Complete One-Loop Renormalization of the Electroweak Chiral Lagrangian

— HEFT 2018, Mainz —

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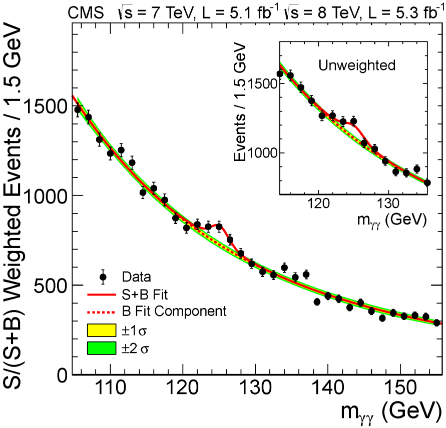
In collaboration with:
Gerhard Buchalla, Oscar Catà, Alejandro Celis, and Marc Knecht,
arXiv:1710.06412, Nucl. Phys. B **928** (2018) 93

Is that the Higgs of the Standard Model?



[1207.7235]

⇒ Answers beyond Yes/No are best addressed using a (model-independent) bottom-up Effective Field Theory.

The interest in EFT techniques increased a lot, recently*.

SMEFT: (weakly-coupled) new physics beyond the (complete) SM

- operator bases at dim 6, 7, ... — Grzadkowski *et al.*; Lehman; Liao/Ma;...
- operator counting — Henning/Lu/Melia/Murayama
- 1-loop renormalization — Alonso/Jenkins/Manohar/Trott
- matching to models — Henning *et al.*; Fuentes-Martin *et al.*
- tools: SMEFTsim, DsixTools, Rosetta — Brivio *et al.*; Celis *et al.*; Falkowski *et al.*

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$ew\chi\mathcal{L}$: (strongly-coupled) new physics in the Higgs sector

- operator basis — Buchalla *et al.*; Alonso *et al.*
- relation between SMEFT and $ew\chi\mathcal{L}$ — Brivio *et al.*; Buchalla *et al.*
- geometric picture — Alonso/Jenkins/Manohar
- renormalization of scalar sector — Gavela *et al.*; Guo *et al.*

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Why is the complete 1-loop renormalization useful?

- to determine the divergence structure of the $ew\chi\mathcal{L}$

  $\Rightarrow$ confirm power counting

  $\Rightarrow$ confirm operator basis

  $\Rightarrow$ get running of LO couplings

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- *"To learn something about Field Theory"* Mike Trott, ca. 2013/14
  $\Rightarrow$ have to tackle ∞ Feynman diagrams

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Abstract

Employing background-field method and super-heat-kernel expansion, we compute the complete one-loop renormalization of the electroweak chiral Lagrangian with a light Higgs boson. Earlier results from purely scalar fluctuations are confirmed as a special case. We also recover the one-loop renormalization of the conventional Standard Model in the appropriate limit.

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Abstract 2 3

Employing background-field method and super-heat-kernel expansion, we compute the complete one-loop renormalization of the electroweak chiral Lagrangian with a light Higgs boson. Earlier results from 4 purely scalar fluctuations are confirmed as a special case. We 1 also recover the one-loop renormalization of the conventional Standard Model in the appropriate limit.

I: ...of the electroweak chiral Lagrangian with a light Higgs.

Ingredients:

- Particles: all SM particles, but we do not assume a relation between the GB and the Higgs
- Symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}$, B, L
  at LO: flavor and custodial symmetry
- Power counting: in terms of chiral dimensions

Buchalla/Catà/CK [1312.5624]

$$2L + 2 = [\text{couplings}]_\chi + [\text{derivatives}]_\chi + [\text{fields}]_\chi$$

$$[\text{bosons}]_\chi = 0,$$
$$[\text{fermion bilinears}]_\chi = [\text{derivatives}]_\chi = [\text{weak couplings}]_\chi = 1$$

I: ...of the electroweak chiral Lagrangian with a light Higgs.

$$\begin{aligned}\mathcal{L}_{\mathrm{LO}} =& \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1+F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h) \\ &+ i\bar{\Psi}_f \not{D}\Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \mathrm{h.c.}) \\ &- \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$

Feruglio[hep-ph/9301281], Bagger *et al.*[hep-ph/9306256], Chivukula *et al.*[hep-ph/9312317], Wang/Wang[hep-ph/0605104], Grinstein/Trott[0704.1505], Contino[1005.4269], Alonso *et al.*[1212.3305], Buchalla/Catà/CK [1307.5017], Buchalla/Catà/Celis/CK [1603.03062], ...

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$$\mathcal{L}_{\mathrm{LO}} = \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \; (1 + F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h)$$

$$+ i\bar{\Psi}_f \not{D} \Psi \ldots$$

$$- \frac{1}{2}\langle \ldots$$

In unitary gauge:

$$\frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle = \frac{g^2 v^2}{4} W^+_\mu W^{\mu -} + \frac{(g^2 + g'^2)v^2}{8} Z_\mu Z^\mu$$

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Properties:

- It has generalized Higgs-couplings compared to the SM.
  $\Rightarrow$ related to the κ-formalism at LO.
- There is a hierarchy to the operators that modify the EWPD.
- It captures the low-energy effects of strongly-coupled new physics.
- It is non-renormalizable at LO.

Complete One-Loop Renormalization of the Electroweak Chiral Lagrangian

Abstract

Employing [2] background-field method and super-heat-kernel expansion [3], we compute the complete one-loop renormalization of the electroweak chiral Lagrangian with a light Higgs boson. Earlier results from [4] purely scalar fluctuations are confirmed as a special case. We [1] also recover the one-loop renormalization of the conventional Standard Model in the appropriate limit.

II: Employing Background-Field Method and ...

starting from the generating functional:

$$Z[j,\rho,\bar{\rho}] = e^{iW[j,\rho,\bar{\rho}]} = \int [d\phi d\psi d\bar{\psi}] \quad e^{i(S[\phi,\psi,\bar{\psi}]+j\phi+\bar{\psi}\rho+\bar{\rho}\psi)},$$

$$\phi = \hat{\phi} + \phi_{qu}, \qquad \psi = \hat{\psi} + \psi_{qu},$$

$$\left(\frac{\delta S}{\delta \phi} + j\right)_{\phi=\hat{\phi}} = 0, \qquad \left(\frac{\delta S}{\delta \bar{\psi}} + \rho\right)_{\bar{\psi}=\hat{\bar{\psi}}} = 0, \qquad \left(\frac{\delta S}{\delta \psi} - \bar{\rho}\right)_{\psi=\hat{\psi}} = 0$$

$$\Rightarrow e^{iW_{L=1}} = \int [d\phi_{qu} d\psi_{qu} d\bar{\psi}_{qu}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi_{qu},\psi_{qu},\bar{\psi}_{qu}]}$$

Abbott '81

II: Employing Background-Field Method and ...

The $ew\chi\mathcal{L}$ is invariant under the transformations T, T_Y

$$\hat{W}_\mu \rightarrow T\left(\hat{W}_\mu - \frac{i}{g}\partial_\mu\right)T^\dagger, \qquad \hat{B}_\mu \rightarrow \hat{B}_\mu - \partial_\mu \alpha_{T_Y}, \qquad \hat{U} \rightarrow T\hat{U}T_Y^\dagger,$$

$$\hat{\psi}_L \rightarrow TT_Y\hat{\psi}_L, \qquad \hat{\psi}_R \rightarrow T_Y\hat{\psi}_R,$$

with the quantum fields transforming as

$$W_\mu \rightarrow TW_\mu T^\dagger, \qquad U \rightarrow T_Y U T_Y^\dagger,$$

$$\psi_L \rightarrow TT_Y\psi_L, \qquad \psi_R \rightarrow T_Y\psi_R$$

$\Rightarrow$ Background gauge invariance for $T \in SU(2)_L$ & $T_Y \in U(1)_Y$!

II: Employing Background-Field Method and ...

Quantum gauge fixing:

$$\mathcal{L}_{\text{gauge-fix}} = -\frac{1}{2\xi}\left(\partial_\mu B^\mu + \frac{\xi}{2} g' v \varphi_3\right)^2 - \frac{1}{\xi}\,\text{Tr}\left\{\left(\hat{D}^\mu_W W_\mu - \frac{\xi}{2} g v \hat{U}\varphi\hat{U}^\dagger\right)^2\right\}$$

- The terms proportional to φ will make the next steps easier.
- Later, we will set $\xi = 1$.

Dittmaier/Grosse-Knetter hep-ph/9505266

Using the background covariant derivative

$$\hat{D}^W_\mu X = \partial_\mu X + ig[\hat{W}_\mu, X]$$

maintains background gauge invariance.

II: Employing Background-Field Method and ...

Further simplification: Stueckelberg transformation

$$\hat{W}_\mu \to \hat{U}\hat{W}_\mu\hat{U}^\dagger - \frac{i}{g}\hat{U}\partial_\mu\hat{U}^\dagger, \qquad W_\mu \to \hat{U}W_\mu\hat{U}^\dagger$$

$$\hat{\psi}_L \to \hat{U}\hat{\psi}_L, \qquad \psi_L \to \hat{U}\psi_L$$

- is equivalent to $\hat{U} \to 1$, but is invertible. (looks like "Background Unitary Gauge")
- simplifies φ-term in $\mathcal{L}_{\text{ghost}}$

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Summary Background Field Method: $\Phi \equiv \hat{\Phi} + \Phi_{qu}$

- easier to handle $U(\varphi)$ and $F(h)$
- gauge choice of background and fluctuating gauge field independent:

$\rightarrow$ manifest background gauge invariance

$\rightarrow$ convenient quantum gauge fixing

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III: ... and Super-Heat-Kernel Expansion, we compute ...

evaluating the one-loop functional — Neufeld/Gasser/Ecker hep-ph/9806436

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi,\psi,\bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi$$

$$W_{L=1} = \frac{i}{2} \operatorname{Tr} \ln A - i \operatorname{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \operatorname{Tr} \left(A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^{T} B^{-1,T} \bar{\Gamma}^{T} \right)^{n}$$

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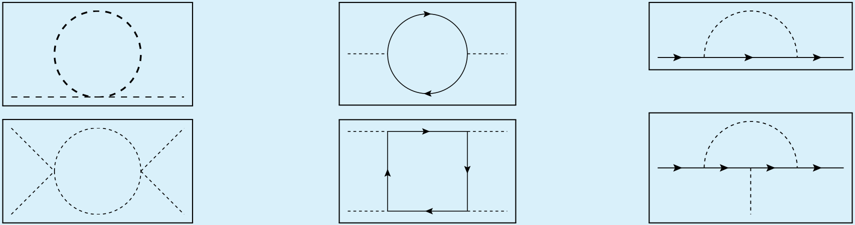

III: ... and Super-Heat-Kernel Expansion, we compute ...

Neufeld/Gasser/Ecker hep-ph/9806436

Introducing supermatrix algebra:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\mathrm{Sdet}\, M = \det\left(A - BD^{-1}C\right) \det D^{-1}$$

$$\mathrm{Str}\, M = \mathrm{Tr}\, A - \mathrm{Tr}\, D$$

$$\mathrm{Sdet}\, M = e^{\mathrm{Str}\ln M}$$

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The one-loop functional becomes:

$$W_{L=1} = \frac{i}{2} \text{Str} \ln K, \qquad K = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{pmatrix}$$

III: ... and Super-Heat-Kernel Expansion, we compute ...

Applying the Heat-Kernel Expansion:

Donoghue/Golowich/Holstein '92

Neufeld/Gasser/Ecker hep-ph/9806436

$$
\begin{aligned}
W_{L=1} &= \frac{i}{2}\,\mathrm{Str}\ln K \\
&= -\frac{i}{2}\int_0^\infty \frac{d\tau}{\tau}\int d^dx\,\mathrm{str}\,\langle x|e^{-\tau K}|x\rangle
\end{aligned}
$$

with the expansion in Seeley-DeWitt coefficients

$$
\langle x|e^{-\tau K}|x\rangle = \frac{i}{(4\pi)^{d/2}}\frac{e^{-\tau m^2}}{\tau^{d/2}}\sum_{n=0}^{\infty} a_n(x)\tau^n
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- The a_n can be computed, knowing the form of K.
- The UV-divergencies of $W_{L=1}$ are the poles in $\frac{1}{\tau}$.

  $\Rightarrow$ only a_2 contributes!

III: ... and Super-Heat-Kernel Expansion, we compute ...

with Donoghue/Golowich/Holstein '92; Neufeld/Gasser/Ecker hep-ph/9806436

$$K = (\partial_\mu + \Lambda_\mu)(\partial^\mu + \Lambda^\mu) + \Sigma$$

we get

$$W_{L=1,div} = \frac{1}{32\pi^2\epsilon} \int d^4x \, \mathrm{str} \left[\frac{1}{12} \Lambda_{\mu\nu} \Lambda^{\mu\nu} + \frac{1}{2} \Sigma\Sigma \right].$$

$$\Lambda_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu + [\Lambda_\mu, \Lambda_\nu]$$

- Specifying the Dirac structure of $S^{(2)}$, we can further evaluate the Dirac-traces.
- The resulting Master-Formula is purely algebraic (Matrix multiplication and traces) 'tHooft '73

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'tHooft '73

Summary Super-Heat-Kernel:

- Supermatrix algebra allows us to treat bosons and fermions together.
- Finding the one-loop divergencies of $S^{(2)}$ becomes an algebraic problem.

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IV: ... the Complete One-Loop Renormalization of the ...

Cross-checks:

- We reproduce previous results of the Scalar sector.
  Guo/Ruiz-Femena/Sanz-Cillero, Phys. Rev. D **92** (2015) 074005, arXiv:1506.04204
- We reproduce the SM-β-functions in the SM-limit.
- We performed 5 independent computations
  with 2 different choices of $\mathcal{L}_{\text{gauge-fix}}$.

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The result:

- confirms the predictions by power counting.
  Buchalla/Catà/CK, Phys. Lett. B **731** (2014) 80, arXiv:1312.5624
- is consistent with the operator basis.
  Buchalla/Catà/CK, Nucl. Phys. B **880** (2014) 552, arXiv:1307.5017

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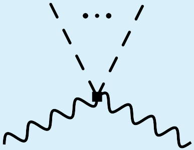
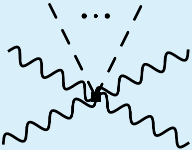
$g^2 U D^2 h$



$$\mathcal{O}_\beta = (g' v)^2 \langle U T_3 D_\mu U^\dagger \rangle^2 \mathcal{F},$$

1/1 operator, $\sim \ (F_U - F_U'^2/4)$

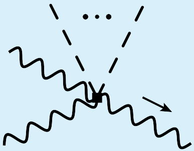
$U D^4 h$



$$\mathcal{O}_{D1} = \langle D_\mu U D^\mu U^\dagger \rangle^2 \mathcal{F},$$

5/15 operators generated

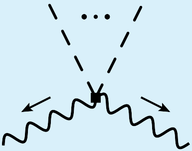
$g U X D^2 h$



$$\mathcal{O}_{XU7} = g' \langle T_3 D_\mu U^\dagger D_\nu U \rangle B^{\mu\nu} \bar{\mathcal{F}},$$

0/8 operators generated

$g^2 U X^2 h$

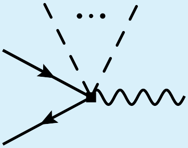


$$\mathcal{O}_{Xh1} = g'^2 B_{\mu\nu} B^{\mu\nu} \bar{\mathcal{F}},$$

0/10 operators, (3 op. $\mathcal{F}(h) = \text{const.} \Rightarrow \mathcal{L}_{LO}$)

IV: ... the Complete One-Loop Renormalization of the ...

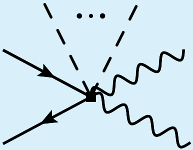
$y^2 UD\Psi^2 h$



$$\mathcal{O}_{\Psi V1} = iy^2 (\bar{q}_L \gamma^\mu q_L) \langle U T_3 D_\mu U^\dagger \rangle \, \mathcal{F},$$

13/13 operators generated

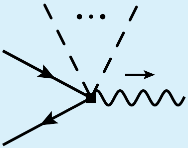
$y UD^2\Psi^2 h$



$$\mathcal{O}_{\Psi S1/2} = y \bar{q}_L U P_\pm q_R \langle D_\mu U D^\mu U^\dagger \rangle \, \mathcal{F},$$

12/30 operators generated (+h.c.)

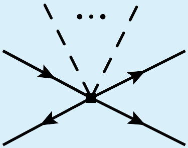
$ygU\Psi^2 Xh$



$$\mathcal{O}_{\Psi X1/2} = yg' \bar{q}_L \sigma_{\mu\nu} U P_\pm q_R B^{\mu\nu} \, \mathcal{F},$$

0/11 operators generated (+h.c.)

$y^2 \Psi^4 Uh$



$$\mathcal{O}_{LL1} = y^2 (\bar{q}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu q_L) \, \mathcal{F},$$

22/60 operators (+h.c.), from $Y \cdot Y$ or $Y^a \cdot Y^a$

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The result is consistent with the operator basis and confirms the power counting. It reproduces the Scalar divergencies and the SM-β-functions in the appropriate limits.