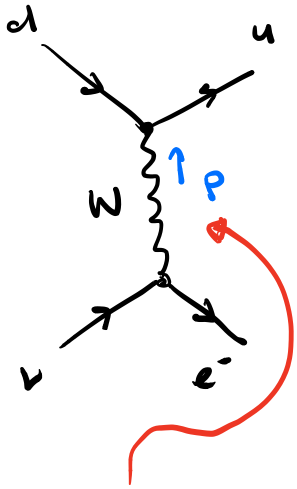
Low-Energy Effective Quantum Field Theories

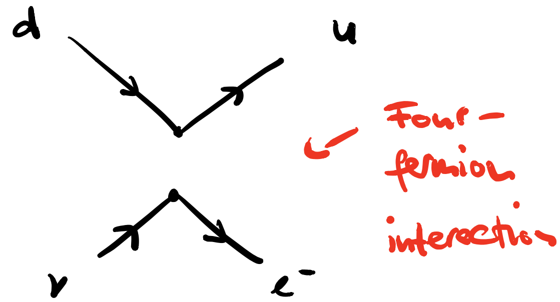
An EFT is a QFT which describes physics below some scale Λ, as opposed to a fundamental (or "full") theory, which is valid up to arbitrary high energies.

Examples: Every QFT known! SM!

More formally, EFTs implement expansion in scale ratio $\lambda = E/\Lambda$: the Taylor series of QFT. As such, EFTs are a fundamental tool of QFT, like perturbation theory, the expansion in the coupling constants.

Perhaps the most famous example of an EFT is the Fermi theory of weak interaction





$$\frac{1}{p^2 - M_W^2} \xrightarrow{\lambda^2 = p^2/M_W^2} -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \dots\right)$$

(derivatives on the fermions)

($\Lambda = M_W$)

Two types of EFTs:

* Wilsonian EFT: integrate out physics above some scale Λ, using path integral
  - $\phi = \phi_L + \phi_H$; ϕ_H contains short distance fluctuation
  - Expand $\mathcal{L}_{eff}(\phi_L) = \mathcal{L}_0 + \sum_{k,i} \frac{C_i^{(k)}}{\Lambda^k} O_i^{(k)}(\phi_L)$ (dimensionless!)
  - top down, difficult in practice due to hard cutoff Λ
  - provides physical picture of renormalization group flow for Wilson coefficients $C_i = C_i(\Lambda)$

Operator dimension

Dimension of fields ($\hbar = c = 1$)

Scalar: $\int d^dx \, \frac{1}{2}(\partial_\mu \phi)^2 \longrightarrow \phi \sim E^{\frac{d-2}{2}}$

$0 = 2 + 2[\phi] - d \qquad [\phi] = 1$ in $d=4$

Fermion: $\int d^dx \, \bar{\psi} i \not{\partial} \psi \qquad \psi \sim E^{\frac{d-1}{2}}$

$0 = 1 + 2[\psi] - d \qquad [\psi] = \frac{3}{2}$ for $d=4$

Gauge Field A_μ: same as scalar but $iD_\mu = i\partial_\mu + g A_\mu \sim E$

$\hat{O}$	$n = [\hat{O}]$	$k = n - d$	$d=4$
ϕ^2	$d-2$	-2	
$\bar{\psi}\psi$	$d-1$	-1	
$\bar{\psi} i \not{\partial} \psi$	d	0	
ϕ^4	$2d-4$	$d-4$	$= 0$
$\bar{\psi}\psi\bar{\psi}\psi$	$2d-2$ $= 6$	$d-2$	$= 2$

Terminology:

$k > 0$: relevant operator

$k = 0$: marginal

$k < 0$: irrelevant
contribution vanishes for
$\Lambda \to \infty$

Four-fermion operator of Fermi theory is irrelevant; prefactor is $\frac{1}{M_W^2} \sim \frac{1}{\Lambda^2}$. This is the reason why the weak interaction is weak!

Q: Now consider SM Lagrangian. How many relevant operators are there?

For $\Lambda \to \infty$ the irrelevant operators fade away: the remaining theory only contains operators which are renormalizable in the traditional sense.

* Continuum EFT ← these lectures

0.) identify low-E degrees of freedom

⇝ introduce appropriate fields

1.) write down most general $\mathcal{L}_{eff}$, identify expansion parameter λ, order terms by λ.

Note: Λ is the physical scale where "new physics", i.e. new degrees of freedom enter

2.) determine coefficients of terms in $\mathcal{L}_{eff}$ by matching

↖ "Wilson coefficient"

↗ compute same quantity in full and effective theory, adjust Wilson coeffs. to reproduce full theory result

usually: dim. reg. instead of hard cutoff.

3.) Renormalize Wilson coefficients

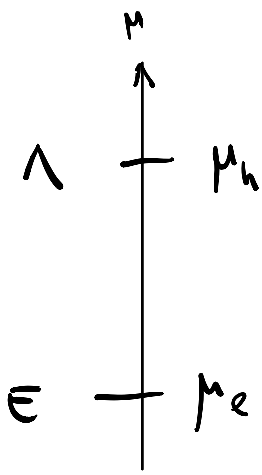
$$C_i^{ren}(\mu) = \lim_{\varepsilon \to 0} \sum_j Z_{ij} C_j^{bare}(\varepsilon)$$

$d = 4 - 2\varepsilon$

Solve RG - equation

$$\frac{d}{d\ln\mu} C_i(\mu) = \sum_j C_j(\mu)\,\gamma_{ji}$$

↑ operator mixing.



$$C_i(\mu_e) = C_i(\mu_h)\,\mathcal{U}_{ij}(\mu_h, \mu_e)$$

4.) Compute low-energy process in EFT

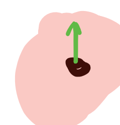
$$\mathcal{A} = \sum_i C_i(\mu_e) \langle O_i \rangle (\mu_e)$$

↑ matrix elements of operators in $\mathcal{L}_{eff}$

Why EFTs ?

- Expansion in scale ratios simplifies computations

- Symmetries
  - approximate : chiral symmetry $m_q \to 0$
  - emergent : heavy quark symm. (spin, flavor)



- Scale separation / Factorization
  - A series of single scale problems
    $\Rightarrow$ can use dimensional analysis
  - Perturbation theory OK for small couplings
    [It breaks down for problems with large scale hierarchies. Problem ($\alpha^n \ln^n(\lambda)$) can be solved using the renormalization group in the EFT.]

- separate perturbative from non-perturbative physics.

- General framework for cases where full theory is not known (e.g. SM) or nonperturbative (CHPT).
  - Model-independent way to parameterize effects of unknown physics at scale Λ.