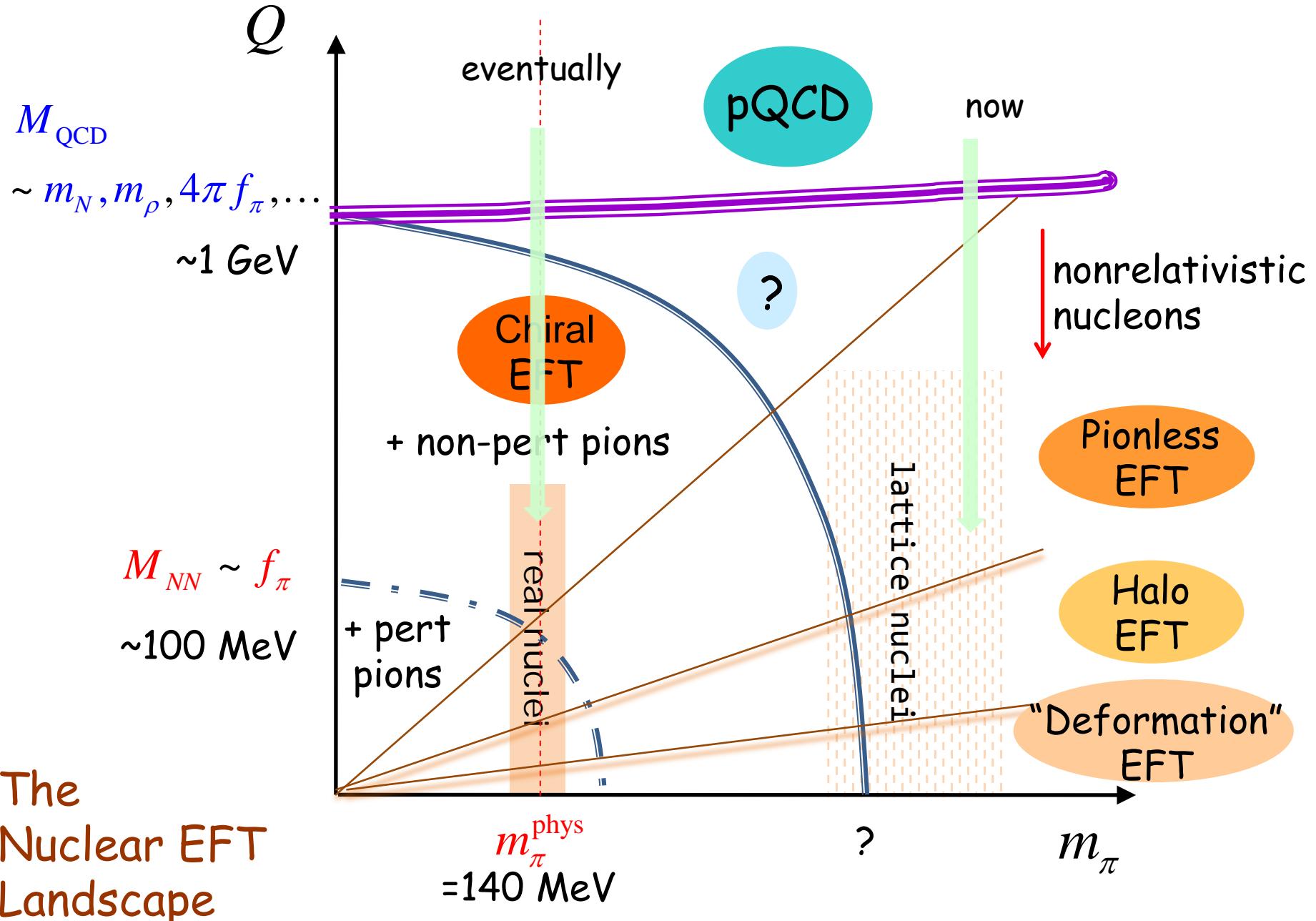
bands (not error estimates):
coordinate-space cutoff variation
0.6 - 0.9 fm
cyan:
NNLO in Weinberg's scheme



Summary

- ◆ A low-energy EFT of QCD **has been** constructed and used to describe nuclear systems
- ◆ Chiral symmetry plays an important role, in particular setting the **scale** for nuclear bound states
- ◆ Nuclear physics canons **emerge** from chiral potential
- ◆ A **new** power counting has been formulated: more counterterms at each order relative to Weinberg's; expect even better description of observables

Outlook



Conclusion

EFT is a **general** framework for theory construction

- ✓ same method across scales
- ✓ model independent
- ✓ controlled expansion

EFT is (very slowly) becoming the paradigm in nuclear physics

- ✓ encodes QCD (and, more generally, B/SM)
- ✓ incorporates hadronic physics
- ✓ generates nuclear structure

The nuclear EFT frontier: many bodies & lattice QCD

- interplay with *ab initio* methods
- new EFTs