Unitary and analytical HEFT for Strongly Interacting Longitudinal Gauge Bosons at the LHC

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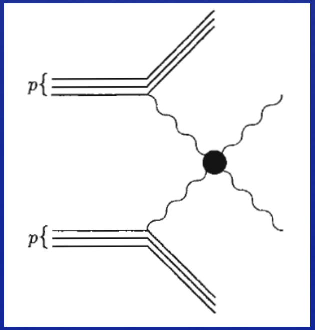
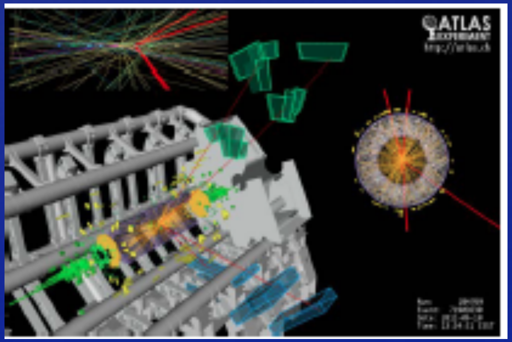
Higgs Effective Field Theory 2018 (HEFT 2018)
Mainz, April 2018





Work done in collaboration with R. Delgado and F. Llanes-Estrada.

Outline

I. Properties of the Higgs boson

II. The Higgs in the Minimal Standard Model

III. Modeling a Strongly Interacting SBS

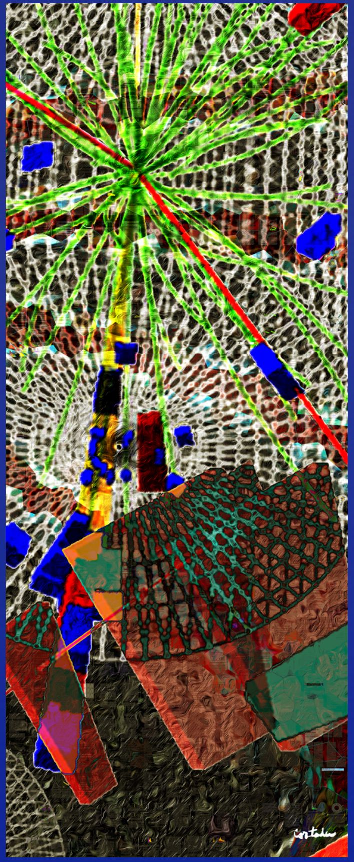
IV. One-loop computations

V. Unitarization methods

VI. Resonances

VII. Conclusions

I. Properties of the Higgs boson



First publications claiming the new-boson discovery by CMS and ATLAS at 2012

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC ☆

CMS Collaboration *

CERN, Switzerland

This paper is dedicated to the memory of our colleagues who worked on CMS but have since passed away. In recognition of their many contributions to the achievement of this observation.

Physics Letters B 716 (2012) 1–29

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC ☆

ATLAS Collaboration *

This paper is dedicated to the memory of our ATLAS colleagues who did not live to see the full impact and significance of their contributions to the experiment.

Higgs-boson properties (PDG 2017)

H^0 $J = 0$

Mass $m = 125.09 \pm 0.24$ GeV
Full width $\Gamma < 0.013$ GeV, CL = 95%

H^0 Signal Strengths in Different Channels

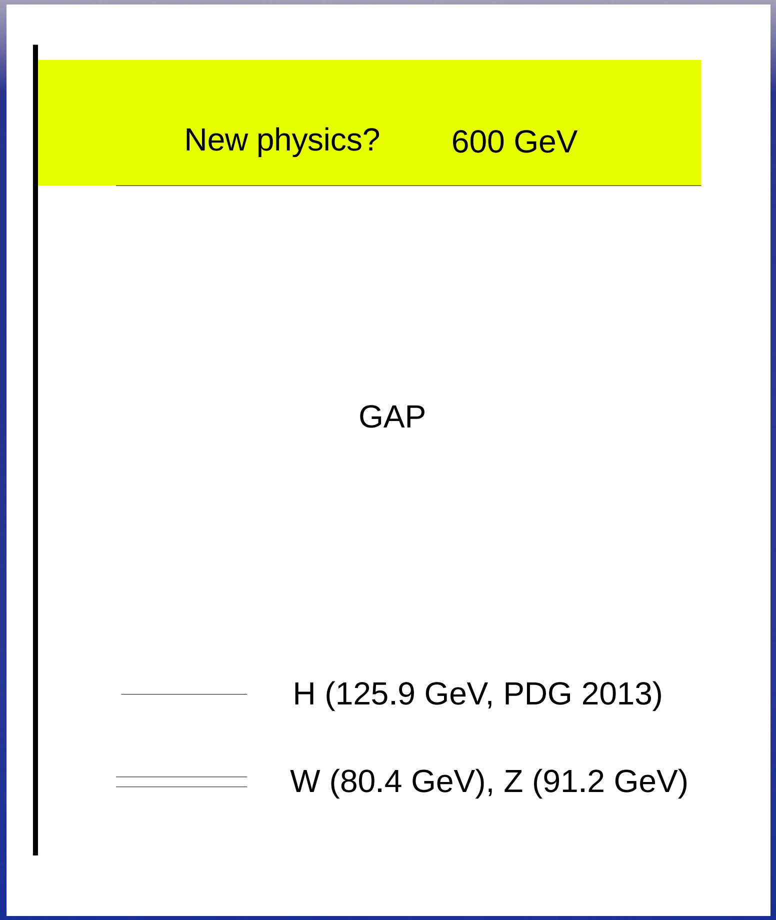
See Listings for the latest unpublished results.

Combined Final States = 1.10 ± 0.11
$W W^* = 1.08^{+0.18}_{-0.16}$
$Z Z^* = 1.29^{+0.26}_{-0.23}$
$\gamma\gamma = 1.16 \pm 0.18$
$b\overline{b} = 0.82 \pm 0.30$ (S = 1.1)
$\mu^+\mu^- = 0.1 \pm 2.5$
$\tau^+\tau^- = 1.12 \pm 0.23$
$Z\gamma < 9.5$, CL = 95%
$t\overline{t}H^0$ Production = $2.3^{+0.7}_{-0.6}$

$$\mu \equiv \sigma \cdot \mathrm{Br}/(\sigma_{\mathrm{SM}} \cdot \mathrm{Br}_{\mathrm{SM}})$$

Signal Strengths

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

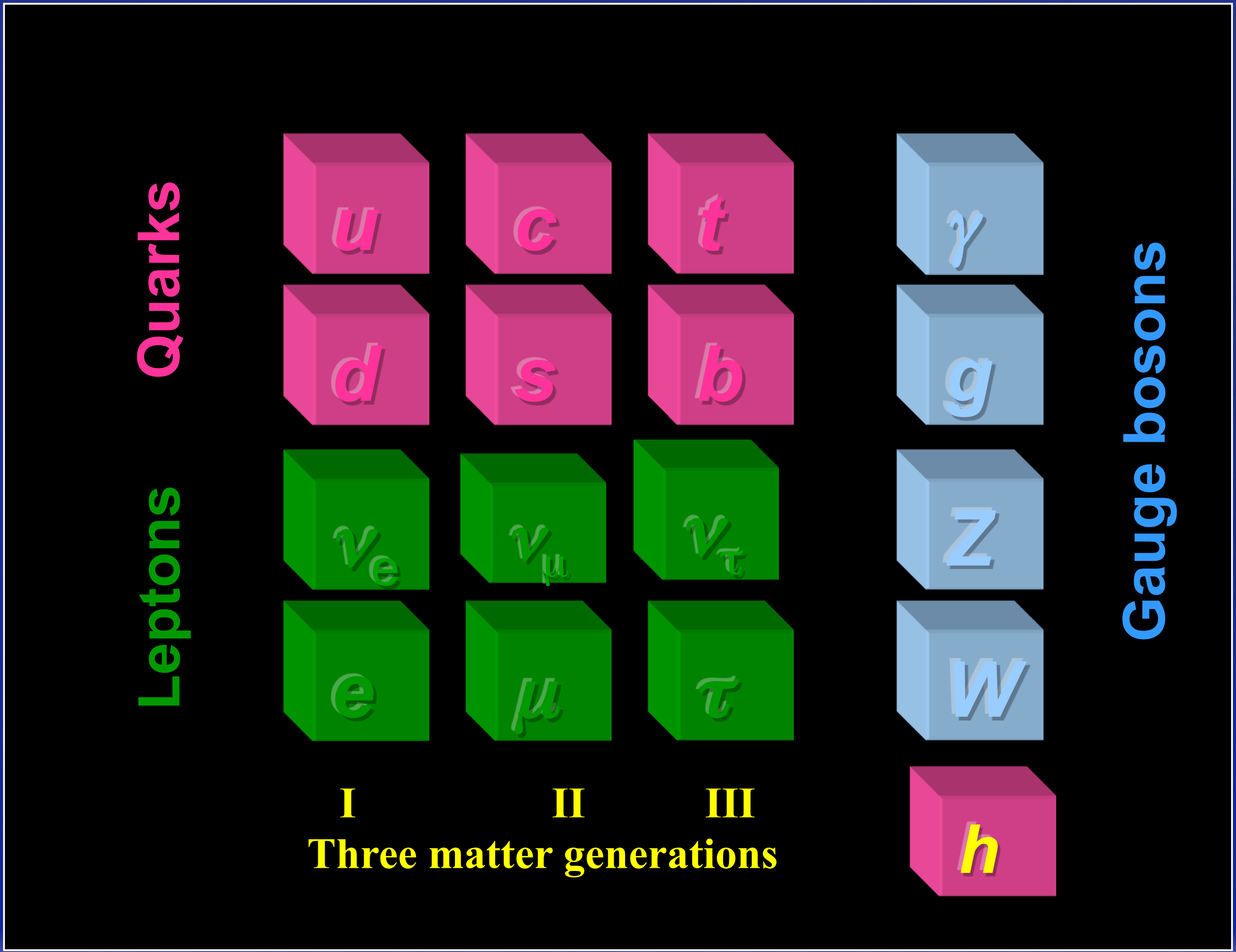


Nothing new at least up to 600 GeV → BIG GAP

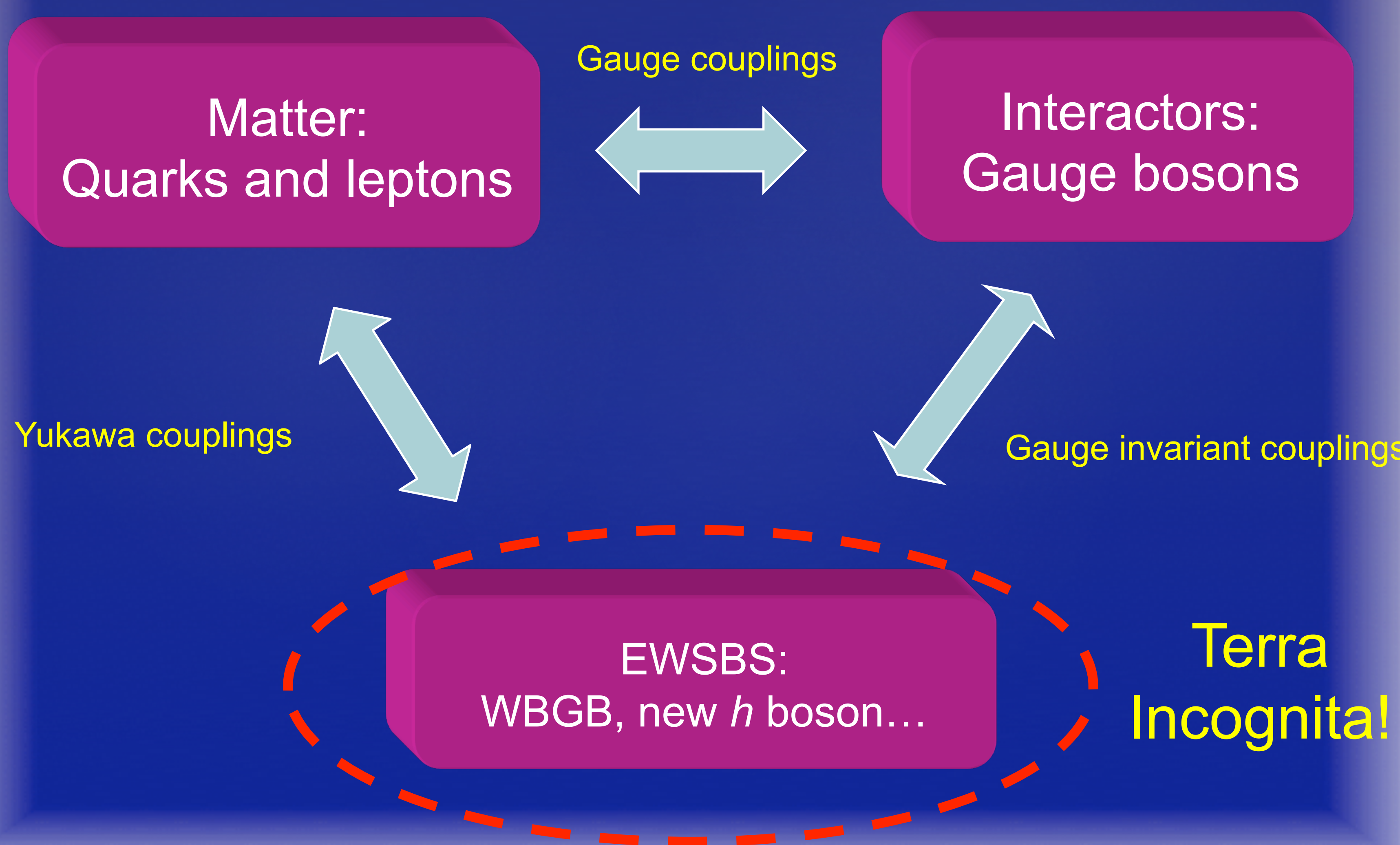
Thus we have ONE and only ONE new SCALAR resonance at *125.5 GeV:* → *h(125)*

So what is this new particle?

II. The Higgs in the Minimal Standard Model



The Standard Model Structure



The EWSBS in the Minimal Standard Model

$$\mathcal{L}_{SBS} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) + \mathcal{L}_{YK}$$

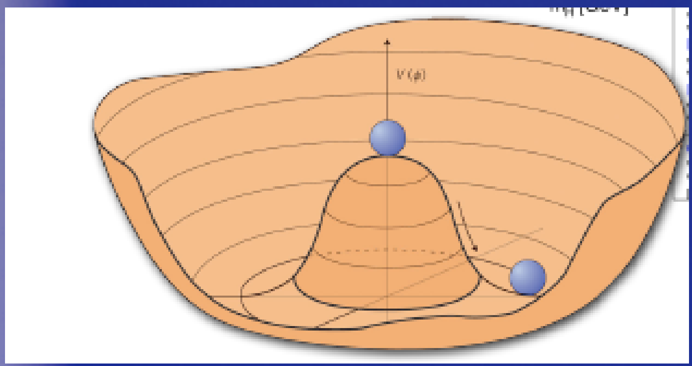
$$D_\mu \phi = \left(\partial_\mu + i\frac{g'}{2} B_\mu - ig\frac{\tau^a}{2} W_\mu^a \right) \phi$$

$$\phi^\dagger \phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

$$\phi^T = (\phi^+, \phi^0)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + h + i\chi \end{pmatrix}$$



$$M_W = \frac{gv}{2}$$

$$M_Z = \frac{M_W}{\cos \theta_W}$$

$$v \simeq 250\,\mathrm{GeV},$$

$$M_H^2 = 2\lambda v^2$$

- We introduce an ad hoc potential to induce the Higgs mechanism.
- We have 4 new degrees of freedom: 3 WBGB and one massive scalar (THE HIGGS BOSON).
- Fermion masses are produced by the Yukawa couplings in a gauge invariant way.
- The theory is unitary and renormalizable.
- The dynamics producing the EWSB is gauge invariant but it is not a gauge interaction
- Light Higgs means weak interactions in the SBS
- The Higgs always appear in the combination *h + v*.

Problems of the Minimal Standard Model

- Origin and nature of the Electroweak Symmetry Breaking

- Light scalars are unnatural because of the big radiative corrections to their masses.

- Vacuum (meta) stability.

- Origin and values of its many parameters
  (masses, elements of the CKM and PMNS* matrices, couplings...)

- The strong CP problem

- Why is $v \ll M_P$?

- Dark matter and dark energy?

- What about gravity?

* Pontecorvo–Maki–Nakagawa–Sakata

So is the new Higgs like particle the MSM Higgs boson?

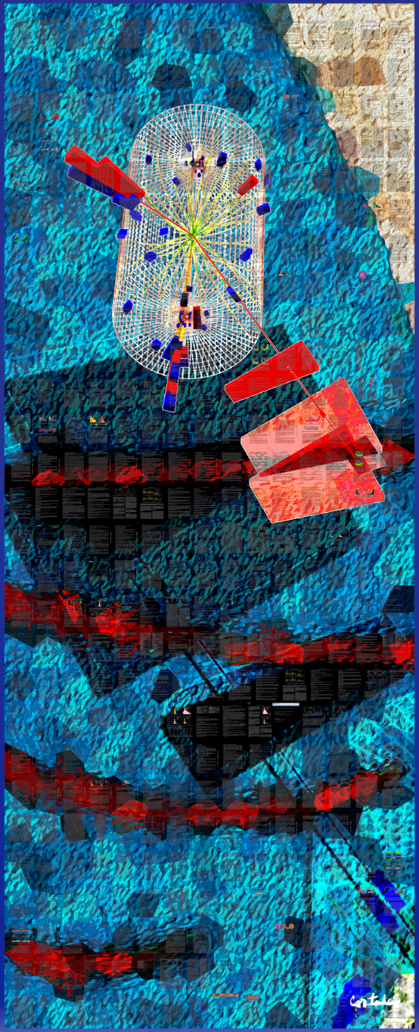
Is it elementary or composite?

The first conclusions after looking at the experimental data is that the new scalar resonance is compatible with the SM Higgs, hence the name of Higgs-like resonance.

However, in there is a lot of room for other possibilities, in particular for a strongly interacting scenario for the EWSBS.

In the following we will concentrate in the compositeness (dynamical symmetry breaking) scenario where the Higgs is a Goldstone boson (GB) associated to some global spontaneous symmetry breaking.

III. Modeling a Strongly Interacting SBS



For describing the physics of the SBS of the SM beyond the MSM under the hypothesis of compositeness at low energies we have to include:

3 WBGB ω^a + one Higgs-like light scalar *h.*

There are at least two possibilities:

a) Linear representation: (SMEFT)

- The ω^a and the h fit in a left $SU(2)$ doublet
- The Higgs always appear in the combination: $h + v$
- Higher symmetry
- Typical situation when h is a fundamental field
- EFT usually based in a cutoff Λ expansion: $O(d) / \Lambda^{d-4}$
  (d = operator dimension, $d = 4,6,8\ldots$)

b) Non-linear representation: (HEFT)

- h is a *SU(2)* singlet and ω^a are coordinates on a coset:

$$SU(2)_L \, X \, SU(2)_R \, / \, SU(2)_V = SU(2) = S^3$$

- Lesser symmetry and more independent higher dimension effective operators but less model depending
- Derivative expansion
- EWChL with *F(h)* insertions
- Appropriate for composite models of the SBS (*h* as a GB)
- Strongly interacting and consistent with the presence of the GAP

SM → SMEFT → HEFT

Alonso, Jenkins and Manohar

Sometimes the difference is difficult to see if we use generalized coordinates.

Important result in QFT: reparametrization invariance of *S* matrix elements

$$G^n(x_1, x_2, \ldots, x_n) = \langle 0|T(\Phi(x_1), \Phi(x_2), \ldots, \Phi(x_n))|0\rangle$$

$$G^n(k_1, k_2, \ldots, k_n) = \int dx_1 dx_2 \ldots dx_n G^n(x_1, x_2, \ldots, x_n) e^{i\sum_{i=1}^n x_i k_i}$$

$$G^n(k_1, k_2, \ldots, k_n) = (2\pi)^4 \delta(k_1 + k_2 + \ldots + k_n) \tilde{G}^n(k_1, k_2, \ldots, k_n)$$

$$\tilde{G}_c^2(k, -k) \simeq \frac{R}{k^2 - M^2}$$

$$\langle q_1, q_2, \ldots, q_m|S|k_1, k_2, \ldots, k_n\rangle = \lim_{k_i^2 \to M^2; q_k^2 \to M^2} \prod_{i=1}^n \frac{i(k_i^2 - M^2)}{R^{1/2}} \prod_{k=1}^m \frac{i(q_k^2 - M^2)}{R^{1/2}} \times G_c^{n+m}(q_1, \ldots, q_m, -k_1, \ldots, -k_n)\,,$$

Lehman-Symanzik-Zimmermann

Field reparametrization:

$$\pi' = \sum_{p=1}^{\infty} a_p \pi^p$$

$$\pi'^\alpha = \pi^\alpha + f^\alpha(\pi)$$

$$G'^n(x_1, x_2, \ldots, x_n) = \langle 0|T(\pi'(x_1), \pi'(x_2), \ldots, \pi'(x_n))|0\rangle$$

$$k_i^2 = M^2$$

$$G'^n(x_1, \ldots, x_n) = \sum_{p_1=1}^{\infty} \cdots \sum_{p_n=1}^{\infty} a_{p_1} \ldots a_{p_n} \langle 0|T(\pi^{p_1}(x_1), \ldots, \pi^{p_n}(x_n))|0\rangle \equiv \sum_{p_1=1}^{\infty} \cdots \sum_{p_n=1}^{\infty} a_{p_1} \ldots a_{p_n} G^n_{p_1, \ldots, p_n}(x_1, \ldots, x_n)$$

$$G^n_{c\ 1,1,\ldots,1}(k_1, k_2, \ldots, k_n) = G_c^n(k_1, k_2, \ldots, k_n)$$

$$G'^n_c(k_1, k_2, \ldots, k_n) \sim G_c^n(k_1, k_2, \ldots, k_n)$$

same *S* matrix elements

In this work we consider the non-linear approach (HEFT) consistent with the GAP and therefore the Higgs will be a GB field associated to some global SSB from a global group G to a subgroup H with the GB living in the coset manifold $M = G / H$.

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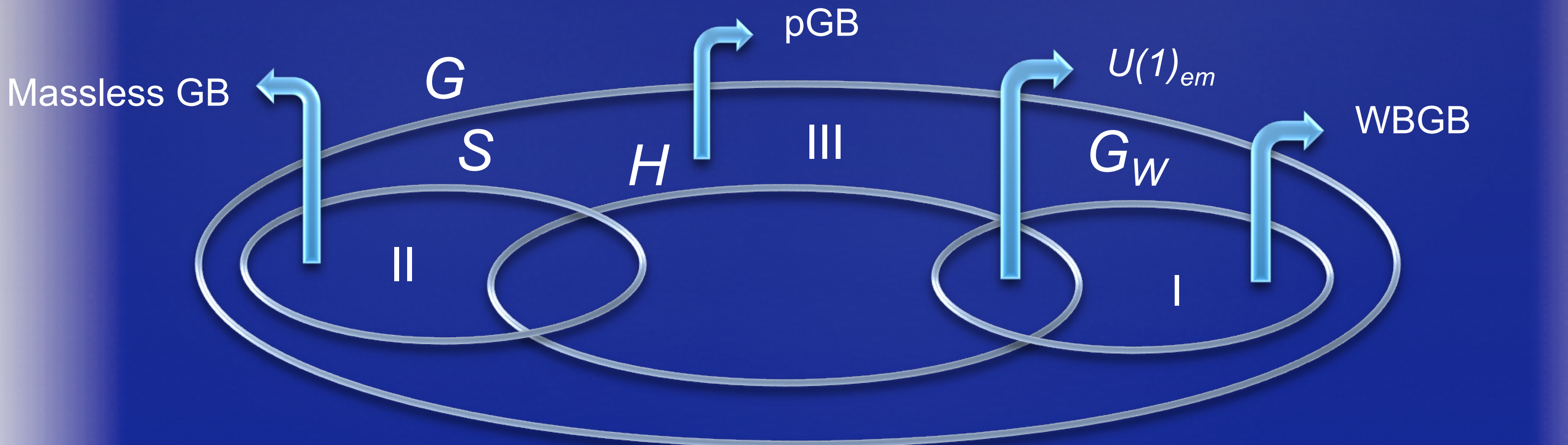
Symmetry breaking pattern for compossite Higgs models

H = spontaneously unbroken subgroup containing custodial group $H_C = SU(2)_{L+R} = SU(2)_C$



G_W = electroweak gauge group = $SU(2)_L \times U(1)_Y$

S = maximal subgroup which commutes with G_W generators

$S \times G_W$ = explicitly broken subgroup

I = W^+, W^- and Z would be GB (WBGB)

II = Massless GB (h,…)

III = massive pseudo GB (extra scalars)

Pokorski

Simplest models (no extra GB but *h* and no pGB)

H = spontaneously unbroken group containing $H_C = SU(2)_{L+R} = SU(2)_C$

G_W = electroweak gauge group = $SU(2)_L \, X \, U(1)_Y$

S = maximal subgroup which commutes with G_W generators

$S \, X \, G_W$ = *explicitly broken subgroup*

I = W^+, W^- and Z would be GB (WBGB)

II = Massless GB (h,…)

The scalar manifold $M = G/H$ with $dim(M) = 3 + 1 = 4$ (3 *WBGB* and the h)

HEFT Lagrangian

Therefore, our effective lagrangian for the EWSBS at low-energy is a gauged NLSM based in the coset *M = G /H (scalar field space)* which has a *h* coordinate with fibre $SU(2)_L \times SU(2)_R/SU(2)_C = SU(2) \simeq S^3$

$$\mathcal{L}_0 = \frac{v^2}{4}\mathcal{F}(h)(D_\mu U)^\dagger D^\mu U + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

$$\mathcal{F}(h) = 1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2 + ..$$

(Gauged) NLSM

U = *WBGB* Fields

$$U = \sqrt{1 - \frac{\omega^2}{v^2}} + i\frac{\bar{\omega}}{v}$$

$$\bar{\omega} = \tau_a \omega^a,$$

$$D_\mu U = \partial_\mu U + W_\mu U - U Y_\mu$$

$$SU(2)_L \times U(1)_Y$$

covariant derivatives

$$\mathcal{L} = \frac{1}{2} G_{\alpha\beta} D_\mu \omega^\alpha D^\mu \omega^\beta$$

$$\omega^\alpha(\pi^a, h)$$

$$a = 1, 2, 3$$

$$\alpha = 1, 2, 3, 4$$

$$\sin\theta = \sqrt{\xi}$$

$$G_{\alpha\beta}(\omega) = \begin{pmatrix} \mathcal{F}(h) g_{ab}(\pi) & 0 \\ 0 & 1 \end{pmatrix}$$

GB space
M = G /H metric

$$g_{ab} = \delta_{ab} + \frac{\pi_a \pi_b}{v^2 - \pi^2}$$

S^3 = metric

If $\mathcal{F}(h^*) = 0$ → $$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^2 + i\phi^1 \\ v + h - i\phi^3 \end{pmatrix}$$ → SMEFT

Alonso, Jenkins and Manohar

NLO-Lagrangian

(extended *Appelquist-Longhitano including h*)

$$\mathscr{L}^h_{\chi=4} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \left(1 + c_G\, \xi\, \mathcal{F}_G(h)\right) - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a \left(1 + c_W\, \xi\, \mathcal{F}_W(h)\right) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \left(1 + c_B\, \xi\, \mathcal{F}_B(h)\right) + \\ + \xi \sum_{i=1}^{5} c_i\, \mathcal{P}_i(h) + \xi^2 \sum_{i=6}^{22} c_i\, \mathcal{P}_i(h) + \xi^3 \sum_{i=23}^{25} c_i\, \mathcal{P}_i(h) + \xi^4\, c_{26}\, \mathcal{P}_{26}(h)\,,$$

$$\begin{aligned}
\mathcal{P}_1(h) &= g\, g'\, B_{\mu\nu} \mathrm{Tr}\left(\mathbf{T}\, W^{\mu\nu}\right) \mathcal{F}_1(h)\\
\mathcal{P}_2(h) &= i\, g'\, B_{\mu\nu} \mathrm{Tr}\left(\mathbf{T}\, [\mathbf{V}^\mu, \mathbf{V}^\nu]\right) \mathcal{F}_2(h)\\
\mathcal{P}_3(h) &= i\, g\, \mathrm{Tr}\left(W_{\mu\nu}\, [\mathbf{V}^\mu, \mathbf{V}^\nu]\right) \mathcal{F}_3(h)\\
\mathcal{P}_4(h) &= i\, g' B_{\mu\nu} \mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\, \partial^\nu \mathcal{F}_4(h)\\
\mathcal{P}_5(h) &= i\, g\, \mathrm{Tr}(W_{\mu\nu} \mathbf{V}^\mu)\, \partial^\nu \mathcal{F}_5(h)\\
\mathcal{P}_6(h) &= \left(\mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}^\mu\right)\right)^2 \mathcal{F}_6(h)\\
\mathcal{P}_7(h) &= \left(\mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}_\nu\right)\right)^2 \mathcal{F}_7(h)\\
\mathcal{P}_8(h) &= g^2\, \left(\mathrm{Tr}\left(\mathbf{T}\, W^{\mu\nu}\right)\right)^2 \mathcal{F}_8(h)\\
\mathcal{P}_9(h) &= i\, g\, \mathrm{Tr}\left(\mathbf{T}\, W_{\mu\nu}\right) \mathrm{Tr}\left(\mathbf{T}\, [\mathbf{V}^\mu, \mathbf{V}^\nu]\right) \mathcal{F}_9(h)\\
\mathcal{P}_{10}(h) &= g\, \epsilon^{\mu\nu\rho\lambda} \mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\mu\right) \mathrm{Tr}\left(\mathbf{V}_\nu\, W_{\rho\lambda}\right) \mathcal{F}_{10}(h)\\
\mathcal{P}_{11}(h) &= \mathrm{Tr}\left((\mathcal{D}_\mu \mathbf{V}^\mu)^2\right) \mathcal{F}_{11}(h)\\
\mathcal{P}_{12}(h) &= \mathrm{Tr}(\mathbf{T}\, \mathcal{D}_\mu \mathbf{V}^\mu)\, \mathrm{Tr}(\mathbf{T}\, \mathcal{D}_\nu \mathbf{V}^\nu)\, \mathcal{F}_{12}(h)\\
\mathcal{P}_{13}(h) &= \mathrm{Tr}([\mathbf{T}\,, \mathbf{V}_\nu]\, \mathcal{D}_\mu \mathbf{V}^\mu)\, \mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu) \mathcal{F}_{13}(h)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{14}(h) &= i\, g\, \mathrm{Tr}(\mathbf{T} W_{\mu\nu}) \mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\, \partial^\nu \mathcal{F}_{14}(h)\\
\mathcal{P}_{15}(h) &= \mathrm{Tr}(\mathbf{T}\, [\mathbf{V}_\mu, \mathbf{V}_\nu]) \mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\, \partial^\nu \mathcal{F}_{15}(h)\\
\mathcal{P}_{16}(h) &= \mathrm{Tr}(\mathbf{V}_\nu\, \mathcal{D}_\mu \mathbf{V}^\mu)\, \partial^\nu \mathcal{F}_{16}(h)\\
\mathcal{P}_{17}(h) &= \mathrm{Tr}(\mathbf{T}\, \mathcal{D}_\mu \mathbf{V}^\mu) \mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\, \partial^\nu \mathcal{F}_{17}(h)\\
\mathcal{P}_{18}(h) &= \mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}^\mu\right)\, \partial_\nu \partial^\nu \mathcal{F}_{18}(h)\\
\mathcal{P}_{19}(h) &= \mathrm{Tr}(\mathbf{V}_\mu\, \mathbf{V}^\mu)\, \partial_\nu \mathcal{F}_{19}(h) \partial^\nu \tilde{\mathcal{F}}_{19}(h)\\
\mathcal{P}_{20}(h) &= \mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}_\nu\right)\, \partial^\mu \mathcal{F}_{20}(h) \partial^\nu \tilde{\mathcal{F}}_{20}(h)\\
\mathcal{P}_{21}(h) &= \left(\mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\mu\right)\right)^2 \partial_\nu \mathcal{F}_{21}(h) \partial^\nu \tilde{\mathcal{F}}_{21}(h)\\
\mathcal{P}_{22}(h) &= \mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\mu\right) \mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\nu\right)\, \partial^\mu \mathcal{F}_{22}(h) \partial^\nu \tilde{\mathcal{F}}_{22}(h)\\
\mathcal{P}_{23}(h) &= \mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}^\mu\right) \left(\mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\nu\right)\right)^2 \mathcal{F}_{23}(h)\\
\mathcal{P}_{24}(h) &= \mathrm{Tr}\left(\mathbf{V}_\mu\, \mathbf{V}_\nu\right) \mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\right) \mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\nu\right) \mathcal{F}_{24}(h)\\
\mathcal{P}_{25}(h) &= \left(\mathrm{Tr}\left(\mathbf{T}\, \mathbf{V}_\mu\right)\right)^2 \partial_\nu \partial^\nu \mathcal{F}_{25}(h)\\
\mathcal{P}_{26}(h) &= \left(\mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\mu\right) \mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\nu\right)\right)^2 \mathcal{F}_{26}(h)\,.
\end{aligned}$$

Alonso, Gavela, Merlo, Rigolin and Yepes

However for *VV* scattering it is enough to consider the much simpler Lagrangian:

LO ECLh (2 derivatives)

$$\mathcal{L}_2 = -\frac{1}{2g^2}\mathrm{Tr}(\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}) - \frac{1}{2g'^2}\mathrm{Tr}(\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}) + \frac{v^2}{4}\left[1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right]\mathrm{Tr}(D^\mu U^\dagger D_\mu U) + \frac{1}{2}\partial^\mu h\,\partial_\mu h + \dots$$

NLO HEFT (4 derivatives) for V_L V_L elastic scattering (V = W, Z))

$$\mathcal{L}_4 = a_4(trV_\mu V_\nu)^2 + a_5(trV_\mu V^\mu)^2 + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2 + \frac{d}{v^2}(\partial_\mu h\partial^\mu h)tr(D_\nu U)^\dagger D^\nu U + \frac{e}{v^2}(\partial_\mu h\partial^\nu h)tr(D^\mu U)^\dagger D_\nu U \qquad V_\mu = D_\mu U U^\dagger$$

One-loop LO and NLO are the same order

It is not consistent to use NLO HEFT without LO one-loop corrections!

Interesting particular cases:

The Minimal Standar Model: $a=b=c=c_i=d_i=1$ $f=v$ $a_i=0$

Linear, renormalizable, unitary and weakly interacting

No Higgs Model $f=v$ $a=b=c=0$ → Old EWCL (ChPT)

Minimal Dilaton Model $h=\varphi$ new scale f $f\neq v$

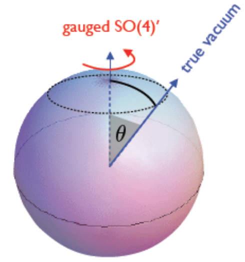
$$V(\varphi)=\frac{M_\varphi^2}{4f^2}(\varphi+f)^2\left[\log\left(1+\frac{\varphi}{f}\right)-\frac{1}{4}\right]$$ $$a^2=b=\frac{v^2}{\hat{f}^2}$$

Halyo, Goldberber, Grinstein, Skiba

Minimal Composite Higgs Model (maximally symmetric spaces)

$$S^4=SO(5)/SO(4)$$ → $$a^2=1-\frac{v^2}{f^2}$$ $$b=1-2\frac{v^2}{f^2}$$

$\xi=v^2/f^2$



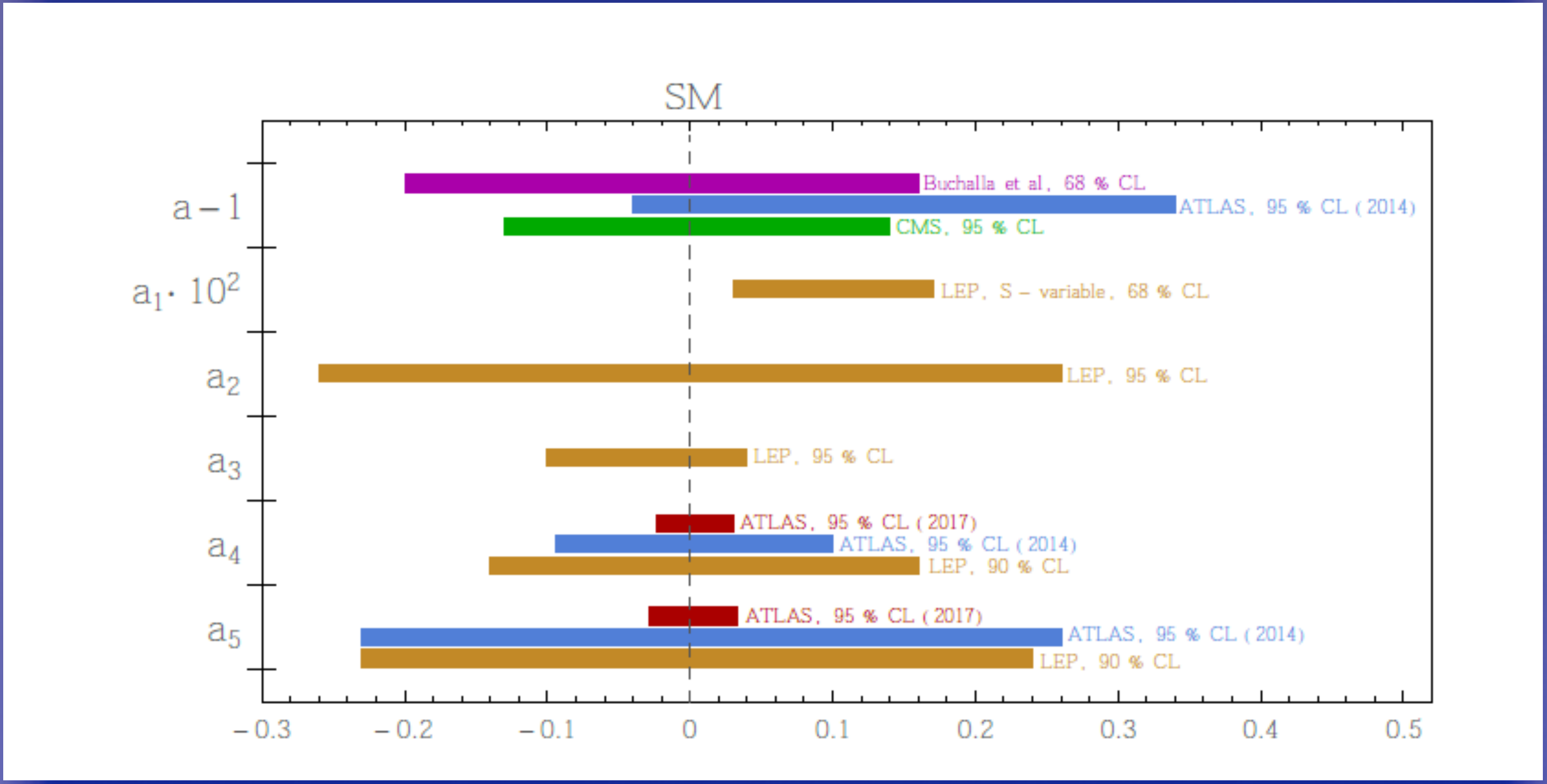
$\sin\theta=\sqrt{\xi}$

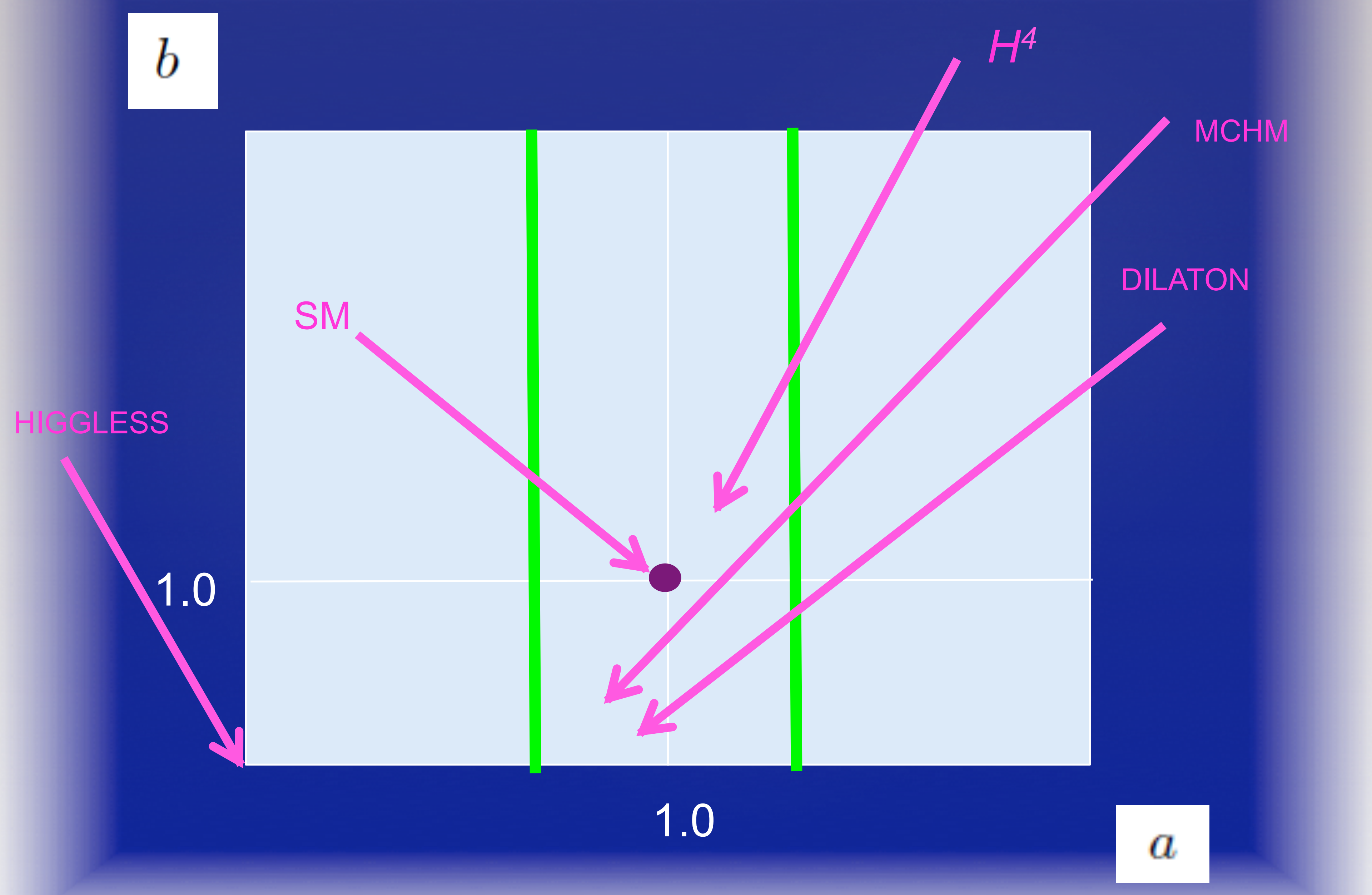
Agashe, Contino, Pomarol, Da Rold

$$\mathcal{H}^4=SO(1,4)/SO(4)$$ → $$a^2=1+\frac{v^2}{f^2}$$ $$b=1+2\frac{v^2}{f^2}$$

Alonso, Jenkins, Manohar

Experimental limits on the HEFT parameters:





b
H^4
MCHM
DILATON
SM
HIGGLESS
1.0
1.0
a

A program for the study a possible strongly interacting scenario for the SBS at the LHC

The only modes at low energies (< 600 GeV) are the WBGB and the Higgs-like particle (most probably composite GB of some highier spontaneously broken symmetry with $dim(G/H) = 4$)

Built an appropriate low-energy HEFT.

Apply the Equivalence Theorem (go to high energies to decople gauge bosons)

Compute the relevant scattering amplitudes at tree level and at the one-loop level (orders s and s^2) ($VV \to VV$, $VV \to hh$, $hh \to hh$, $\gamma\gamma \to VV$, $VV \to t\underline{t}$…)

Unitarize the amplitudes to extrapolate to higher energies (generate resonances dynamically)

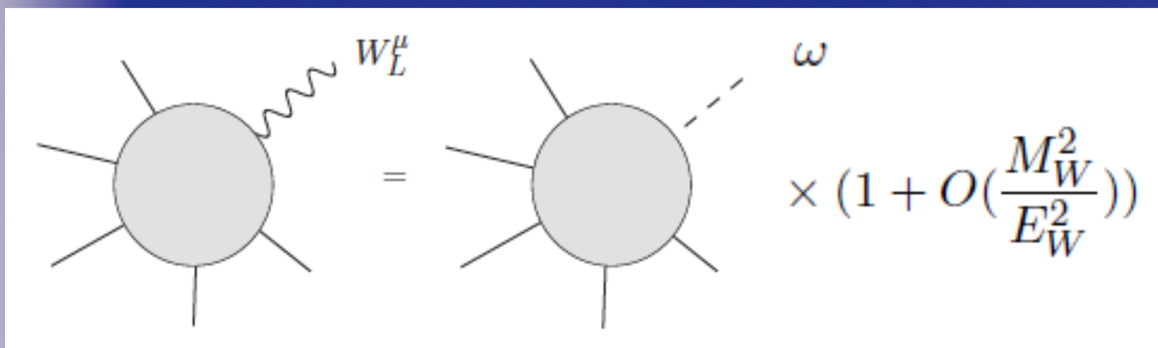
Study the properties of the emerging resonances in terms of the low-energy couplings (make predictions for other processes)

Compare with next year LHC results when possible.

Perform more accurate computations not using the ET or the Equivalent-*W* approximation, include other radiative corrections, the top quark, QCD corrections… to make the results realistic for comparison with data (MC)

The EWSBS dynamics could be studied at the LHC through the High Energy Longitudinal Electroweak Boson Scattering

The Equivalence Theorem (for *R* gauges)



$$T(\omega^a\omega^b \to \omega^c\omega^d) = T(W_L^aW_L^b \to W_L^cW_L^d) + O(\frac{M_W}{\sqrt{s}})$$

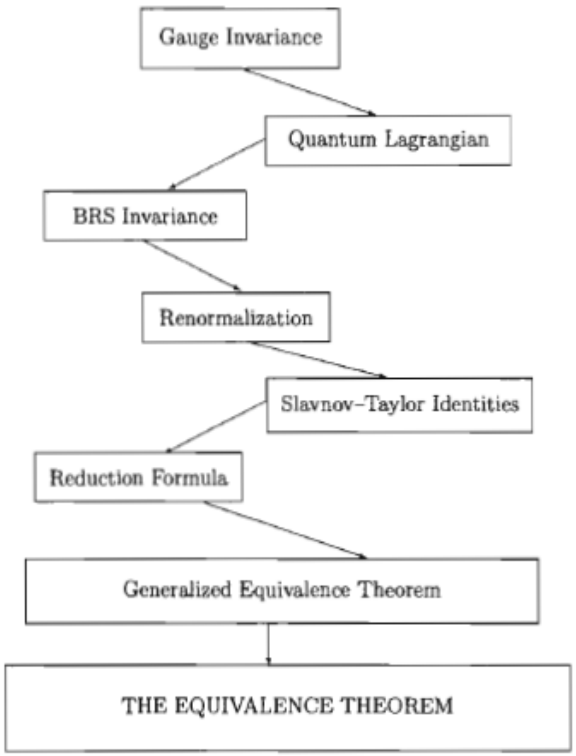


Fig. 7.3. The proof of the equivalence theorem

At high energies the LCGB could become strongly interacting and the TC decouple from the LC which become Goldstone Bosons

The low-energy Effective Lagrangian for $W_L W_L$, $Z_L Z_L$ and hh one-loop scattering

$$M_W^2, M_Z^2, M_h^2 << s << \Lambda^2$$

$$g = g' = H_{YK} = 0$$

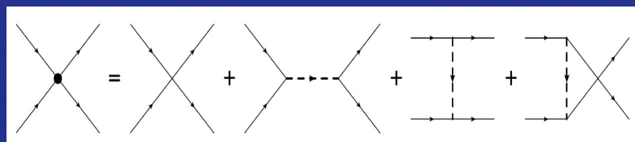
$$\mathcal{L} = \frac{1}{2}\left(1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab} + \frac{\omega^a\omega^b}{v^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b$$
$$+ \frac{4a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2 + \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^a\partial_\nu\omega^a.$$

$$U(x) = \sqrt{1 - \frac{\omega^2}{v^2}} + i\frac{\tilde{\omega}}{v}$$

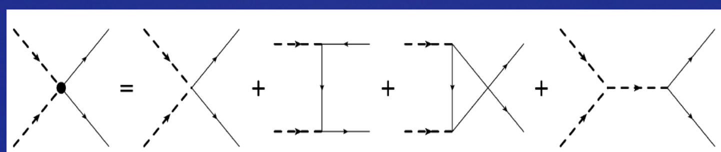
LO amplitudes:
low-energy theorems

$$M_h^2 \ll s < 4\pi v \simeq 3\,\mathrm{TeV}.$$

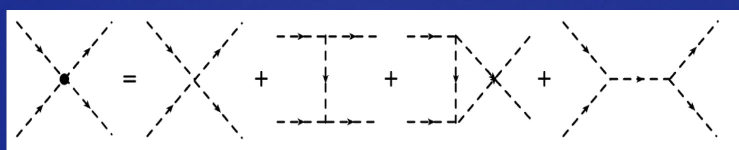
$$M_h = 0$$



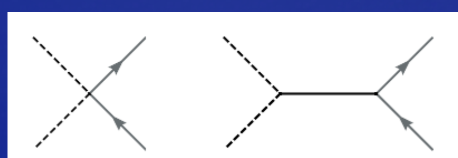
$$T(\omega^+\omega^- \to \omega^+\omega^-) = \frac{s+t}{v^2}(1-a^2)$$



$$T(\omega^a\omega^b \to hh) = \frac{s}{v^2}(a^2-b)\delta_{ab}$$



$$T(hh \to hh) = 0$$



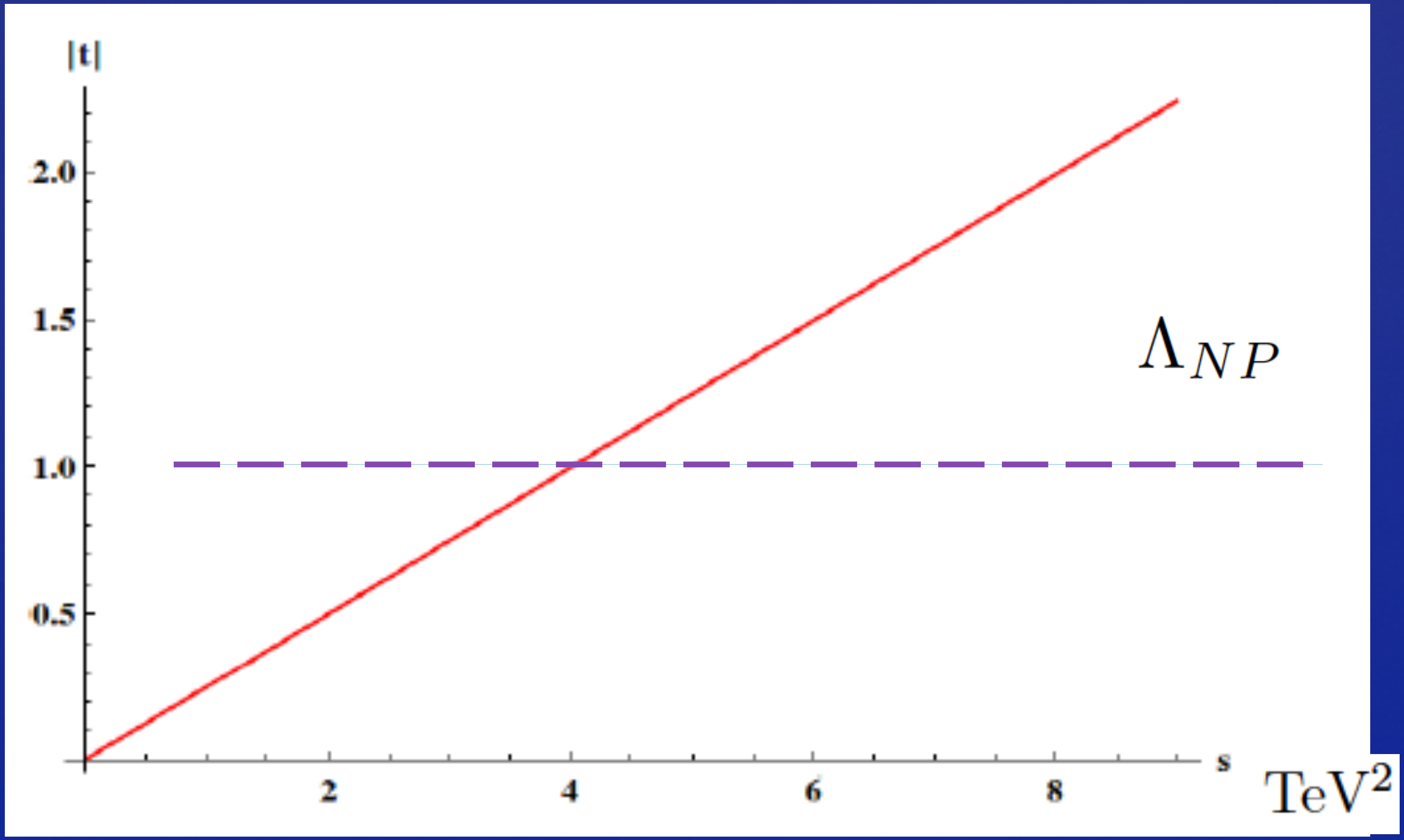
$$T(\omega^a\omega^b \to t_L\bar{t}_R) = \frac{M_t}{v^2}(1-ac)\sqrt{s-4M_t^2}\,\delta_{ab}$$

Those are the generalization of the Weinberg low-energy theorems for pion scattering
The amplitudes generically strongly interacting, grow with the energy and then they badly violate unitarity at some new physics scale: Λ_{NP}

The only exception occurs for *a = b = c = 1* which is the case of the MSM

Contino, Grojean, Moretti, Piccinini, Ratazzi

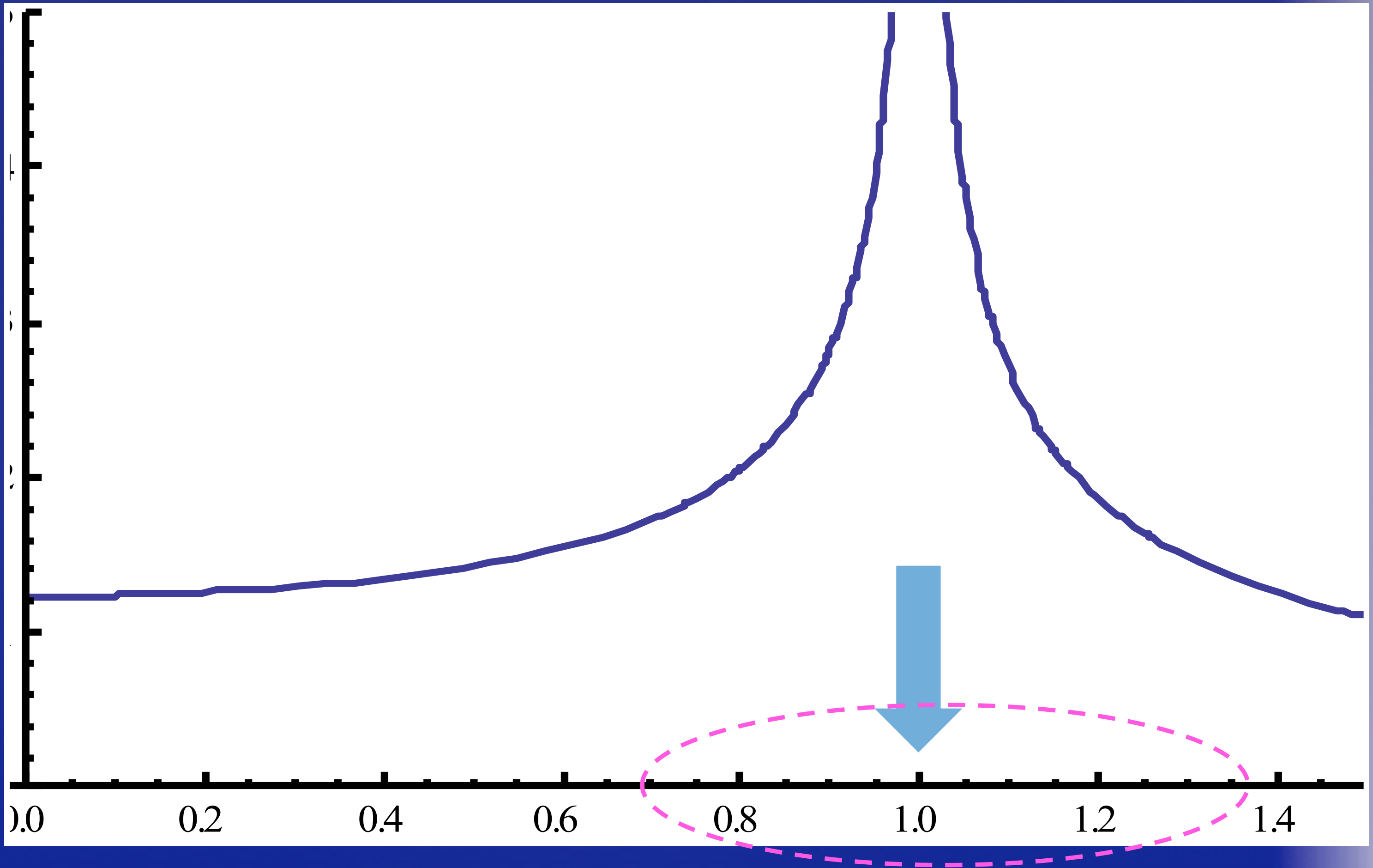
All of these amplitudes violate badly unitarity at some point



New physics scale:

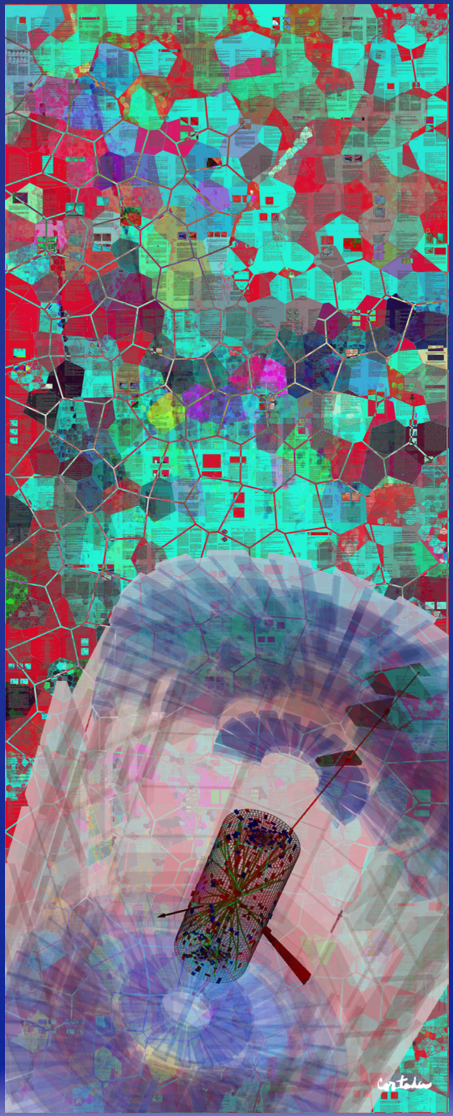
Λ_{NP}

TeV



$a \simeq \kappa_W \simeq \kappa_Z \simeq \kappa_V$

IV. One-loop computations



Electroweak Chiral Perturbation Theory with a light Higgs-like boson up to one-loop for :

$VV \rightarrow VV$, $VV \rightarrow hh$, $hh \rightarrow hh$, $Vh \rightarrow Vh$… $(V=W, Z)$

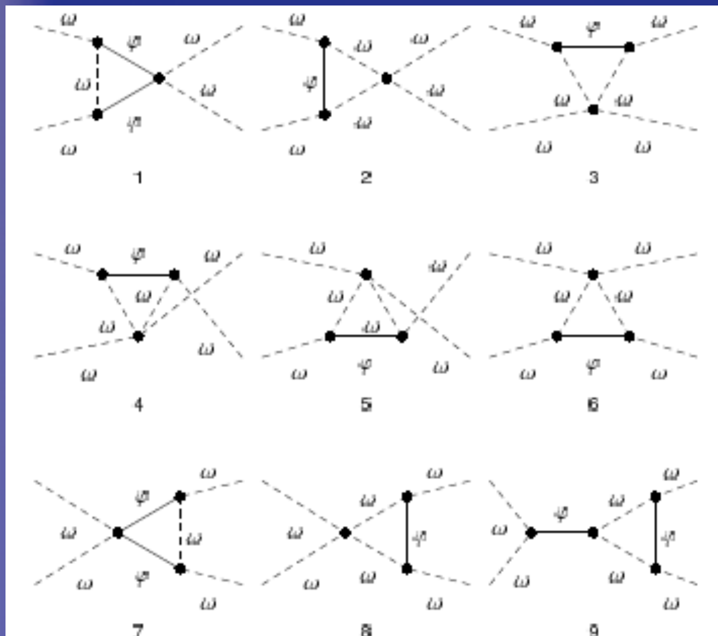
- Equivalence Theorem
- Landau Gauge (massless WBGB and no ghosts at this level)
- No fermions and $g = g' = 0$ (custodial isospin)
- Dimensional regularization
- $\overline{\text{MS}}$ scheme for the NLO derivatives couplings bellow
(no other renormalization is needed for vanishing *h* mass)

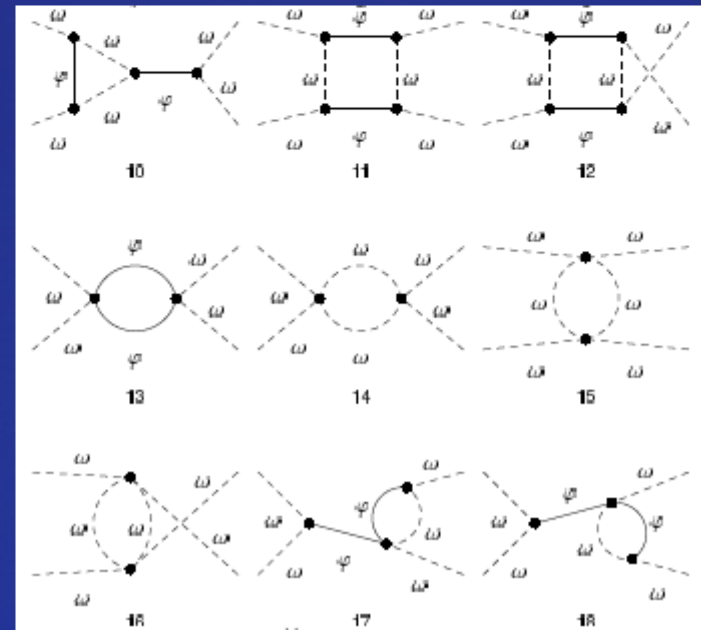
FeynRules: Generates Feynman rules from the Lagrangian as an output produces the input for FeynArts.

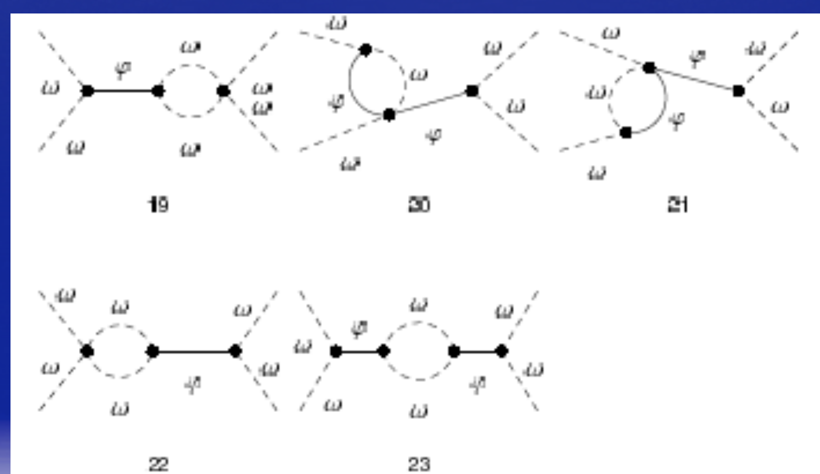
FeynArts: Obtains the Feynman diagrams to some given order. Introduces ”symbolically” the vertices generated by FeynRules.

FormCalc: Simplifies the output by FeynArts and generates an analytical output (and also a FORTRAN output for MC)

One-loop Feynman diagrams for $\omega_a\omega_b \to \omega_c\omega_d$







Electroweak Chiral Perturbation Theory with a light Higgs-like scalar up to one-loop

$\omega\ \omega \longrightarrow \omega\ \omega$ (elastic scattering)

$\omega_a\omega_b \to \omega_c\omega_d$

$$T_{abcd} = A(s,t,u)\delta_{ab}\delta_{cd} + B(s,t,u)\delta_{ac}\delta_{bd} + C(s.t.u)\delta_{ad}\delta_{bc}$$

$$A(s,t,u) = \frac{s}{v^2}(1-a^2) + \frac{4}{v^4}[2a_5^r(\mu)s^2 + a_4^r(\mu)(t^2+u^2)] + \frac{1}{16\pi^2 v^4}\left(\frac{1}{9}(14a^4 - 10a^2 - 18a^2b + 9b^2 + 5)s^2\right.$$
$$+ \frac{13}{18}(a^2-1)^2(t^2+u^2) - \frac{1}{2}(2a^4 - 2a^2 - 2a^2b + b^2 + 1)s^2 \log\frac{-s}{\mu^2} + \frac{1}{12}(1-a^2)^2(s^2 - 3t^2 - u^2)\log\frac{-t}{\mu^2}$$
$$\left. + \frac{1}{12}(1-a^2)^2(s^2 - t^2 - 3u^2)\log\frac{-u}{\mu^2}\right), \quad \text{(A}$$

$$a_4^r(\mu) = a_4^r(\mu_0) - \frac{1}{192\pi^2}(1-a^2)^2 \log\frac{\mu^2}{\mu_0^2},$$
$$a_5^r(\mu) = a_5^r(\mu_0) - \frac{1}{768\pi^2}[3(a^2-b)^2 + 2(1-a^2)^2]\log\frac{\mu^2}{\mu_0^2},$$

Espriu, Yencho, Mescia, A.D., Delgado, Llanes-Estrada

Typical results

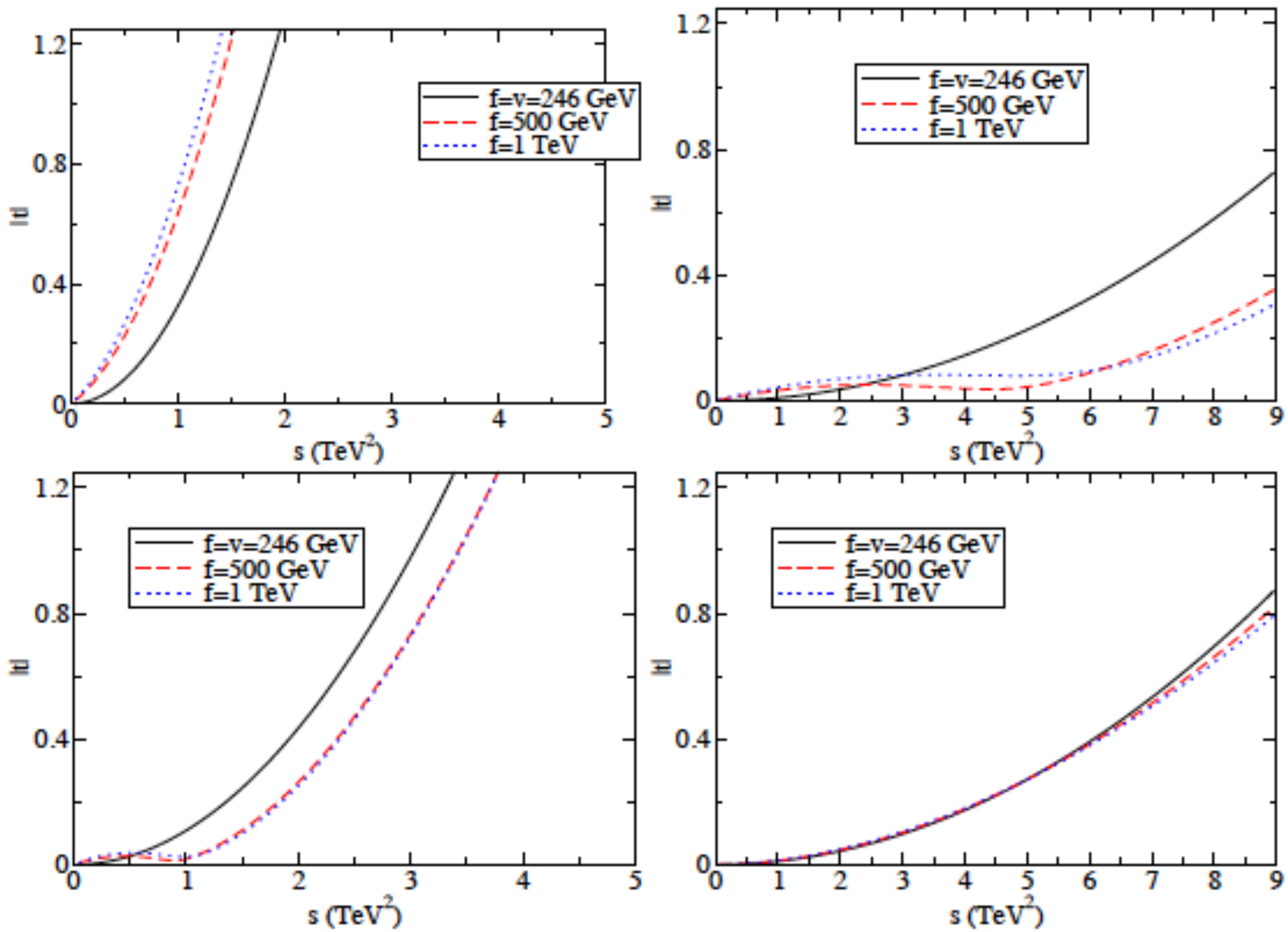
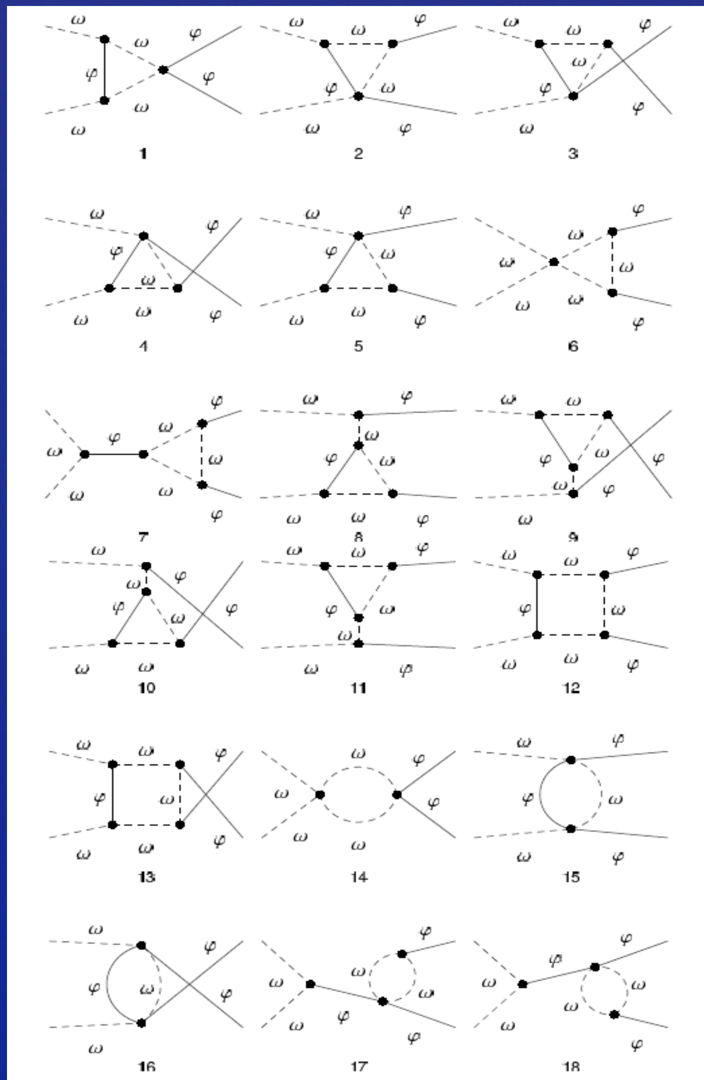
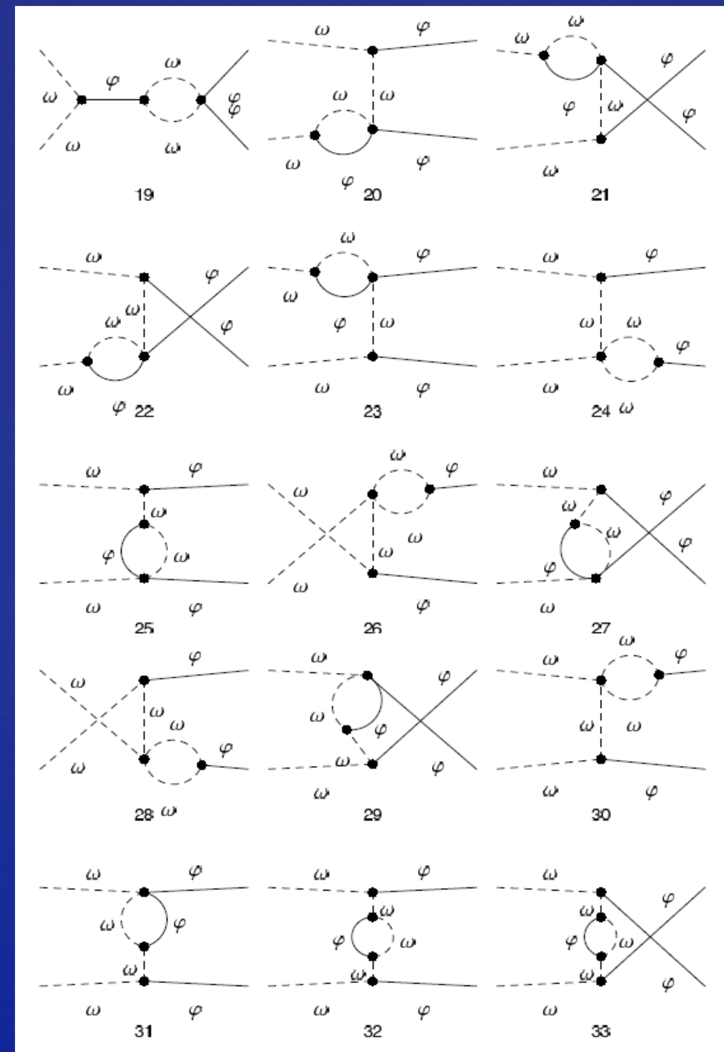


Figure 3. For fixed $a_4 = a_5 = 0.0025$ at $\mu = 1$ TeV, we vary f as indicated and plot the modulus of the perturbative partial wave amplitudes for elastic $ww \to ww$ scattering. In clockwise sense from the top left, we show $|A_{00}|$, $|A_{11}|$, $|A_{02}|$, $|A_{20}|$.

One-loop Feynman diagrams for $\omega_a \omega_b \to hh$





Electroweak Chiral Perturbation Theory with a light Higgs-boson up to one-loop

$$\omega\ \omega \longrightarrow h\ h$$

$$\omega_a \omega_b \to hh \qquad \mathcal{M}_{ab}(s,t,u) = M(s,t,u)\delta_{ab}$$

$$M(s,t,u) = \frac{a^2-b}{v^2}s + \frac{2d^r(\mu)}{v^4}s^2 + \frac{e^r(\mu)}{v^4}(t^2+u^2) + \frac{(a^2-b)}{576\pi^2 v^4}\left\{\left[72 - 88a^2 + 16b + 36(a^2-1)\log\frac{-s}{\mu^2}\right.\right.$$
$$\left. + 3(a^2-b)\left(\log\frac{-t}{\mu^2} + \log\frac{-u}{\mu^2}\right)\right]s^2 + (a^2-b)\left(26 - 9\log\frac{-t}{\mu^2} - 3\log\frac{-u}{\mu^2}\right)t^2$$
$$\left. + (a^2-b)\left(26 - 9\log\frac{-u}{\mu^2} - 3\log\frac{-t}{\mu^2}\right)u^2\right\},$$

$$d^r(\mu) = d^r(\mu_0) + \frac{1}{192\pi^2}(a^2-b)[(a^2-b) - 6(1-a^2)]\log\frac{\mu^2}{\mu_0^2},$$
$$e^r(\mu) = e(\mu_0) - \frac{1}{48\pi^2}(a^2-b)^2\log\frac{\mu^2}{\mu_0^2}.$$

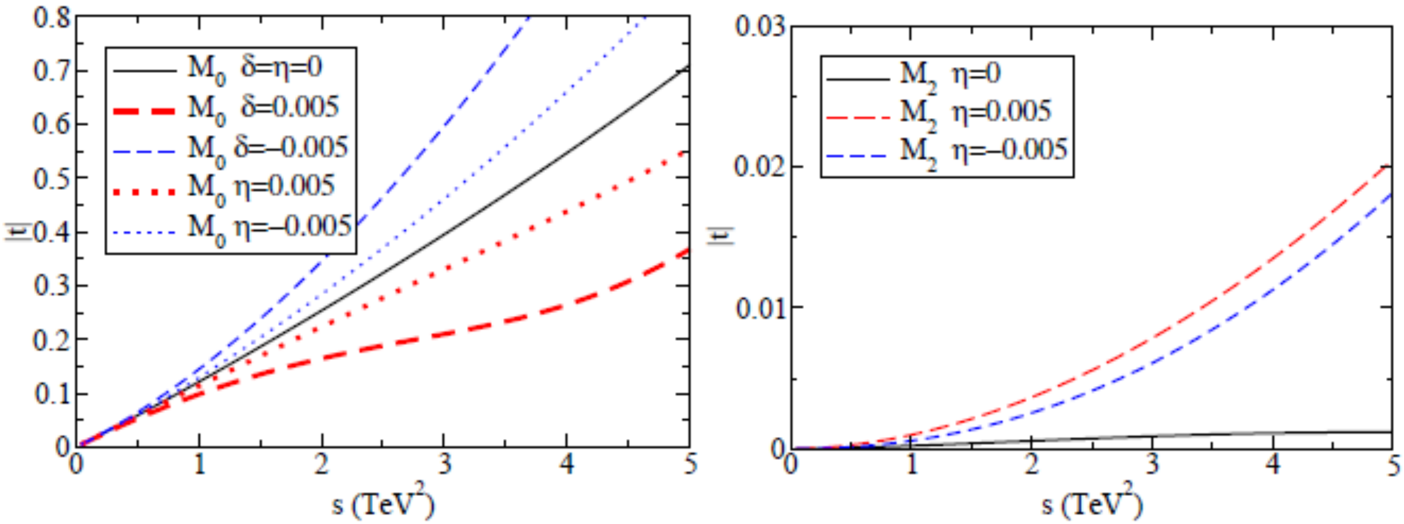
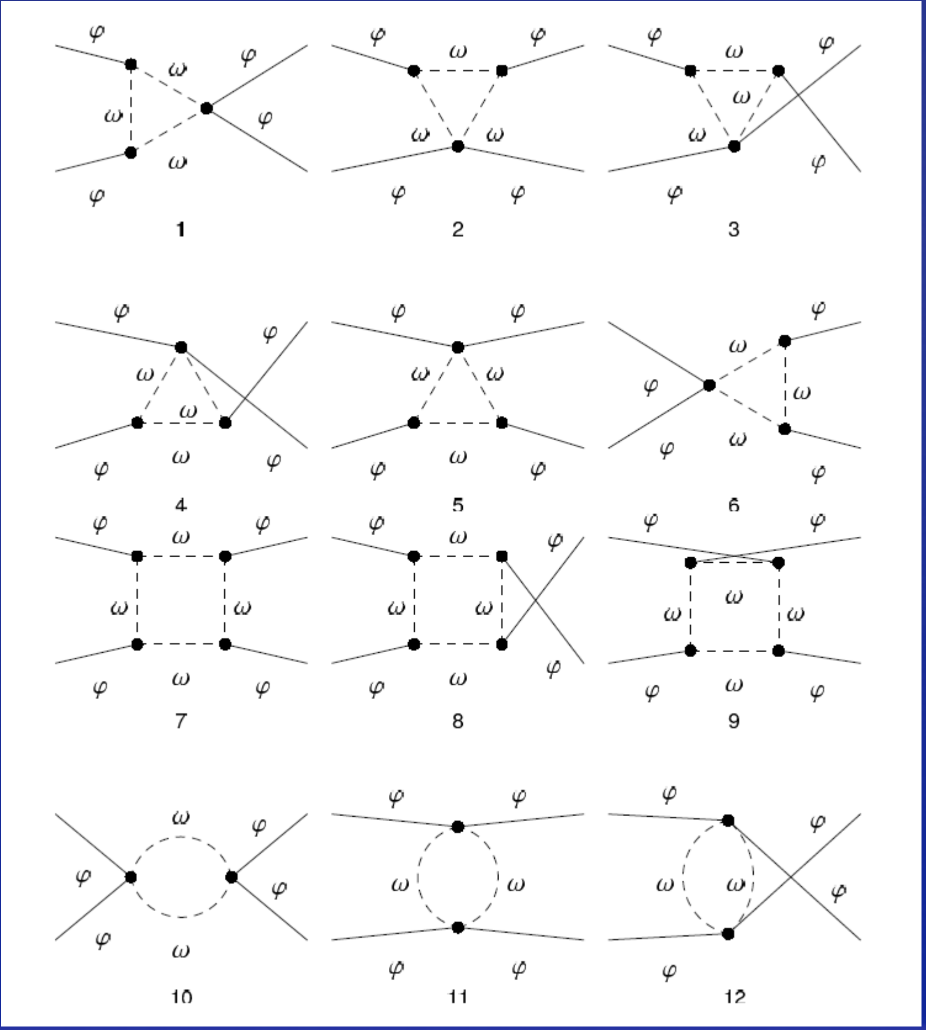


Figure 6. $\omega\omega \to \varphi\varphi$ channel-coupling amplitude in the presence of the NLO δ and η parameters taken at $\mu = 1$ TeV, alternatively. Left: modulus of the scalar partial-wave. Right: modulus of the tensor partial-wave. Note the very different scale.

One-loop Feynman diagrams for $hh \to hh$



Electroweak Chiral Perturbation Theory (with a light Higgs-like scalar) up to one-loop

$$h\,h \Longrightarrow h\,h$$

$$hh \to hh$$

$$T(s,t,u) = \frac{2g^r(\mu)}{v^4}(s^2+t^2+u^2) + \frac{3(a^2-b)^2}{32\pi^2 v^4}\left[2(s^2+t^2+u^2) - s^2\log\frac{-s}{\mu^2} - t^2\log\frac{-t}{\mu^2} - u^2\log\frac{-u}{\mu^2}\right]$$

$$g^r(\mu) = g^r(\mu_0) - \frac{3}{64\pi^2}(a^2-b)^2\log\frac{\mu^2}{\mu_0^2},$$

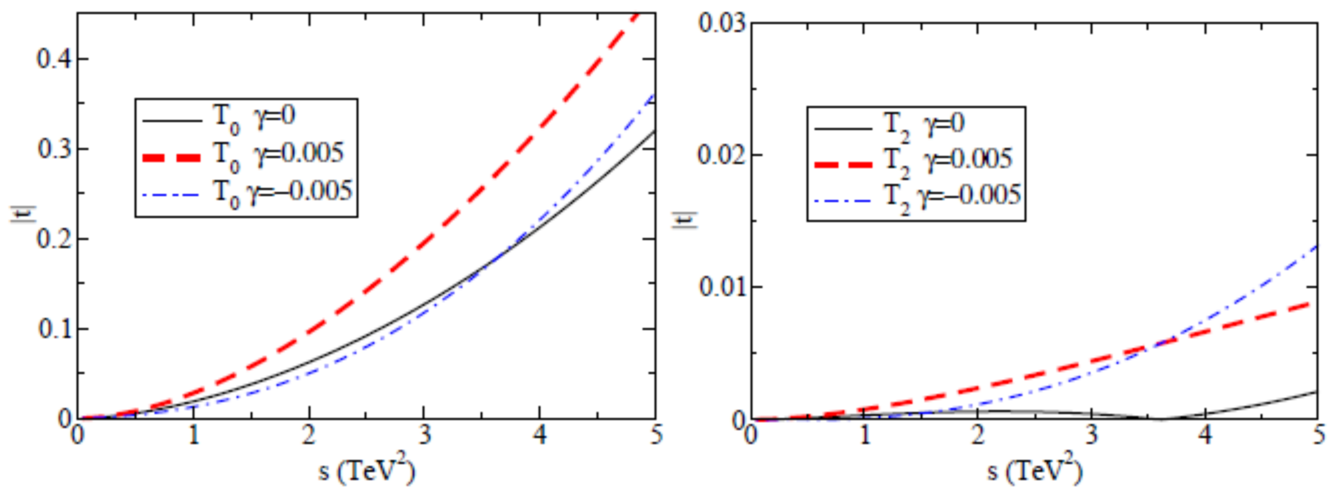


Figure 5. $\varphi\varphi$ elastic scattering in the presence of the NLO γ parameter with $\mu = 1$ TeV. Left: modulus of the scalar partial-wave. Right: modulus of the tensor partial-wave. Note the very different scale.

Electroweak Chiral Perturbation Theory (with a light Higgs-like scalar) up to one-loop

$\omega\, h \Longrightarrow \omega\, h$

$\omega^a h \to \omega^a h$

$T_{II_z}(\omega^{I_z} h \to \omega^{I'_z} h) = M(s,t,u)\delta_{I_z I'_z}$

$$
\begin{aligned}
M(s,t,u) = {} & \frac{a^2-b}{v^2}t + \frac{2d^r(\mu)}{v^4}t^2 + \frac{e^r(\mu)}{v^4}(s^2+u^2) \\
& + \frac{a^2-b}{576\pi^2 v^4}\Bigg[\left(72 - 88a^2 + 16b + 36(a^2-1)\log\frac{-t}{\mu^2}\right. \\
& \left. +3(a^2-b)\left(\log\frac{-s}{\mu^2} + \log\frac{-u}{\mu^2}\right)\right)t^2 \\
& +(a^2-b)\left(26 - 9\log\frac{-s}{\mu^2} - 3\log\frac{-u}{\mu^2}\right)s^2 \\
& +(a^2-b)\left(26 - 9\log\frac{-u}{\mu^2} - 3\log\frac{-s}{\mu^2}\right)u^2\Bigg]
\end{aligned}
$$

The scattering amplitudes need to fulfill a number of properties such as unitarity and analyticity to be physically acceptable,

This is best seen in the partial waves: $\omega\,\omega \longrightarrow \omega\,\omega$

$$\begin{aligned} A_0(s,t,u) &= 3A(s,t,u) + A(t,s,u) + A(u,t,s) \\ A_1(s,t,u) &= A(t,s,u) - A(u,t,s) \\ A_2(s,t,u) &= A(t,s,u) + A(u,t,s)\ . \end{aligned}$$

custodial isospin amplitudes *I = 0 ,1 ,2*

$$A_{IJ}(s) = \frac{1}{64\pi}\int_{-1}^{1} d(\cos\theta)\, P_J(\cos\theta)\, A_I(s,t,u)$$

$I \neq 0$

$$A_{IJ}(s) = A^{(0)}_{IJ}(s) + A^{(1)}_{IJ}(s) + \ldots,$$

$$\mathrm{Im}\, A_{IJ} = |A_{IJ}|^2 \quad I \neq 0$$

exact elastic unitarity on the RC

$$\begin{aligned} A^{(0)}_{IJ}(s) &= K s \\ A^{(1)}_{IJ}(s) &= \left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{-s}{\mu^2}\right) s^2 \end{aligned}$$

LO

NLO

$$\mathrm{Im}\, A^{(1)}_{IJ} = |A^{(0)}_{IJ}|^2 \quad I \neq 0$$

perturbative unitarity

$\longrightarrow$

$$|\, A_{IJ}\, |^2 \leq 1$$

necessary condition
(not sufficient)

$IJ = 00$

$$K_{00} = \frac{1}{16\pi v^2}(1-a^2),$$

$$B_{00}(\mu) = \frac{1}{9216\pi^3 v^4}[101(1-a^2)^2 + 68(a^2-b)^2 + 768\{7a_4(\mu) + 11a_5(\mu)\}\pi^2],$$

$$D_{00} = -\frac{1}{4608\pi^3 v^4}[7(1-a^2)^2 + 3(a^2-b)^2],$$

$$E_{00} = -\frac{1}{1024\pi^3 v^4}[4(1-a^2)^2 + 3(a^2-b)^2].$$

$\omega_a\omega_b \to \omega_c\omega_d$

$$K'_0 = \frac{\sqrt{3}}{32\pi v^2}(a^2-b),$$

$$B'_0(\mu) = \frac{\sqrt{3}}{16\pi v^4}\left[d(\mu) + \frac{e(\mu)}{3}\right] + \frac{\sqrt{3}}{18432\pi^3 v^4}(a^2-b) \times [72(1-a^2) + (a^2-b)],$$

$$D'_0 = -\frac{\sqrt{3}(a^2-b)^2}{9216\pi^3 v^4},$$

$$E'_0 = -\frac{\sqrt{3}(a^2-b)(1-a^2)}{512\pi^3 v^4}, \qquad \text{(A}$$

$\omega\omega \to hh$

$$K'_2 = 0,$$

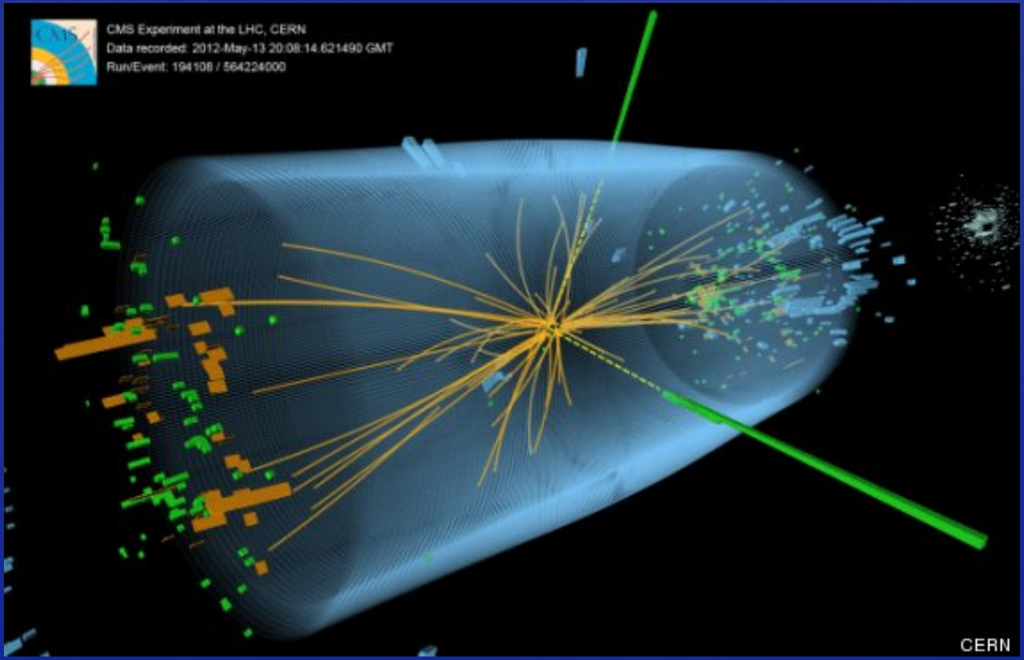
$$B'_2(\mu) = \frac{e(\mu)}{160\sqrt{3}\pi v^4} + \frac{83(a^2-b)^2}{307200\sqrt{3}\pi^3 v^4},$$

$$D'_2 = -\frac{(a^2-b)^2}{7680\sqrt{3}\pi^3 v^4},$$

$$E'_2 = 0.$$

$hh \to hh$

V. Unitarization methods



Properties of the partial waves

- IR, UV finite and μ independent
- Unitary
- Right low-energy limit matching the NLO results
- Proper analytical structure (right (*R*) and (*L*) cuts)
- No poles in the first Riemann sheet
- The poles in the second Riemann sheet can be understood as dynamically generated resonances
- Admit extensions for coupled channels (*hh*, $\gamma\gamma$, or $t\,\underline{t}$)

Perturbative one-loop amplitudes have *L* and R cut, no poles and are unitary only at low energies.

Thus they must be complemented with dispersion relations to be physically acceptable!!!

The Inverse Amplitude Method

A.D. , Herrero, Truong, Pelaez…

$$A(s) = A^{NLO}(s) + O(s^3)$$

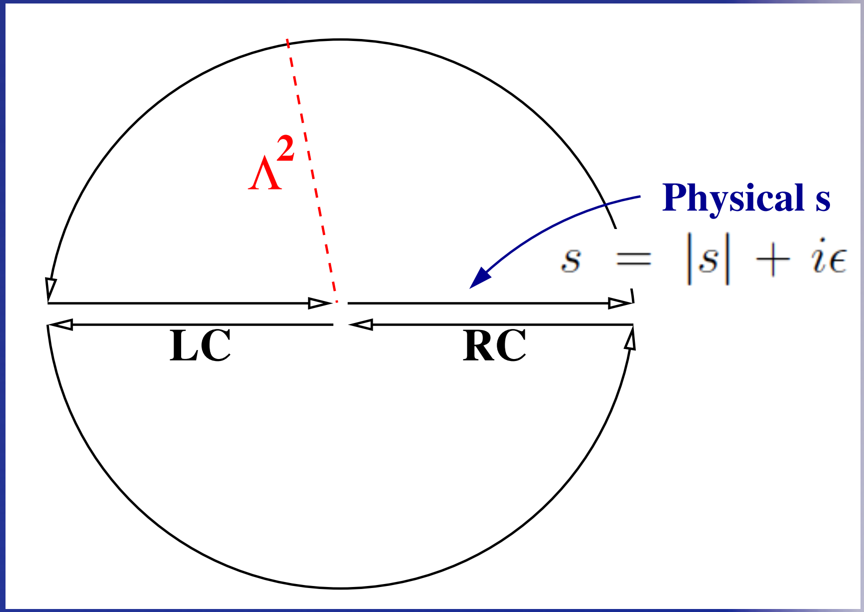
$$I \neq 0$$

$$A^{NLO}(s) = A^{(0)}(s) + A^{(1)}(s)$$

$$A^{(0)}(s) = Ks$$
$$A^{(1)}(s) = s^2\left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{-s}{\mu^2}\right)$$

$$\mathrm{Im}\,A^{(1)} = (A^{(0)})^2 \qquad K^2 = -E\pi$$

$$A^{\mathrm{NLO}}(s) = Ks + \frac{s^2}{\pi}\int_0^{\Lambda^2}\frac{ds'\,\mathrm{Im}\,A^{(1)}(s')}{s'^2(s'-s-i\epsilon)} + \frac{s^2}{\pi}\int_{-\Lambda^2}^{0}\frac{ds'\,\mathrm{Im}\,A^{(1)}(s')}{s'^2(s'-s-i\epsilon)} + \frac{s^2}{2\pi i}\int_{C_\Lambda}\frac{ds'\,A^{(1)}(s')}{s'^2(s'-s)}.$$



$$B(\mu) = B(\mu_0) + (D+E)\log\frac{\mu^2}{\mu_0^2}$$

$$A^{NLO}(s) = b_1 s + b_2 s^2 + Ds^2\log\frac{s}{\mu^2} + Es^2\log\frac{-s}{\mu^2} \Rightarrow \begin{cases} b_1 = K \\ b_2 = B(\mu^2) \end{cases}$$

inverse amplitude

$$w(s)\equiv\frac{(A^{(0)}(s))^2}{A(s)}$$

$$w(s)=Ks+O(s^2)$$

$$w(s)=Ks+\frac{s^2}{\pi}\int_0^{\Lambda^2}\frac{ds'\mathrm{Im}w(s')}{s'^2(s'-s-i\epsilon)}+\frac{s^2}{\pi}\int_{-\Lambda^2}^{0}\frac{ds'\mathrm{Im}w(s')}{s'^2(s'-s-i\epsilon)}+\frac{s^2}{2\pi i}\int_{C_\Lambda}\frac{ds'w(s')}{s'^2(s'-s)}$$

RC $\mathrm{Im}w(s)=-(A^{(0)}(s))^2$

LC $\mathrm{Im}w(s)\simeq-\mathrm{Im}A^{(1)}(s)$

$$w(s)\simeq A^{(0)}(s)-A^{(1)}(s)$$

$$A(s)\simeq A^{IAM}(s)=\frac{(A^{(0)}(s))^2}{A^{(0)}(s)-A^{(1)}(s)}$$

$$\mathrm{Im}A^{IAM}=A^{IAM}(A^{IAM})^*$$

$$A^{IAM}(s)=A^{NLO}(s)+O(s^3)$$

The IAM method produces:

Unitary amplitudes with the same low energy limit as the NLO, the proper analytical structure which can have poles in the second Riemann sheet reproducing new resonances. Extension to coupled channels.

$$F_{IJ}^{IAM}=F_{IJ}^{(0)}(F_{IJ}^{(0)}-F_{IJ}^{(1)})^{-1}F_{IJ}^{(0)}$$

$$\mathrm{Im}\,F_{IJ}^{IAM}=F_{IJ}^{IAM}(F_{IJ}^{IAM})^\dagger$$

The N/D (adapted) Method

$$A(s) = \frac{N(s)}{D(s)}$$

RC $\mathrm{Im}\,A(s) = |A(s)|^2$ → $\mathrm{Im}\,D(s) = -N(s)$

LC → $\mathrm{Im}\,N(s) = D(s)\mathrm{Im}\,A(s)$

$$D(s) = 1 - \frac{s}{\pi}\int_0^\infty \frac{ds' N(s')}{s'(s'-s-i\epsilon)},$$

$$N(s) = \frac{s}{\pi}\int_{-\infty}^0 \frac{ds' D(s')\mathrm{Im}\,A(s')}{s'(s'-s-i\epsilon)}.$$

$$A^{\mathrm{NLO}}(s) = Ks + \left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{-s}{\mu^2}\right)s^2$$ →

left-right spliting

$$A_L(s) \equiv \left(\frac{B(\mu)}{D+E} + \log\frac{s}{\mu^2}\right)Ds^2,$$

$$A_R(s) \equiv \left(\frac{B(\mu)}{D+E} + \log\frac{-s}{\mu^2}\right)Es^2.$$

$$g(s) = \frac{1}{\pi}\left(\frac{B(\mu)}{D+E} + \log\frac{-s}{\mu^2}\right)$$

one-loop function
(μ independent)

$A^{(1)}(s) = A_L(s) + A_R(s)$ →

$$A_L(s) = \pi g(-s)Ds^2,$$

$$A_R(s) = \pi g(s)Es^2,$$

$N_0(s) \equiv A^{(0)}(s) + A_L(s)$ → $D_0(s) = 1 - \frac{A_R(s)}{A^{(0)}(s)} + \frac{\pi}{2}[(g(s)]^2 Ds^2.$ → $A(s) \simeq A^{\mathrm{N/D}}(s) = \frac{N_0(s)}{D_0(s)},$

$$A^{\mathrm{N/D}}(s) = A^{(0)}(s) + A^{(1)}(s) + O(s^3)$$

proper low energy behavior

$$\mathrm{Im}A^{\mathrm{N/D}}(s) = |A^{\mathrm{N/D}}(s)|^2$$

elastic unitarity

$$\mathrm{Im}F^{\mathrm{N/D}} = F^{\mathrm{N/D}}(F^{\mathrm{N/D}})^\dagger$$

coupled channels

The (improved) K Matrix Method

$$S = \frac{1 - iK/2}{1 + iK/2}.$$

S unitary

$$K = \frac{i(S-1)}{1 + (S-1)/2}$$

K hermitian

$$K = K^{(1)} + K^{(2)} + \cdots,$$

any hermitian expansion

$$S = 1 + \tilde{S}^{(1)} + \tilde{S}^{(2)} + \cdots,$$

order by order unitary expansion

$$A_0(s)$$

approximate real partial wave
(typically tree level result)

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)}$$

$$\mathrm{Im}\ A_0^K = |A_0^K|^2 = \frac{A_0^2}{1 + A_0^2}$$

unitary but not analytical!!
no Right cut!

$$g(s) = \frac{1}{\pi}\left(\frac{B(\mu)}{D+E} + \log\frac{-s}{\mu^2}\right)$$

$$-i \to g(s)$$

$$\mathrm{Im}\, g(s) = -1$$

$$A_0(s) = A^{(0)}(s) + A_L(s)$$

$$A^{\mathrm{IK}}(s) = \frac{A_0(s)}{1 + g(s)A_0(s)}$$

$$A^{\mathrm{IK}}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}} - \frac{A_L(s)A_R(s)}{(A^{(0)})^2}}$$

proper low energy behavior, elastic unitarity, analytical and extensible to coupled channels

Summary of the various unitarization methods (IAM, N/D and IK matrix)

All the partial waves are:

- IR, UV finite and μ independent
- Unitary
- Right low energy limit matching the effective theory
- Proper analytical structure (cuts and poles)
- The poles in the second Riemann sheet can be understood as dynamically generated resonances
- Admit extensions for coupled channels
- (*hh*, $\gamma\gamma$, or *t* anti *t*)

$$
\begin{aligned}
A^{\mathrm{IAM}}(s) &= \frac{[A^{(0)}(s)]^2}{A^{(0)}(s)-A^{(1)}(s)} \\
&= \frac{A^{(0)}(s)+A_L(s)}{1-\frac{A_R(s)}{A^{(0)}(s)}-\left(\frac{A_L(s)}{A^{(0)}(s)}\right)^2+g(s)A_L(s)} \\
A^{\mathrm{N/D}}(s) &= \frac{A^{(0)}(s)+A_L(s)}{1-\frac{A_R(s)}{A^{(0)}(s)}+\frac{1}{2}g(s)A_L(-s)} \\
A^{\mathrm{IK}}(s) &= \frac{A^{(0)}(s)+A_L(s)}{1-\frac{A_R(s)}{A^{(0)}(s)}+g(s)A_L(s)}.
\end{aligned}
$$

But:

- They have different contributions from the LC
- They can be different at high energies
- N/D and IK requires R/L splitting not possible when $D + E = 0$. This is the case of the vector channel when $a^2 = b$ (QCD like models).
- IAM is not defined for $K = E = 0$. ($J = 2$).

$$
\begin{aligned}
A^{\mathrm{NLO}}(s) &= A^0(s)+A^{(1)}(s) = A^{\mathrm{IAM}}(s)+O(s^3) \\
&= A^{\mathrm{N/D}}(s)+O(s^3) = A^{\mathrm{IK}}(s)+O(s^3).
\end{aligned}
$$

Whenever $E + D$ is not close to 0, $A_L << A_R$ and the three methods produce similar results. IAM can be applied in this case too so it becomes the method of choice provided K is different from zero

IJ	00	02	11	20	22
Method of choice	Any	N/D IK	IAM	Any	N/D IK

The IAM, N/D and IIK produce similar resuls qualitatively and in many times also quantitatively, at least in the scalar channels

This is not the case of the naive K matrix because it is not analitical

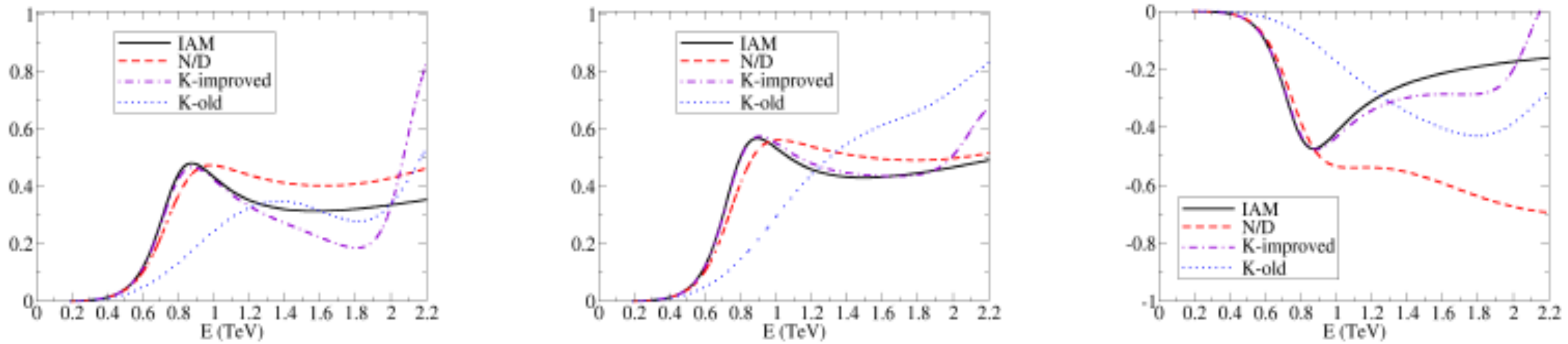
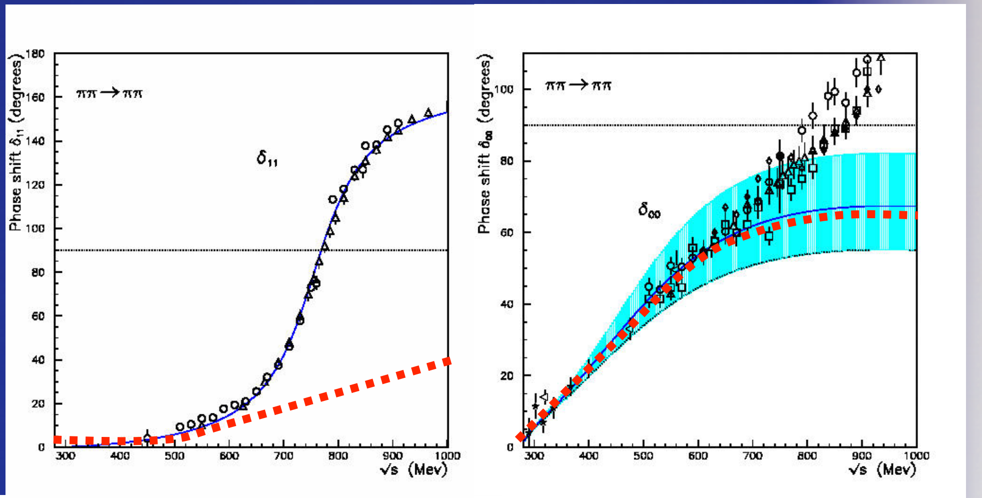


FIG. 2: Scalar-isoscalar amplitudes (from left to right, elastic $\omega\omega$, elastic hh, and cross-channel $\omega\omega \to hh$), for $a = 0.88$, $b = 3$, and all NLO parameters set to 0 at a scale $\mu = 3$ TeV. Note that, as explained on sec. VI A, the old K-matrix method gives different results because its complex-s plane analytic structure is not the correct one. It will be discarded from now on.

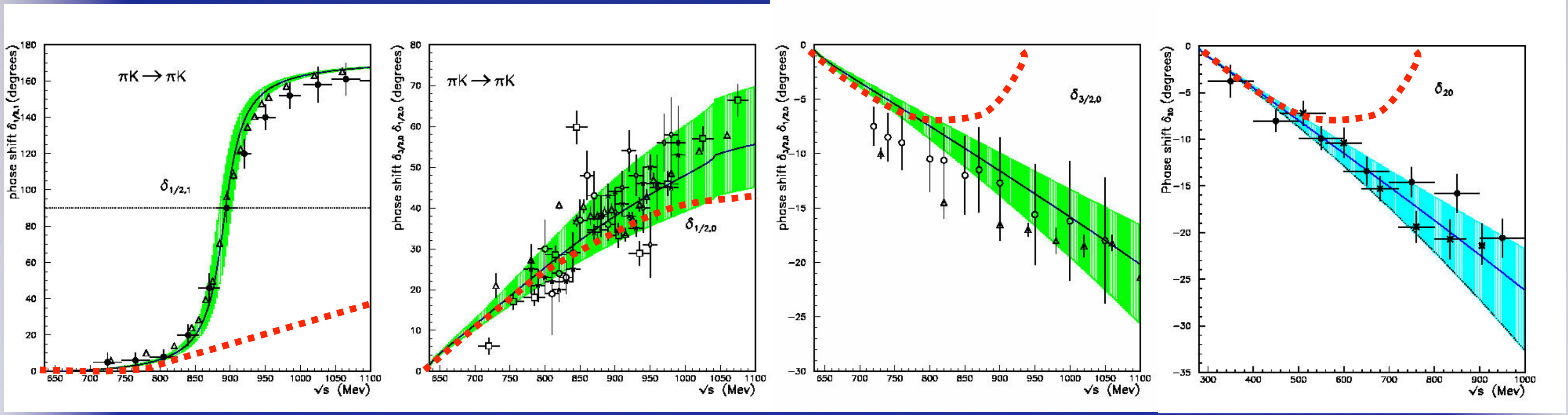
The Inverse Amplitude Method in ChPT

Chiral Perturbation Theory plus Dispersion Relations

Simultaneous description of $\pi\pi \to \pi\pi$ and $\pi K \to \pi K$ up to 800-1000 *MeV* including resonances

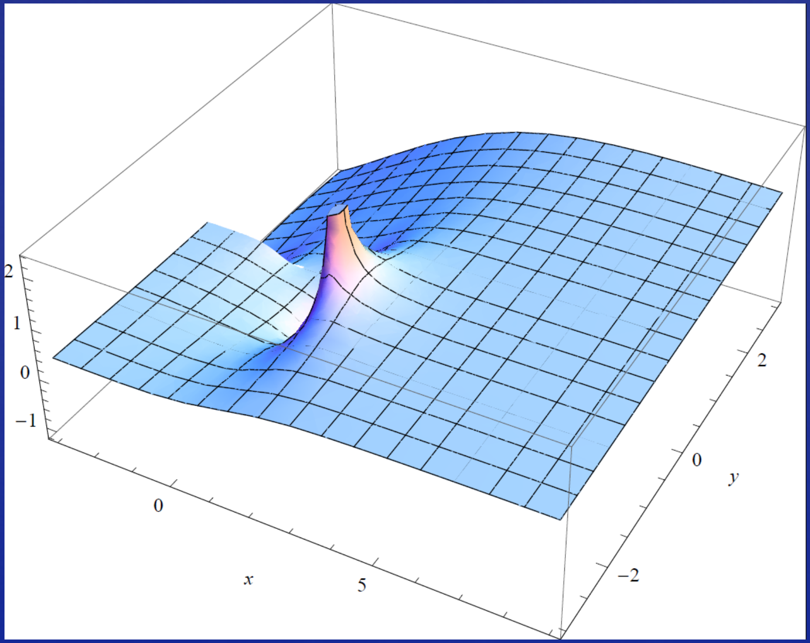


Lowest order ChPT (Weinberg Theorems) and even one-loop computations are only valid at very low energies



A.D., Pelaez

VI. Resonances



Strongly interacting systems are expected to have resonances!

They can be included explicitly in the effective theory

(Ecker, Gasser, Pich, de Rafael)

$$\mathscr{L}_2 = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U\rangle$$

$$\mathscr{L}_4 = L_1 \langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U\rangle \langle D^\mu U^\dagger D^\nu U\rangle + \ldots$$

$$\mathscr{L}_R = \sum_i \left\{ \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\mu\nu}\rangle + \frac{iG_{V_i}}{\sqrt{2}} \langle V_i^{\mu\nu} u_\mu u_\nu\rangle + \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\mu\nu}\rangle + c_{d_i} \langle S_i u^\mu u_\mu\rangle + c_{m_i} \langle S_i \chi_+\rangle + i d_{m_i} \langle P_i \chi_-\rangle \right\},$$

$$2L_1 = L_2 = \frac{1}{4}L_9 = -\frac{1}{3}L_{10} = \frac{f^2}{8M_V^2},$$

$$L_3 = -\frac{3f^2}{8M_V^2} + \frac{f^2}{8M_S^2}, \qquad L_5 = \frac{f^2}{4M_S^2}, \qquad L_8 = \frac{3f^2}{32M_S^2}$$

assuming dominance of the first resonance

For QCD this approximation works very well phenomenologically

Thus the chiral parameters carry information about the resonances (mass, width and coupling)

Therefore one could try another approach. By unitarizating the effective theory results to higher energies in a way compatible with unitarity and analiticity, one can obtain amplitudes which may show poles in the second Riemann sheet. Those poles can be understood as dinamically generated Resonances. Their location and residues (mass, width and coupling) are a function of the low-energy couplings.

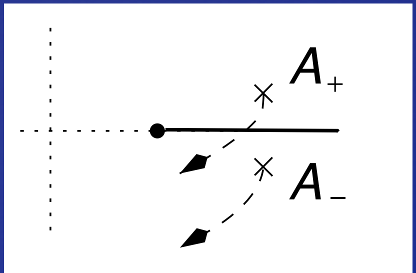
The position of the poles in the second Riemann sheet, understood as dynamically generated resonances, is relatated with the resonance parameters as:

$$s_R = M^2 - i\Gamma M$$

By using the Schwarz reflexion principle

$$A^{II}(s) = \frac{A(s)}{1 - 2iA(s)}$$

$$A(s + i\epsilon) \;=\; F\left(A(s - i\epsilon)\right)$$



Gribov

$$A(s_R) + \frac{i}{2} = 0$$

$$A^{(0)}(s_R) - A^{(1)}(s_R) - 2i[A^{(0)}(s_R)]^2 = 0$$

IAM

$$\begin{aligned} M &= M(a, b, a_4(\mu), a_4(\mu), d(\mu), e(\mu), g(\mu); \mu) \\ \Gamma &= \Gamma(a, b, a_4(\mu), a_5(\mu), d(\mu), e(\mu), g(\mu); \mu). \end{aligned}$$

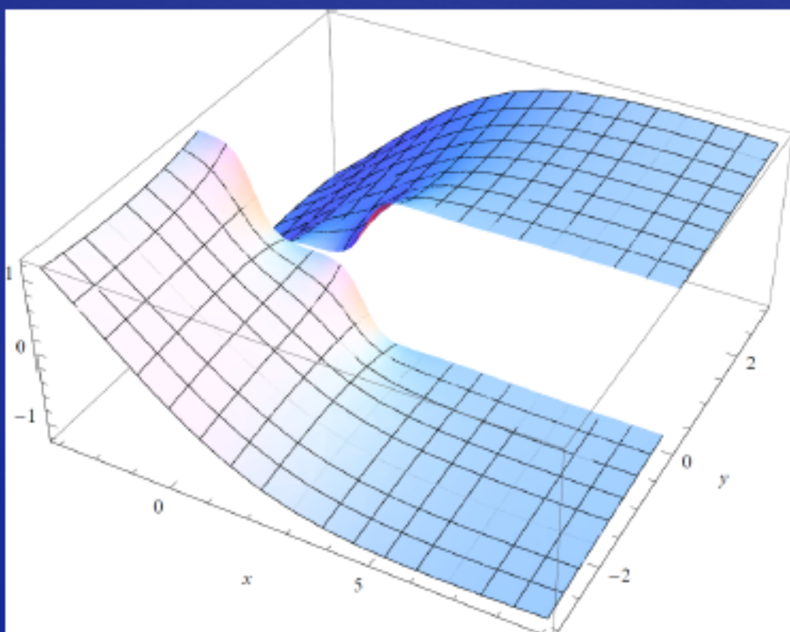
$$\begin{aligned} \frac{dM}{d\mu} &= \frac{\partial M}{\partial \mu} + \frac{\partial M}{\partial a_4}\frac{da_4}{d\mu} + \frac{\partial M}{\partial a_5}\frac{da_5}{d\mu} + \ldots = 0 \\ \frac{d\Gamma}{d\mu} &= \frac{\partial \Gamma}{\partial \mu} + \frac{\partial \Gamma}{\partial a_4}\frac{da_4}{d\mu} + \frac{\partial \Gamma}{\partial a_5}\frac{da_5}{d\mu} + \ldots = 0. \end{aligned}$$

$$a_4 = a_4(\mu_0),\; a_5 = a_5(\mu_0), \ldots,$$

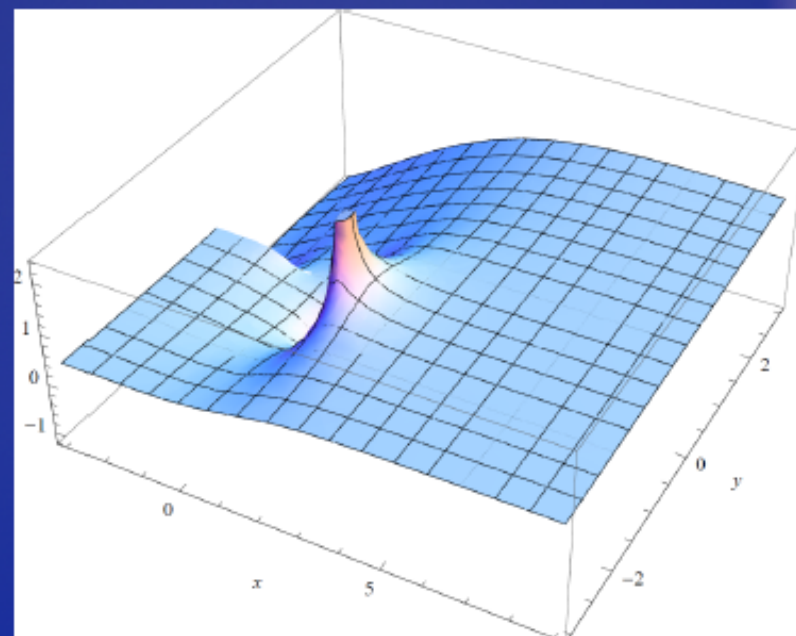
$$\begin{aligned} M &= M(a, b, a_4, a_4, d, e, g) \\ \Gamma &= \Gamma(a, b, a_4, a_5, d, e, g)\ . \end{aligned}$$

The IAM, N/D and IK methods considered here produce UV completions of the partial waves which are unitary and have the proper analytical structure:

For example: IAM: $a = 0.9$, $b = 1$, $\boldsymbol{a_4} = 0.005$ $I = J = 0$ channel



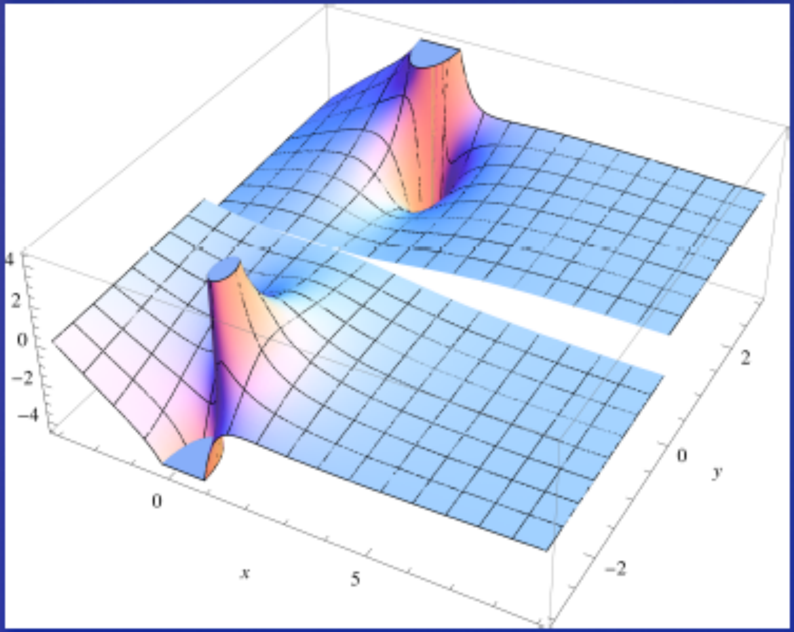
Im A: 1º Riemann Sheet



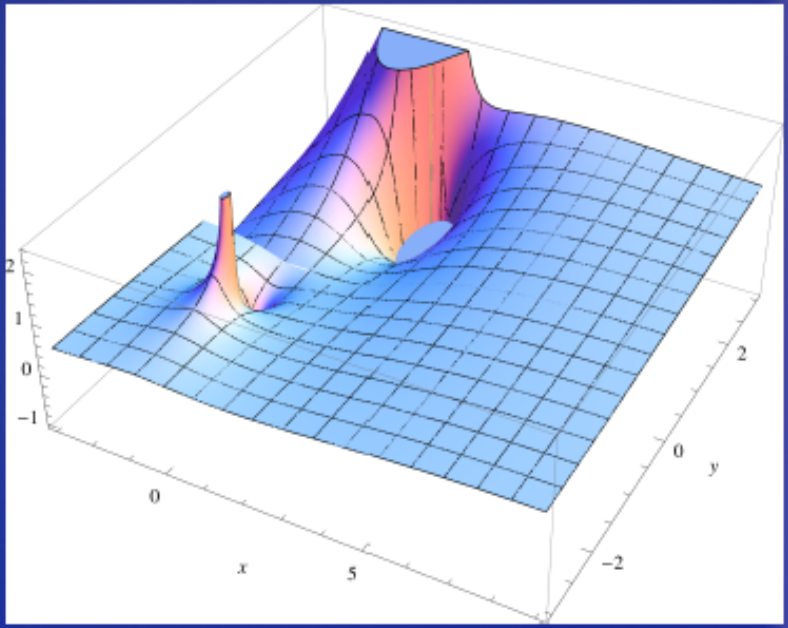
Im A: 2º Riemann Sheet

However, for some values of the couplings the poles can appear in the first Riemann sheet indicating that the model is inconsistent in that case (violation of analiticity and causality)

For example: IAM: $a = 0.9$, $b = 1$, $a_4 = -0.005$ $I = 2$, $J = 0$ channel



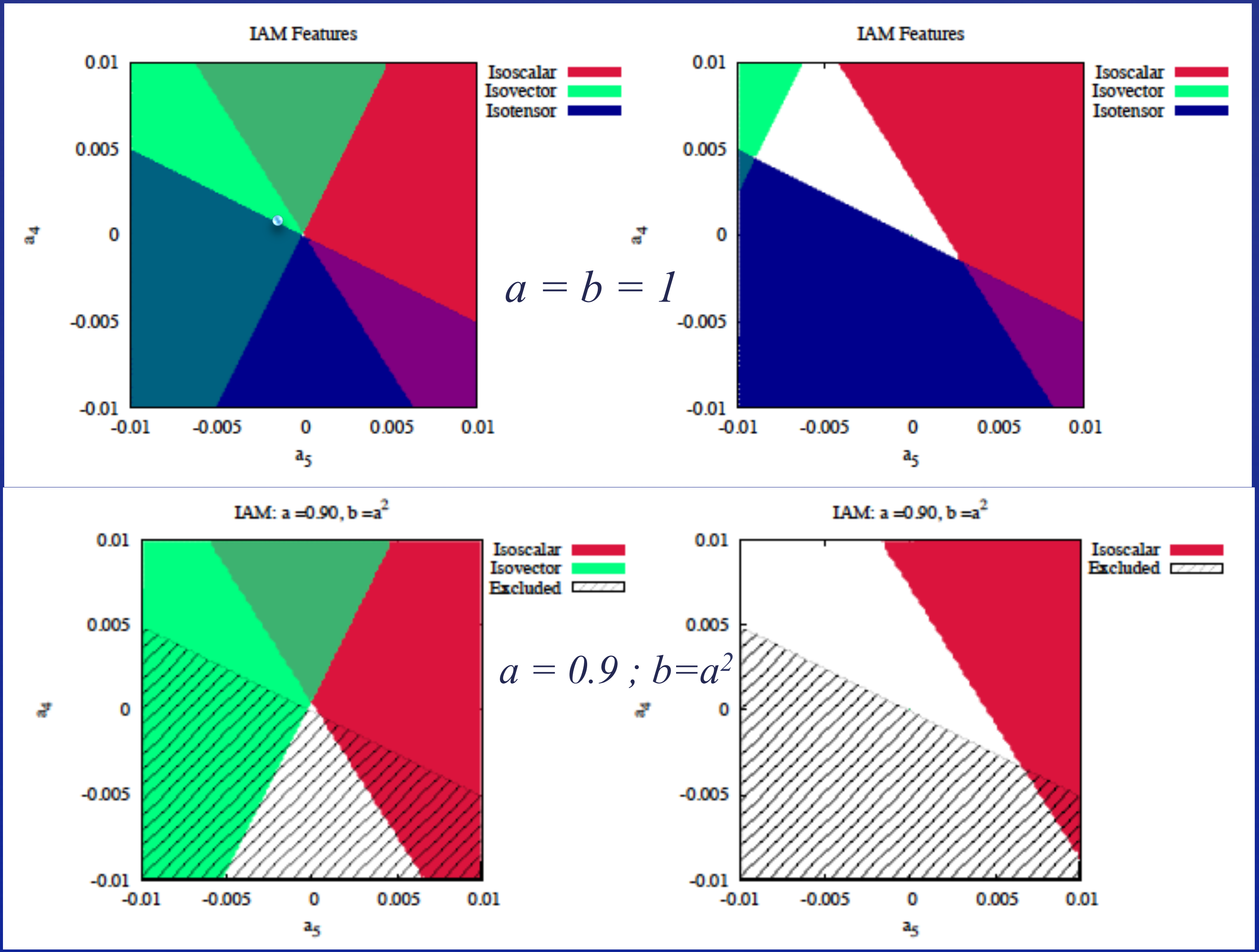
Im A: 1º Riemann Sheet



Im A: 2º Riemann Sheet

This fact can provide powerful constraints on potentially interesting effective theories!

Resonance spectrum (elastic case)



$a = b = 1$

$a = 0.9\ ;\ b = a^2$

Espriu, Yencho, Mescia

Narrow resonances: $\omega_a\omega_b \to \omega_c\omega_d$ $\Gamma << M$

Amplitude phase ishift in the physical region

$$\delta(s) = \arctan \frac{ImA(s)}{Re(s)} |_{RC}$$

$$\delta(M^2) = \frac{\pi}{2}$$ Mass

$$\Gamma = \frac{1}{M}(\frac{d\delta}{ds}|_{s=M^2})^{-1}$$ Width

$$\gamma = \frac{\Gamma}{M} = \frac{K^2}{B(M)+D+E}$$

$$M^2 = \frac{K}{B(M)}$$

IAM

$$A^{(0)}_{IJ}(s) = Ks$$
$$A^{(1)}_{IJ}(s) = s^2\left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{-s}{\mu^2}\right)$$

$$M = M(a, b, a_4, a_4, d, e, g)$$
$$\Gamma = \Gamma(a, b, a_4, a_5, d, e, g) \; .$$

Mass and width of different resonances in terms of the couplings *a, b, a_4* and *a_5*

Scalar resonances *I* = *J* = *0*

$\omega_a \omega_b \to \omega_c \omega_d$

$$M_S^2 = \frac{576\pi^2 v^2 (1-a^2)}{101(1-a^2)^2 + 68(a^2-b)^2 + 768\pi^2(7a_4 + 11a_5)}$$

$$\gamma_S = \frac{36\pi(1-a^2)^2}{51(1-a^2)^2 + 35(a^2-b)^2 + 768\pi^2(7a_4 + 11a_5)}$$

$$\gamma = \frac{\Gamma}{M}$$

Vector resonances *I* = *J* = *1*

$$M_V^2 = \frac{1152\pi^2 v^2 (1-a^2)}{8(1-a^2)^2 - 75(a^2-b)^2 + 4608\pi^2(a_4 - 2a_5)}$$

$$\gamma_V = \frac{12\pi(1-a^2)^2}{8(1-a^2)^2 - 48(a^2-b)^2 + 4608\pi^2(a_4 - 2a_5)}$$

Isotensor resonances *I* = *2, J* = *0*

$$M_{IT}^2 = -\frac{576\pi^2 v^2 (1-a^2)}{91(1-a^2)^2 + 28(a^2-b)^2 + 3072\pi^2(2a_4 + a_5)}$$

$$\gamma_{IT} = \frac{18\pi(1-a^2)^2}{51(1-a^2)^2 + 16(a^2-b)^2 + 3072\pi^2(2a_4 + a_5)}$$

Curvature of the GB space and resonances: $a_4 = a_5 = 0$

Maximally symmetric spaces (constant curvature):

SM, flat space: $a = b = 1$ $\quad R = 0$ $\quad A = M = 0$

MCHM, *M* = *S*^{4} (positive curvature):

$1 - a^2 = a^2 - b = \frac{v^2}{f^2}$ $\quad R = \frac{12}{f^2} > 0$ curvature $\quad M = \frac{s}{f^2}$ $\quad A = \frac{s}{f^2}$

low energy theorem

$M_S^2 = \frac{576}{169}\pi^2 f^2 > 0$ $\quad \gamma_S = \frac{36}{86}\pi > 0$

broad scalar resonance *I* = *0*, *J* = *0*
(enhanced ZZ and W$^+$ *W*$^-$ *production)*

$M_V^2 = -\frac{1152}{67}\pi^2 f^2 < 0$ $\quad M_{IT}^2 = -\frac{576}{119}\pi^2 f^2 < 0$

Hyperbolic, *M* = *H*^{4} (negative curvature):

low energy theorem

$1 - a^2 = a^2 - b = -\frac{v^2}{f^2}$ $\quad R = -\frac{12}{f^2} < 0$ curvature $\quad A = -\frac{s}{f^2}$ $\quad M = -\frac{s}{f^2}$

ghost

$M_S^2 = -\frac{576}{169}\pi^2 f^2 < 0$ $\quad M_V^2 = \frac{1152}{67}\pi^2 f^2 > 0$ $\quad \gamma_V = -\frac{36}{86}\pi < 0$ $\quad \Lambda_G^2 = \frac{1152}{67}\pi^2 f^2$ uper bound

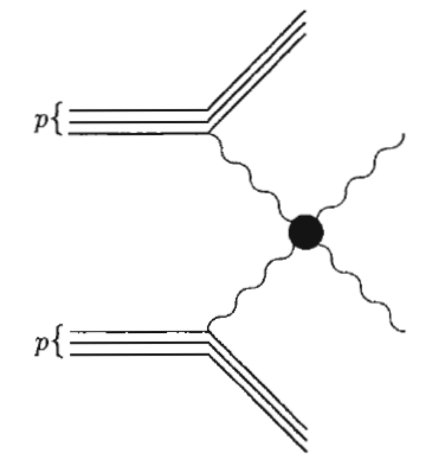
$M_{IT}^2 = \frac{576}{119}\pi^2 f^2 > 0$ $\quad \gamma_{IT} = \frac{18}{67}\pi > 0$

broad isotensor resonance *I* = *2*, *J* = *0*
(enhanced W$^+$*W*$^+$ *production)*

Resonances at the LHC

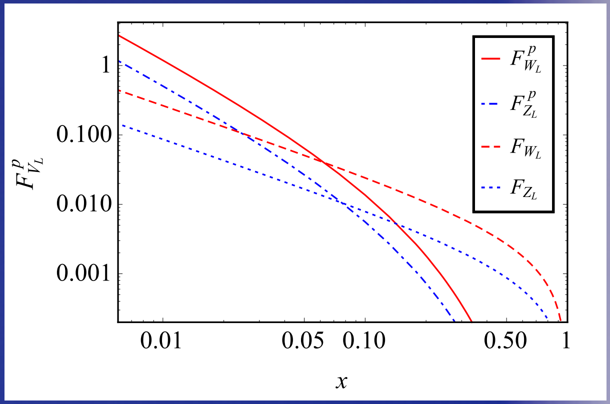
VV and Vh scattering at the LHC:

Convolution with Parton Distribution Functions (pdf)



$$F^p_{W_L}(x) \equiv \int_x^1 \frac{dy}{y} \sum_i f_i(y) \times F^{q_i}_{W_L}\left(\frac{x}{y}\right)$$

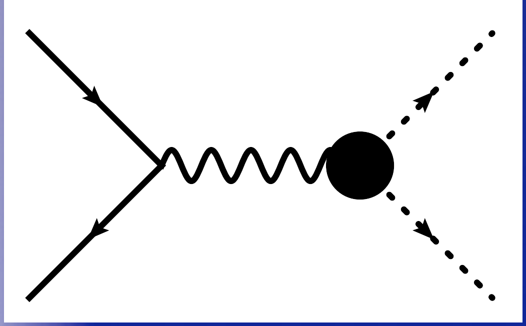
$$g_Z^u = \frac{\alpha[1+(1-\frac{8}{3}\sin\theta_W^2)^2]}{16\pi\sin\theta_W^2\cos\theta_W^2}, \qquad g_Z^d = \frac{\alpha[1+(1-\frac{4}{3}\sin\theta_W^2)^2]}{16\pi\sin\theta_W^2\cos\theta_W^2}$$



In addition we have the process:

$$\sigma(pp \to W + X \to wz + X)$$

$$\frac{d\sigma}{ds}(pp \to w^+z + X) = \int_0^1 dx_u \int_0^1 dx_{\bar{d}}\, \delta\left(s - x_u x_{\bar{d}} E^2_{\text{tot}}\right) \hat{\sigma}(u\bar{d} \to w^+z) f(x_u) f(x_{\bar{d}}) \,,$$



$$\frac{d\hat{\sigma}(u\bar{d} \to w^+z)}{d\Omega_{\text{CM}}} = \frac{1}{64\pi^2 s}\left(\frac{1}{4}\right)\left(\frac{g^4}{8}\right) |\, F_V(s)\,|^2 \sin^2\theta$$

$$F_V(s) = F_{11}(s) = \left[1 - \frac{A^{(1)}_{11}(s)}{A^{(0)}_{11}(s)}\right]^{-1}$$

IAM form factor

This process dominates the vector and axial channesl when vector resonances are present

VII. Conclusions:

The Higgs boson found at CERN in 2012 has the same quantum numbers and a behaviour compatible with the MSM Higgs.

However assuming only custodial symmetry, the existence of the Higgs-like light boson and the huge gap, makes it possible to write a HEFT, containing the SMEFT and SM as particular cases.

By using this Lagrangian at the one-loop level, complemented with dispersion relations and the ET, it possible to study the scattering of the longitudinal components of the EWGB related with the underlying unknown EWSBS dynamics in terms of a small number of parameters.

In the parameter space, $Z_L Z_L$, $W_L W_L$ and $W_L h$ scattering is generically strongly interacting, gives rise to new resonant states in many cases and also to other processes which are suppressed in the MSM as $\gamma\gamma \to Z_L Z_L$ and $W_L W_L$ and $Z_L Z_L$, $W_L W_L \to t\,\underline{t}$.

Thus strongly interacting $V_L V_L$ scattering would be a signal of new physics BSM. Much more work is needed for making realistic predictions.

Wait for more LHC data to see!

Thank you for your attention