# (a) <br>  <br>  <br> (b) <br> (c) <br>  <br> (d) <br>  <br> (a) <br>  <br> (b) <br>  <br> (c) <br> (d) <br> <br> The SM as an EFT <br> <br> The SM as an EFT <br> (supplementary slides to lecture 2) 

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## SM EFT

SM is not the ultimate theory of nature, so it must be viewed as an EFT. We should add higher-dim operators to its Lagrangian:

$$
\mathcal{L}_{\mathrm{S} M}=\mathcal{L}_{\mathrm{S} M}^{(4)}+\frac{1}{\Lambda} \sum_{k} C_{k}^{(5)} Q_{k}^{(5)}+\frac{1}{\Lambda^{2}} \sum_{k} C_{k}^{(6)} Q_{k}^{(6)}+\mathcal{O}\left(\frac{1}{\Lambda^{3}}\right),
$$

- We don't know the value of $\wedge$ (and different new physics could arise at different $\wedge$ 's),
- Naturalness $C_{k} \sim 1$
- Model independent way to search for New Physics


## Operator basis

Conceptually, writing down the operators is not more difficult than what we did for Euler Heisenberg, but in practice, things become much more involved

- 3 gauge groups: $\mathrm{SU}_{\mathrm{C}}(3) \times S \mathrm{~L}_{\mathrm{L}}(2) \times \mathrm{U}_{\mathrm{Y}}(1)$
- Complicated matter sector

|  | fermions |  |  |  |  | scalars |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| field | $l_{L p}^{j}$ | $e_{R p}$ | $q_{L p}^{\alpha j}$ | $u_{R p}^{\alpha}$ | $d_{R p}^{\alpha}$ | $\varphi^{j}$ |
| hypercharge $Y$ | $-\frac{1}{2}$ | -1 | $\frac{1}{6}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $\frac{1}{2}$ |

## $d=5$ operator

Only a single operator arises at $d=5$

$$
Q_{\nu \nu}=\varepsilon_{j k} \varepsilon_{m n} \varphi^{j} \varphi^{m}\left(l_{p}^{k}\right)^{T} C l_{r}^{n} \equiv\left(\widetilde{\varphi}^{\dagger} l_{p}\right)^{T} C\left(\widetilde{\varphi}^{\dagger} l_{r}\right)
$$

- Violates lepton number
- After EW symmetry breaking, this term gives Majorana masses to $v$ 's and causes $v$-mixing.
- $\wedge$ is very large $\sim 10^{14} \mathrm{TeV}$, not relevant for LHC.


## $d=6$ operators

## Buchmüller and Wyler, Nucl.Phys. B268 (1986) 621 Grzadkowski, Iskrzyński, Misiak, Rosiek JHEP 1010 (2010) 085

Many operators at $d=6$ ! Construction of operator basis is nontrivial, use

- Fierz identities (Dirac and Color), ...
- integration by part, classical EOM, ...
to reduce operators to a minimal set
- 59 operators (compared to 14 at $d=4$ ) which conserve baryon number $B$
-     + 5 additional ones if $B$ is violated
- flavor indices: 2499 parameters with B conserved


## $d=6$ operators

| $X^{3}$ |  | $\varphi^{6}$ and $\varphi^{4} D^{2}$ |  | $\psi^{2} \varphi^{3}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{G}$ | $f^{A B C} G_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi}$ | $\left(\varphi^{\dagger} \varphi\right)^{3}$ | $Q_{e \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{l}_{p} e_{r} \varphi\right)$ |
| $Q_{\widetilde{G}}$ | $f^{A B C} \widetilde{G}_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi \square}$ | $\left(\varphi^{\dagger} \varphi\right) \square\left(\varphi^{\dagger} \varphi\right)$ | $Q_{u \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} u_{r} \widetilde{\varphi}\right)$ |
| $Q_{W}$ | $\varepsilon^{I J K} W_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ | $Q_{\varphi D}$ | $\left(\varphi^{\dagger} D^{\mu} \varphi\right)^{\star}\left(\varphi^{\dagger} D_{\mu} \varphi\right)$ | $Q_{d \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} d_{r} \varphi\right)$ |
| $Q_{\widetilde{W}}$ | $\varepsilon^{I J K} \widetilde{W}_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ |  |  |  |  |
|  |  |  |  |  |  |
| $X^{2} \varphi^{2}$ |  |  | $\psi^{2} X \varphi$ |  | $\psi^{2} \varphi^{2} D$ |
| $Q_{\varphi G}$ | $\varphi^{\dagger} \varphi G_{\mu \nu}^{A} G^{A \mu \nu}$ | $Q_{e W}$ | $\left(\bar{l}_{p} \sigma^{\mu \nu} e_{r}\right) \tau^{I} \varphi W_{\mu \nu}^{I}$ | $Q_{\varphi l}^{(1)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{l}_{p} \gamma^{\mu} l_{r}\right)$ |
| $Q_{\varphi \widetilde{G}}$ | $\varphi^{\dagger} \varphi \widetilde{G}_{\mu \nu}^{A} G^{A \mu \nu}$ | $Q_{e B}$ | $\left(\bar{l}_{p} \sigma^{\mu \nu} e_{r}\right) \varphi B_{\mu \nu}$ | $Q_{\varphi l}^{(3)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu}^{I} \varphi\right)\left(\bar{l}_{p} \tau^{I} \gamma^{\mu} l_{r}\right)}\right.$ |
| $Q_{\varphi W}$ | $\varphi^{\dagger} \varphi W_{\mu \nu}^{I} W^{I \mu \nu}$ | $Q_{u G}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} T^{A} u_{r}\right) \widetilde{\varphi} G_{\mu \nu}^{A}$ | $Q_{\varphi e}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu} \varphi\right)\left(\bar{e}_{p} \gamma^{\mu} e_{r}\right)}\right.$ |
| $Q_{\varphi \widetilde{W}}$ | $\varphi^{\dagger} \varphi \widetilde{W}_{\mu \nu}^{I} W^{I \mu \nu}$ | $Q_{u W}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} u_{r}\right) \tau^{I} \widetilde{\varphi} W_{\mu \nu}^{I}$ | $Q_{\varphi q}^{(1)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu} \varphi\right)\left(\bar{q}_{p} \gamma^{\mu} q_{r}\right)}\right.$ |
| $Q_{\varphi B}$ | $\varphi^{\dagger} \varphi B_{\mu \nu} B^{\mu \nu}$ | $Q_{u B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} u_{r}\right) \widetilde{\varphi} B_{\mu \nu}$ | $Q_{\varphi q}^{(3)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu}^{I} \varphi\right)\left(\bar{q}_{p} \tau^{I} \gamma^{\mu} q_{r}\right)}\right.$ |
| $Q_{\varphi \widetilde{B}}$ | $\varphi^{\dagger} \varphi \widetilde{B}_{\mu \nu} B^{\mu \nu}$ | $Q_{d G}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} T^{A} d_{r}\right) \varphi G_{\mu \nu}^{A}$ | $Q_{\varphi u}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} u_{r}\right)}\right.$ |
| $Q_{\varphi W B}$ | $\varphi^{\dagger} \tau^{I} \varphi W_{\mu \nu}^{I} B^{\mu \nu}$ | $Q_{d W}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \tau^{I} \varphi W_{\mu \nu}^{I}$ | $Q_{\varphi d}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu} \varphi\right)\left(\bar{d}_{p} \gamma^{\mu} d_{r}\right)}\right.$ |
| $Q_{\varphi \widetilde{W} B}$ | $\varphi^{\dagger} \tau^{I} \varphi \widetilde{W}_{\mu \nu}^{I} B^{\mu \nu}$ | $Q_{d B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \varphi B_{\mu \nu}$ | $Q_{\varphi u d}$ | $i\left(\widetilde{\varphi}^{\dagger} D_{\mu} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} d_{r}\right)$ |

plus four-fermion operators

## $d=6$ operators

| $X^{3}$ |  | $\varphi^{6}$ and $\varphi^{4} D^{2}$ |  | $\psi^{2} \varphi^{3}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{G}$ | $f^{A B C} G_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi}$ | $\left(\varphi^{\dagger} \varphi\right)^{3}$ | $Q_{e \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{l}_{p} e_{r} \varphi\right)$ |
| $Q_{\widetilde{G}}$ | $f^{A B C} \widetilde{G}_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi \square}$ | $\left(\varphi^{\dagger} \varphi\right) \square\left(\varphi^{\dagger} \varphi\right)$ | $Q_{u \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} u_{r} \widetilde{\varphi}\right)$ |
| $Q_{W}$ | $\varepsilon^{I J K} W_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ | $Q_{\varphi D}$ | $\left(\varphi^{\dagger} D^{\mu} \varphi\right)^{\star}\left(\varphi^{\dagger} D_{\mu} \varphi\right)$ | $Q_{d \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} d_{r} \varphi\right)$ |
| $Q_{\widetilde{W}}$ | $\varepsilon^{I J K} \widetilde{W}_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ |  |  |  |  |
| $X^{2} \varphi^{2}$ |  |  | $\psi^{2} X \varphi$ |  | $\psi^{2} \varphi^{2} D$ |
| $Q_{\varphi G}$ | $\varphi^{\dagger} \varphi G_{\mu \nu}^{A} G^{A \mu \nu}$ |  |  |  |  |
| $Q_{\varphi \widetilde{G}}$ | $\varphi^{\dagger} \varphi \widetilde{G}_{\mu \nu}^{A} G^{A \mu \nu}$ |  | Zero in Euler-Heisenberg, but |  |  |
| $Q_{\varphi W}$ | $\varphi^{\dagger} \varphi W_{\mu \nu}^{I} W^{I \mu \nu}$ |  | there in non-abelian theories |  |  |
| $Q_{\varphi \widetilde{W}}$ | $\varphi^{\dagger} \varphi \widetilde{W}_{\mu \nu}^{I} W^{I \mu \nu}$ | $Q_{2}$ | there in | $Q_{\varphi q}^{(\stackrel{)}{\prime}}$ | $\left(\varphi^{\top} i D_{\mu}^{1} \varphi\right)\left(\bar{q}_{p} \tau^{1} \gamma^{\mu} q_{r}\right)$ |
| $Q_{\varphi B}$ | $\varphi^{\dagger} \varphi B_{\mu \nu} B^{\mu \nu}$ | $Q_{u B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} u_{r}\right) \widetilde{\varphi} B_{\mu \nu}$ |  |  |
| $Q_{\varphi \widetilde{B}}$ | $\varphi^{\dagger} \varphi \widetilde{B}_{\mu \nu} B^{\mu \nu}$ | $Q_{d G}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} T^{A} d_{r}\right) \varphi G_{\mu \nu}^{A}$ | $Q_{\varphi u}$ | $\left(\varphi^{\dagger} i \overleftrightarrow{B}_{\mu} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} u_{r}\right)$ |
| $Q_{\varphi W B}$ | $\varphi^{\dagger} \tau^{I} \varphi W_{\mu \nu}^{I} B^{\mu \nu}$ | $Q_{d W}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \tau^{I} \varphi W_{\mu \nu}^{I}$ | $Q_{\varphi d}$ | $\left(\varphi^{\dagger} i \overleftrightarrow{D}_{\mu} \varphi\right)\left(\bar{d}_{p} \gamma^{\mu} d_{r}\right)$ |
| $Q_{\varphi \widetilde{W} B}$ | $\varphi^{\dagger} \tau^{I} \varphi \widetilde{W}_{\mu \nu}^{I} B^{\mu \nu}$ | $Q_{d B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \varphi B_{\mu \nu}$ | $Q_{\text {¢ud }}$ | $i\left(\widetilde{\varphi}^{\dagger} D_{\mu} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} d_{r}\right)$ |

plus four-fermion operators

## $d=6$ four fermion operators

| $(\bar{L} L)(\bar{L} L)$ |  | $(\bar{R} R)(\bar{R} R)$ |  | $(\bar{L} L)(\bar{R} R)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} Q_{l l} \\ Q_{q q}^{(1)} \\ Q_{q q}^{(3)} \\ Q_{l q}^{(1)} \\ Q_{l q}^{(3)} \end{gathered}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{l}_{s} \gamma^{\mu} l_{t}\right)$ | $\begin{aligned} & Q_{e e} \\ & Q_{u u} \\ & Q_{d d} \\ & Q_{e u} \\ & Q_{e d} \\ & Q_{u d}^{(1)} \\ & Q_{u d}^{(8)} \end{aligned}$ |  | $Q_{l e}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{e}_{s} \gamma^{\mu} e_{t}\right)$ |
|  | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} q_{t}\right)$ |  |  | $Q_{l u}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ |
|  | $\left(\bar{q}_{p} \gamma_{\mu} \tau^{I} q_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} \tau^{I} q_{t}\right)$ |  |  | $Q_{l d}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ |
|  | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} q_{t}\right)$ |  |  | $Q_{q e}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{e}_{s} \gamma^{\mu} e_{t}\right)$ |
|  | $\left(\bar{l}_{p} \gamma_{\mu} \tau^{I} l_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} \tau^{I} q_{t}\right)$ |  |  | $Q_{q u}^{(1)}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ |
|  |  |  |  | $Q_{q u}^{(8)}$ | $\left(\bar{q}_{p} \gamma_{\mu} T^{A} q_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} T^{A} u_{t}\right)$ |
|  |  |  |  | $Q_{q d}^{(1)}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ |
|  |  |  |  | $Q_{q d}^{(8)}$ | $\left(\bar{q}_{p} \gamma_{\mu} T^{A} q_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} T^{A} d_{t}\right)$ |
| $(\bar{L} R)(\bar{R} L)$ and $(\bar{L} R)(\bar{L} R)$ |  | $B$-violating |  |  |  |
| $Q_{l e d q}$ | $\left(\bar{l}_{p}^{\bar{j}} e_{r}\right)\left(\bar{d}_{s} q_{t}^{j^{j}}\right)$ | $Q_{d u q}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k}$ | ${ }^{\text {Cu}}$ | ( $\left.\left.{ }_{s}^{\gamma j}\right)^{T} C l_{t}^{k}\right]$ |
| $Q_{\text {quad }}^{(1)}$ | $\left(\bar{q}_{p}^{j} u_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} d_{t}\right)$ | $Q_{q q u}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k}\left[\left({ }^{\text {d }}\right.\right.$ | ${ }^{T} C q$ | $\left[\left(u_{s}^{\gamma}\right)^{T} C e_{t}\right]$ |
| $Q_{\text {quqd }}^{(8)}$ | $\left(\bar{q}_{p}^{j} T^{A} u_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} T^{A} d_{t}\right)$ | $Q_{q q q}^{(1)}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k} \varepsilon_{m n}[$ | $)^{T} C$ | $]\left[\left(q_{s}^{\gamma m}\right)^{T} C l_{t}^{n}\right]$ |
| $Q_{\text {lequ }}^{(1)}$ | $\left(\bar{l}_{p}^{j} e_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} u_{t}\right)$ | $Q_{q q q}^{(3)}$ | $\varepsilon^{\alpha \beta \gamma}\left(\tau^{I} \varepsilon\right)_{j k}\left(\tau^{I} \varepsilon\right)$ | ${ }^{\alpha j}{ }^{\text {j }}$ | $\left.C_{r}^{\beta k}\right]\left[\left(q_{s}^{\gamma^{\prime m}}\right)^{T} C l_{t}^{n}\right]$ |
| $Q_{\text {lequ }}^{(3)}$ | $\left(\bar{l}_{p}^{\bar{j}} \sigma_{\mu \nu} e_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} \sigma^{\mu \nu} u_{t}\right)$ | $Q_{\text {duu }}$ | $\varepsilon^{\alpha \beta \gamma}\left[\left(d_{p}^{\alpha}\right)\right.$ | $\left.{ }^{\text {c }} u_{r}^{\beta}\right]$ | $\left.\left.u_{s}^{\gamma}\right)^{T} C e_{t}\right]$ |

Flavor indices: $3^{4}=81 \ldots$ Many couplings!

## EFT for Higgs physics

Not all of the 59 operators are important for Higgs physics, but a significant number of them is.

- A lot of recent work to identify the most important NP effects in Higgs physics and to parameterize possible effects in a model independent way.
- Deviations are often parameterized as deviations from SM coupling strengths to given particle type.


## Value of $\wedge$

We have a plethora of SM measurements which impose constraints on $\wedge$

- Neutrino masses $\wedge \sim 10^{14} \mathrm{TeV}$
- Flavor physics $\wedge$ ¿ 1 - 100 TeV
- Most stringent bounds: FCNC's
- EW precision physics $\wedge \approx 2 \mathrm{TeV}$

Absence of New Physics signals can indicate either a high scale, or a special form which suppresses (e.g. flavor physics) signals.

## Naturalness Problem

The SM contains a single relevant operator

$$
\mu^{2} \varphi^{\dagger} \varphi=C^{(2)} \Lambda^{2} \varphi^{\dagger} \varphi
$$

with $2 \mu^{2}=m_{H^{2}}$. Naturalness:

- Expect New Physics at $\wedge \sim m_{H}$.
- Should protect the Higgs mass from contributions from higher scales (such as $\mathrm{Mpl}_{\mathrm{p}}$ ).
- SUSY, compositeness, ... ?

Naturalness problem: we do not see any effects of the higher-dim. operators: $\wedge \gg m_{H}$

- Important: $\wedge$ is scale of new physics, not unphysical cutoff!

